Tunable Radiation Source through Upshifting without Ionization

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(Received 28 July 1997)

A mechanism for generating electromagnetic wakes of infrared radiation by a short laser pulse, propagating through an underdense plasma in the presence of a magnetostatic undulator, is described. As opposed to the undulator radiation, produced when a charged particle bunch propagates in a periodic magnetic field, here a laser pulse is used instead of the particle bunch. This mechanism for a tunable, plasma-based radiation source does not rely on ionization or photoswitching. It is also found that, in the presence of a static periodic magnetic field, long laser pulses become modulationally unstable, generating an electromagnetic wake. [S0031-9007(98)05667-1]

PACS numbers: 52.40.Nk, 52.75.Va

High-power tunable far infrared (FIR) radiation is an important research tool in various scientific applications, such as atomic physics, nonlinear dynamics of semiconductors, and biophysics. Relativistic electron beams propagating through periodic magnetic fields (undulators) are used to generate FIR radiation [1] to fill in the "technology gap" in the spectrum left by the traditional laser sources. To make a beam radiate coherently in the FIR, it should either initially consist of subpicosecond microbunches or become bunched further along the undulator via a self-consistent interaction with amplified spontaneous emission (FEL). The FEL requires a high-current high-quality beam. In either case, Doppler upshift, which depends on the electron energy, provides tunability.

An effective Doppler upshift can be achieved in a stationary medium with appropriately introduced spatiotemporal correlations [2]. This concept is realized in several radiation sources, which are based on free electrons, but do not require relativistic electron beams [3–5]. In these devices the dc electric field is converted into radiation by rapidly changing the number of free carriers. In a gaseous medium this is accomplished by laser-induced ionization [4], while in a semiconductor the electron-hole pairs are generated by absorbing a short laser pulse [3,5]. The required spatiotemporal correlation in the plasma is created by making a dc electric field periodic in space, and by employing a rapidly moving, e.g., laser-induced, ionization front [4]. All of these schemes share similar limitations: First, the output radiation power is limited by the dc electric field (which must not exceed the breakdown threshold); second, the repetition rate is limited by the longest of the recombination (or removal) time of the carriers and the recharging time of the capacitors.

In this Letter, we identify and analyze a way of generating tunable laser-driven undulator radiation in the Infrared (LURI). We show how a picosecond compact FIR source can be based on the LURI concept. The schematic of such a source is shown in Fig. 1. It consists of a short intense laser pulse, propagating through a *preformed* plasma in the presence of a static undulator. Plasma electrons experience a ponderomotive kick from the laser, giving them axial momentum. The magnetic field of the wiggler couples with the longitudinal motion of the electrons to produce transverse acceleration, causing them to radiate in the axial direction. As in undulator radiation, a "radiation zone," roughly defined as the extent of the laser pulse, propagates with relativistic speed in the direction of the undulator periodicity. Here, however, the Doppler upshift is caused by the group velocity of the laser pulse instead of the, typically small, axial velocity of the plasma electrons. LURI is emitted by mostly stationary (although transversely accelerated) plasma electrons. The practical viability of LURI as a future table-top radiation source will depend critically on the availability of ultrahigh power, high-repetition rate short-pulse lasers.

This Letter describes, for the first time, how *electromagnetic* wakes can be generated by a laser pulse in a plasma-filled undulator. In contrast with the *electrostatic* wake at ω_p in unmagnetized plasma [6], or Cerenkov radiation at a frequency close to ω_p in uniformly magnetized plasma [7], the frequency of the LURI radiation is given by $\omega_1 = \omega_p^2/2k_w c$, where $\omega_p = \sqrt{4\pi n e^2/m}$ is the plasma

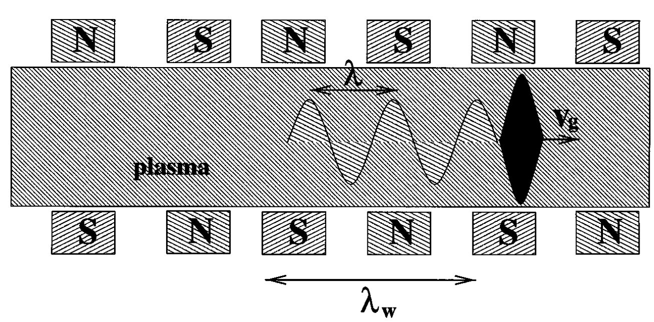

FIG. 1. Schematic for generation of a laser-driven undulator radiation. The resonant wavelength $\lambda = 2\lambda_p^2/\lambda_w$.

frequency, $\lambda_w = 2\pi/k_w$ is the undulator periodicity, n is the plasma density, and $-e$ and m are electron charge and mass, respectively. In addition, we demonstrate that a long intense laser pulse becomes Raman unstable in the presence of periodic magnetic field. Unlike the usual Raman instability in unmagnetized plasma, where the internal mode is an electrostatic plasma wave, in our case the internal mode is the LURI.

To proceed, assume a linearly polarized magnetic wiggler with $\vec{B}_w = B_w \cos(k_w z)\vec{e}_x$ and a linearly polarized laser pulse with vector potential $\vec{A}_0 = \vec{e}_x a_0(\zeta) mc^2/e \cos(\omega_0 t - k_0 z)$, where $\zeta = v_g t - z$. Plasma is assumed to be very underdense, so that $\omega_0 \gg \omega_p$, and the group velocity $v_g \approx c$. It will be shown below that LURI is most efficiently generated by laser pulses of duration $\tau_L \approx \lambda_1/2 \ll \lambda_p$, so that the influence of the plasma waves on electron motion can be neglected. In the nonrelativistic limit of $a_0^2 \ll 1$ the longitudinal electron velocity is given by $v_z = ca_0^2(\zeta)/4$. Hence, only the plasma electrons overlapping with the laser pulse are moving and can, in principle, radiate. The high-frequency ($2\omega_0$) component of the longitudinal electron velocity also leads to radiation generation; this effect is neglected because the focus of this Letter is on producing infrared radiation. In addition, we limit our calculation to the coherent radiation generation and assume plasma to be cold since it can be shown that the incoherent emission is very small and the thermal effects are negligible for subrelativistic plasma temperature.

Influenced by the magnetostatic field, electrons experience acceleration $\dot{v}_y = -[ea_0^2(\zeta)B_w/4m]\cos(k_w z)$. The effect of the magnetic field on the longitudinal motion of plasma electrons was neglected since $eB_w/mc\tau_L \ll 1$ is always satisfied for magnetic fields not exceeding 10 T and subpicosecond laser pulses. Periodic magnetic field is important for two reasons: (i) It enables plasma electrons to radiate in the forward direction by imparting a finite transverse acceleration $\dot{v}_y$, and (ii) periodicity provides phase matching for the emitted radiation, enabling constructive interference from different regions along the wiggler. The combination of a laser pulse, moving at a speed approaching c, and a periodic magnetic field introduces the necessary spatiotemporal correlation between different regions of the plasma, resulting in a Doppler-upshifted radiation [2].

In 1D, condition (ii) is equivalent to requiring that emitted radiation slips one wavelength ahead as the laser pulse moves by one wiggler period: $k_z \Delta z - \omega \Delta t = -2\pi$, where $\Delta z = \lambda_w$ and $\Delta t = \lambda_w/v_g$. This can be recast in a form familiar from FEL physics, with the group velocity of the laser $v_g = c\sqrt{1 - \omega_p^2/\omega_0^2}$ assuming the role of the beam velocity,

$$(k_z + k_w)v_g = \omega\,, \tag{1}$$

where ω and k_z are related through the dispersion relation $\omega^2 = k^2c^2 + \omega_p^2$. Equation (1) determines the frequency of the emitted radiation, and can have two or no solutions. In the most interesting case,

$$k_w c \ll \omega_p \ll \sqrt{\omega_0 k_w c}\,, \tag{2}$$

two solutions can be found at $\omega_1 \approx \omega_p^2/2k_w c$ and $\omega_2 \approx 2k_w c\omega_0^2/\omega_p^2$. Note that under condition (2) $\omega_1 \ll \omega_0 \ll \omega_2$. For example, for $n_0 = 4.0 \times 10^{15}$ cm^{-3}, $\lambda_w = 1$ cm, $\lambda_0 = 1.0$ μm, we find $\lambda_1 = 50$ μm, $\lambda_2 = 200$ Å. As we explain below, the rate of emission at λ_2 is negligible. However, by appropriately shaping the driver laser pulse, a considerable amount of far infrared/THz radiation at λ_1 can be obtained.

If the Rayleigh length of the emitted radiation is much larger than $1/k_w$, electric field can be assumed in a form $\vec{E} = \vec{e}_y E_y(\zeta)\exp(ik_w z)/2 + \text{c.c.}$ Substituting this expression into the wave equation and combining it with the Lorentz equation of motion for the plasma electrons, we obtain, after some algebra,

$$\left(\frac{\partial}{\partial\zeta} + i\,\frac{\omega_1}{c} - \frac{i}{2k_w}\,\nabla_\perp^2\right)E_y = -i\,\frac{k_p^2}{8k_w}\,B_w a_0^2(\zeta, \vec{x}_\perp)\,. \tag{3}$$

Equation (3) can be integrated in 1D, neglecting the $\nabla_\perp^2$ term, and yielding

$$E_y = -i\,\frac{k_p^2 B_w}{8k_w}\,e^{-i\omega_1\zeta/c}\int_0^\zeta d\zeta' e^{i\omega_1\zeta'/c}a_0^2(\zeta')\,. \tag{4}$$

Equation (4) indicates that the radiation behind a finite duration laser pulse has a well-defined frequency ω_1, and its amplitude is proportional to the Fourier transform of the laser intensity profile. Hence, LURI is most efficiently generated by a square laser pulse of duration $\tau_L = \pi/\omega_1$, resulting in

$$|E_y| = a_0^2 B_w/2\,. \tag{5}$$

Note that, if the laser pulse is much longer than half of the wavelength of LURI, the amplitude of the radiation is exponentially small. In contrast, the radiation amplitude of a dc to ac radiation converter (DARC) is insensitive to the width of the ionization front [4]. An equation similar to Eq. (5) can be derived for the radiation amplitude at the higher frequency solution of Eq. (1). Since λ_2 is very short (shorter than λ_0), no appreciable radiation at wavelength λ_2 is expected. Another approach to generating LURI is to use an electromagnetic beat wave of two long laser pulses instead of a single ultrashort one. When the frequency detuning of the two lasers matches the LURI frequency ω_1, LURI is resonantly driven, and its amplitude is proportional to the duration of the beat wave. Thus, the LURI source can also be viewed as an efficient plasma-based frequency-difference generator.

As an example of a picosecond LURI source at 50 μm, based on the present technology, consider a $N_w = 50$ period undulator with periodicity $\lambda_w = 1$ cm and magnetic field strength $B_w = 1.0$ T, filled with $n_0 = 4.0 \times 10^{15}$ cm^{-3} $T_e = 25$ eV plasma. The LURI is driven by a 3 J, $\tau_L = 85$ fs micron laser focused to a $\sigma = 200$ μm radius. Such a laser pulse has a peak intensity $I_0 = 3 \times 10^{16}$ W/cm^2 (corresponding to $a_0 = 0.15$) and is assumed to be generated at a $f_0 = 1$ Hz repetition rate. The basis for choosing σ is explained below, after the two-dimensional effects are discussed. Such a source produces $\tau_{\mathrm{FIR}} = N_w \lambda_1 / c = 8$ ps long pulses with about 1 kW peak power. A source of preformed plasma of the required size and plasma density could be, for example, a lithium oven described in Ref. [8]. Incidentally, the plasma does not have to be preformed for the LURI scheme to work (in contrast to a DARC source, where plasma needs to recombine or be removed from the interaction region in order to charge the capacitors for the next shot). For example, the driving laser may tunnel ionize the gas when its intensity is high enough.

Using this numerical example, we discuss some advantages of the LURI source over DARC. In general, utilizing static magnetic field avoids the breakdown and recharging limitations of the dc to ac radiation conversion schemes, and having a steady-state plasma eliminates the idle time of recombination. From Eq. (5) the peak electric field $E_y = 30$ kV/cm. To generate FIR radiation of such intensity using DARC requires dc electric field E_s of the same strength $E_s = E_y$. Increasing the laser power and the magnetic field of an undulator to increase the output power of a LURI source may be much easier technologically than further increasing the static electric field in a DARC because of the breakdown limitations.

The repetition rate of LURI source can, in principle, be very high since the driving laser pulse can be cycled through the undulator with arbitrary frequency. In practice, of course, the repetition rate will be limited by the heating of the plasma, as discussed below. Since a 3 J laser cannot be fired very frequently, the plasma-filled undulator needs to be placed inside a high-quality optical cavity. The device would then operate in a burst mode, with its output consisting of macropulses (pulsed at frequency f_0), each consisting of Q micropulses separated by the cavity roundtrip time, thereby producing $f = Qf_0$ bursts per second. Reusing the laser pulse by confining it in a high-Q cavity is also necessary to improve the efficiency of a LURI source since only a small fraction of the laser energy is lost in a single pass.

Note that the laser pulse leaves a wake of static magnetic field behind. To see this, note that electrons retain the finite velocity in the y direction (gained inside the laser pulse due to $v_z B_w$ force) after they fall behind the laser pulse. This velocity is proportional to the local undulator strength, i.e., is periodic with the undulator period λ_w. Thus it can shown that two different modes are excited in the plasma: (i) the earlier discussed undulator radiation mode, with frequency $\omega = \omega_1$ and amplitude $B_x \approx E_y$, and (ii) a magnetostatic mode, with frequency $\omega = 0$ and amplitude $|B_x^{(s)}| \approx 0.5 k_w B_w \int d\zeta\, a_0^2(\zeta)$. Clearly, $|B_x^{(s)}|$ behind the laser pulse depends only on the total laser energy per unit area, and not on the duration of the pulse, in contrast to the DARC, which generates appreciable magnetostatic field only when the ionization front is shorter than λ_1 [4]. We estimate that the energy left behind in the static magnetic field and the kinetic energy of the plasma electrons is much smaller than the LURI energy, if Eq. (2) is satisfied.

Another source of energy loss, not taken into account by the present calculation, is to electrostatic plasma waves. Despite the fact that the laser pulse is much shorter than the plasma period, the ratio of the LURI energy U_L to the energy of electrostatic waves U_w is, typically, small: $\eta = U_L / U_w \approx (\omega_B / k_w c)^2 \lambda_w / \lambda_1$, where $\omega_B = e B_w / mc$ is the electron cyclotron frequency in the wiggler field. Thus the efficiency η is about 0.5% for this numerical example. However, the energy lost to the electrostatic wake can be recovered, in principle, by launching laser pulses in pairs, separated by a half-integer number of plasma wavelengths. An unrecoverable source of energy loss is inverse bremsstrahlung. It can be shown that for the parameters of a picosecond LURI source, presented above, bremsstrahlung losses are negligible. For the same parameters, plasma heating also turns out to be small (about 0.5 eV per shot). Another effect that the laser pulse might have on the plasma is to blow it up via the transverse ponderomotive force. This effect turns out to be insignificant because the driving laser pulses are pancake shaped—the transverse size is larger than then longitudinal. The fractional change in the electron density (neglecting the ion response) is indeed very small: $\delta n / n \approx 0.5 a_0^2 c^2 \tau_L^2 / \sigma^2 = 3 \times 10^{-3}$.

Focusing the incident laser to a tight spot is advantageous for obtaining the largest LURI power for the given laser power. The one-dimensional calculation presented above fails when the spot size becomes small. Using the results of the three-dimensional theory of undulator radiation by electron bunches [9], we estimate the peak LURI power, given the incident laser energy and assuming the optimal pulse duration $\tau_L = \pi / \omega_1$. The rms angular spread of the undulator radiation depends on the spectral width of the collected radiation, and is given by $(\delta\theta) = \sqrt{(\Delta\omega/\omega)\lambda/\lambda_w}$. This angular spread roughly corresponds to a focal waist size $\Delta x = \lambda / (\delta\theta)$. If the laser spot size $\sigma \gg \Delta x$, the one-dimensional treatment is accurate, and further focusing of the laser increases the total power radiated into the $(\Delta\omega/\omega)$ bandwidth. This can be qualitatively understood by noting that the radiation from a wide beam $\sigma \gg \Delta x$ is transversely incoherent, with the transverse coherence length roughly equal to Δx. The total radiated power is proportional to the product of the square of the number of electrons inside the coherence zone (independent of σ) and the number of coherence zones $\mathcal{N}_{\mathrm{coh}} = (\sigma / \Delta x)^2$. Also, the electron acceleration scales as $a_0^2 \sim \sigma^{-2}$ for a fixed laser power, so that the total radiated power scales as σ^{-2}. However, as σ approaches Δx, radiation acquires transverse coherence, and its intensity scales as the square of the number of electrons inside the laser pulse $\mathcal{N}^2 \propto \sigma^4$, becoming independent of σ.

To quantify this argument, we use the three-dimensional Eq. (3) to estimate the total power of LURI. Assuming a transversely Gaussian longitudinally flat-top incident laser pulse $a_0^2 \propto \exp(-r^2/\sigma^2)$, we obtain

$$P_{\rm tot} = \frac{e^4 P_L^2 B_w^2 \lambda_0^4}{16\pi^4 c^3 (mc^2)^4} \int_0^\infty dk\, k e^{-k^2\sigma^2/2} \times \frac{\cos^2(k^2 \pi c/4\omega_1 k_w)}{1 + k^2 c/(2k_w \omega_1)}, \quad (6)$$

where P_L is the incident laser power. Equation (6) loses validity for transverse numbers $k > \sqrt{2k_w \omega_1/c}$.

Radiation emitted at different angles to the direction of the laser propagation (different k) has different frequencies, approximated by $\omega = \omega_1(1 + k^2 c/2k_w \omega_1)$. Coherence properties of radiation are determined by the spot size σ, and can be classified into three regimes: (i) $\sigma > \sqrt{\lambda_1 \lambda_w N_w/8}$; (ii) $\sqrt{\lambda_1 \lambda_w/8} < \sigma < \sqrt{\lambda_1 \lambda_w N_w/8}$; (iii) $\sigma < \sqrt{\lambda_1 \lambda_w/8}$. In the regime (i), radiation has a very narrow bandwidth determined by the number of undulator cycles $\Delta\omega/\omega \approx 1/N_w$; in the regime (ii), radiation bandwidth is limited by the spot size: $\Delta\omega/\omega \approx \lambda_w \lambda_1/8\sigma^2$; in the regime (iii), radiation is broadband $\Delta\omega/\omega \approx 1$, and the total power is independent of σ. The laser spot size in the numerical example of a picosecond LURI source was chosen $\sigma = \sqrt{\lambda_1 \lambda_w/8} \approx 200\ \mu$m since further decrease in the spot size does not result in a higher power in the infrared. Equation (6) confirms that approximately a kilowatt of far-infrared light is generated. Additional information provided by the three-dimensional treatment is that this radiation is emitted into a range of wavelengths $25 < \lambda < 50\ \mu$m.

It has long been established that intense laser pulses, longer than plasma wavelength, are unstable to Raman forward scattering (RFS) [10,11]. In a simple three-wave picture, a plasma wave is generated by the ponderomotive beating of the pump a_0 and the Stokes component a_S, which is frequency downshifted by ω_p. The Stokes component is then reinforced by the pump scattering off the plasma wave. In very underdense plasmas the anti-Stokes component is also excited, reducing the growth rate of the instability. The generated wake can be utilized for particle acceleration in a self-modulated laser wakefield accelerator [12].

A similar phenomenon occurs in a periodically magnetized plasma: Stokes and anti-Stokes components a_- and a_+ of the incident laser, shifted by ω_1 and polarized *collinearly* with the pump, generate an intensity modulation which drives the undulator radiation E_y according to Eq. (4). Driven by the undulator radiation, plasma electrons acquire velocity in the y direction $v_y = -[ieE_y/2m\omega_1] \exp(ik_w z) +$ c.c. and, under the $\vec{v} \times \vec{B}_w$ force, are longitudinally bunched: $\delta n/n_0 = e^2 B_w E_y/4m^2c^2\omega_1^2 +$ c.c. Equation (4) then yields

$$\left(\frac{\partial^2}{\partial \zeta^2} + \frac{\omega_1^2}{c^2}\right) \frac{\delta n}{n_0} = -\frac{\omega_B^2 (a_0 a_- + a_0 a_+)}{32 c^2}. \quad (7)$$

The laser pump then scatters off this density perturbation, reinforcing the Stokes and anti-Stokes lines. Combining the nonrelativistic 1D wave equation $\Box a = k_p^2(1 + \delta n/n_0)a$ with Eq. (7) results in a dispersion relation,

$$(\omega^2 - \omega_1^2) = \frac{\omega_B^2 \omega_p^2 |a_0|^2}{32} \left(\frac{1}{D_-} + \frac{1}{D_+}\right), \quad (8)$$

where $D_\pm = c^2(k_0 \pm k)^2 + \omega_p^2 - (\omega_0 \pm \omega)^2$ are the dispersion functions for the anti-Stokes (Stokes) lines, and (ω, k) are the frequency and wave number of the undulator radiation. Equation (8) describes the spatiotemporal evolution (in one spatial dimension) of a novel undulator radiation instability of long intense laser pulses in the presence of a periodic magnetic field. A detailed spatiotemporal analysis can be carried out following Refs. [11,13]. Here we calculate a purely temporal growth rate of this four-wave instability (of relevance to very long laser pulses) using Eq. (8): $\gamma_{4w} = \omega_p \omega_B |a_0|/8\omega_0$.

In conclusion, a new method for generating coherent tunable radiation in the plasma is described. Radiation is emitted when an ultrashort laser pulse propagates through the plasma in the presence of a static periodic magnetic field. This radiation can be viewed as an electromagnetic wake, propagating along with the pulse. It is also demonstrated that, in the presence of static periodic magnetic field, long laser pulses become modulationally unstable, generating electromagnetic wakes.

This work was supported by the United States Department of Energy (U.S. DOE) Contract No. DE-AC02-CHO-3073. We gratefully acknowledge discussions with Dr. M. Zolotarev.

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