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Atomic Physics Effects on Dissipative Toroidal Drift Wave Stability

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Abstract

The effects of atomic physics processes such as ionization, charge exchange, and radiation on the linear stability of dissipative drift waves are investigated in toroidal geometry both numerically and analytically. For typical Tokamak Fusion Test Reactor (TFTR) [*Plasma Physics and Controlled Nuclear Fusion Research 1986* (IAEA, Vienna, 1987), Vol. 1, p. 51] and Texas Experimental Tokamak (TEXT) [Nucl. Technol./Fusion **1**, 479 (1981)] edge parameters, overall linear stability is determined by the competition between the destabilizing influence of ionization and the stabilizing effect due to the electron temperature gradient. An analytical expression for the linear marginal stability condition, η_e^{crit}, is derived. The instability is most likely to occur at the extreme edge of tokamaks with a significant ionization source and a steep electron density gradient.

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I. Introduction

Tokamak edge plasmas are characterized by large density and potential fluctuations and large particle diffusivities, which typically increase toward the edge as a function of minor radius.[1−3] Most previous drift wave turbulence models have not been successful in explaining the radial profiles of the diffusivities and the large fluctuation levels.[4] Another widely studied edge turbulence candidate is resistivity gradient driven turbulence[5] which evolves from the rippling instability[6] in the linear regime. Although this model predicts larger fluctuation levels and radially increasing particle diffusivities, it suffers from the explicit dependence on the edge current density, which has been called into question by a number of current ramp experiments.[7,8] However, the radial gradient of the toroidal electric field which develops during the ramp could drive some residual instability. Including realistic effects, such as impurity dynamics[9,10] and radiation,[11] tends to make the theoretical predictions closer to experimental observations.

Recently, drift wave turbulence has been reconsidered including atomic physics effects with a hope that the aforementioned problems could be remedied.[12−14] Most of these efforts have been performed in a simplified geometry, with a focus on the nonlinear regime.

In this paper, we consider the effects of atomic physics on the linear stability of dissipative drift waves in toroidal geometry. It has been shown that in toroidal geometry, the coupling of adjacent poloidal harmonics renders magnetic shear induced damping ineffective.[15] Therefore, the instability with toroidal mode structure is more likely to be excited than one with Pearlstein-Berk mode structure.[16]

For typical Tokamak Fusion Test Reactor (TFTR)[17−19] and Texas Experimental Tokamak (TEXT)[20−22] edge parameters, atomic physics effects are small, and are most important for long wavelength modes. We therefore treat atomic physics and collisions perturbatively. For typical parameters, the toroidal mode structure[15] is maintained in the presence of atomic physics effects. The electron dynamics are nearly adiabatic (Boltzmann-like), rather than hydrodynamic, over most of the width of the eigenmode, although the deviation from adiabaticity is crucial and non-negligible. Various asymptotic regimes in parameter space are identified and fluctuation characteristics are discussed for each regime. Relations to previous works are elucidated.

It is shown that stability is typically determined by competition between ionization (destabilizing) and collisions (stabilizing for typical η_e). To properly evaluate the collisional effects and overall stability, *nonlocal* analysis is necessary.

The organization of the rest of this paper is as follows. In Sec. II, we present the basic model and obtain the dispersion relation in the local limit. The basic trends atomics physics effects have on stability are indicated by local analysis. We turn to the nonlocal analysis in Sec. III, where we derive the eigenmode equation using the ballooning transformation, and solve it both numerically and analytically. Conclusions and discussion of these results are presented in Sec. IV.

II. Theoretical Model and Local Analysis

The basic model for dissipative drift waves can be derived from the Braginskii fluid equations.[23] To properly include atomic physics effects, we take moments of the Boltzmann equation:

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\nabla_x f + \frac{q}{m}\left(\mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c}\right)\cdot\nabla_v f = \frac{\delta f}{\delta t}.$$

obtaining, for the zeroth, first, and second velocity moments:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{u}) = \int_{-\infty}^{\infty} d\mathbf{v}\frac{\delta f}{\delta t}, \tag{1}$$

$$\begin{aligned} nm\frac{d\mathbf{u}}{dt} &= -\nabla p + qn\left(\mathbf{E} + \frac{\mathbf{u}\times\mathbf{B}}{c}\right) &(2)\\ &+ m\int_{-\infty}^{\infty} d\mathbf{v}\frac{\delta f}{\delta t}(\mathbf{v}-\mathbf{u}), &(3)\\ \frac{3}{2}n\frac{dT}{dt} &= -p\nabla\cdot\mathbf{u} - \nabla\cdot\mathbf{q} &(4)\\ &+ \frac{m}{2}\int_{-\infty}^{\infty} d\mathbf{v}\frac{\delta f}{\delta t}(\mathbf{v}-\mathbf{u})^2 - T\int_{-\infty}^{\infty} d\mathbf{v}\frac{\delta f}{\delta t}. &(5) \end{aligned}$$

Atomic physics processes and collisions enter these moment equations through the velocity space integrals of

$$\frac{\delta f}{\delta t} = \left(\frac{\delta f}{\delta t}\right)_{coll} + \left(\frac{\delta f}{\delta t}\right)_{ion} + \left(\frac{\delta f}{\delta t}\right)_{cx} + \left(\frac{\delta f}{\delta t}\right)_{rad}.$$

Therefore, the usual cold ion Braginskii equations are modified in the following way:

$$\frac{\partial n_e}{\partial t} + \nabla\cdot(n_e\mathbf{v}_e) = S_e, \tag{6}$$

$$\begin{aligned} m_e n_e\frac{d\mathbf{v}_e}{dt} &= -\nabla p_e - en_e(\mathbf{E} + \frac{\mathbf{v}_e\times\mathbf{B}}{c}) &(7)\\ &+ \mathbf{R}_{ei} + m_e S_e(\mathbf{v}_N - \mathbf{v}_e), &(8)\\ \frac{3}{2}n_e\frac{dT_e}{dt} &= -p_e\nabla\cdot\mathbf{v}_e - \nabla\cdot\mathbf{q}_e + Q_{ei} &(9)\\ &- P_{rad} - T_e S_e + \frac{m_e}{2}S_e(\mathbf{v}_N - \mathbf{v}_e)^2, &(10) \end{aligned}$$

$$\frac{\partial n_i}{\partial t} + \nabla\cdot(n_e\mathbf{v}_i) = S_i, \tag{11}$$

$$\begin{aligned} m_i n_i\frac{d\mathbf{v}_i}{dt} &= en_i(\mathbf{E} + \frac{\mathbf{v}_i\times\mathbf{B}}{c}) - \mathbf{R}_{ei} &(12)\\ &+ m_i D_i(\mathbf{v}_N - \mathbf{v}_i) + m_i S_i(\mathbf{v}_N - \mathbf{v}_i). &(13) \end{aligned}$$

The atomic physics effects are contained in:

$$\begin{aligned} S_e = S_i &= n_e n_N\langle\sigma v\rangle_{ion}\\ D_i &= n_i n_N\langle\sigma v\rangle_{cx}\\ P_{rad} &= n_e n_z L_z(T_e) \end{aligned}$$

for ionization, charge exchange, and radiation, respectively. Physically ionization is a source in the electron and ion continuity equations, charge exchange causes drag on the ion parallel velocity, and radiation cools the electrons. Note that while ionization does not affect the fluid momentum ($n\mathbf{v}$) or energy (nT), it appears as a drag in the velocity equation, and also enters the temperature equation.

For electrostatic perturbations with $\omega \ll \Omega_i$, the perpendicular dynamics are governed by:

$$\begin{aligned} &\mathbf{v}_{\perp e} = \mathbf{v}_{E\times B} + \mathbf{v}_{de} \qquad &\mathbf{v}_{E\times B} = -\frac{c}{B^2}\nabla_\perp\Phi\times\mathbf{B}\\ &\mathbf{v}_{\perp i} = \mathbf{v}_{E\times B} + \mathbf{v}_{pi} \qquad &\mathbf{v}_{de} = -\frac{c}{eB^2 n_e}\mathbf{B}\times\nabla_\perp p_e\\ & &\mathbf{v}_{pi} = -\frac{c}{B\Omega_i}\frac{\partial}{\partial t}\nabla_\perp\Phi. \end{aligned}$$

Here we have used $m_i \gg m_e$ and assumed $T_i \ll T_e$ for simplicity.

In a toroidal system, the divergence of the $\mathbf{E}\times\mathbf{B}$ drift in the ion continuity equation does not vanish, and leads to coupling between different poloidal harmonics. For a large aspect ratio torus with concentric circular flux surfaces, we have $\nabla_\perp\cdot\mathbf{v}_{E\times B} = -2i\omega_{de}\left(\cos\theta - \frac{i}{k_\theta}\sin\theta\frac{\partial}{\partial r}\right)\frac{e\Phi}{T}$, where $\omega_{de} = \frac{k_\theta\rho_s c_s}{R}$.

We linearize Eqs. (4)-(8) with $n_e = n_0 + n_{e1}$, $n_i = n_0 + n_{i1}$, $p_e = p_0 + p_{e1}$, $T_e = T_0 + T_{e1}$, $v_\parallel = v_{\parallel 1}$, assuming there is no mean flow ($v_{\parallel 0} = 0$). The neutral velocity is assumed to be stationary compared to the perturbed ion and electron velocities. We also assume that the wavelengths of the fluctuations are much shorter than the equilibrium gradient scale lengths ($L_n, L_T, \ldots$) but comparable to the ion gyroradius at the electron temperature, $\rho_s = \frac{c_s}{\Omega_i}$. Defining the following dimensionless field quantities,

$$\tilde{n}_e = \frac{n_{e1}}{n_0},\ \tilde{n}_i = \frac{n_{i1}}{n_0},\ \tilde{v}_i = \frac{v_{\parallel i}}{c_s},\ \tilde{v}_e = \frac{v_{\parallel e}}{c_s},\ \tilde{T} = \frac{T_{e1}}{T_0},\ \tilde{\Phi} = \frac{e\Phi}{T_0}$$

and the following atomic process rates,

$$
\begin{aligned}
\beta_n &= \frac{\partial S_e}{\partial n_e} = n_N \left<\sigma v\right>_{ion} \\
\beta_T &= \frac{T_0}{n_0}\frac{\partial S_e}{\partial T_e} = n_N T_0 \frac{\partial \left<\sigma v\right>_{ion}}{\partial T_e} \\
\gamma_v &= \frac{D_i}{n_0} = n_N \left<\sigma v\right>_{cx} \\
\gamma_n &= \frac{2}{3}\frac{1}{T_0}\frac{\partial P_{rad}}{\partial n_e} = \frac{2}{3}\frac{n_z}{T_0}L_z \\
\gamma_T &= \frac{2}{3}\frac{1}{n_0}\frac{\partial P_{rad}}{\partial T_e} = \frac{2}{3}n_z\frac{\partial L_z}{\partial T_e}
\end{aligned}
$$

we have,

$$
\tilde{n}_e = \frac{\omega_*}{\omega}\tilde{\Phi} + \frac{k_\| c_s}{\omega}\tilde{v}_e + i\frac{\beta_n}{\omega}\tilde{n}_e + i\frac{\beta_T}{\omega}\tilde{T} \tag{14}
$$

$$
0.51\,\nu_{ei}(\tilde{v}_i - \tilde{v}_e) = i\frac{k_\| v_{te}^2}{c_s}(\tilde{n}_e - \tilde{\Phi} + 1.71\tilde{T}) + \beta_n\tilde{v}_e + i\omega\tilde{v}_e \tag{15}
$$

$$
\begin{aligned}
\tilde{T} &= \eta_e\frac{\omega_*}{\omega}\tilde{\Phi} + \frac{2}{3}\frac{k_\| c_s}{\omega}\tilde{v}_e - i\frac{2}{3}3.16\frac{k_\|^2 v_{te}^2}{\omega\nu_{ei}}\tilde{T} \\
&- \frac{2}{3}0.71\frac{k_\| c_s}{\omega}(\tilde{v}_i - \tilde{v}_e) - i\frac{\gamma_n}{\omega}\tilde{n}_e - i\frac{\gamma_T}{\omega}\tilde{T} \\
&- i\frac{2}{3}\left(\frac{\beta_n}{\omega}\tilde{n}_e + \frac{\beta_n}{\omega}\tilde{T} + \frac{\beta_T}{\omega}\tilde{T}\right) \qquad (16) \\
\tilde{n}_i &= \left(\frac{\omega_*}{\omega} - 2\frac{\omega_{de}}{\omega}(\cos\theta - \frac{i}{k_\theta}\sin\theta\frac{\partial}{\partial r}) + \rho_s^2(\frac{\partial^2}{\partial r^2} - k_\theta^2)\right)\tilde{\Phi} \\
&+ \frac{k_\| c_s}{\omega}\tilde{v}_i + i\frac{\beta_n}{\omega}\tilde{n}_e + i\frac{\beta_T}{\omega}\tilde{T} \qquad (17) \\
\tilde{v}_i &= \frac{k_\| c_s}{\omega}\left(\tilde{n}_e + \tilde{T}\right) - i\frac{\gamma_v}{\omega}\tilde{v}_i - i\frac{\beta_n}{\omega}\tilde{v}_i \qquad (18)
\end{aligned}
$$

where the numerical factors come from the Braginskii coefficients ($\eta_\| = 0.51\frac{m_e\nu_{ei}}{ne^2}$, $\kappa_\| = 3.16\frac{nT}{m_e\nu_{ei}}$, $\alpha = 1.71$) and $\eta_e = \frac{L_n}{L_T}$, $\omega_* = \frac{k_\theta\rho_s c_s}{L_n}$. We note that density and temperature fluctuations affect the ionization source (β_n, β_T), and the radiated power (γ_n, γ_T).

Before presenting the solution of the eigenmode equation (in Sec. III), which is necessary to accurately determine the stability of this system, we begin by examining the local dispersion relation. For the purposes of this discussion, we treat the ω_{de} term as a constant, evaluated at $\theta = 0$ (bad curvature side), and treat $-\frac{\partial^2}{\partial r^2} - k_\theta^2$ as a constant, $k_\perp^2$. In this local limit,

$$
\tilde{n}_i = \left(\frac{\omega_*}{\omega} - 2\frac{\omega_{de}}{\omega} - k_\perp^2\rho_s^2\right)\tilde{\Phi} + \frac{k_\| c_s}{\omega}\tilde{v}_i + i\frac{\beta_n}{\omega}\tilde{n}_e + i\frac{\beta_T}{\omega}\tilde{T}. \tag{19}
$$

Using quasineutrality and Eqs. (9)-(11) and (13), we can obtain the local dispersion relation.

Two dimensionless parameters play crucial roles in characterizing the properties of the fluctuations: $\frac{k_\|^2 v_{te}^2}{\omega\nu_{ei}}$ and $\frac{k_\|^2 c_s^2}{\omega^2}$. In the first, $\frac{k_\|^2 v_{te}^2}{\nu_{ei}}$ comes from the $\kappa_\|\nabla_\|^2 T_e$ term in $\nabla\cdot\mathbf{q}_e$, and measures electron thermal conduction along $\mathbf{B}$. Thus the parameter $\frac{k_\|^2 v_{te}^2}{\omega\nu_{ei}}$ compares the parallel electron thermal conduction rate to the mode frequency. The second, $\frac{k_\| c_s}{\omega}$ is the ratio of the sound wave propagation rate along $\mathbf{B}$ to the mode frequency. When $\frac{k_\|^2 v_{te}^2}{\omega\nu_{ei}} \gg 1$, electron thermal conduction is strong enough to smooth temperature fluctuations along $\mathbf{B}$, so $\tilde{T}_e \to 0$. This is known as the adiabatic limit, and electrons have a Boltzmann-like distribution. When $\frac{k_\|^2 c_s^2}{\omega^2} \gg 1$, sound waves smooth pressure perturbations along $\mathbf{B}$, so $\tilde{p}_e \to 0$. This will be called the pressure balance limit in this paper.

As will be shown in Sec. III using the nonlocal mode structure, the adiabatic regime is relevant for typical tokamak edge parameters. We discuss several other limits to relate this work to previous studies.

i). In the adiabatic regime, $\frac{k_\|^2 v_{te}^2}{\omega\nu_{ei}} \gg 1$, $\frac{k_\|^2 c_s^2}{\omega^2}$:

$$
\tilde{n}_e \simeq \tilde{\Phi}, \qquad \tilde{T} \simeq 0, \qquad \tilde{v}_i \simeq \frac{k_\| c_s}{\omega + i\gamma_v + i\beta_n}\tilde{\Phi}. \tag{20}
$$

Quasi-neutrality gives the following dispersion relation to the leading order in $\frac{\omega\nu_{ei}}{k_\|^2 v_{te}^2}$:

$$
\begin{aligned}
&\omega^2(1+b) - \omega\left(\omega_* - 2\omega_{de} + i\beta_n - i(1+b)(\gamma_v + \beta_n)\right) - k_\|^2 c_s^2 \\
&\quad - i(\gamma_v + \beta_n)(\omega_* - 2\omega_{de} + i\beta_n) = 0,
\end{aligned} \tag{21}
$$

where $b = k_\perp^2\rho_s^2$. When $k_\| c_s$, γ_v, $\beta_n \ll \omega$, the electron root further reduces to:

$$
\begin{aligned}
\omega_0 \simeq\ & \frac{\omega_* - 2\omega_{de} + i\beta_n}{1+b} + \frac{k_\|^2 c_s^2}{(\omega_* - 2\omega_{de})^2}\left[\omega_* - 2\omega_{de}\right. \\
&\left. - i\beta_n - i(\beta_n + \gamma_v)(1+b)\right].
\end{aligned} \tag{22}
$$

In the local approximation, the destabilizing effect of ionization (β_n) comes from the inverse dissipation in the ion continuity equation. The stabilizing influence due to the ionization and charge-exchange drag on the parallel ion velocity is smaller roughly by a factor of $\frac{k_\|^2 c_s^2}{\omega^2}$. When spatially dependent $k_\|$ is actually taken into account, nonlocal analysis in Sec. III shows that the overall effect of ionization is destabilizing. Therefore, the ionization term in the ion velocity equation, often neglected in previous studies, seems to be important only when $\frac{k_\parallel^2 c_s^2}{\omega^2} \gtrsim 1$. We also note that the real frequency is shifted below ω_* by ω_{de} and finite Larmor radius effects. Because $\tilde{T}$ is small in this regime, radiation effects are subdominant.

Collisional effects appear as a first order ($\sim \frac{\omega\nu_{ei}}{k_\parallel^2 v_{te}^2}$) correction to ω_0 in Eq. (16);

$$\omega \simeq \omega_0 + i\frac{0.812}{(1+b)^2}\frac{\omega_*^2\nu_{ei}}{k_\parallel^2 v_{te}^2}\left[1.77(1-\frac{\omega_0}{\omega_*}) - \eta_e\right]. \quad (23)$$

Collisional effects are destabilizing when the down-shift of ω overcomes the stabilizing effects of η_e, i.e., $\eta_e < 1.77(1-\frac{\omega}{\omega_*})$. When the spatial dependence of $k_\parallel$ is treated nonlocally in the next section, this scaling with ν_{ei} changes to $\nu_{ei}^{\frac{1}{2}}$, increasing the relative importance of collisional effects. Radiation effects also enter at this order in $\frac{\omega\nu_{ei}}{k_\parallel^2 v_{te}^2}$, but only shift the real frequency, and do not affect stability directly.

ii). In the hydrodynamic regime, $\frac{k_\parallel^2 v_{te}^2}{\omega\nu_{ei}}, \frac{k_\parallel^2 c_s^2}{\omega^2} \ll 1$:

$$\tilde{n}_e \simeq \frac{\omega_*}{\omega}\tilde{\Phi}, \qquad \tilde{T}_e \simeq \eta_e\frac{\omega_*}{\omega}\tilde{\Phi}.$$

Here the dynamics along $\mathbf{B}$ are insignificant, and the electron density and temperature are mainly $\mathbf{E}\times\mathbf{B}$ convected. Note that if $\eta_e \gg 1$, we recover the limit considered by Ware, *et al.*[12] where $\omega \sim \eta_e\omega_* = \omega_{*T}$ and $\tilde{T} \sim \tilde{\Phi} \gg \tilde{n}_e$. In this limit, radiation effects could be significant.

iii). In the pressure balance regime, $\frac{k_\parallel^2 c_s^2}{\omega^2} \gg 1, \frac{k_\parallel^2 v_{te}^2}{\omega\nu_{ei}}$:

$$\tilde{n}_e \simeq -\tilde{T} \simeq \left[\frac{\frac{\omega_*}{\omega}(\frac{2}{3}-\eta_e) + i\,1.82\,\frac{k_\parallel^2 v_{te}^2}{\omega\nu_{ei}}}{\frac{5}{3} + i\,5.51\,\frac{k_\parallel^2 v_{te}^2}{\omega\nu_{ei}}}\right]\tilde{\Phi}$$

This limit is similar to the one investigated by Drake, *et al.*[24] for the case $\tilde{\mathbf{J}} \simeq 0$. In contrast to the near adiabatic regime, the pressure balance regime permits large electron temperature fluctuations, and consequently, radiation could become an important destabilizing effect.

III. Nonlocal Analysis

We derive the linear eigenmode equation using the ballooning formalism.[25–27] This procedure reduces the two dimensional problem in (r,θ) to a one dimensional eigenmode equation in the ballooning coordinate (η), which can be regarded as the coordinate along the field lines. The simplifying assumption is that for large mode numbers m, different poloidal harmonics have nearly identical structures centered at each rational surface.

Using quasineutrality, Eqs. (12) and (13) become:

$$\left(1 - \frac{k_\parallel^2 c_s^2}{\omega^2} - \frac{\omega_*}{\omega} + 2\frac{\omega_{de}}{\omega}(\cos\theta - \frac{i}{k_\theta}\sin\theta\frac{\partial}{\partial r})\right.$$
$$\left.-\rho_s^2(\frac{\partial^2}{\partial r^2} - k_\theta^2)\right)\tilde{\Phi} = -\left(1 - \frac{k_\parallel^2 c_s^2}{\omega^2}\right)\delta\tilde{n}_e$$
$$+\left(i\frac{\beta_n}{\omega} - \frac{i(\gamma_v+\beta_n)k_\parallel^2 c_s^2}{\omega^2(\omega+i\gamma_v+i\beta_n)}\right)\tilde{n}_e$$
$$+\left(i\frac{\beta_T}{\omega} + \frac{k_\parallel^2 c_s^2}{\omega(\omega+i\gamma_v+i\beta_n)}\right)\tilde{T}. \quad (24)$$

Here we have broken the perturbed electron density into its adiabatic and non-adiabatic parts: $\tilde{n}_e = \tilde{\Phi} + \delta\tilde{n}_e$. The right-hand side of Eq. (24) contains the non-adiabatic electron response and the atomic physics effects. Because the electrons are nearly adiabatic and atomic physics effects are small for typical TFTR and TEXT parameters considered here, we treat the right hand side of Eq. (24) perturbatively.

We use the usual (r,θ,ξ) coordinates, corresponding to the minor radial, poloidal, and toroidal directions. In a large aspect-ratio axisymmetric torus, the perturbed potential can be expressed as:

$$\tilde{\Phi}(r,\theta,\xi) = \sum_j \hat{\Phi}_j(s)\exp[i(m_0\theta + j\theta - n\xi - \omega t)], \quad (25)$$

where $|j| \ll m_0$, $s = \frac{(r-r_0)}{\Delta r_s}$, r_0 is the reference rational surface $m_0 = nq(r_0)$, $\Delta r_s = \frac{1}{k_\theta\hat{s}}$, $k_\theta = \frac{m_0}{r_0}$, and $\hat{s} = \frac{rq'}{q}$ at $r = r_0$. Ignoring for the moment the right-hand side, Eq. (24) is:

$$\left[1 - \frac{1}{\Omega} - \frac{(s-j)^2}{\eta_s^2\Omega^2} - b_\theta(\hat{s}^2\frac{\partial^2}{\partial s^2} - 1)\right]\hat{\Phi}_j$$
$$+\frac{\epsilon_n}{\Omega}\left[\hat{\Phi}_{j-1} + \hat{\Phi}_{j+1} - \hat{s}\frac{\partial}{\partial s}(\hat{\Phi}_{j-1} - \hat{\Phi}_{j+1})\right] = 0 \quad (26)$$

where we have used $k_\parallel = \frac{(s-j)}{qR}$, $b_\theta = k_\theta^2\rho_s^2$, $\Omega = \frac{\omega}{\omega_*}$, $\eta_s = \frac{qb_\theta^{\frac{1}{2}}}{\epsilon_n}$, $\epsilon_n = \frac{L_n}{R}$. Using the ballooning mode formalism, for $|m_0| \sim |n| \gg 1$, $\hat{\Phi}_j(s) = \Phi(z)$, and $\hat{\Phi}_{j\pm1}(s) = \Phi(z\mp1)$, with $z = s - j$; i.e., the eigenmodes are composed of identical structures centered at each rational surface. The eigenmode equation is:

$$\left[1-\frac{1}{\Omega}-\frac{z^2}{\eta_s^2\Omega^2}-b_\theta(\hat{s}^2\frac{d^2}{dz^2}-1)\right]\Phi(z)$$

$$+\frac{\epsilon_n}{\Omega}\left[\Phi(z+1)+\Phi(z-1)-\hat{s}\frac{d}{dz}[\Phi(z+1)-\Phi(z-1)]\right]=0 \tag{27}$$

We now Fourier transform this equation:

$$\Phi(\eta)=\int_{-\infty}^{\infty} dz\, e^{i\eta z}\,\Phi(z). \tag{28}$$

The eigenmode equation is now a one-dimensional differential equation in the ballooning coordinate, η:[15]

$$\left[\frac{d^2}{d\eta^2}+Q(\eta,\Omega)\right]\Phi(\eta)=0, \tag{29a}$$

where

$$Q(\eta,\Omega)=\eta_s^2\Omega^2\left[1-\frac{1}{\Omega}+b_\theta(1+\hat{s}^2\eta^2)+\frac{2\epsilon_n}{\Omega}(\cos\eta+\hat{s}\eta\sin\eta)\right]. \tag{29b}$$

The $\frac{d^2}{d\eta^2}$ term is the ion sound wave contribution. In the expression for Q, 1 is the Boltzmann electron response, $-\frac{1}{\Omega}$ is the $\mathbf{E}\times\mathbf{B}$ convection of the ion density, $\hat{s}^2\eta^2$ is the ion polarization drift, and $\frac{2\epsilon_n}{\Omega}(\cos\eta+\hat{s}\eta\sin\eta)$ is the toroidal coupling term. We repeat this procedure including the right-hand side of Eq. (24), and obtain:

$$\left[\frac{d^2}{d\eta^2}+Q(\eta,\Omega)+\delta Q(\eta,\Omega)\right]\Phi(\eta)=0, \tag{30a}$$

$$\begin{aligned}\delta Q(\eta,\Omega)=\frac{\eta_s^2\Omega^2}{\Phi(\eta)}\int_{-\infty}^{\infty} dz\, e^{i\eta z}\,\Phi(z)\Bigg[&\left(1-\frac{z^2}{\eta_s^2\Omega^2}\right)\delta\tilde{n}_e\\ &-i\left(\frac{\frac{\beta_n}{\omega_*}}{\Omega}-\frac{\frac{\gamma_v}{\omega_*}+\frac{\beta_n}{\omega_*}}{\Omega+i\frac{\gamma_v}{\omega_*}+i\frac{\beta_n}{\omega_*}}\frac{z^2}{\eta_s^2\Omega^2}\right)\tilde{n}_e\\ &-\left(\frac{i\frac{\beta_T}{\omega_*}}{\Omega}+\frac{\Omega}{\Omega+i\frac{\gamma_v}{\omega_*}+i\frac{\beta_n}{\omega_*}}\frac{z^2}{\eta_s^2\Omega^2}\right)\tilde{T}\Bigg],\end{aligned} \tag{30b}$$

where $\Phi(z)=\frac{1}{2\pi}\int_{-\infty}^{\infty} dz\, e^{-i\eta z}\,\Phi(\eta)$.

We solve this equation as follows. First, we find the lowest order eigenfunction using adiabatic electrons ($\delta Q=0$). Then using this lowest order eigenfunction, we evaluate $\delta Q(\eta)$ in Eq. (25b). We find that $\delta Q(\eta)$ is constant over the eigenmode $\Phi(\eta)$ to a very good approximation. For small and constant δQ, the effect on stability is simply $\mathrm{Im}(\omega/\omega_*)\simeq -\mathrm{Im}(\delta Q)/\eta_s^2\Omega^2$. As far as linear analysis is concerned, TFTR and TEXT parameters justify the perturbative treatment, since $\gamma \ll \omega_r$, as will be shown. Equation (24) is the eigenmode equation of Ref. 15, and is obtained from our equations neglecting the non-adiabatic electron response and atomic physics. We solve Eq. (24) numerically using an interactive WKB shooting code.[28] We are interested in the most unstable modes for this system, which are the neutrally stable toroidicity induced eigenmodes. Although TFTR and TEXT differ greatly in size, the dimensionless parameters in Eq. (24) are very similar for these two tokamaks, as shown in Table I ($q=4$, $\hat{s}=2$, $\epsilon_n=0.04$). Figures 1 and 2 show numerical solutions to this equation, for an extremely long wavelength eigenmode, $k_\theta\rho_s=0.02$ (Fig. 1), and for a shorter wavelength eigenmode, $k_\theta\rho_s=0.1$ (Fig. 2). The effective potential, $-Q(\eta)$, is shown in (a). The toroidal coupling terms create the local potential well which inhibits the magnetic shear induced damping, thus making these modes neutrally stable. The eigenmode structure in the ballooning coordinate, $\Phi(\eta)$, is shown in (b). The eigenmode structure in the radial coordinate, $\Phi(z)$, obtained by Fourier transforming $\Phi(\eta)$, is shown in (c). The eigenfrequencies for these two cases are $\Omega=0.87$ for $k_\theta\rho_s=0.02$ and $\Omega=0.67$ for $k_\theta\rho_s=0.1$.

To proceed, we solve Eqs. (9)-(11) and (13) for $\tilde{n}_e$, $\delta\tilde{n}_e$, and $\tilde{T}$ in terms of $\tilde{\Phi}$, and obtain:

$$\tilde{n}_e=\frac{ae+bf}{ad-cf}\tilde{\Phi} \qquad \tilde{T}=\frac{bd+ce}{ad-cf}\tilde{\Phi} \tag{31}$$

$$\delta\tilde{n}_e=\frac{a(e-d)+(b+c)f}{ad-cf}\tilde{\Phi}$$

where:

$$\begin{aligned}
a &= i\frac{2}{3}(3.16+\frac{(1.71)^2}{0.51})\frac{x^2}{\overline{\nu}\Omega}-\frac{2}{3}\frac{x^2}{\Omega(\Omega+i\frac{\gamma_v}{\omega_*}+i\frac{\beta_n}{\omega_*})}+1\\
&\quad + i(\frac{\gamma_T}{\omega}+\frac{2}{3}\frac{\beta_n}{\omega}+\frac{2}{3}\frac{\beta_T}{\omega})\\
b &= i\frac{2}{3}\frac{1.71}{0.51}\frac{x^2}{\overline{\nu}\Omega}+\frac{\eta_e}{\Omega}\\
c &= -i\frac{2}{3}\frac{1.71}{0.51}\frac{x^2}{\overline{\nu}\Omega}+\frac{2}{3}\frac{x^2}{\Omega(\Omega+i\frac{\gamma_v}{\omega_*}+i\frac{\beta_n}{\omega_*})}-i(\frac{\gamma_n}{\omega}+\frac{2}{3}\frac{\beta_n}{\omega})\\
d &= i\frac{x^2}{0.51\,\overline{\nu}\Omega}-\frac{x^2}{\Omega(\Omega+i\frac{\gamma_v}{\omega_*}+i\frac{\beta_n}{\omega_*})}+1-i\frac{\beta_n}{\omega}\\
e &= i\frac{x^2}{0.51\,\overline{\nu}\Omega}+\frac{1}{\Omega}\\
f &= -i\frac{1.71}{0.51}\frac{x^2}{\overline{\nu}\Omega}+\frac{x^2}{\Omega(\Omega+i\frac{\gamma_v}{\omega_*}+i\frac{\beta_n}{\omega_*})}+i\frac{\beta_T}{\omega}.
\end{aligned}$$

We have normalized $z=x\eta_s$, and $\overline{\nu}=\frac{\nu_{ei}m_e}{\omega_* m_i}$. These terms in δQ are ratios of fourth order polynomials in z, and do not Fourier transform nicely into ballooning space as the z^2 and

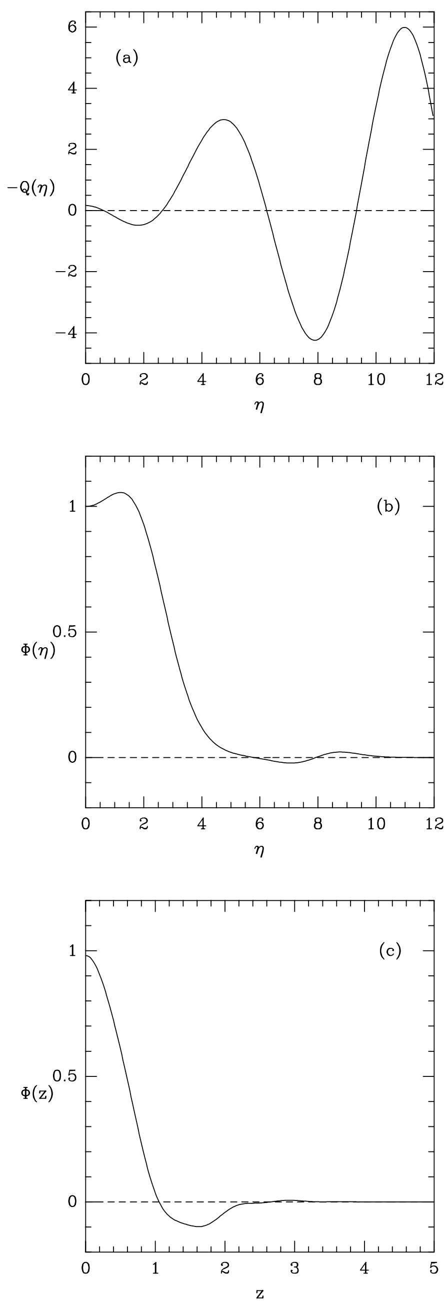

Figure 1: Numerical solution of Eq. (24) for extremely long wavelength ($k_\theta \rho_s = 0.02$), with $q = 4$, $\epsilon_n = 0.04$, and $\hat{s} = 2$. (a) Effective potential $-Q(\eta)$ in the ballooning coordinate. (b) Lowest order eigenmode in the ballooning coordinate, $\Phi(\eta)$. (c) Lowest order eigenmode in the radial coordinate, $\Phi(z)$.

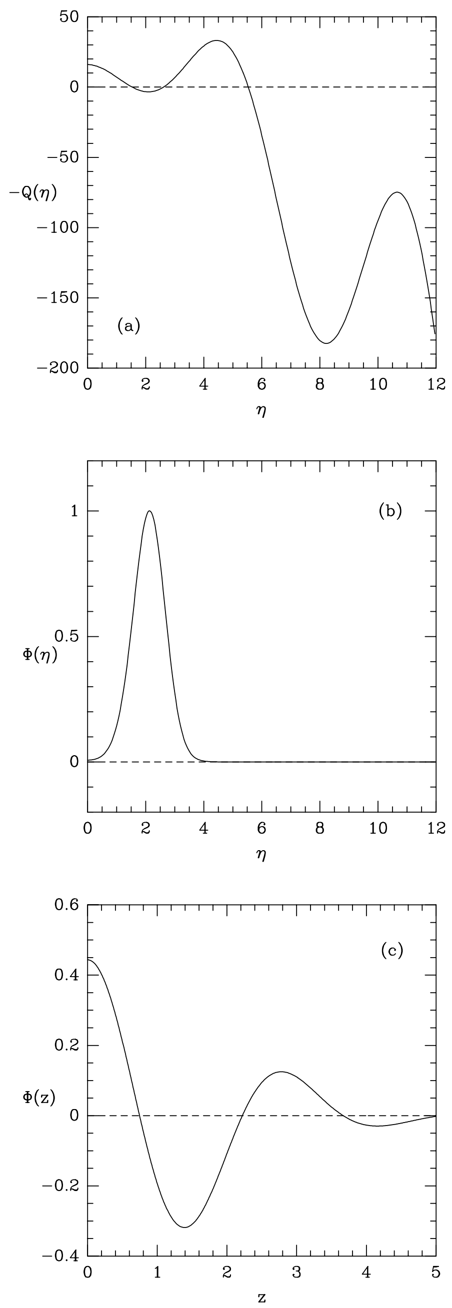

Figure 2: Numerical solution of Eq. (24) for $k_\theta \rho_s = 0.1$, with $q = 4$, $\epsilon_n = 0.04$, and $\hat{s} = 2$. (a) Effective potential $-Q(\eta)$ in the ballooning coordinate. (b) Lowest order eigenmode in the ballooning coordinate, $\Phi(\eta)$. (c) Lowest order eigenmode in the radial coordinate, $\Phi(z)$.

$\frac{d^2}{dz^2}$ terms do in the unperturbed eigenmode equation, so we evaluate them in configuration space, and then transform to ballooning space.

Typical edge parameters for TFTR[18,19] and TEXT[21] are

Table I. Typical Edge Parameters
r=0.95 a

	TFTR	TEXT
n_e (cm^{-3})	8×10^{12}	8×10^{12}
T_e (eV)	150	40
n_N (cm^{-3})	3×10^9	3×10^{10}
P_{rad} (W/cm^3)	0.03	0.1
L_n (cm)	10	4
L_T (cm)	5	2
η_e	2	2
a (cm)	80	27
R_0 (cm)	245	100
B_0 (T)	4.5	2
q	4	4
$\hat{s}$	2	2
ϵ_n	.04	.04
c_s (cm/s)	8.5×10^6	4.4×10^6
ν_{ei} (s^{-1})	1.9×10^5	1.4×10^6
ρ_s (cm)	3.9×10^{-2}	4.6×10^{-2}
ω_{*e} (s^{-1}) for $k_\theta \rho_s = 0.1$	8.5×10^4	1.1×10^5
	$(m = 195)$	$(m = 56)$
ω_{*e} (s^{-1}) for $k_\theta \rho_s = 0.02$	1.7×10^4	2.2×10^4
	$(m = 39)$	$(m = 11)$
β_n (s^{-1})	100	800
β_T (s^{-1})	-10	300
γ_v (s^{-1})	200	1000
γ_n (s^{-1})	100	1300
γ_T (s^{-1})	-200	1400

shown in Table I. γ_n is calculated using experimentally measured P_{rad}.[18,22] To estimate γ_T, we use coronal equilibrium cooling rates,[29] normalized to match P_{rad}. Since the actual $L_z(T_e)$ will be smoother than the coronal model, this is an upper bound on γ_T. All atomic physics rates are much smaller than ω_* for m values of interest. Using the lowest order eigenfunctions, we can now evaluate $\frac{k_\parallel^2 v_{te}^2}{\omega \nu_{ei}}$ and $\frac{k_\parallel^2 c_s^2}{\omega^2}$, and identify the parameter regime for these fluctuations. For the toroidicity induced modes, $k_\parallel \sim \frac{\Delta x}{L_s} k_\theta$, where $\Delta x \sim \frac{1}{k_\theta \hat{s}}$ and $L_s = \frac{qR}{\hat{s}}$, so

$$\frac{k_\parallel^2 c_s^2}{\omega^2} \sim (\frac{L_n}{qR})^2 (\frac{\omega_*}{\omega})^2 \frac{1}{(k_\theta \rho_s)^2}. \tag{32}$$

In the TFTR and TEXT edge, this quantity varies from 10^{-3} for short wavelengths ($k_\theta \rho_s = 1$) to 1 for longer wavelengths ($k_\theta \rho_s = 0.05$). Therefore, we have $\frac{k_\parallel^2 c_s^2}{\omega^2} \lesssim 1$ for parameters of interest, and pressure balance is not likely to be achieved, even for rather low $k_\theta \rho_s$ ($m \sim 50$).

More importantly, since

$$\frac{k_\parallel^2 v_{te}^2}{\omega \nu_{ei}} = \frac{1}{\overline{\nu}} \frac{k_\parallel^2 c_s^2}{\omega^2} \frac{\omega}{\omega_*}, \tag{33}$$

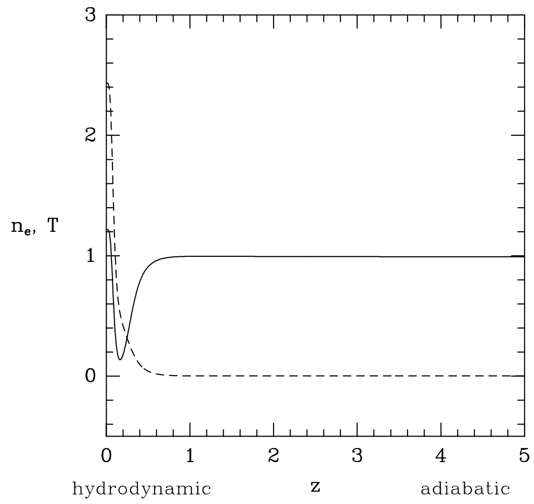

Figure 3: Spatial dependence of $\tilde{n}/\tilde{\Phi}$ (solid) and $\tilde{T}/\tilde{\Phi}$ (dashed) for $\overline{\nu} \ll 1$.

and typically $\overline{\nu} \ll 1$, even for rather low $k_\theta \rho_s$, we have $\frac{k_\parallel^2 v_{te}^2}{\omega \nu_{ei}} \gg 1 \gtrsim \frac{k_\parallel^2 c_s^2}{\omega^2}$. In summary, these fluctuations are typically in the adiabatic regime.

Because of the spatial dependence of $k_\parallel$, the electrons are hydrodynamic only within a narrow region near the rational surface, and adiabatic outside this region, as shown in Fig. 3. For illustration, Fig. 4 shows the case $\overline{\nu} \gg 1$, where the electrons are hydrodynamic near the rational surface, and pressure balance is enforced outside this region. The typical parameter regimes for toroidal drift waves in TFTR and TEXT are shown in Fig. 5.

We can now evaluate the integral for δQ using the lowest order eigenfunctions $\Phi(z)$, as shown in Figs. 1(c) and 2(c), for example. Since $\overline{\nu} \ll 1$, the electrons are non-adiabatic only within a narrow region near the rational surface, and the integrand for the perturbed potential δQ looks like a delta function in z. When transformed into ballooning space, $\delta Q(\eta)$ is roughly constant over the width of the eigenmode, as shown for these two cases in Figs. 6 and 7. This remains true when atomic physics effects are included.

If the atomic physics rates are η-dependent due to specific experimental asymmetries such as the limiter configuration and gas puff location, the imaginary (dissipative) part of δQ would be η-dependent and could induce toroidal coupling.[30] For the parameters considered in this paper, the toroidal coupling in the real (reactive) part of Q caused by ion drifts is a larger effect since $\omega_{de} \gg \gamma$.

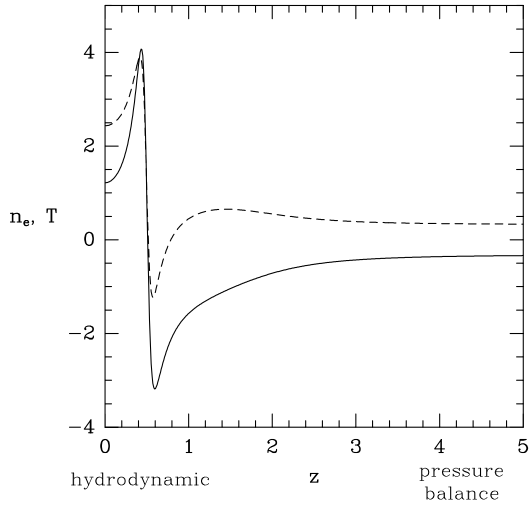

Figure 4: Spatial dependence of $\tilde{n}/\tilde{\Phi}$ (solid) and $\tilde{T}/\tilde{\Phi}$ (dashed) for $\overline{\nu} \gg 1$.

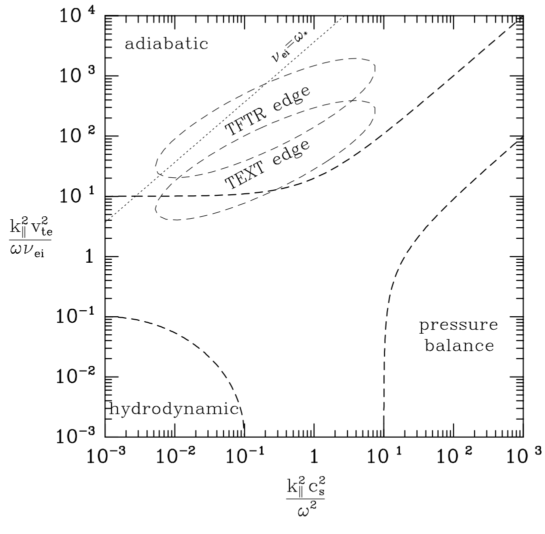

Figure 5: Typical parameter regimes for toroidal drift waves in TFTR and TEXT (electron response).

In the limit $\overline{\nu} \ll 1$, we can evaluate δQ analytically without knowing the detailed eigenmode structure, keeping terms up to $O(\overline{\nu}^{\frac{1}{2}})$. The lowest order piece of the $\delta \tilde{n}_e$ term in Eq. (25b) contributes

$$\delta Q = \eta_s^3 \overline{\nu}^{\frac{1}{2}} \Omega^{\frac{3}{2}} \frac{\sqrt{\pi}}{2}(1-i)\,[2.51(1-\Omega) - 1.06\,\eta_e] \tag{34}$$

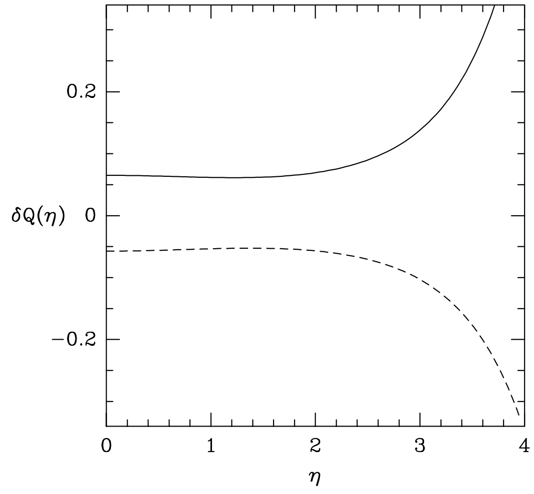

Figure 6: Perturbed potential for the eigenmode in Fig. 1, with $\overline{\nu} = 0.001$, $\eta_e = 0$, and no atomic physics.

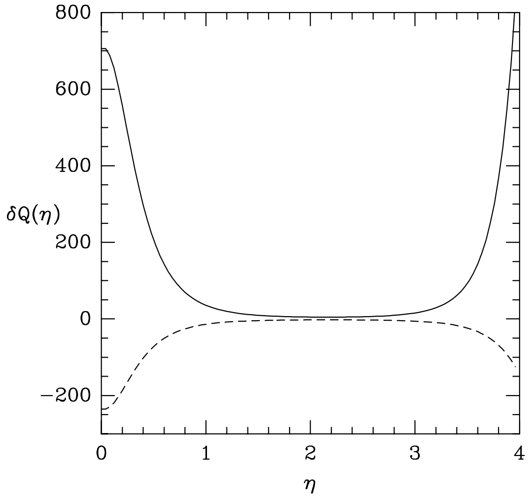

Figure 7: Perturbed potential for the eigenmode in Fig. 2, with $\overline{\nu} = 0.001$, $\eta_e = 0$, and no atomic physics.

$$+\eta_s^3 \overline{\nu}^{\frac{1}{2}} \Omega^{\frac{1}{2}} \frac{\sqrt{\pi}}{2}(1+i)\,[\,\frac{\gamma_T}{\omega_*}(0.24\,\Omega + 0.33(1-\Omega) + 0.19\,\eta_e)$$

$$+\frac{\gamma_n}{\omega_*}(1.13\,\Omega + 0.49(1-\Omega) - 0.16\,\eta_e)$$

$$+\frac{\beta_T}{\omega_*}(0.35\,\Omega + 0.02(1-\Omega) + 0.87\,\eta_e)\,]\,.$$

For the second term in Eq. (25b), $\tilde{n}_e = \tilde{\Phi}$ to lowest order. The β_n term is trivial to evaluate, but for the γ_v term we approximate $\int_{-\infty}^{\infty} z^2\, \Phi(z)\, dz \simeq \frac{2}{\pi^2}$. The dominant contribution is:

$$\delta Q = -i\eta_s^2 \Omega^2 \frac{\beta_n}{\omega} + i\frac{2}{\pi^2}\left(\frac{\gamma_v}{\omega} + \frac{\beta_n}{\omega}\right). \tag{35}$$

The lowest order contribution from the $\tilde{T}$ term is proportional to $\frac{\beta_T}{\omega_*}$:

$$\delta Q = \eta_s^3 \overline{\nu}^{\frac{1}{2}} \Omega^{\frac{1}{2}} \frac{\sqrt{\pi}}{2}(1+i)\frac{\beta_T}{\omega_*}(0.35\,\Omega + 0.16(1-\Omega) - 0.14\,\eta_e). \tag{36}$$

Combining these terms, the growth rate is:

$$\begin{aligned} \frac{\gamma}{\omega_*} &= \frac{\beta_n}{\omega} - \frac{2}{\pi^2 \eta_s^2 \Omega^2}\left(\frac{\gamma_v}{\omega} + \frac{\beta_n}{\omega}\right) \\ &+ \frac{\sqrt{\pi}}{2}\eta_s\left(\frac{\overline{\nu}}{\Omega}\right)^{\frac{1}{2}}[2.51\,(1-\Omega) - 1.06\,\eta_e] \\ &- \frac{\sqrt{\pi}}{2}\eta_s\left(\frac{\overline{\nu}}{\Omega}\right)^{\frac{1}{2}}\Big[\,\frac{\gamma_T}{\omega}(0.24\,\Omega + 0.33(1-\Omega) + 0.19\,\eta_e) \\ &+ \frac{\gamma_n}{\omega}(1.13\,\Omega + 0.49(1-\Omega) - 0.16\,\eta_e) \\ &+ \frac{\beta_T}{\omega}(0.69\,\Omega + 0.17(1-\Omega) + 0.74\,\eta_e)\,\Big]. \end{aligned} \tag{37}$$

The basic trends of the local analysis remain, with some important modifications. The destabilization from the ionization term in the ion continuity equation is reduced by charge-exchange and ionization drag on the ion velocity, but the stabilizing term is smaller by a factor of $\frac{2}{\pi^2\eta_s^2\Omega^2}$, which depends on the wavelength ($\eta_s \sim k_\theta \rho_s$). As $k_\theta\rho_s$ gets smaller, the stabilizing term becomes competitive. In the third term, the collisional effects scale like $\overline{\nu}^{\frac{1}{2}}$ instead of $\overline{\nu}$ predicted from local theory. Therefore, this term is not negligible compared to $\overline{\nu}$-independent terms. The radiation effects are very weak, since $\overline{\nu}$, γ_n, and γ_T are all small. Similarly, the dependence of the growth rate on β_T is weak.

We test the validity of the analytical estimate of Eq. (33) by comparing to direct numerical evaluation of the integral in Eq. (25b), using the parameters and unperturbed eigenfunction in Fig. 1. Figures 8(a)-(c) compare numerical and analytical growth rates as $\overline{\nu}$, η_e, and β_n are varied. The effects of the parameters β_T, γ_v, γ_n, and γ_T are much weaker, and the good agreement remains as they are varied. Figure 8(a) shows the small $\overline{\nu}$ expansion beginning to break down for larger $\overline{\nu}$.

Typically, the growth rate is determined by the competition between ionization and collisions, the latter is stabilizing for large η_e, and overall stability is achieved when:

$$\eta_e > \eta_e^{crit} = 2.37(1-\Omega) + \frac{2}{\sqrt{\pi}}\frac{\beta_n}{\omega_*}\frac{1}{(\overline{\nu}\Omega)^{\frac{1}{2}}}\frac{1}{\eta_s}. \tag{38}$$

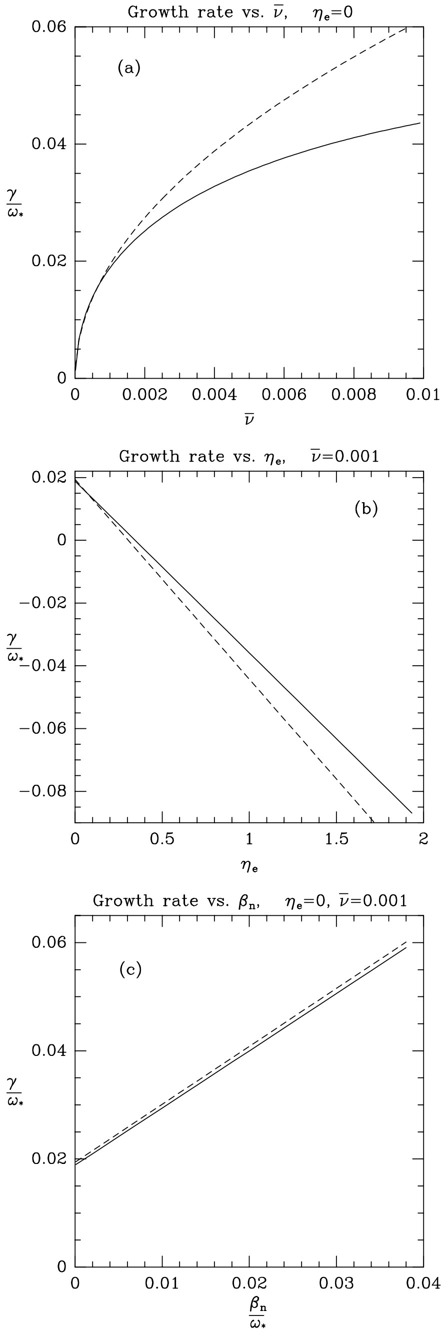

Figure 8: Comparison of analytically (dashed) and numerically (solid) computed growth rates as (a) $\overline{\nu}$, (b) η_e, (c) β_n are varied, using the parameters in Fig. 1.

For the cases considered in Table I, collisional stabilization is stronger than the destabilization from ionization, rendering these modes stable. However, at the extreme edge of tokamaks, steeper density gradients and higher ionization rates may destabilize these modes.

IV. Conclusions and Discussion

This paper has focused on the effects of various atomic physics processes on dissipative drift wave stability in toroidal geometry. The principal conclusions of this paper are as follows.

i). For typical parameters of the TFTR and TEXT edge, the electron dynamics are nearly adiabatic.

ii). The deviation from the adiabatic response which is required for instability comes from ionization, collisions, charge exchange, and radiation, roughly in order of importance.

iii). For an instability, the inverse dissipation from ionization or from the downshift of the real frequency below ω_{*e} should overcome the stabilizing effects due to the electron temperature gradient, quantified by η_e. Therefore, for long wavelength modes, an instability is likely to occur at the extreme edge in minor radius with significant ionization source and rather steep electron density gradient (low η_e).

iv). Ionization acts as an inverse dissipation in the density continuity equation, but also appears as an effective drag in the ion *velocity* equation. This effect, although ignored in previous studies,[13] can quantitatively affect stability if $\frac{k_\parallel^2 c_s^2}{\omega^2} \gtrsim 1$.

v). The magnitudes of two dimensionless parameters, $\frac{k_\parallel^2 v_{te}^2}{\omega \nu_{ei}}$ and $\frac{k_\parallel^2 c_s^2}{\omega^2}$ determine the asymptotic regimes in parameter space and the characteristics of the fluctuations. Relevances of the previous studies[12,24] on radiation-driven edge turbulence models are discussed for certain regimes.

The detailed linear analyses and results presented in this paper indicate that the edge drift instability has a growth rate which is smaller than the real frequency for $m > 10$. However, we would like to point out that this does not necessarily imply small fluctuation levels at saturation. Since the longer wavelength fluctuations (with $k_\perp \rho_s \ll 1$) are more likely to be destabilized by the ionization, weak turbulence analysis based upon nonlinear wave-particle interactions (including ion Compton scattering) is not likely to be justified. In the strong turbulence regime, the Hasegawa-Mima nonlinearity[31] is also negligible because it relies on the nonlinear polarization drift. Therefore, the nonlinear saturation could occur through a nonlinear process which is insensitive to the strength of finite Larmor radius effects. One possibility would be the dissipative correction to $\mathbf{E} \times \mathbf{B}$ convection.[32] Since this nonlinearity is also weak, it is possible to have a large fluctuation level at saturation.[13]

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