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Variational formulation of the Gardner's restacking algorithm

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Abstract

The incompressibility of the phase flow of Hamiltonian wave-plasma interactions restricts the class of realizable wave-driven transformations of the particle distribution. After the interaction, the distribution remains composed of the original phase-space elements, or local densities, which are only rearranged ("restacked") by the wave. A variational formalism is developed to study the corresponding limitations on the energy and momentum transfer. A case of particular interest is a toroidal plasma immersed in a dc magnetic field. The restacking algorithm by Gardner [Phys. Fluids 6 (1963) 839] is formulated precisely. The minimum energy state for a plasma with a given current is determined.

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1. Introduction

The incompressibility of the phase flow of Hamiltonian wave-plasma interactions restricts the class of realizable wave-driven transformations of the particle distribution [1]. This restricts the energy in a plasma available for extraction [2]. In the case where the particle interactions with waves are diffusive, the energy available for extraction is further constrained by the consideration of only diffusive phase-space rearrangements [3]. This limits, for example, the amount of energy that can be extracted from α particles in a tokamak [4].

A related problem has also been addressed recently in Refs. [5,6] in connection with generating plasma current by means of an asymmetric ponderomotive barrier for thermal particles. In this case, a one-way rf barrier is set up that can reflect particles coming from one direction, while being transparent to particles coming from the other direction. The barrier must of necessity heat the particles that pass through it in order to conserve the phase space density. This means that the current can be generated by these barriers in plasma, but only at the price of some energy expended.

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In general, all these limitations can be attributed to the existence of what can be called the "plasma ground state" for a given one-particle distribution f_1. By ground state, we mean such a distribution of particles f_2, which minimizes the total plasma energy on the manifold of all Hamiltonian transformations $f_1 \to f_2$. As reported in the pioneering paper by Gardner [1], the ground state plasma energy $W_{\min}$ is generally nonzero, which can be explained as follows. Suppose that a bounded plasma particles having the initial phase-space distribution f_1 are introduced into an electromagnetic field for a limited time, which eventually results in bringing the plasma to some final state f_2. Imagine that we partition the plasma phase space into small cells of equal volume $\Delta\Gamma_i = \Delta\Gamma$, and to each cell attach a certain value of the distribution function $f(\Gamma_i)$. As the number of cells that have a given value of f is conserved throughout the interaction (as follows from the Liouville theorem), the distribution f_2 may not be arbitrary, but rather will represent a result of reordering ("restacking") of the original phase-space elements $\Delta\Gamma_i$, regardless of the spatial and temporal structure of the external fields. Alternatively, this fact can be expressed as conservation of the so-called Casimir invariants, which determine the distribution of the values $f(\Gamma_i)$ (see, e.g., Ref. [7]) and whose existence is an intrinsic property of any Hamiltonian system.

The plasma ground state will correspond to the distribution f_2, such that the elements $\Delta\Gamma_i$ with larger $f(\Gamma_i)$ occupy the states with lower particle energy $\mathcal{E}$. In a bounded plasma, only a finite phase volume is allotted to the states with $\mathcal{E}$ below a given value. Hence, from incompressibility of the phase flow, it follows that after the interaction the plasma will be left with the total energy $W \geqslant W_{\min}$, where $W_{\min}$ is a nonzero quantity defined as the minimum of W over all possible ways of restacking the elements $\Delta\Gamma_i$.

While chopping phase space into discrete elements is pictorial, it is fairly artificial in case of a continuous function f_1. Hence, solving the "restacking problem" must be possible in terms of a differential formulation. The purpose of the present Letter is to derive such a formulation and apply it to a number of cases of interest, not previously considered.

The Letter is organized as follows: in Section 2, we generalize the Gardner's problem by putting it into a variational form for an abstract dynamical system. We determine the condition under which a Hamiltonian transformation of the system phase space yields the maximum or minimum of a given functional (such as the plasma energy in Ref. [1]). In the framework of this formalism, we reproduce the results given in Ref. [1] and, in Section 3, solve a similar, yet different problem of finding the minimum energy state at given plasma current. In Section 4, we apply our formalism to magnetized toroidal plasmas and derive a reduced variational principle. In Section 5, we summarize our main ideas.

2. Variational formalism

Let us first restate the Gardner's problem in its original form [1]. Suppose that a bounded plasma with the initial distribution $f(\mathbf{r}_1, \mathbf{p}_1)$ is introduced into external fields for a limited time, which eventually results in bringing the plasma to some final state $f(\mathbf{r}_2, \mathbf{p}_2)$. The particle distribution is conserved: $f(\Gamma_2) = f(\Gamma_1)$, where $\Gamma_2 \equiv (\mathbf{r}_2, \mathbf{p}_2)$ is a single-valued reversible function of $\Gamma_1 \equiv (\mathbf{r}_1, \mathbf{p}_1)$. Thus, the total energy left inside the plasma after the interaction equals

$$W = \int \mathcal{E}(\Gamma_2) f(\Gamma_1)\, d\Gamma, \tag{1}$$

where $\mathcal{E}$ is the individual particle energy, and where we made use of phase space conservation: $d\Gamma \equiv d\Gamma_1 = d\Gamma_2$, $d\Gamma_i \equiv d^3 r_i\, d^3 p_i$. Suppose that the particles initially occupy a nonzero phase volume. In a bounded plasma, only a finite phase volume is allotted to the states with $\mathcal{E}(\Gamma_2)$ below a given value. Hence, from incompressibility of the phase flow it follows that after the interaction the plasma will be left with the total energy $W \geqslant W_{\min}$, where

$$W_{\min} = \min_{\Gamma_1 \to \Gamma_2} \int \mathcal{E}(\Gamma_2) f(\Gamma_1)\, d\Gamma \tag{2}$$

is a nonzero quantity defined as the minimum of W over all possible Hamiltonian (canonical) phase-space transformations $(\mathbf{r}_1, \mathbf{p}_1) \to (\mathbf{r}_2, \mathbf{p}_2)$. Hence, determination of $W_{\min}$ can be considered as a variational problem of searching for the canonical transformation $\Gamma_1 \to \Gamma_2$, which minimizes the functional (1).

Treated like that, the Gardner's problem yields a natural generalization as follows. Suppose one is given a function $\phi(\Gamma_1)$ defined in a $2N$-dimensional phase space Γ_1. Suppose also that $\Gamma_1 \equiv (\mathbf{q}, \mathbf{p})$ undergoes Hamiltonian evolution into some phase space $\Gamma_2 \equiv (\mathbf{Q}, \mathbf{P})$. The generalized Gardner's problem then consists of determining the connection between Γ_1 and Γ_2, which provides the minimum or the maximum of the functional

$$G = \int \psi(\Gamma_2)\phi(\Gamma_1)\, d\Gamma, \tag{3}$$

where Γ_2 is considered a function of Γ_1 (or vice versa), ψ is a known function of Γ_2, and $d\Gamma \equiv d\Gamma_1 = d\Gamma_2$ is a phase space element

$$d\Gamma \equiv d^N q\, d^N p = d^N Q\, d^N P \tag{4}$$

conserved by the canonical transformation.

Suppose $\Gamma_1 \to \Gamma_2$ is an extremizing transformation and consider a small canonical transformation $\Gamma_2 \to \Gamma_2'$ determined by an arbitrary trial Hamiltonian $\mathcal{H}$. Since ϕ is defined as a given function of the initial variables, which remain unchanged by the transformation, one gets

$$\Delta G = \int \Delta\psi(\Gamma_2)\phi(\Gamma_1)\, d\Gamma_1. \tag{5}$$

The change of ψ, $\Delta\psi = \psi(\Gamma_2') - \psi(\Gamma_2)$, can be expressed as

$$\Delta\psi \approx \frac{\partial\psi}{\partial\mathbf{Q}} \cdot \Delta\mathbf{Q} + \frac{\partial\psi}{\partial\mathbf{P}} \cdot \Delta\mathbf{P} + \frac{1}{2}\Delta\mathbf{Q} \cdot \frac{\partial^2\psi}{\partial\mathbf{Q}\partial\mathbf{Q}} \cdot \Delta\mathbf{Q} + \Delta\mathbf{Q} \cdot \frac{\partial^2\psi}{\partial\mathbf{Q}\partial\mathbf{P}} \cdot \Delta\mathbf{P} + \frac{1}{2}\Delta\mathbf{P} \cdot \frac{\partial^2\psi}{\partial\mathbf{P}\partial\mathbf{P}} \cdot \Delta\mathbf{P}, \tag{6}$$

where the dot product stands for summation over repeating indices. The changes of $\Delta\mathbf{Q}$ and $\Delta\mathbf{P}$ are derivable from canonical equations and can be put in the form

$$\Delta\mathbf{Q} = \Delta t \frac{\partial\mathcal{H}}{\partial\mathbf{P}} + \frac{\Delta t^2}{2}\left\{\mathcal{H}, \frac{\partial\mathcal{H}}{\partial\mathbf{P}}\right\} + o\big(\Delta t^2\big), \tag{7a}$$

$$\Delta\mathbf{P} = -\Delta t \frac{\partial\mathcal{H}}{\partial\mathbf{Q}} - \frac{\Delta t^2}{2}\left\{\mathcal{H}, \frac{\partial\mathcal{H}}{\partial\mathbf{Q}}\right\} + o\big(\Delta t^2\big), \tag{7b}$$

where Δt is the time interval, on which the evolution generated by $\mathcal{H}$ is considered, and $\{\cdot,\cdot\}$ stands for Poisson brackets

$$\{f, g\} \equiv \frac{\partial f}{\partial\mathbf{P}} \cdot \frac{\partial g}{\partial\mathbf{Q}} - \frac{\partial f}{\partial\mathbf{Q}} \cdot \frac{\partial g}{\partial\mathbf{P}}. \tag{8}$$

Substituting the above equations into Eq. (5) and integrating by parts, one gets that

$$\Delta G = \delta G + \delta^2 G + o\big(\Delta t^2\big), \tag{9a}$$

$$\delta G = \Delta t \int \{\psi, \phi\}\mathcal{H}\, d\Gamma, \tag{9b}$$

$$\delta^2 G = -\frac{\Delta t^2}{2} \int \{\psi, \mathcal{H}\}\{\phi, \mathcal{H}\}\, d\Gamma, \tag{9c}$$

assuming that the surface integrals are equal to zero. (In case if ψ or ϕ stands for a distribution function, the surface integrals vanish, e.g., if the system is localized within a finite phase volume.) Because of the invariance of Poisson brackets, the above expressions equally apply to any space Γ canonically obtainable from Γ_1 or Γ_2 (Γ may also coincide with one of the two), if the functions are understood as

$$\phi = \phi\big[\Gamma_1(\Gamma)\big], \qquad \psi = \psi\big[\Gamma_2(\Gamma)\big]. \tag{10}$$

From Eqs. (9) it follows that the necessary condition for an extremizer, that is $\delta G = 0$ for an arbitrary $\mathcal{H}$, can be put in the form

$$\{\psi, \phi\} = 0. \tag{11}$$

In turn, the minimum and maximum of G are realized when $\delta^2 G$ has a definite sign regardless of $\mathcal{H}$. Noting that

$$\delta\psi = \Delta t\{\mathcal{H}, \psi\}, \qquad \delta\phi = \Delta t\{\mathcal{H}, \phi\}, \tag{12}$$

one can rewrite the expression for $\delta^2 G$ as

$$\delta^2 G = -\frac{1}{2} \int \delta\psi\, \delta\phi\, d\Gamma. \tag{13}$$

From Eq. (13) it can be concluded that the minimum of G is achieved if

$$\phi = \phi(\psi), \quad \text{or} \quad \psi = \psi(\phi) \tag{14}$$

is a single-valued monotonically decreasing function; the maximum of G corresponds to a single-valued monotonically increasing function (14). (Note also that Eq. (14) automatically satisfies Eq. (11).)

The function (14) can be determined using the phase-space conservation imposed by the Liouville theorem. With the density of states Ω defined for an arbitrary function $\xi(\Gamma)$ according to

$$\Omega(\tilde{\xi}) \equiv \int \delta\big[\tilde{\xi} - \xi(\Gamma)\big]\, d\Gamma, \tag{15}$$

the phase-space conservation requires that

$$\left|\frac{d\phi}{d\psi}\right| = \frac{\Omega(\psi)}{\Omega(\phi)}, \qquad \phi(\psi_0) = \phi_0. \tag{16}$$

The sign of the derivative and the constant of integration must be chosen correspondingly, depending on whether the maximum or the minimum of G is required: taking $d\phi/d\psi \geqslant 0$ with $\phi(\psi_{\max}) = \phi_{\max}$ yields the maximizer, while $d\phi/d\psi \leqslant 0$ with $\phi(\psi_{\max}) = \phi_{\min}$ corresponds to the minimizer.

If $\phi(\Gamma_1)$ and $\psi(\Gamma_2)$ are continuous functions, then the transformation, which yields the absolute minimum or the absolute maximum of G, is at least piecewise continuous. In this case, the differential formulation as presented here is more natural than the "Gardner's restacking algorithm" described in Ref. [3]. Making use of the differential formulation in certain cases can yield an analytical solution of the restacking problem or, at least, allow a solution by quadratures, hence simplifying the numerical procedure of finding the extremized functional value. On the other hand, our results can also be formulated in terms of reordering of discrete phase space elements as follows: To obtain an extremizing transformation, first, chop the phase space Γ into differentially small elements with equal volume $d\Gamma^{(i)} = d\Gamma$, each confined between the neighboring isosurfaces of ϕ. Numerate the elements in ascending order with respect to $\phi(\Gamma^{(i)})$. Then prepare the new "sites" for these elements – the phase space bins of the same volume, $d\tilde{\Gamma}^{(j)} = d\Gamma$, each located between the neighboring isosurfaces of ψ. Numerate them in ascending order with respect to $\psi(\tilde{\Gamma}^{(j)})$ and allocate $\Gamma^{(i)}$ at $\tilde{\Gamma}^{(j)}$. The maximizing transformation requires that $i(j)$ be an increasing function, while the minimizing transformation requires that $i(j)$ be decreasing.

Let us apply the obtained results to reproduce the solution of the original Gardner's problem. To put the energy functional (1) into the form (3), take

$$\phi(\Gamma_1) = f(\mathbf{r}_1, \mathbf{p}_1), \qquad \psi(\Gamma_2) = \mathcal{E}_2(\mathbf{r}_2, \mathbf{p}_2). \tag{17}$$

If $\mathcal{E}_2$ includes only the kinetic energy of a particle ($\mathcal{E}_2 = p_2^2/2m$), then the final particle distribution $f(\mathbf{r}_2, \mathbf{p}_2)$ corresponding to the minimum plasma energy cannot depend on $\mathbf{r}_2$, as follows already from Eq. (11). From the subsequent arguments, one gets that the final distribution must be a single-valued decreasing function of $\mathcal{E}_2$ and satisfy the equation

$$\frac{df}{d\mathcal{E}_2} = -\frac{\Omega(\mathcal{E}_2)}{\Omega(f)}, \tag{18}$$

where $f(0) = \max f(\mathbf{r}_1, \mathbf{p}_1)$. The density of states $\Omega(f)$ can be calculated knowing the initial distribution $f(\mathbf{r}_1, \mathbf{p}_1)$, and $\Omega(\mathcal{E}_2)$ is given by

$$\Omega(\mathcal{E}_2) = 4\pi m\sqrt{2m\mathcal{E}_2}. \tag{19}$$

The same analysis, including Eq. (18), applies if $\mathcal{E}_2$ contains also a potential energy of a particle in a static background field U,

$$\mathcal{E}_2(\mathbf{r}_2, \mathbf{p}_2) = \frac{p_2^2}{2m} + U(\mathbf{r}_2), \tag{20}$$

if the density of states $\Omega(\mathcal{E}_2)$ is modified correspondingly. Hence, one can see that the results of Ref. [1] can be naturally obtained in the framework of the proposed formalism. Yet, the latter also yields other results of interest, as we show in the next sections.

3. Conditional extremum

The formalism presented in Section 2 yields a natural generalization to the case of a conditional restacking problem. Consider finding an extremum of the functional (3) under the condition

$$\mathbf{R} = \mathbf{R}^{(0)}, \qquad \mathbf{R} = \{R_i \mid i = 1, \ldots, K\}, \tag{21}$$

where

$$R_i = \int \tilde{\psi}_i(\Gamma_2)\tilde{\phi}_i(\Gamma_1)\, d\Gamma, \tag{22}$$

assuming that $\tilde{\psi}_i(\Gamma_2)$, $\tilde{\phi}_i(\Gamma_1)$ are given functions, and $\mathbf{R}^{(0)} = \{R_i^{(0)} \mid i = 1, \ldots, K\}$ is a set of constants. Conditional extrema of G are realized at unconditional extremizers of the functional

$$\tilde{G} = G + \boldsymbol{\lambda} \cdot \mathbf{R}, \tag{23}$$

where $\boldsymbol{\lambda} = \{\lambda_i \mid i = 1, \ldots, K\}$ are indefinite Lagrange multipliers to be found. As seen from the previous analysis, the extrema of $\tilde{G}$ are realized under the condition

$$\{\psi, \phi\} + \sum_{i=1}^{K} \lambda_i \{\tilde{\psi}_i, \tilde{\phi}_i\} = 0. \tag{24}$$

If $\tilde{\phi}_i$ are all equal to ϕ (alternatively, all $\tilde{\psi}$ may be equal to ψ), Eq. (24) is simplified:

$$\{\Psi, \phi\} = 0, \tag{25}$$

where $\Psi = \psi + \boldsymbol{\lambda} \cdot \tilde{\boldsymbol{\psi}}$. Eq. (25) has the form of Eq. (11) and hence can be solved by the method proposed in Section 2, with λ_i to be determined from Eq. (21).

An illustrative example of how this formalism can be applied is the problem of finding (again, assuming phase-space conservation) the minimum energy state of a plasma with a given current. The problem has a definite practical value, as its solution determines how much energy is required for generating a given amount of plasma current. (Note, however, that this is not the problem that determines the "efficiency" of maintaining a current, since the maximum efficiency may not occur for the minimum energy distribution [8].) To get the minimum energy current, consider the frame of reference moving with the velocity $\mathbf{v}_0 = \mathbf{j}/en$, where $\mathbf{j}$ is the current density, e is the charge of an individual particle, and n is the particle number density. Solve the unconditional energy minimization problem for the new frame, as shown in Section 2. The absolute minimum of the total particle energy is achieved at an isotropic distribution, which carries no current. On the other hand, in the moving frame, the current density *must* be zero by definition. Thus, the solution of the unconditional problem in the new frame satisfies the requirements of the original conditional problem. Hence, the minimum energy state at given current is realized at particle distribution isotropic and monotonically decreasing with energy in the frame of reference where the net current is zero.

Note that the same result can be obtained formally as follows. Consider the functional

$$\tilde{G} = W + \boldsymbol{\lambda} \cdot \mathbf{j}, \tag{26}$$

where the plasma current, assuming given initial distribution $f(\mathbf{p}_1)$, equals

$$\mathbf{j} = e \int \mathbf{v}_2 f(\mathbf{p}_1)\, d^3 p_1. \tag{27}$$

Rewrite Eq. (26) as

$$\tilde{G} = \int \frac{(\mathbf{p}_2 + \boldsymbol{\lambda} e)^2}{2m} f(\mathbf{p}_1)\, d^3 p_1 - \frac{\lambda^2 e^2}{2m}, \tag{28}$$

where the value of the second term is fixed, and take

$$\phi(\Gamma_1) = f(\mathbf{p}_1), \qquad \Psi(\Gamma_2) = \frac{(\mathbf{p}_2 + \boldsymbol{\lambda} e)^2}{2m}. \tag{29}$$

Hence, one can see that, to minimize $\tilde{G}$, the particle distribution must become a function of energy in the frame of reference moving with $\mathbf{v}_0 = -\boldsymbol{\lambda}/me$, and thus

$$\tilde{G}_{\min} = W'_{\min} - \frac{\lambda^2 e^2}{2m}. \tag{30}$$

Here $W'_{\min}$ is the minimized energy in the moving frame, where the particle distribution must be isotropic (as follows from Section 2), i.e., carry no current. The total current then equals $en\mathbf{v}_0$, hence

$$\boldsymbol{\lambda} = -m\mathbf{j}/ne^2. \tag{31}$$

On the other hand, by definition,

$$\tilde{G}_{\min} = W_{\min} + \boldsymbol{\lambda} \cdot \mathbf{j}. \tag{32}$$

Using Eqs. (30)–(32), one has

$$W_{\min} = \frac{mj^2}{2ne^2} + W'_{\min}, \tag{33}$$

where the first term represents the energy of the average flow, while the second term stands for the minimum thermal energy of the original distribution $f(\mathbf{p}_1)$, which cannot be reduced further by Hamiltonian transformations of the original particle distribution.

4. Restacking algorithm for magnetized toroidal plasmas

Consider now the formalism developed in Section 2 in application to magnetized plasmas. Assume that a plasma has a toroidal geometry, so that inhomogeneities along the magnetic field are smoothed out and the plasma becomes uniform along a flux surface on time scales large compared to the period of particle rotation along the torus. Similarly, assume uniform distribution over gyrophases, plus assume homogeneous plasma profile across flux surfaces.

Suppose now that the plasma, having an initial distribution $f(\mathbf{r}_1, \mathbf{p}_1)$, undergoes Hamiltonian interaction with an electromagnetic field for a limited time, which eventually results in bringing the plasma to some final state $f(\mathbf{r}_2, \mathbf{p}_2)$. The number of particles within each phase space element is conserved:

$$f(\Gamma_1)\, d\Gamma_1 = f(\Gamma_2)\, d\Gamma_2, \tag{34}$$

as well as conserved is the distribution function itself, as follows from the Liouville theorem. Assuming spatially uniform both initial and final distributions and neglecting the dependence on the gyrophase, obtain

$$f(\epsilon_2, u_2) = f(\epsilon_1, u_1), \tag{35}$$

where ϵ is the energy of the particle motion transverse to the dc magnetic field, and u is the particle velocity along the field. Since $d\Gamma_i = m^2\, du_i\, d\epsilon_i\, d\theta_i\, dV_i$, where θ_i is the gyrophase and dV_i is the element of a spatial volume, from Eqs. (34), (35) one has

$$du_2\, d\epsilon_2 = du_1\, d\epsilon_1 \tag{36}$$

after integrating over θ and V. Eq. (36) can be considered as the requirement of space conservation on an effective phase plane (u, ϵ), whose evolution can hence be considered a Hamiltonian process with a single degree of freedom ($N = 1$).

A variational formalism, like in Section 2, can be readily restated for the reduced system. Hence, the absolute maximum and the absolute minimum of the functional

$$G = \int \psi(u_2, \epsilon_2)\phi(u_1, \epsilon_1)\, d\tilde{\Gamma} \tag{37}$$

($d\tilde{\Gamma} \equiv d\tilde{\Gamma}_1 = d\tilde{\Gamma}_2$, $d\tilde{\Gamma}_i = du_i\, d\epsilon_i$) are realized by phase space transformations, which map the surfaces of constant ϕ on the plane (u_1, ϵ_1) to the surfaces of constant ψ on the plane (u_2, ϵ_2) and provide that $\phi(\psi)$ becomes a monotonic function given by Eq. (16). If $d\phi/d\psi \geqslant 0$, the absolute maximum is realized; if $d\phi/d\psi \leqslant 0$, then the absolute minimum is obtained.

5. Conclusions

We developed a variational formalism to study the phase space limitations on the Hamiltonian interaction between plasmas and electromagnetic fields. The solution of the so-called Gardner's restacking problem [1] was given a precise mathematical formulation over a class of piecewise continuous phase space transformations. The analysis was extended to the conditional restacking problem, through which we found the minimum energy state of a plasma with a given current. We also showed how the formalism could be applied to toroidal plasmas in a dc magnetic field.

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