Chapter 7

Conclusions

SEVERAL ADVANCES toward an understanding of turbulent transport in tokamaks are made in this thesis. The primary thrust of this work is the development of nonlinear toroidal simulations which predict fluctuation and transport levels that compare favorably with experiment. These simulations rely on more accurate fluid equations for the ions and new bounce averaged trapped electron equations. Together, these equations provide an accurate description of most of the physics considered relevant for microinstability driven turbulence. This reduced fluid model is sufficiently simple to solve directly in high resolution numerical simulations. These simulations implement a reduced flux tube geometry for further numerical efficiency, and fully incorporate toroidal effects. Toroidal effects are found to significantly enhance thermal transport and fluctuation levels over sheared slab predictions, bringing the predictions up to experimentally measured levels. In addition, the nonlinear fluctuation spectrum is peaked at long wavelengths, and is anisotropic in k_r and k_θ due to the ballooning nature of this turbulence, in agreement with experiment. Finally, the trapped electron fluid equations provide the first high resolution toroidal nonlinear simulations which simultaneously include toroidal ITG modes and trapped electron modes, and allow calculation of the full transport matrix: ion and electron heat fluxes and particle fluxes.

7.1 Summary

Toroidal ion gyrofluid equations are derived with improved models of the important kinetic effects associated with toroidicity. Special care is taken to derive closure approximations which, though similar to those of [Waltz *et al.*, 1992], are well behaved in the mixed limit where both toroidal drifts and parallel free streaming are important, i.e. where both $k_\parallel$ and ω_d are non-zero. The four moment toroidal gyrofluid model of [Waltz *et al.*, 1992] is extended to six moments, including the $\mu\hat{\mathbf{b}} \cdot \nabla B$ mirroring terms. This keeps the parallel velocity equation exact, important for accurate poloidal flow damping rates. Including the $\mu\hat{\mathbf{b}} \cdot \nabla B$ terms also incorporates trapped ion effects to some level of approximation (the growth rate in the very low $k_\theta \rho_i$ trapped ion mode regime is within a factor of two of full gyrokinetics). New toroidal FLR terms are treated which arise from the variation of B (in the argument of J_0) with major radius, and generalize the FLR model of [Dorland and Hammett, 1993] to toroidal geometry. An improved four moment model is also presented, which is simpler and numerically less demanding than the six moment model. Impurity and (Maxwellian) beam dynamics are equally well described by these toroidal gyrofluid equations, and have been incorporated into the code by Dorland.

New bounce average fluid equations for trapped electrons are derived, incorporating sophisticated models of the trapped electron toroidal precession resonance and pitch-angle scattering from collisions. Because these equations are bounce averaged, the fast parallel electron time scale is removed, allowing high resolution toroidal simulations simultaneously including drive from toroidal ITG modes and trapped electron modes. Including nonadiabatic electron dynamics also allows predictions of electron heat transport and particle transport. Both the toroidal ion gyrofluid and trapped electron equations are carefully benchmarked against fully kinetic linear theory, in the local limit and in fully nonlocal eigenmode calculations.

A reduced flux tube coordinate system is presented which exploits the elongated nature of microinstability driven turbulence. A slab gyrofluid code [Dorland, 1993] is extended to toroidal geometry using this coordinate system and the comprehensive toroidal gyrofluid equations derived here. These simulations find that the effects of toroidal geometry are quite important. The fluctuation levels and transport are about 25 times larger than sheared slab simulations, bringing the predicted transport up to measured levels. The fluctuation spectra from these simulations are peaked at $k_\theta \rho_i \approx 0.1-0.2$ for typical parameters and are anisotropic in k_r and k_θ, similar to BES fluctuation measurements on TFTR [Fonck *et al.*, 1993].

The importance of turbulence-generated small-scale sheared $\mathbf{E}\times\mathbf{B}$ flows on toroidal ITG turbulence is demonstrated. The damping of the (large) poloidal component of these flows is shown to be a sensitive control of the turbulence level. The damping of these flows within the six moment gyrofluid model is carefully investigated. For the relevant short time scales of the turbulence, the toroidal gyrofluid model is shown to be accurate.

Direct comparison of these toroidal simulation results with L-mode experiments are found to be encouraging. The predicted ion and electron heat transport in the core, $r/a < 0.7-0.8$, are in reasonable agreement with those calculated from power balance. The transport in the edge, $r/a > 0.7-0.8$, is too low, and possible mechanisms to explain this discrepancy are discussed.

7.2 Future Directions

The gyrofluid equations are an approximation to the full nonlinear gyrokinetic equation, and break down in some regimes. For example, the weak turbulence wave-kinetic equation derived from the gyrofluid equations successfully reproduces the gyrokinetic wave-kinetic equation in the limit $\omega \gg k_\| v_{ti}$, but fails to recover the ion-Compton scattering rate very near marginal stability, in the limit $\gamma \ll \omega \ll k_\| v_{ti}$ [Mattor, 1992; Dorland, 1993]. The nonlinear validity of the gyrofluid equations in strong turbulence regimes has not yet been unambiguously verified on fundamental grounds. However, gyrofluid simulations have been compared against technically more accurate gyrokinetic particle simulations, finding similar behavior in a sheared slab [Parker *et al.*, 1994; Dorland, 1993]. The toroidal simulations developed in this thesis have been benchmarked with toroidal gyrokinetic particle simulations (though not as extensively as the sheared slab simulations), and find reasonable agreement [Parker *et al.*, 1994]. Very recently, the toroidal gyrokinetic particle simulations of [Dimits *et al.*, 1994] appear to predict lower transport by about a factor of 2.5. While in principle gyrokinetic simulations are more accurate, since they solve the more fundamental gyrokinetic equation directly, there are a number of issues which need investigation: particle noise, particle filtering, resolution, and geometry (we implement field-line coordinates in a somewhat different way than [DIMITS *et al.*, 1994], which tends to emphasize resolution in different parts of k-space). Detailed comparisons with gyrokinetics is worthy of further study, to track down the causes of this discrepancy.

Closing the fluid hierarchy with linear closures naively appears to introduce an error of $\mathcal{O}(\Delta\omega_{NL}/k_\parallel v_{ti})$, which is typically $\mathcal{O}(1)$. Here $\Delta\omega_{NL} \sim \mathbf{v}_E \cdot \nabla$ is some measure of the nonlinear decorrelation rate. In this sense it is very interesting that gyrofluid models work as well as they do nonlinearly, but there are physical reasons behind this. Each gyrofluid equation, as a moment of the gyrokinetic equation, is an exact nonlinear conservation law: closure approximations are introduced into higher moment equations in a way which preserves the conservative form the equations. As more moments are retained, more details of the distribution function are accurately described. Smith has demonstrated convergence in the number of moments for the nonlinear plasma echo problem [HAMMETT *et al.*, 1993], though it required many moments in that case. In the strong turbulence limit, it seems unlikely that many moments need to be kept, since the broad spectrum of modes should average out sharp velocity space variations in the distribution function. Our equations retain the dominant ($\mathbf{E} \times \mathbf{B}$) nonlinearities and provide accurate physics based models of the linear drive and dissipation. Future work should continue to test the validity of the gyrofluid approximation, both through comparisons with kinetic simulations and through purely theoretical simplified problems.

The toroidal ion gyrofluid equations and trapped electron fluid equations presented here are both derived in the electrostatic limit. Recent work has begun including electromagnetic effects [WALTZ *et al.*, 1994b; HAMMETT *et al.*, 1994]. The main difficulty here is that magnetic fluctuations are driven by parallel current fluctuations, and since trapped particles do not carry current, passing electrons need to be evolved (they can no longer be considered adiabatic). Resolving the fast electron parallel motion seriously slows down the numerical calculations. Some trick analogous to bounce averaging would be useful.

The nonlinear simulation results in this thesis are in axisymmetric, low-β, high aspect ratio magnetic geometry. Including the effects of general magnetic geometry is a straightforward next step. It is of great interest to investigate the effects of elongation and triangularity on the transport, both to compare with existing non-circular tokamaks and to optimize the design of future experiments. The derivation of both the toroidal ion gyrofluid equations and the trapped electron fluid equations is valid for general geometry. Further, the flux tube simulation geometry in Chapter 4 is formulated in general geometry. All that is required is a pre-processor which would take the equilibrium magnetic field described by Eq. (4.1) and calculate the metric coefficients in Eqs. (4.5) and (4.6). These metric coefficients would then be used as further input for the toroidal gyrofluid code.

Although these toroidal gyrofluid simulations are relatively fast, an analytic formula for χ_i, χ_e, and D would be more desirable. In addition to aiding comparisons with experiment, analytic formulas (even if they are approximate) usually offer more insight than purely numerical results. Our focus to date has been to add more physics to our equations and simulations until we are confident that they are experimentally relevant. Now that it appears we are reaching this point, it would be very interesting to investigate the nonlinear dynamics in the simulations in more detail, to try to develop an analytic model for the transport. While the interpolation formula for χ_i in [Dorland *et al.*, 1994b] represents a significant step towards a reduced description of the simulation results, a model for the transport in terms of a simplified renormalized dispersion relation would be more satisfying. [Dorland *et al.*, 1994b] have found that much of the variation in χ_i is captured by $\max(\gamma/k_\perp^2)$ (different from $\max(\gamma)/k_\perp^2$ because of the $k_\perp$ dependence of γ), but the residual variation is still described by a numerical fit to the simulation results. This residual variation presumably includes physics involving sheared-flow generation and damping, and it would be nice to have an analytic model of this.

The electron equations have been implemented in the nonlinear simulations only quite recently; a more careful investigation of electron heat fluxes and particle fluxes is clearly called for. Further study of TEM driven turbulence is required to accurately describe supershots and ohmic plasmas. We can now study several important questions: Why is the convective multiplier as low as 3/2 in supershots? When are there particle pinches? What is the helium ash diffusion coefficient? We can also study advanced tokamak configurations with trapped electron mode stabilization due to reversed shear or high-β, which are predicted to have improved confinement. A complete transport model must be able to predict density as well as temperature profiles. Nonadiabatic electron dynamics are thus an essential ingredient.