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Effect of quantum uncertainty on the rate of nuclear reactions in the Sun

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Abstract

Galitskii and Yakimets [ZhETF 51 (1966) 957] showed that the particle distribution function over momenta acquires a power-like tail even under conditions of thermodynamic equilibrium. The question whether this tail leads to acceleration of nuclear reaction rates is addressed by considering nuclear reactions in the Sun. The rates of the *pp* reaction as well as all decay and electron capture reactions are unchanged, while other reactions experience significant acceleration. These new rates meet important constraints such as the solar luminosity and the sound speed. Most interesting is that the ^{7}Be neutrino flux is decreased by the amount necessary to agree well with experimental data. © 2000 Elsevier Science B.V. All rights reserved.

A rather interesting effect in dense or low temperature media [1] is that the distribution function over momenta of particles can acquire a non-Maxwellian tail due to Lorentz-type dispersion relation between interacting particles. A provocative suggestion was made in [2,3] that such a tail may lead to a dramatic increase in the rate of nuclear and other reactions.

The purpose of this letter is to examine whether these calculations can be verified or falsified by analysing solar data. The theory of non-Maxwellian tails implies rates of fusion in the Sun for a number of reactions that are quite enhanced over what a Maxwellian distribution would give. The resulting observables appear not to be inconsistent with actual observations; in fact, the enhanced fusion rates calculated here do give a ^{7}Be neutrino flux that much more closely matches the solar data than what flows from conventional models of fusion [4]. There were earlier attempts at solving the neutrino problems through a modified distribution function [5], but no microscopic derivation of such a non-Maxwellian distribution function was given.

The argument for enhanced fusion rates arises from the form in dense media of the generalized particle distribution function $G^{-+}(\epsilon,\boldsymbol{p};\boldsymbol{R},t)$, where $\epsilon,\boldsymbol{p}$ are the energy and momentum of the particle [6]. For free particles $G^{-+}=i2\pi n_F(\epsilon)\delta(\epsilon-p^2/2m)$, where $n_F(\epsilon)$ is the Fermi distribution function. However, in dense media $G^{-+}\sim\delta_\gamma(\epsilon)$, with

$$\delta_\gamma(\epsilon)=\frac{\gamma(\epsilon,\boldsymbol{p})}{\pi\left[\left(\epsilon-\epsilon_p-\Delta(\epsilon,\boldsymbol{p})\right)^2+\gamma^2(\epsilon,\boldsymbol{p})\right]} \quad (1)$$

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instead of the delta function [7]. Here the shift $\Delta(\epsilon,\boldsymbol{p})$ and the width $\gamma(\epsilon,\boldsymbol{p})$ of the particle energy are equal to the real and imaginary parts of the retarded mass operator $\Sigma^R(\epsilon,\boldsymbol{p})$ respectively [7]; $\epsilon_{\boldsymbol{p}} = p^2/2m$.

The distribution over momenta $f(\boldsymbol{p}) = \int_{-\infty}^{+\infty} d\epsilon G^{-+}(\epsilon,\boldsymbol{p})$ becomes, at large momenta:

$$f(\boldsymbol{p}) = f_M(\boldsymbol{p}) + \frac{h\nu_{\boldsymbol{p}}T}{2\pi\epsilon_{\boldsymbol{p}}^2}\mathrm{e}^{\mu/T} \tag{2}$$

where $f_M(\boldsymbol{p})$ is the Maxwell distribution over momenta, $\nu_{\boldsymbol{p}}$ is the collision frequency with all background species, and T, μ are the temperature and chemical potential respectively [1]. The collision frequency with species j may be taken as $\nu_{pj} = 1.8 \times 10^{-7} n_j Z_\mathrm{p}^2 Z_j^2 \Lambda/(m_\mathrm{p}^{1/2}\epsilon_{\boldsymbol{p}}^{3/2})$, where Z_j is the charge state of species j, m_p is the particle mass normalized by the proton mass, Λ is the Coulomb logarithm, and density is given in cm^{-3} and energy in eV [8]. Note that the first term in Eq. (2) is just the Maxwellian distribution over momenta at thermal equilibrium, while the second term corresponds to the quantum tail [1].

Since it is the momentum rather than energy that enters into the scattering amplitude in the gaseous approximation [9], it has been speculated [2] that as a first approximation fusion reaction rates in equilibrium systems can be calculated by averaging the reaction cross-section

$$\sigma_{ij}(\epsilon_{\boldsymbol{p}}) = \frac{S_{ij}(\epsilon_{\boldsymbol{p}})}{\epsilon_{\boldsymbol{p}}}\exp\left[-\pi\left(\epsilon_G/\epsilon_{\boldsymbol{p}}\right)^{1/2}\right] \tag{3}$$

over the momentum distribution function given in Eq. (2). It is not our intent to enter into the subtleties of this speculation, but we do want to see if solar neutrino data stand in contradiction.

To do so, consider the ratio $r_{ij}(n,T) = \langle\sigma_{ij}v\rangle_\Delta/\langle\sigma_{ij}v\rangle_M$, where $\sigma_{ij}(\epsilon)$ is the cross-section of a nuclear reaction between species i and j, and the subscripts Δ, M indicate averaging with respect to the power-like part of the distribution (2) and Maxwell distribution respectively, giving

$$r_{ij} = 4.8\cdot 10^{-3}\,\frac{\rho\dfrac{\Lambda}{10}}{T_k^{5/2}}\,\frac{\mathrm{e}^{\tau_{ij}}}{\tau_{ij}^8}\sum_b \frac{Z_i^2 Z_b^2 X_b}{A_{ib}^{1/2}A_b} \tag{4}$$

Here

$$\tau_{ij} = 3(\pi/2)^{2/3}\left(\frac{100 Z_i^2 Z_j^2 A_{ij}}{T_k}\right)^{1/3},$$

$$A_{ij} = A_i\cdot A_j/\left(A_i + A_j\right),$$

T_k is the temperature in keV, ρ is the total density, X_b is the mass fraction of the background species.

Using temperature and density profiles calculated in the standard solar model [4], and using Eq. (4) to calculate the nuclear reaction rates, we can estimate abundances of certain elements, neutrino fluxes, luminosity and sound speed. The new nuclear reaction rates are likely to change the solar profiles, so full solar model calculations are necessary to get precise numbers. But these profiles may not change significantly, because the theory does not change the rate of the *pp* reaction. The rates of all electron capture and decay reactions are also unchanged.

Consider first neutrino fluxes. The *pp* reaction rate is unchanged leading to the same neutrino flux predicted by solar models. To see that the power-like tail of the distribution function hardly changes the rate of the *pp* reaction, note that, at the radius of the highest neutrino flux from this reaction, $r = 0.0759R_\odot$, the ratio r_{11} is

$$r_{11}(r = 0.0759R_\odot) = 3.5\cdot 10^{-3} \tag{5}$$

where parameters from the standard solar model are used [4].

On the other hand, the reaction $p + {}^7\mathrm{Be}$ is accelerated, so that the neutrino flux from the ${}^7\mathrm{Be}(e,\nu_\mathrm{e})\mathrm{Li}^7$ reaction is decreased. Interestingly, it is decreased by the amount required to agree well with the data [10]. The discrepancy between predictions of the standard solar models and the observations of this neutrino flux has been an outstanding puzzle for many years [10].

If we use the new reaction rate for ${}^7\mathrm{Be}(p,\gamma)^8\mathrm{B}$ but take into account only the decay ${}^8\mathrm{B}(\mathrm{e}^+\nu_\mathrm{e})^8\mathrm{Be}$, then the density of ${}^8\mathrm{B}$ is increased by three orders of magnitude. However, there exist other nuclear reactions in which ${}^8\mathrm{B}$ is destroyed, which are generally assumed unimportant due to very low rates. Our estimates indicate that these reactions will be accelerated significantly, so that the concentration of ${}^8\mathrm{B}$ may not increase by quite so much.

To estimate the solar luminosity we need to know the rates for two other reactions $^3\mathrm{He}+{}^3\mathrm{He}$ and $^4\mathrm{He}+{}^3\mathrm{He}$. Using the data from Ref. [4] and the astrophysical parameters $S_{ij}(\epsilon)$ from [11] we find the ratios r_{ij} and the rates $k_{ij}^{\mathrm{new}} = r_{ij}\langle\sigma_{ij}v\rangle_M$-$(\mathrm{cm}^3/\mathrm{s})$ for these reactions at the radius $0.0759R_\odot$ $r_{34}=3.1\cdot10^9, k_{34}^{\mathrm{new}}=3.2\cdot10^{-30}, r_{33}=4.5\cdot10^8, k_{33}^{\mathrm{new}}=5.1\cdot10^{-26}$. Since the rate k_{11} is unchanged and because the abundances of other elements are much smaller than 1, the abundancy of ^{4}He is also unchanged, $X_4\approx1-X_1$. The density of ^{3}He, however, is changed. We find it by solving the usual rate equation with new k_{ij}^{new}:

$$\dot{n}_3 = k_{11}n_1^2 - 2k_{33}n_3^2 - k_{34}n_3n_4 \tag{6}$$

The steady state solution is

$$n_3 = -\frac{k_{34}}{4k_{33}}n_4 + \sqrt{\left(\frac{k_{34}}{4k_{33}}\right)^2 + \frac{k_{11}}{2k_{33}}n_1^2} \tag{7}$$

If we evaluate n_3 with the classical rates, then the third term in Eq. (7) is much greater than the second one, so $n_3\approx n_1\sqrt{k_{11}/(2k_{33})}$. If we substitute new rates, then the third term is less then the second one, so $n_3\approx k_{11}n_1^2/(k_{34}n_4)$. This leads to ^{3}He density, $n_3^{\mathrm{new}}=3.5\cdot10^{12}\,\mathrm{cm}^{-3}$, as opposed to the value $n_3=6.3\cdot10^{20}$ obtained for the Maxwell distribution in [4].

The solar luminosity then can be estimated as

$$L_\odot \propto n_1^2k_{11}Q_{11} + n_3^2k_{33}Q_{33} + n_3n_4k_{34}Q_{34} \tag{8}$$

where Q_{ij} is the energy release per reaction. If we use Maxwell rates in Eq. (8), the dominant contribution comes from the first two terms and the luminosity is $L_\odot(0.0759R_\odot)=1.16\cdot10^9V$ MeV/s where V is the total volume of the Sun. With the new rates, the dominant terms are the first and the third ones, but the luminosity is almost unchanged, $L_\odot(0.0759R_\odot)=0.76\cdot10^9V$ MeV/s.

Neutrino flux from electron capture by ^{7}Be is proportional to the product of the rate of the electron capture, k_{e7}, and the density of ^{7}Be, n_7. Since the quantum tail increases the rates of the reactions occuring via the tunnelling through a Coulomb barrier, the rate of the electron capture, k_{e7}, is not changed. To find $n_{^7\mathrm{Be}}^{\mathrm{new}}$ we plug the new n_3^{new} density, $n_3^{\mathrm{new}}\approx k_{11}n_1^2/(k_{34}n_4)$, into the steady state solution

$$n_{^7\mathrm{Be}} = \frac{n_3n_4k_{34}}{n_{e7}k_{e7}+n_1k_{17}} \tag{9}$$

Since the classical rates give $n_{^7\mathrm{Be}}^M = n_3^Mn_4k_{34}^M/n_ek_{e7}$, we find the ratio

$$\frac{n_{^7\mathrm{Be}}^{\mathrm{new}}}{n_{^7\mathrm{Be}}^M} = \frac{n_1k_{11}}{n_4k_{34}^{\mathrm{new}}}\frac{r_{34}}{r_{17}}\frac{k_{e7}}{k_{17}}\sqrt{\frac{2k_{33}^M}{k_{11}}} \tag{10}$$

We find the following rates and ratios at $r=0$: $r_{34}=5.9\cdot10^8, k_{34}^{\mathrm{new}}=4.1\cdot10^{-30}, r_{17}=4.2\cdot10^5, k_{33}^M=6.9\cdot10^{-34}$. Substituting them into Eq. (10), we find that the ratio of the ^{7}Be neutrino fluxes is

$$\frac{\phi^{\mathrm{new}}(^7\mathrm{Be})}{\phi^M(^7\mathrm{Be})} = \frac{n_{^7\mathrm{Be}}^{\mathrm{new}}}{n_{^7\mathrm{Be}}^M} = \frac{1}{50} \tag{11}$$

This is roughly the value necessary to explain one of the neutrino puzzles [10].

The density of ^{8}B is $n_8=n_1n_{^7\mathrm{Be}}k_{17}/k_{\beta^8}$. Using Eq. (11) we find

$$\frac{n_8^{\mathrm{new}}}{n_8^M}(r=0) = r_{17}\frac{n_{^7\mathrm{Be}}^{\mathrm{new}}}{n_{^7\mathrm{Be}}^M} = 8.4\cdot10^3 \tag{12}$$

This value exceeds the desired one [10] by about 10^3. Certain other reactions, $^8\mathrm{B}(d,2p)2\alpha$, $^8\mathrm{B}(^3\mathrm{He},3p)2\alpha$, $^8\mathrm{B}(\alpha,p)C^{11}$, do contribute to the burning of ^{8}B. According to Eq. (4) the ratios for these reactions at $r=0$ are respectively as follows: $r_{28}=1.2\cdot10^{12}, r_{38}=2.6\cdot10^{29}, r_{48}=1.9\cdot10^{32}$. Due to the large value of β-decay rate of ^{8}B, they lead to a density ratio which differs only slightly from Eq. (12), $n_8^{\mathrm{new}}/n_8^M(r=0)=7\cdot10^3$.

However, there are uncertainties large enough to account for this discrepancy: one, the S-factors for these rare reactions have not been measured, while the theoretical uncertainty is high [12]; and, two, the enhancement of 10^{32} is so high that small corrections in the exponent, 32 (for example, due to non-equilibration) could have a large effect. Note, by way of comparison, the enhancement for ^{7}Be, leading to Eq. (11), is only a factor of 10^8, so corrections are likely smaller.

Many of the reactions in the CNO cycle will experience large acceleration. This will change the abundances but will not lead to the accelerated burn of all of the elements. The speed of stages I, II, III as well as the loss channel of the whole CNO cycle $F^{19}(p,\gamma)Ne^{20}$ [11] will be determined by the β^+ decay reactions rates, which are unchanged. This also means that the mean average molecular weight, A, and hence the sound speed will be left unchanged from standard solar model predictions.

Our theory applies to the case of thermal equilibrium, but becomes invalid when $h\nu_{ij}/T \gg 1$. This parameter is $h\nu_{ij}/T \ll 1$ everywhere in the Sun. So the theory applies almost everywhere, except the very outer region $r > 0.8\,R_\odot$, where the ions are not fully stripped because of recombination at low temperature. The physics of nuclear reactions is more complicated there and does not reduce to a simple picture of quantum tunneling through a Coulomb barrier, which we used to determine the reaction rates. Yet if we still assume fully stripped ions in this region, our theory leads to high enough reaction rates to change the surface abundances. This question requires further study.

It is interesting to consider also nuclear fusion under laboratory conditions, for example, the reaction D + T. In order to maximize the effect of the power-like distribution function, one can increase the collision rate ν_{ib} by mixing D, T with higher Z elements. Consider the mixture $g_{\mathrm{D}} = n_{\mathrm{D}}/n_{\mathrm{e}} = g_T = 0.25$, with $Z = 50$, $A = 100$, and with the density n_A being found from quasineutrality. For temperature $T = 0.1\,\mathrm{keV}$ and total density $\rho = 10^2\ \mathrm{g/cm^2}$, we find $r_{\mathrm{DT}} = 8.6 \cdot 10^7, k_{\mathrm{DT}}^M = 6.1 \cdot 10^{-29}\mathrm{cm^3/s}$, $k_{\mathrm{DT}}^{\mathrm{new}} = 5.3 \cdot 10^{-21}\mathrm{cm^3/s}$. This, however, is still much less than the required rate $k_{\mathrm{DT}}^M = 1.1 \cdot 10^{-16}\mathrm{cm^3/s}$, expected to be achieved in a tokamak at $T = 10\mathrm{keV}$.

Note that this rate $k_{\mathrm{DT}}^{\mathrm{new}}$ is calculated for the plasma in the strongly coupled regime, with the non-ideality parameter being equal to $\Gamma = ((4\pi/3)n\lambda_d^3)^{-2/3} = 37.2$. The Coulomb logarithm was put to $\Lambda = 1$ in Eq. (4). This regime is already beyond the applicability of our theory, since it is only valid when $\Gamma \le 1$. However, if we try to extrapolate the theory even further into the strongly coupled regime, the ratio becomes very high. An important contribution to the acceleration of the nuclear reaction rate in this regime is also made by the effects of screening [13,14], which are not very important in the Sun interior. For example, for the same mixture of D,T at density $\rho = 10^2\ \mathrm{g/cm^2}$ and temperature $T = 0.041\,\mathrm{keV}$ the results with screening included are $k_{\mathrm{DT}}^M = 4.6 \cdot 10^{-31}\ \mathrm{cm^3/s}, k_{\mathrm{DT}}^{\mathrm{new}} = 1.02 \cdot 10^{-16}\mathrm{cm^3/s}, \Gamma = 90.8$. Although, these figures are not to be trusted quantitatively, they indicate that we may expect interesting physics in this regime.

If the theory is extrapolated beyond its limit of applicability, namely to a very high density low temperature plasma of reacting nuclei $h\nu_{ij}/T \gg 1$, it predicts a very large acceleration of nuclear reaction rates. For certain types of nuclear mixtures, this acceleration leads indeed to high enough reaction rates that one may speculate on possibilities for controlled nuclear fusion.

We note that recent attempts were made to consider the influence of the quantum uncertainty on the nuclear reaction rates in Ref. [15,18]. Although close in spirit, these treatments are different from ours, which takes into account quantum kinetic effects. The influence of classical kinetic effects on the reaction rate was considered in [16].

In conclusion, we showed that solar data apparently does not contradict theories of enhanced nuclear reaction rates based on quantum uncertainty. On the contrary, the ^{7}Be neutrino flux now matches the experimental data, while luminosity and sound speed appear to be unaltered. Although the ^{8}B neutrino flux does not appear to match experimental data, uncertainties in the calculation of this flux could well account for the mismatch. Of course, precise solar model calculations are necessary to validate all of the speculations posed here. But the encouraging match to solar neutrino data presented here has already motivated more rigorous consideration of the speculations [2,3] that quantum tails might lead to a dramatic increase in the rate of nuclear and other reaction rates (see [17]).

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