Appendix A

Old Toroidal Gyrofluid Equations

The equations used in the nonlinear simulations in Chapter 4 are briefly summarized here, since they are an earlier version of those derived in Chapter 2. Since the aim of the simulations in Chapter 4 is to test various assumptions implicit in flux tube simulation, the simpler four moment model is used, using adiabatic electrons and ignoring collisions and particle trapping (i.e. $\nabla_\parallel B = 0$). The four moment equations here use less accurate toroidal closure coefficients and FLR approximations than those in Section 2.7. Using the normalizations and definitions in Chapter 2, the dynamical equations are:

$$
\begin{aligned}
\frac{dn}{dt} &+ \nabla_\parallel u_\parallel - \left(1 + \frac{\eta_\perp}{2}\hat{\nabla}_\perp^2\right) i\omega_* \Psi + \left(\frac{1}{2}\hat{\nabla}_\perp^2 \mathbf{v}_\Psi\right) \cdot \nabla T_\perp = -i\omega_d \left(p_\parallel + p_\perp + 2\Psi\right), \\
\frac{du_\parallel}{dt} &+ \nabla_\parallel (p_\parallel + \Psi) = -4i\omega_d u_\parallel - 2i\omega_d \nu_{5i} u_\parallel - 2|\omega_d|\nu_{5r} u_\parallel, \\
\frac{dT_\parallel}{dt} &+ 2\nabla_\parallel u_\parallel + \sqrt{2}|k_\parallel|\chi_\parallel T_\parallel - \eta_\parallel i\omega_* \Psi = \\
&- 2i\omega_d \left[(3 + \nu_{1i}) T_\parallel + \nu_{2i} T_\perp + n + \Psi\right] - 2\nu_{1r}|\omega_d| T_\parallel - 2\nu_{2r}|\omega_d| T_\perp, \\
\frac{dT_\perp}{dt} &+ \sqrt{2}|k_\parallel|\chi_\perp \left(T_\perp + \frac{1}{2}\hat{\nabla}_\perp^2 \Psi\right) - \left[\frac{1}{2}\hat{\nabla}_\perp^2 + \eta_\perp (1 + \hat{\hat{\nabla}}_\perp^2)\right] i\omega_* \Psi \\
&+ \left(\frac{1}{2}\hat{\nabla}_\perp^2 \mathbf{v}_\Psi\right) \cdot \nabla n + (\hat{\hat{\nabla}}_\perp^2 \mathbf{v}_\Psi) \cdot \nabla T_\perp = -2\nu_{3r}|\omega_d| T_\parallel - 2\nu_{4r}|\omega_d| T_\perp \\
&- 2i\omega_d \left[\nu_{3i} T_\parallel + (2 + \nu_{4i}) T_\perp + \frac{n}{2} + \frac{1}{2}(1 + \frac{3}{2}\hat{\nabla}_\perp^2)\Psi\right].
\end{aligned}
$$

The total time derivative includes the main $\mathbf{E} \times \mathbf{B}$ nonlinearities. The parallel closure coefficients are $\chi_\parallel = 2/\sqrt{\pi}$ and $\chi_\perp = 1/\sqrt{\pi}$. The toroidal closure coefficients have both dissipative and reactive pieces, and written in the form $\nu = (\nu_r, \nu_i) =$

$\nu_r + i\nu_i|\omega_d|/\omega_d$, they are $\nu_1 = (1.93, -.39)$, $\nu_2 = (.24, 1.29)$, $\nu_3 = (-1.40, .47)$, $\nu_4 = (-.14, -1.75)$, and $\nu_5 = (.76, -.98)$.

The adiabatic electron response is given by $n_e = \tau\,(\Phi - \langle\Phi\rangle)$, where $\langle\Phi\rangle(\psi) = (\int d\alpha dz J|\nabla\psi|\Phi)/\ (\int d\alpha dz J|\nabla\psi|)$ is a flux surface average. In circular concentric geometry, this becomes $\langle\Phi\rangle = (4\Delta y\, z_0)^{-1} \int dy\, dz(R/R_0)\Phi(x,y,z)$, and is only non-zero for the $k_y = 0$ components. The gyrokinetic quasineutrality constraint is $n_e = \bar{n}_i + (\Gamma_0 - 1)\Phi$, where the expression used for $\bar{n}_i$ is related to the ion guiding center density and perpendicular temperature by the FLR closure relation in [Dorland and Hammett, 1993], yielding:

$$\tau\,(\Phi - \langle\Phi\rangle) = \frac{\Gamma_0^{1/2}}{D(b)}\left[N(b)\,n + \frac{1}{2}\hat{\nabla}_\perp^2 T_\perp\right] + (\Gamma_0 - 1)\,\Phi.$$

Where $\tau = T_i/T_e$, and explicit forms for the functions $N(b)$, $D(b)$, are given in [Dorland and Hammett, 1993].

Since this equation involves both Φ and $\langle\Phi\rangle$, it is woth noting the procedure used to determine Φ, given n and $T_\perp$. In general, the coefficients in this equation can be functions of the field line coordinate, so writing $\Phi = \langle\Phi\rangle + \delta\Phi$, and solving for $\delta\Phi$ gives:

$$\delta\Phi = \frac{\bar{n}_i + (\Gamma_0 - 1)\langle\Phi\rangle}{\tau + 1 - \Gamma_0}.$$

Averaging both sides, since $\langle\delta\Phi\rangle = 0$, and solving for $\langle\Phi\rangle$ gives:

$$\langle\Phi\rangle = \langle\frac{\bar{n}_i}{\tau + 1 - \Gamma_0}\rangle/\langle\frac{(1 - \Gamma_0)}{\tau + 1 - \Gamma_0}\rangle.$$

Now that $\langle\Phi\rangle$ is determined, we use this expression in the quasineutrality constraint to obtain Φ.