The Dynamics of Small-Scale Turbulence Driven Flows

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The dynamics of small-scale fluctuation driven flows are of great interest for micro-instability driven turbulence, since nonlinear toroidal simulations have shown that these flows play an important role in the regulation of the turbulence and transport levels. The gyrofluid treatment of these flows was shown to be accurate for times shorter than a bounce time.[1] Since the decorrelation times of the turbulence are generally shorter than a bounce time, our original hypothesis was that this description was adequate. Recent work[2] pointed out possible problems with this hypothesis, emphasizing the existence of a linearly undamped component of the flow which could build up in time and lower the final turbulence level. While our original gyrofluid model reproduces some aspects of the linear flow, there are differences between the long time gyrofluid and kinetic linear results in some cases. On the other hand, if the long time behavior of these flows is dominated by nonlinear damping (which seems reasonable), then the existing nonlinear gyrofluid simulations may be sufficiently accurate. We test these possibilities by modifying the gyrofluid description of these flows and diagnosing the flow evolution in nonlinear simulations.

[1] Beer, M. A., Ph. D. thesis, Princeton University (1995).
[2] Hinton, F. L., Rosenbluth, M. N., and Waltz, R. E., Sherwood (1997).

Outline

- Linear flow damping tests:
  - linear initial value comparisons: gyrokinetic Vlasov code vs. gyrofluid
  - good agreement on fast linear damping rates for all k_r
  - standard gyrofluid model does not model residual component accurately
- Correlation function for radial modes from gyrofluid simulations
  - correlation time on the order of damping time, but not tied to damping time (different k_r dependence)
- Nonlinear flow damping tests
  - nonlinear effects appear to dominate evolution of residual flow component
- Nonlinear Gyrokinetic Particle vs. Gyrofluid comparisons
  - removing residual flow component does not reduce GF vs. GKP discrepancy
  - removing flows altogether does not reduce GF vs. GKP discrepancy
  - good agreement for NTP test case at $\hat{s} = 0$
  - factor of 2-3 difference for other parameters

Background and Motivation

- Early toroidal gyrofluid simulations showed that ITG turbulence can drive fluctuating sheared $\mathbf{E} \times \mathbf{B}$ flows which play an important role in regulating the turbulence, by breaking up radially elongated eddies which cause large transport [Beer, Hammett, et al., BAPS (1992); Hammett, Beer, et al., PPCF (1993)]. Also seen in GKP simulations [Dimits, et al., PRL (1996); Z. Lin].
- Added $\mu \nabla_{\|} B$ parallel acceleration terms to accurately model linear $\mathbf{E} \times \mathbf{B}$ flow damping. Showed that gyrofluid equations accurately model fast collisionless damping for times less than an ion bounce time, $t < \tau_{\mathrm{bounce}}$ [Beer, Ph.D. Thesis (1995)]. Argued that long time linear flow dynamics are not important, as nonlinear effects will dominate long term nonlinear flow evolution.
- Hinton and Rosenbluth, submitted (1997), emphasized that there is a linearly undamped component of the flow. This "residual" flow is damped by collisional effects. Argued that this residual component should grow in time $\sim \sqrt{t}$ in the collisionless limit. Modeled nonlinear drive term as a white noise source.
- Since present gyrofluid eqns underestimate residual component, if residual component is important nonlinearly, gyrofluid simulations would underestimate $\mathbf{E} \times \mathbf{B}$ flow levels and overpredict χ_i.
- Results here indicate that residual flow component is not very important nonlinearly (except perhaps near marginal stability), and that differences between gyrokinetic and gryofluid flow damping is not dominant cause of discrepancy between GyroKinetic Particle (GKP) simulations and GyroFluid (GF) simulations.

Toroidal Gyrofluid Equations for Ion Species

[**Beer & Hammett, PoP 3, 4046 (1996)**]

For ions, evolve moments of nonlinear electrostatic toroidal gyrokinetic eqn. $(n, u_{\|}, T_{\|}, T_{\perp}, q_{\|}, q_{\perp})$: [**Frieman&Chen, Lee, Dubin, Krommes, Hahm**]

$$\frac{\partial f}{\partial t}+(v_{\|}\hat{\mathbf{b}}+\bar{\mathbf{v}}_E+\mathbf{v}_d)\cdot\nabla f+\left(\frac{e}{m}\bar{E}_{\|}-\mu\nabla_{\|}B+v_{\|}(\hat{\mathbf{b}}\cdot\nabla\hat{\mathbf{b}})\cdot\bar{\mathbf{v}}_E\right)\frac{\partial f}{\partial v_{\|}}=C(f)$$

$$\begin{aligned}
\frac{\partial n}{\partial t} &+ \bar{\mathbf{v}}_E\cdot\nabla n + B\nabla_{\|}\frac{u_{\|}}{B} - \left(1+\frac{\eta_\perp}{2}\hat{\nabla}_\perp^2\right)i\omega_*\bar{\Phi} \\
&+ \left(2+\frac{3}{2}\hat{\nabla}_\perp^2-\hat{\hat{\nabla}}_\perp^2\right)i\omega_d\bar{\Phi}+i\omega_d(p_{\|}+p_\perp)=0 \\
\frac{\partial u_{\|}}{\partial t} &+ \bar{\mathbf{v}}_E\cdot\nabla u_{\|}+B\nabla_{\|}\frac{p_{\|}}{B}+\nabla_{\|}\bar{\Phi}+\left(p_\perp+\frac{1}{2}\hat{\nabla}_\perp^2\bar{\Phi}\right)\frac{1}{B}\nabla_{\|}B \\
&+ i\omega_d(q_{\|}+q_\perp+4u_{\|})=0 \\
&\vdots \\
\frac{\partial q_{\|}}{\partial t} &+ \bar{\mathbf{v}}_E\cdot\nabla q_{\|}+(3+\beta_{\|})\nabla_{\|}T_{\|}+\sqrt{2}D_{\|}|k_{\|}|q_{\|} \\
&+ i\omega_d(-3q_{\|}-3q_\perp+6u_{\|})+i|\omega_d|(\nu_5u_{\|}+\nu_6q_{\|}+\nu_7q_\perp)=-\nu_{ii}q_{\|}
\end{aligned}$$

- each moment equation has $\mathbf{E}\times\mathbf{B}$ nonlinear term
- toroidal terms: $i\omega_d\equiv(cT/eB^3)\mathbf{B}\times\nabla B\cdot\nabla$
- H&P type parallel and toroidal closures: $|k_{\|}|$, $|\omega_d|$
- trapped ion CGL terms, ion-ion collisions (ν_{ii})
- FLR closures, $\hat{\nabla}_\perp$, $\hat{\hat{\nabla}}_\perp$

Gyrokinetic Vlasov Code for Linear Damping Comparisons

- made slight modifications to Liu's initial value Vlasov code [**Liu & Cheng, BAPS 38, 2102 (1993)**]
- solves for nonadiabatic part of distribution function on a velocity space grid, $h(v_{\|}, \mu)$, in flux-tube coordinates:

$$f = g + \frac{e\Phi}{T_i} F_0,$$

$$h = g - J_0 \frac{e\Phi}{T_i} F_0,$$

$$\frac{\partial h}{\partial t} + \frac{v_{\|}}{qR}\frac{\partial h}{\partial \theta} + i\omega_d h + a_{\|}\frac{\partial g}{\partial v_{\|}} =$$

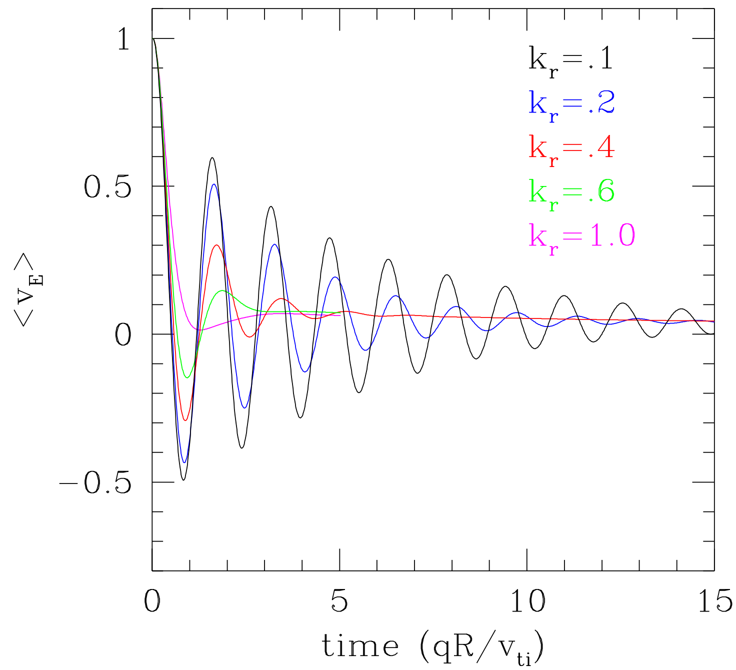
$$\left(-\frac{v_{\|}}{qR}\frac{\partial}{\partial \theta} - i\omega_d h + m v_{\|} a_{\|}/T_i - i\omega_*^T \right) F_0 J_0 e\Phi/T_i,$$

  and quasineutrality.
- For flow damping tests, initial condition is $\Phi(r)$ and $h(v_{\|}, \mu)$ is a Maxwellian.
- Typical numerical parameters: 30 θ grid pts, 60 $v_{\|}$ grid pts, 60 μ grid pts, $v_{\|}^{\max} = v_{\perp}^{\max} = 7 v_{ti}$

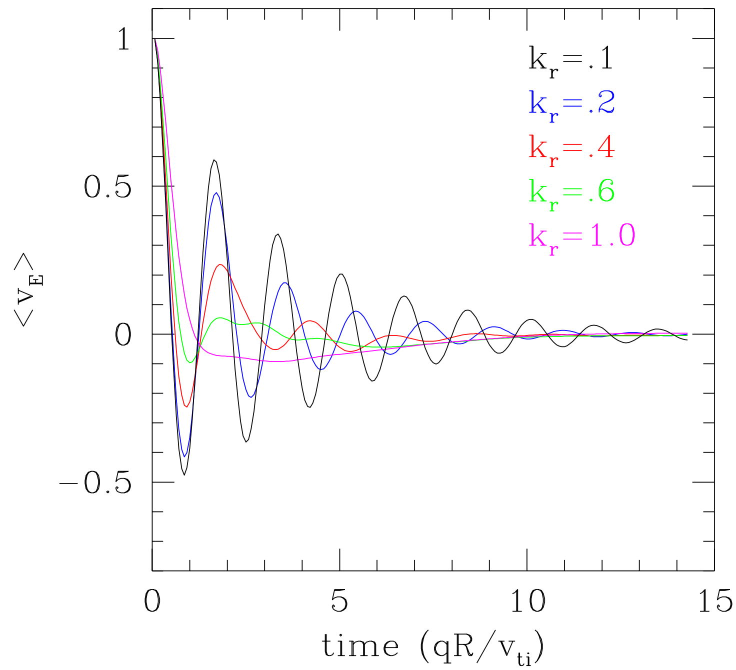
Comparison of Gyrokinetic and Gyrofluid Flow Damping

Fast linear damping rates agree very well
Varying k_r:

Gyrokinetic:



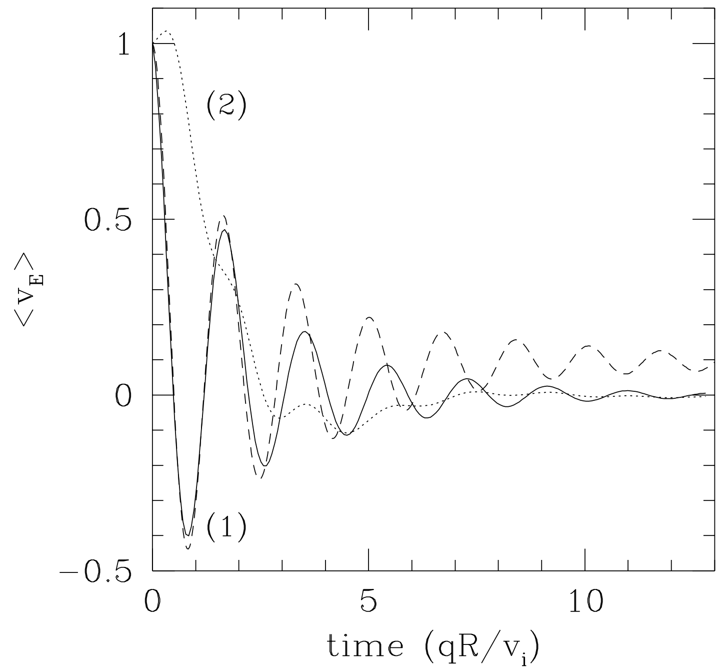
Gyrofluid:



Residual flow not modeled accurately by standard gyrofluid equations, as trapped particle dynamics are important, and our models of trapped ion dynamics are approximate.

When there is a gyrofluid residual perpendicular flow, it is usually in $u_{\|}$, not $\mathbf{v}_E$.

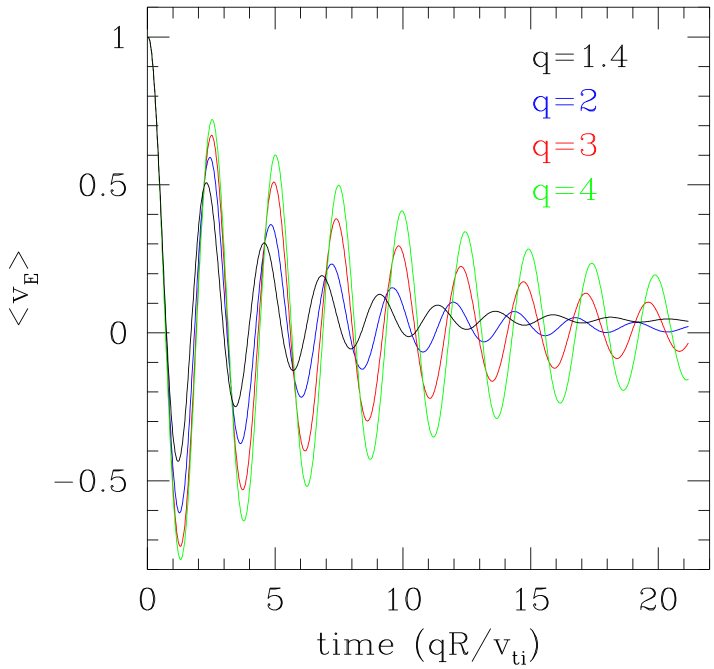
Different initial conditions:
(1) $\Phi(r)$ only (and comparison with Z. Lin's GKP code,dashed)
(2) $\Phi(r)$ and a small parallel flow, $u_{\|}(\theta)$, to avoid exciting $\|$ sound waves



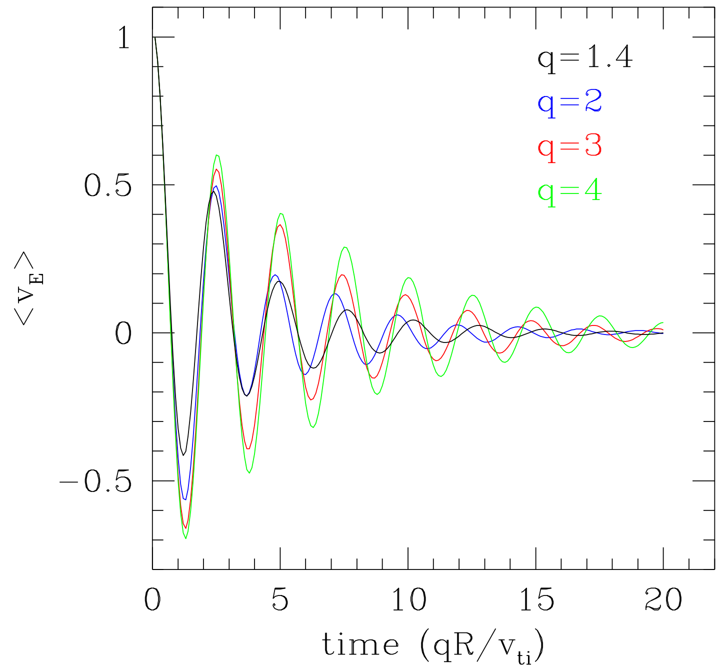
Evidence of two eigenmodes: one with zero real frequency, the other with finite real frequency (Global Acoustic Mode) [Winsor, Johnson, Dawson, PF (1968)]. Both eigenmodes have damping time on the order of a few transit times. Similar results found in [Novakovskii, et al., submitted (1997), Lebedev, et al. PoP (1996)].

Varying q:

Gyrokinetic:

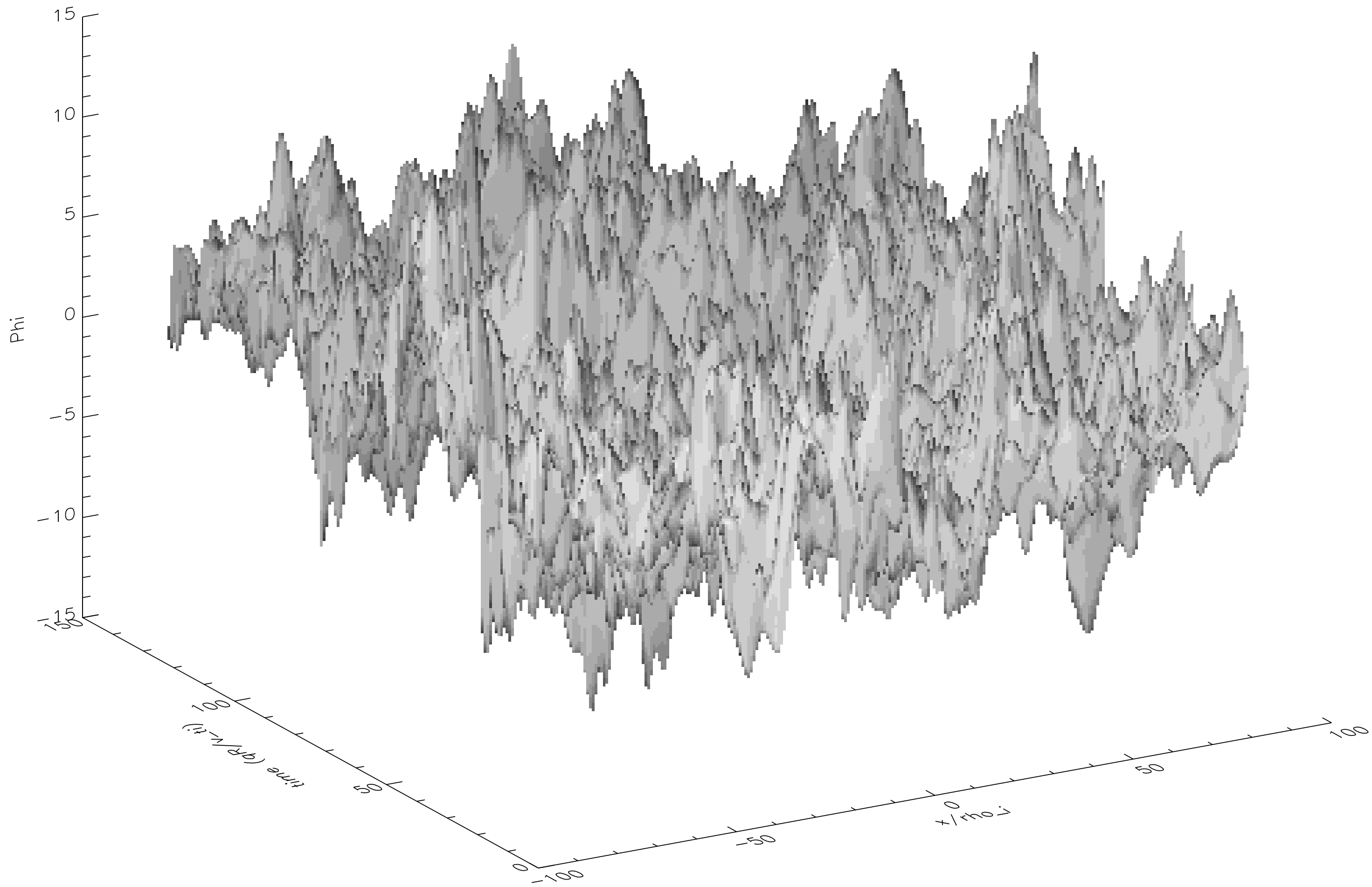


Gyrofluid:



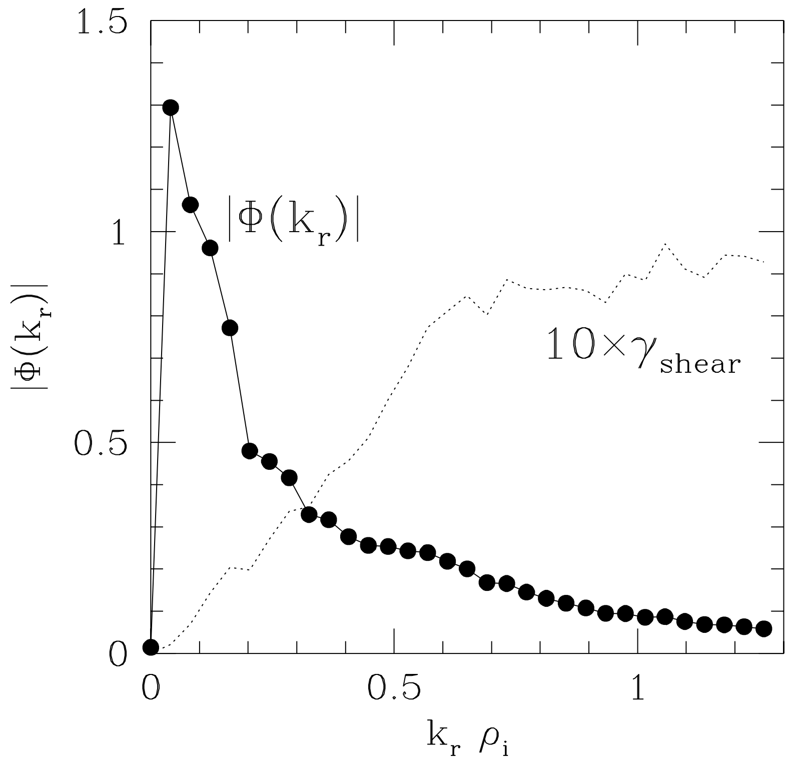
Calculation of Flow Spectra and Correlation Functions

Flow spectra and correlation functions are calculated from the time history of the flux surface averaged potential, $\langle\Phi(r,t)\rangle$, from the saturated phase of a nonlinear run for DIIID #81499 parameters at $\rho = 0.5$: $\hat{s} = .776$, $q = 1.4$, $\eta_i = 3.11$, $\epsilon_n = 0.45$, $T_i = T_e$.



Time Averaged Flow Spectrum

Spectrum of saturated flux surface averaged potential $|\Phi(k_r)|$ obtained by Fourier Transforming in r and averaging in t.



Shearing rate peaks at high k_r: $\gamma_{\text{shear}} = k_r^2|\Phi(k_r)|$

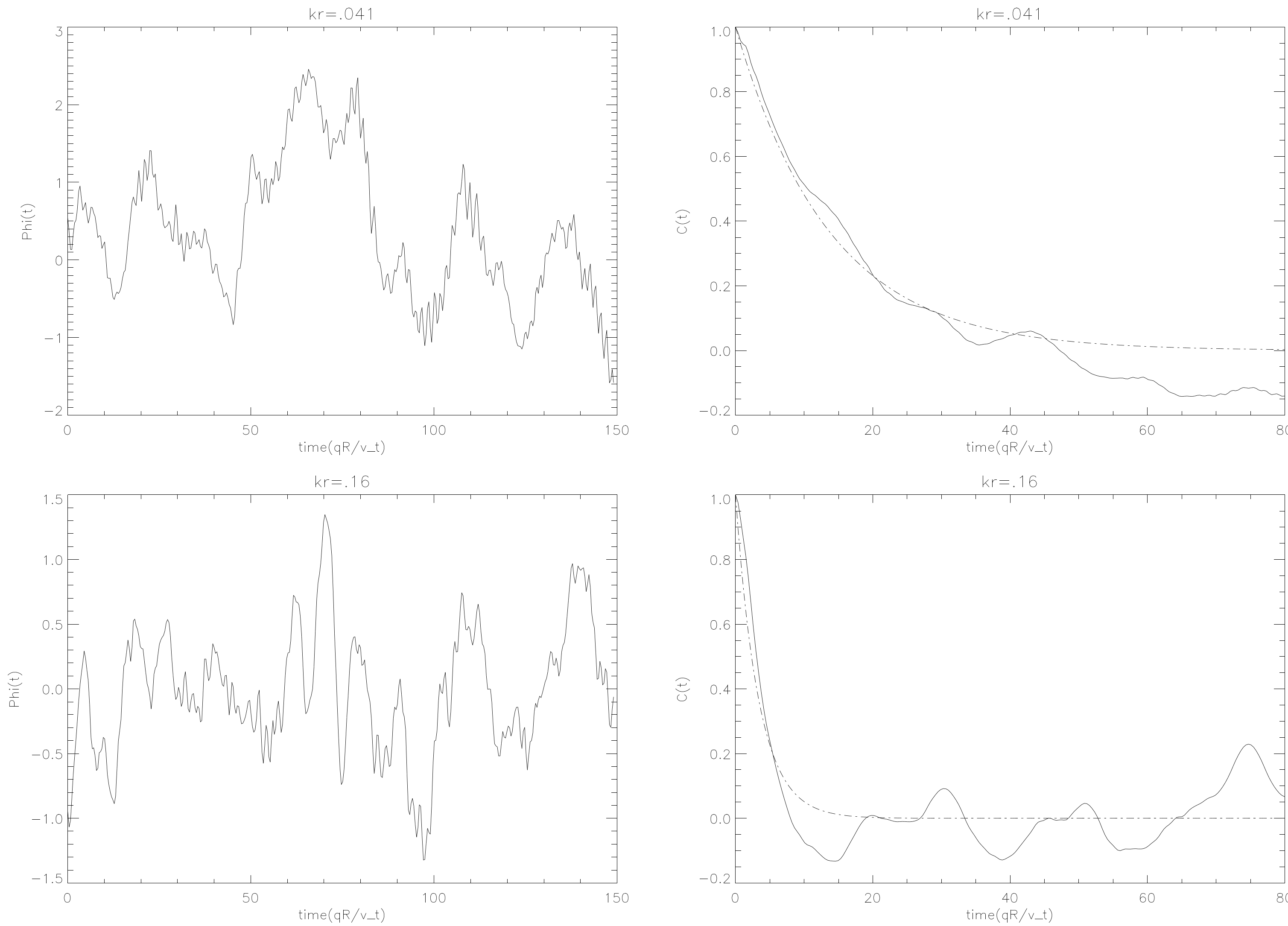
While highest k_r shearing rates are large, they have small correlation times. Maximum $\gamma_{\text{lin}} \approx 0.1$

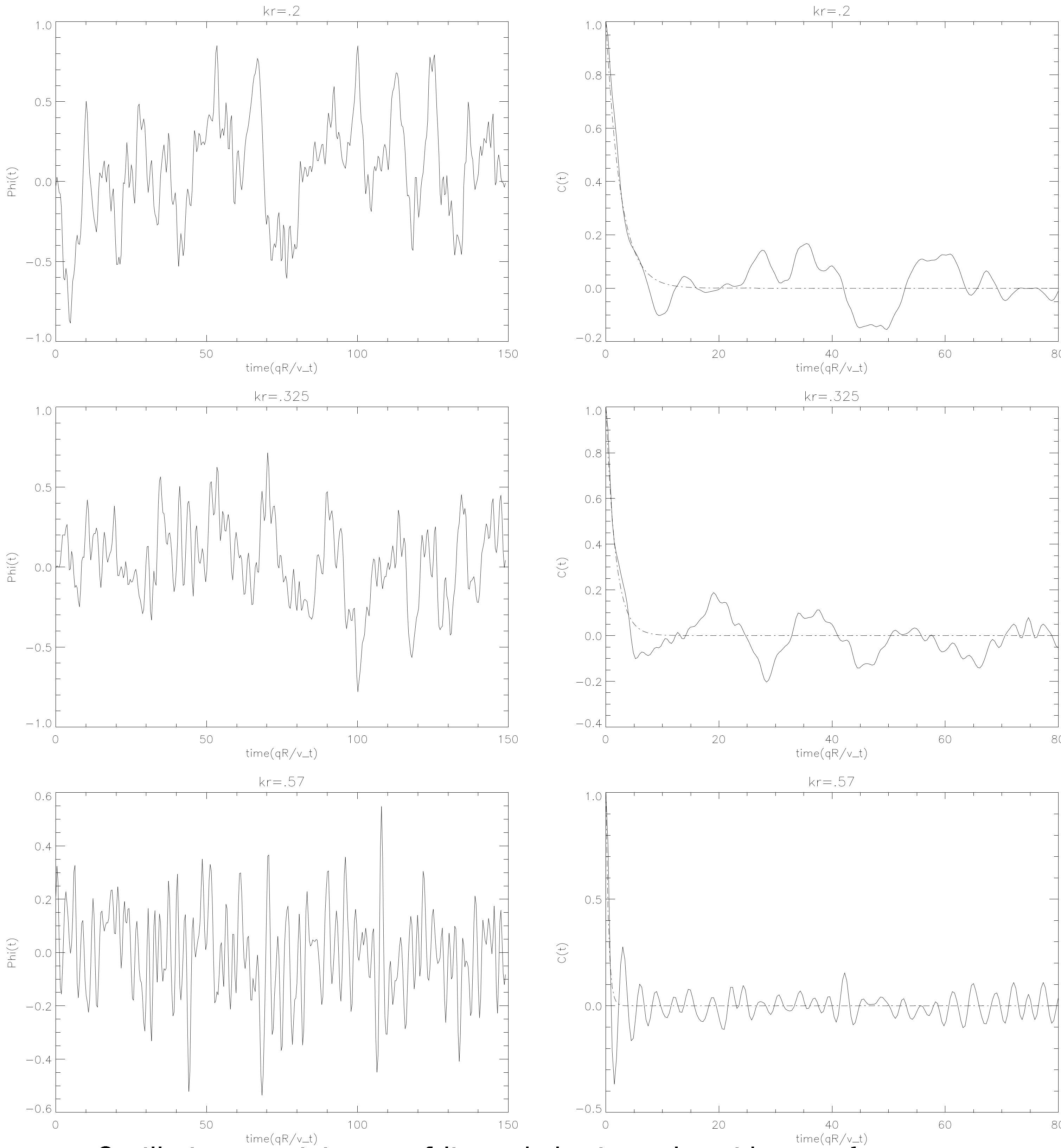
Flow Correlation Functions

After transforming in r, the correlation function can be obtained from the time series $\Phi(k_r, t)$:

$$C(t) = \int dt\, e^{-i\omega t} \Phi^* \Phi(\omega)$$

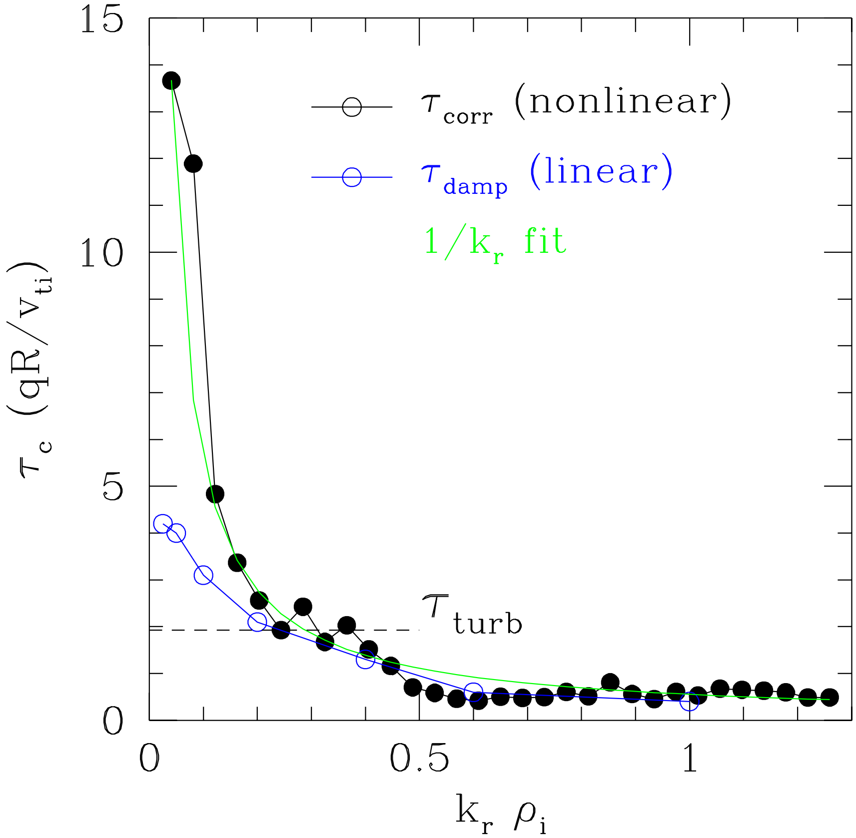
A least squares fit to the numerical data of the form $C(t) = e^{-t/\tau_c}$ is also shown





Oscillations reminiscent of linear behavior only evident at $k_r > 0.5$, where correlation time becomes very short.

$\tau_{\rm corr}$ vs. k_r similar to measurements by Coda [this meeting (1997)]:



$\tau_c \approx \tau_{\rm damp}$ except for small k_r, where $\tau_c > \tau_{\rm damp}$.

$\tau_c > \tau_{\rm damp}$ implies that finite spectral width of nonlinear source $S(\omega)$ is dominating τ_c:

$$\frac{\partial \Phi}{\partial t} + i\omega_r \Phi = -\nu \Phi + S \quad \Rightarrow \quad |\Phi^2(\omega)| = \frac{|S^2(\omega)|}{(\omega - \omega_r)^2 + \nu^2}$$

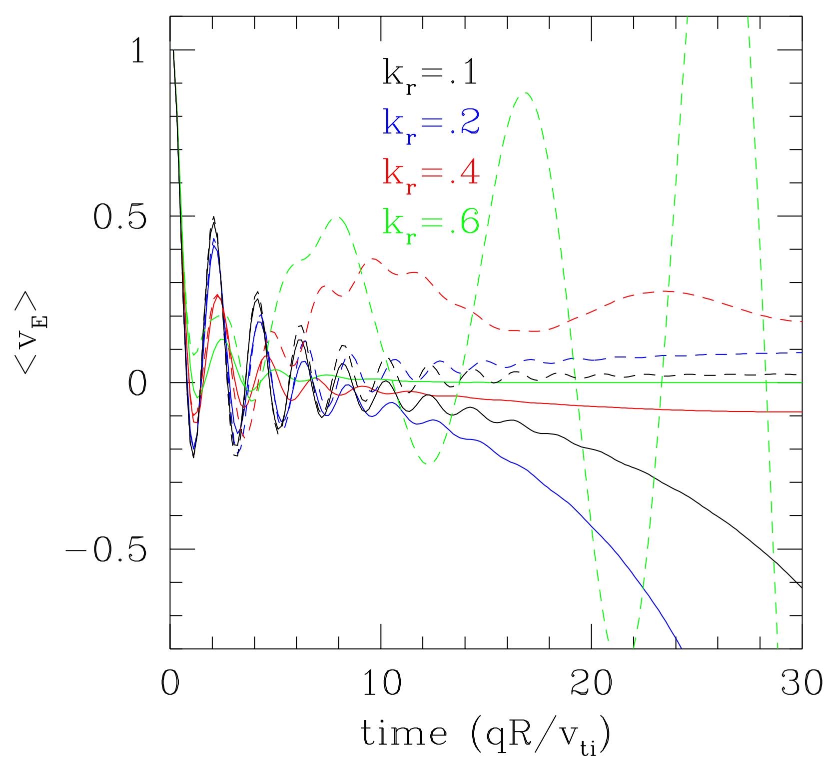
$\Rightarrow$ NL Source is not white at low k_r.

Tests of Importance of Residual Flow

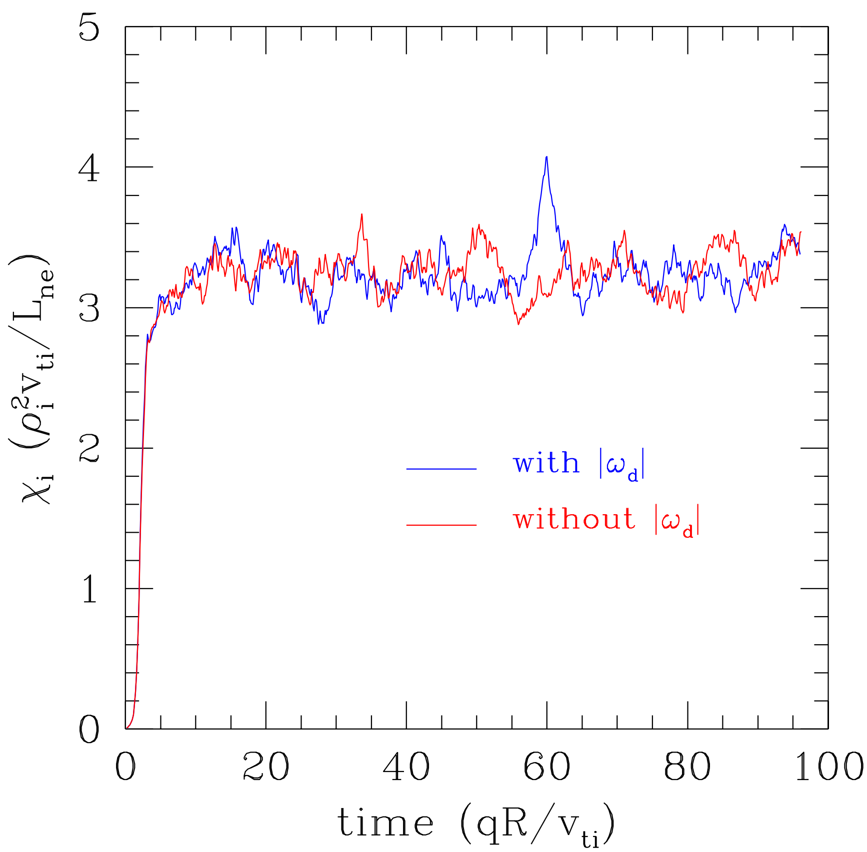
Since the fast damping is controlled by parallel damping (the $|k_{\parallel}|$ closure terms) and the long time residual is controlled by the toroidal terms ($|\omega_d|$ closure terms), we can change the long time linear flow behavior without changing the fast linear damping by changing the toroidal closure coefficients or turning off the $|\omega_d|$ terms only for the flows ($k_y = 0$ components).

These tests are not using our standard closure coefficients, which do give a long time flow instablilty.

Linear damping of flows:
(1) with $|\omega_d|$ closure terms for $k_y = 0$ (solid)
(2) without $|\omega_d|$ closure terms for $k_y = 0$ (dashed)



Then for the same two cases, nonlinear runs:

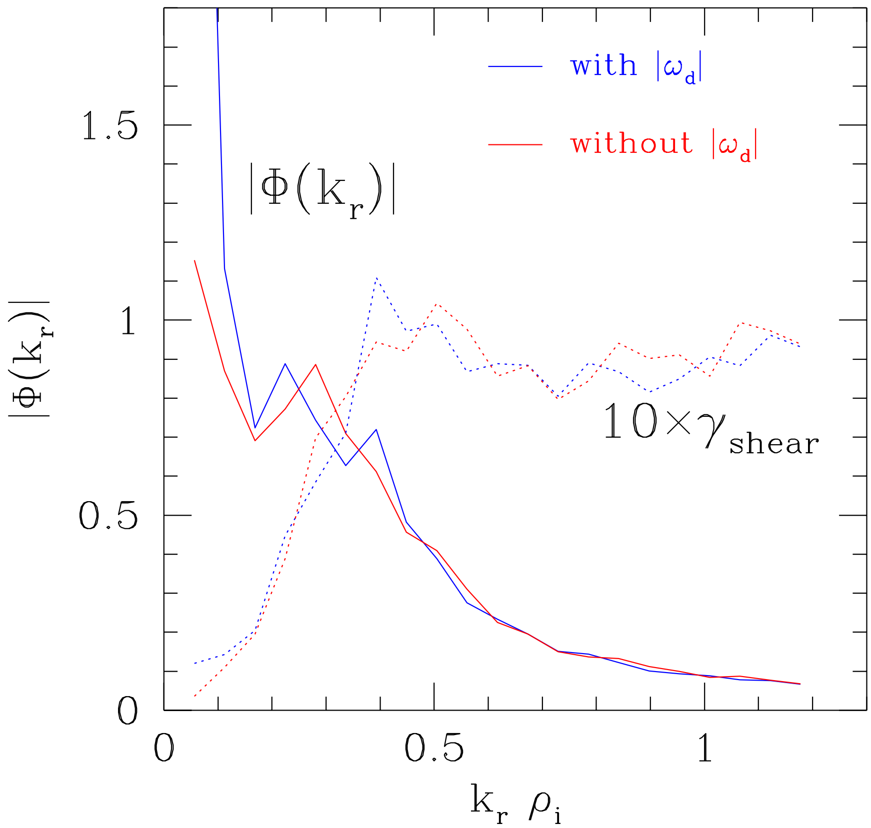


χ_i's are similar, implying that long time differences in the linear flow behavior are not affecting transport

Parameters for these tests are $\hat{s} = 1$, $q = 1$, $\epsilon_n = 0.1$, $\eta_i = 3$, $\epsilon = 0.1$.

For other parameters, with weaker turbulence, the flow instabilities can grow up and $\chi_i \rightarrow 0$. This never happens with our standard closure terms, which do not lead to a flow instability.

Flow spectra are similar, implying that flow saturation levels are determined by the balance between nonlinear drive/damping and the fast linear damping.



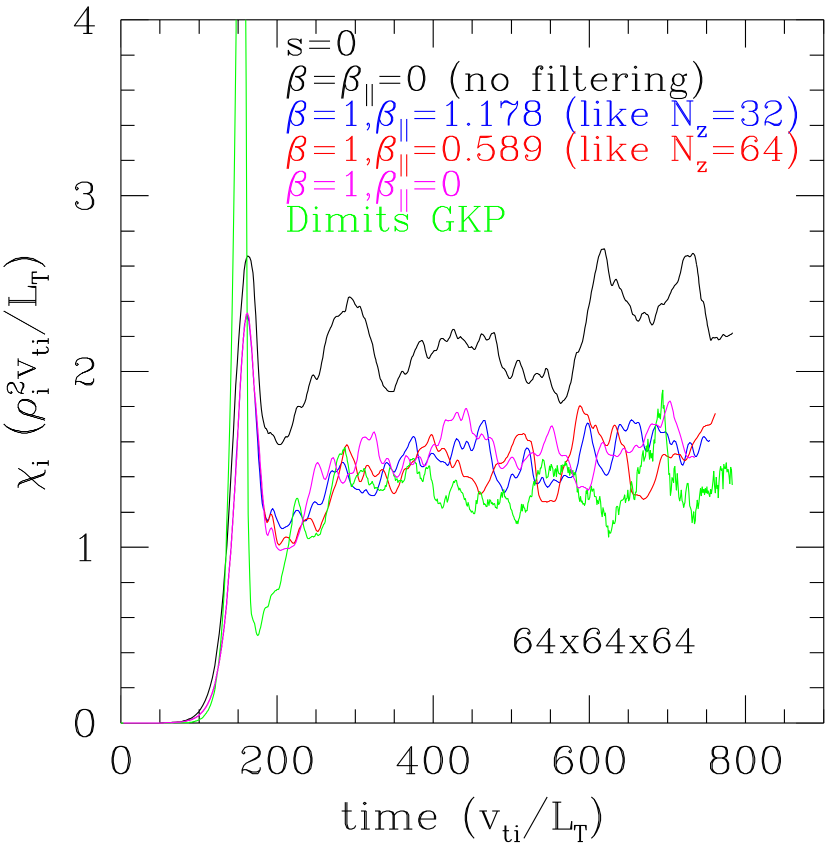
Gyrofluid/Gyrokinetic Flux-tube Simulation Comparisons: NTP test case with $\hat{s} = 0$

Thanks to Andris Dimits for GKP results in comparison sections.

Earlier results by Hammett showed that a Φ filter of the form:

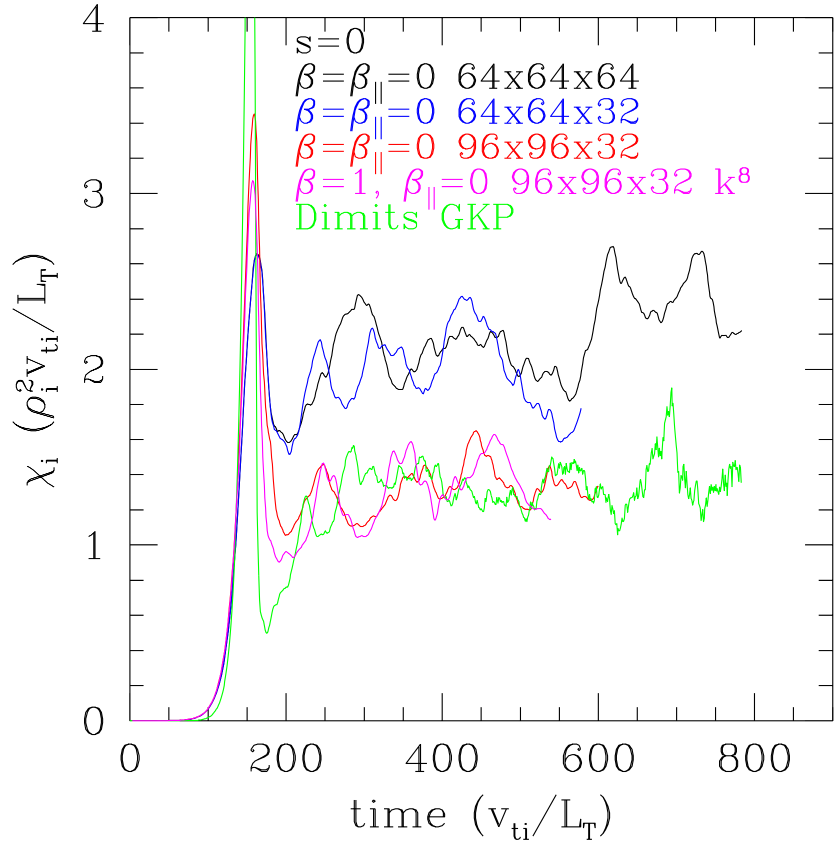
$$\frac{e^{-(\beta k_y \rho_i)^4}}{[1+(\beta k_x \rho_i)^4][1+(\beta_\parallel k_\parallel \Delta_\parallel)^4]},$$

as used in GKP simulations, brought GF and GKP into agreement. These runs used a $64 \times 64 \times 64$ grid.



Perpendicular filter has largest effect.

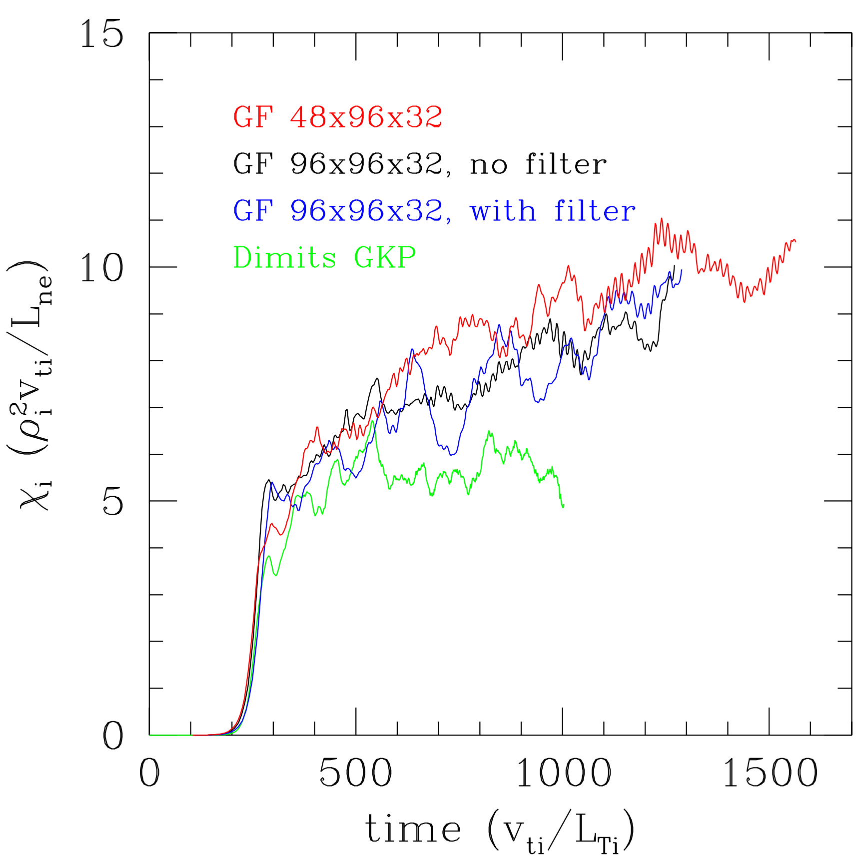
Running with higher resolution ($96 \times 96 \times 32$) brings GF results into agreement with GKP with or without filter.



Gyrofluid/Gyrokinetic Flux-tube Simulation Comparisons: NTP test case with $\hat{s} \neq 0$

For NTP test case with $\hat{s} = 1.5$, GF χ_i is about $2\times$ GKP χ_i, even at high resolution, with and without filter.

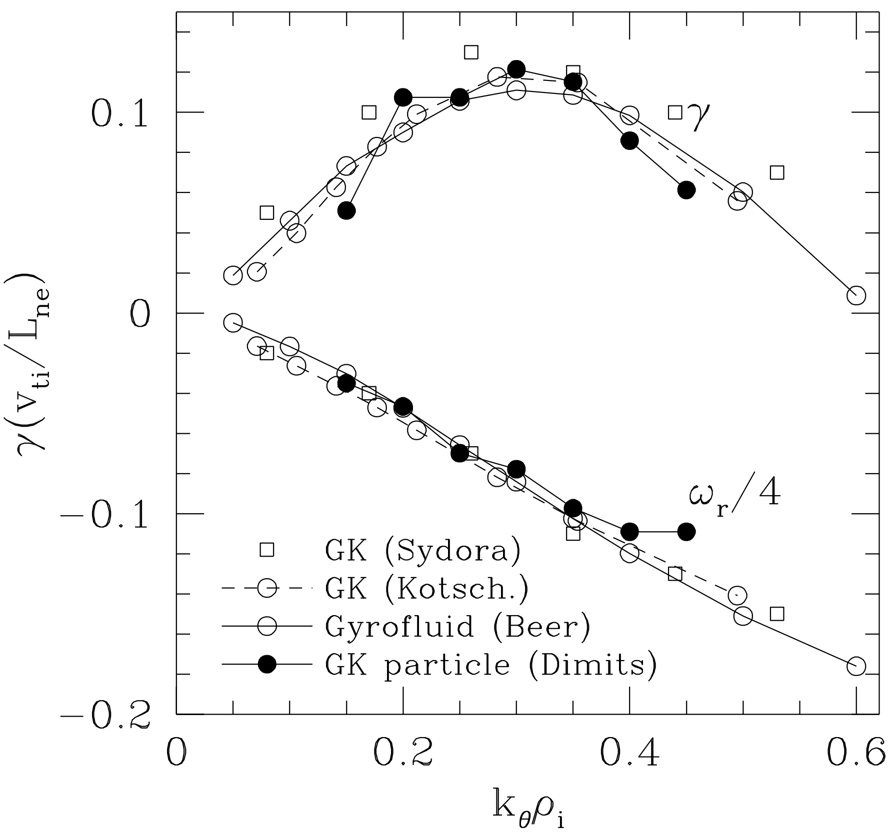
Parameters taken from TFTR L-mode:
$\hat{s} = 1.5$, $q = 2.4$, $\eta_i = 4$, $\epsilon_n = 0.4$, $\epsilon = 0.2$, $T_i = T_e$



Numerical resolution issues require further investigation.

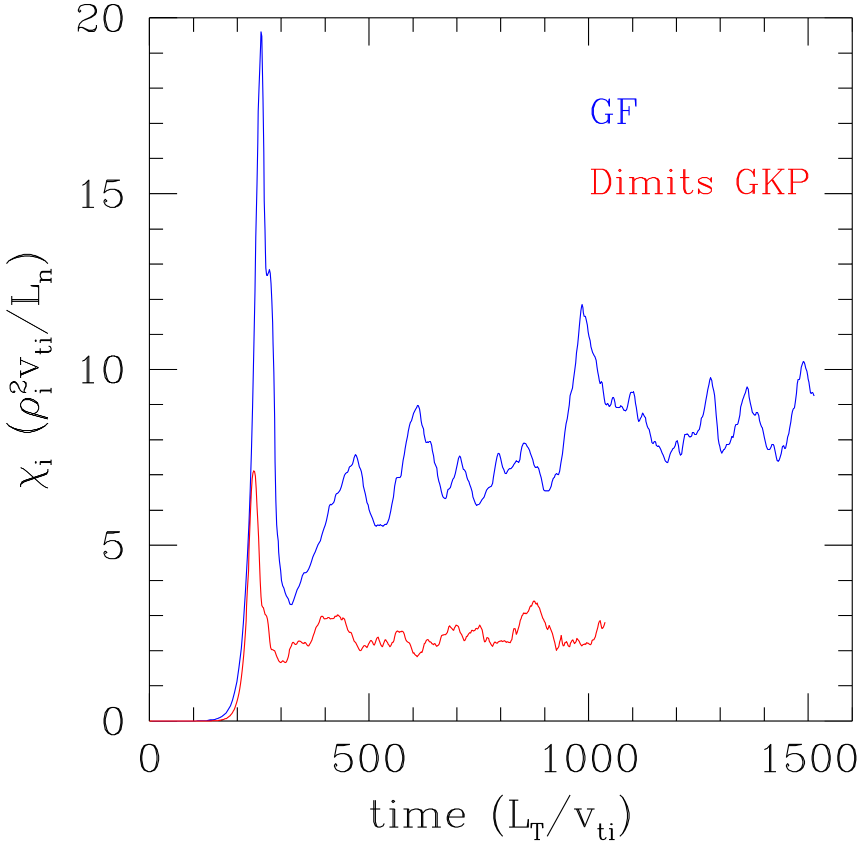
Linear Comparisons for DIIID #81499 parameters ar $\rho = 0.5$

$\hat{s} = 0.78$, $q = 1.4$, $\eta_i = 3.11$, $\epsilon_n = 0.45$, $\epsilon = 0.18$, $T_i = T_e$



Overall good agreement in linear growth rates. Possible discrepancies at low k_θ?

Nonlinear Comparisons for DIIID #81499 parameters ar $\rho = 0.5$



For this case, GKP χ_i is lower than GF χ_i by a factor of 3.3.

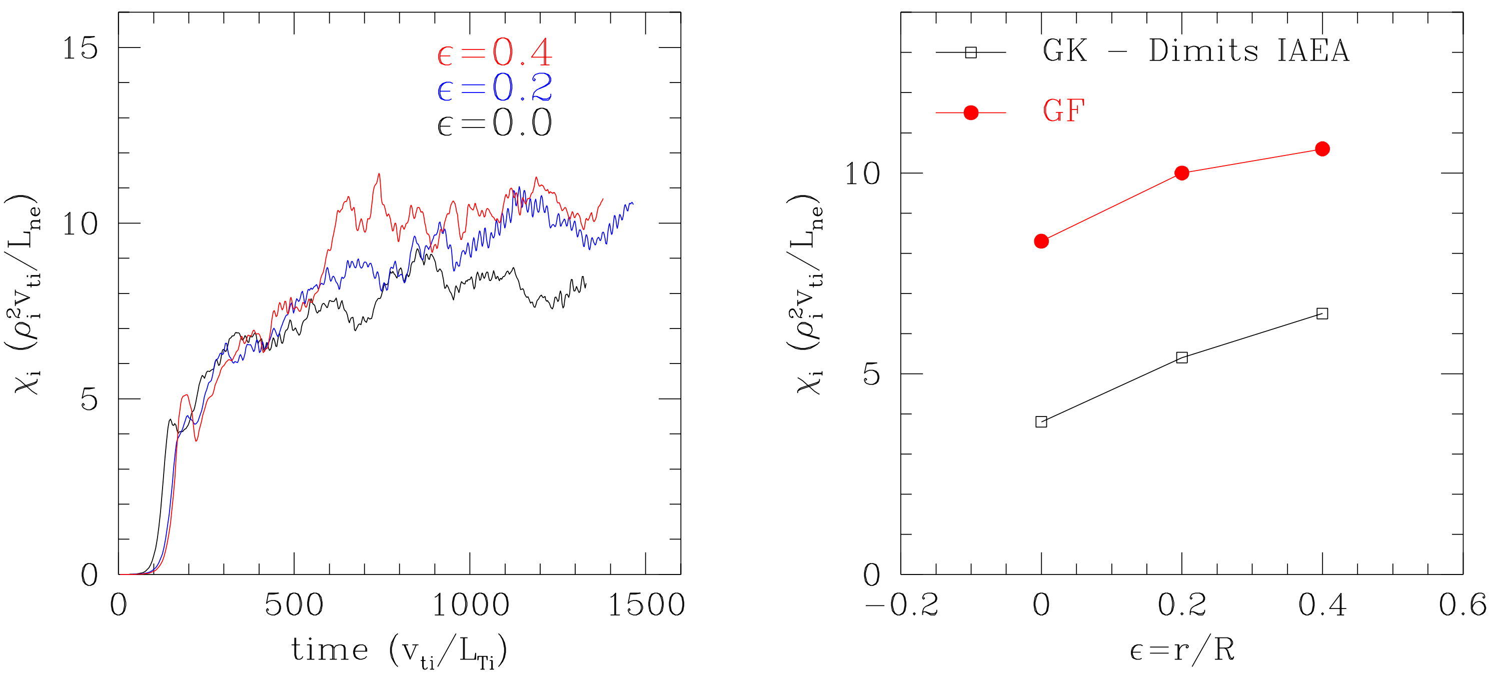
Gyrofluid/Gyrokinetic Comparisons: ϵ scan to test Hinton-Rosenbluth effect

The amount of residual flow after an initial flow perturbation has damped away is controlled by $\epsilon = r/R$, as given by Hinton & Rosenbluth (HR) and verified by Dimits (who found $c = 0.6$, HR found $c = 1/1.6 = 0.625$):

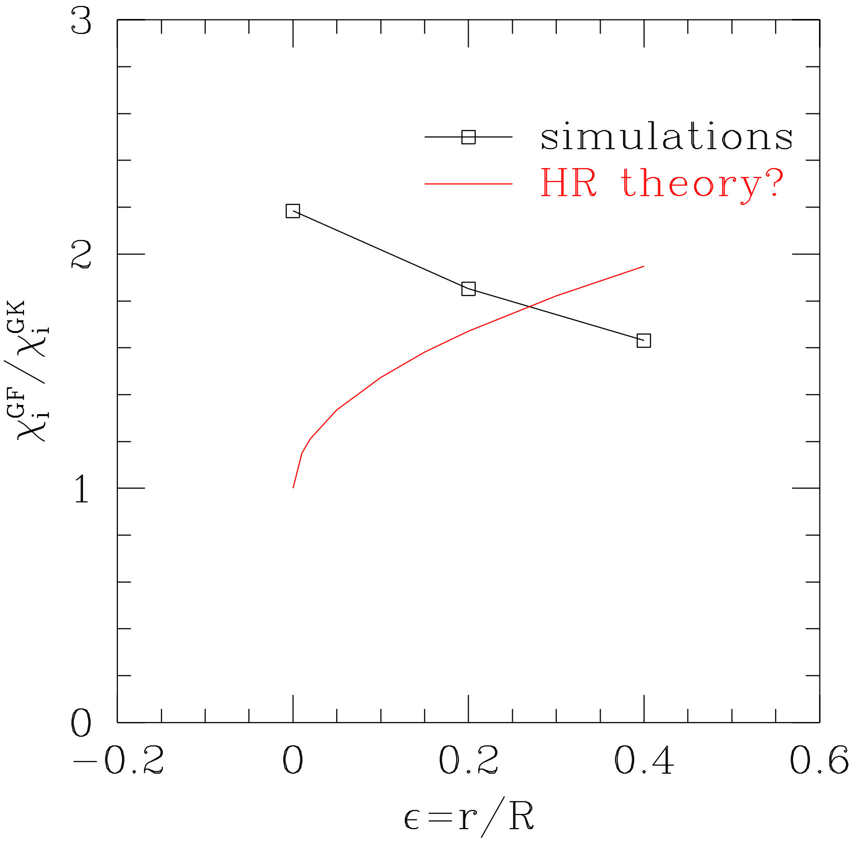
$$\frac{\mathbf{v}_{Ef}}{\mathbf{v}_{Ei}} = \frac{c\sqrt{\epsilon}/q^2}{1 + c\sqrt{\epsilon}/q^2}$$

Residual flow component and HR effect can be turned off by taking $\epsilon \rightarrow 0$.

Dimits reported an ϵ scan for the NTP test case parameters in his IAEA (1994) paper which we repeated with GF simulations.

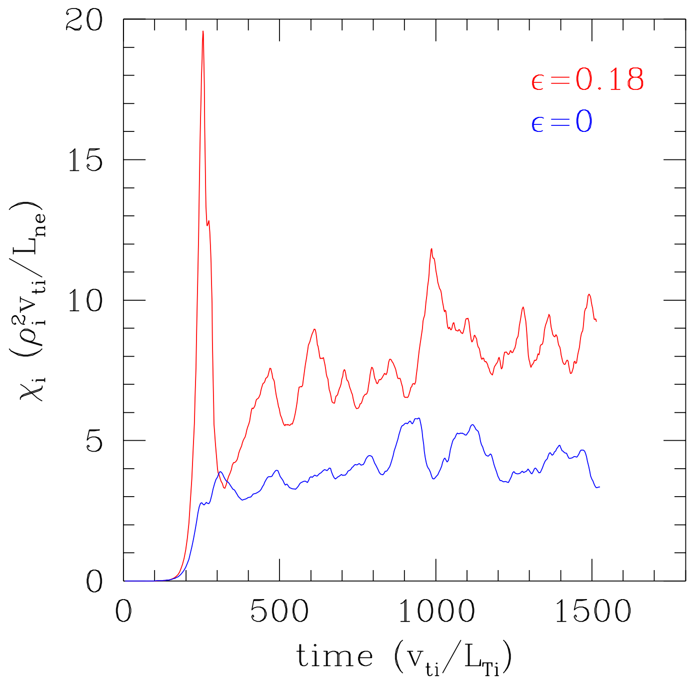


Ratio of χ_{GF}/χ_{GK} does not go to zero as residual flow and HR effect is turned off.



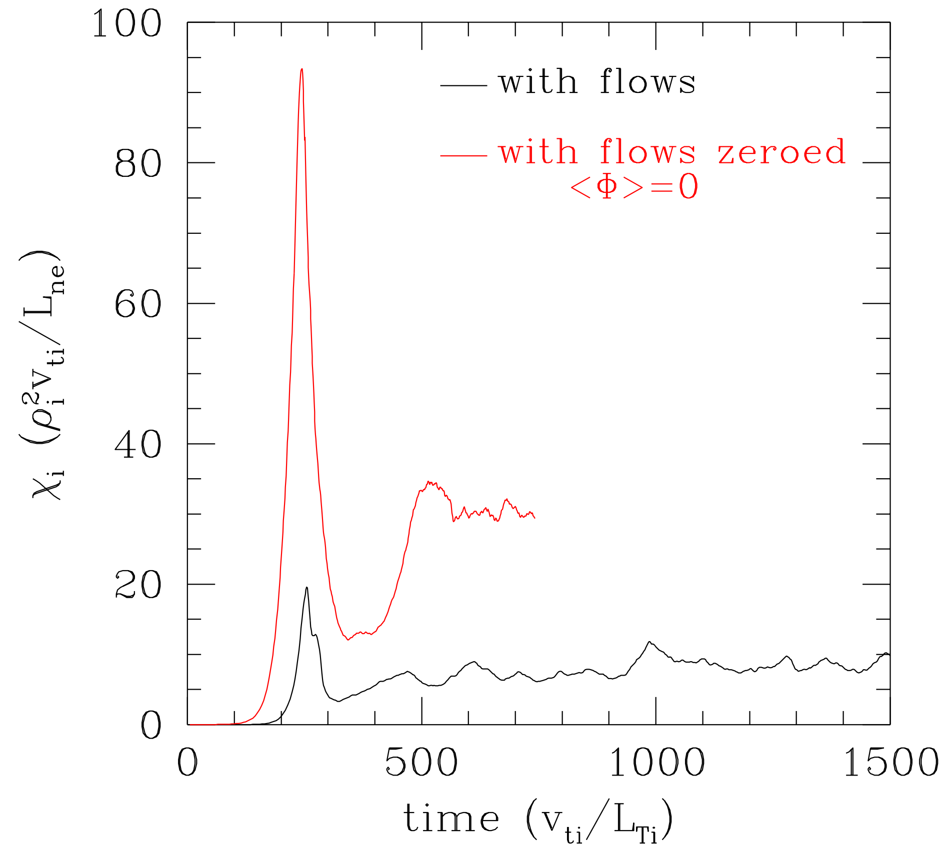
Indicates that HR effect is not dominant source of GF vs. GKP discrepancy

Similar results for DIIID #81499 parameters. Dimits also sees χ_i drop by 2 when $\epsilon = 0$.



Effect of Zeroing Flows: $\langle\Phi\rangle$=0

For DIIID #81499 parameters:



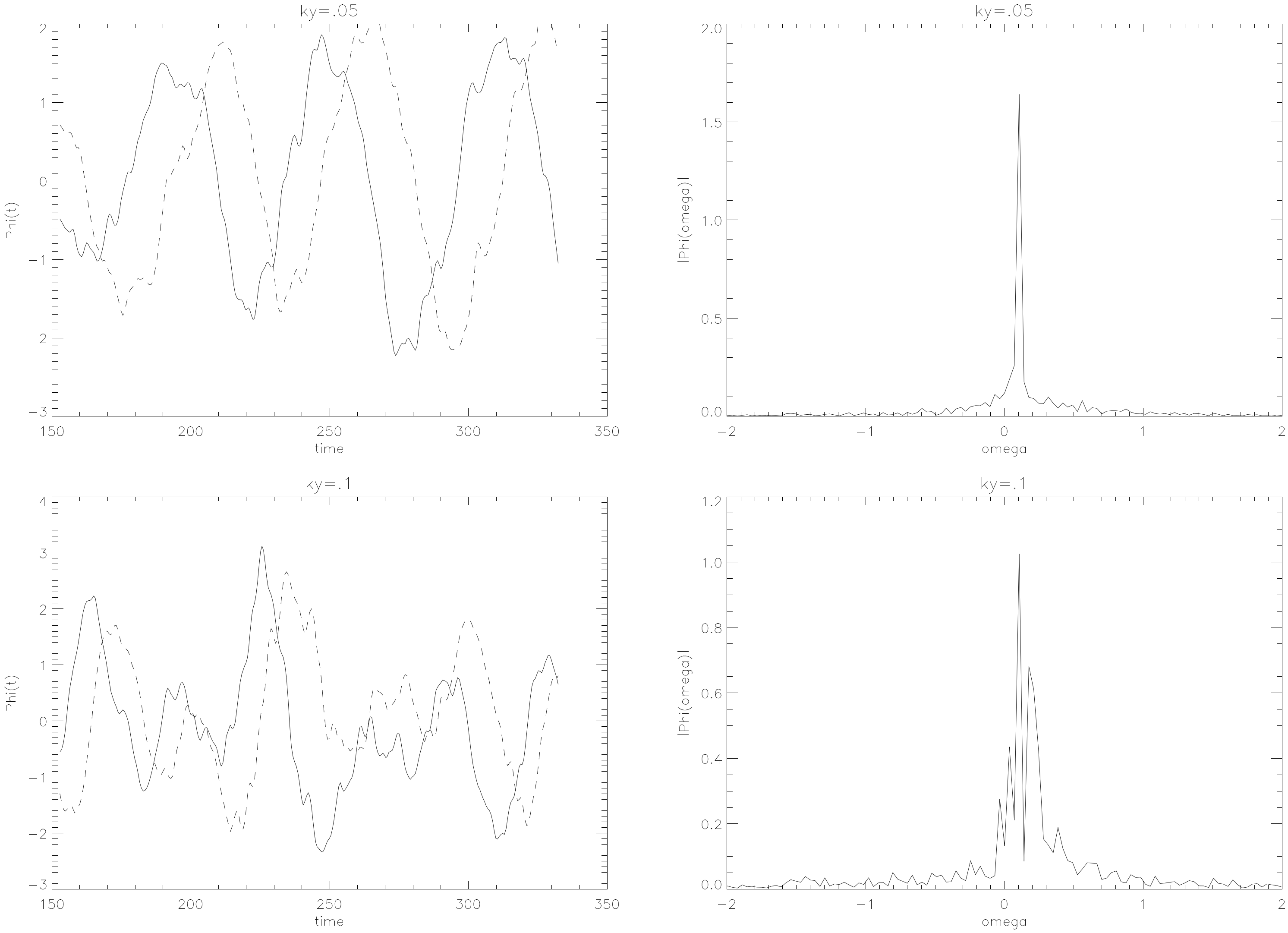
Dimits sees χ_i increase from 2.5 to 20 when $\langle\Phi\rangle$=0. Indicates that differences in flow dynamics cannot explain full GF vs. GKP discrepancy

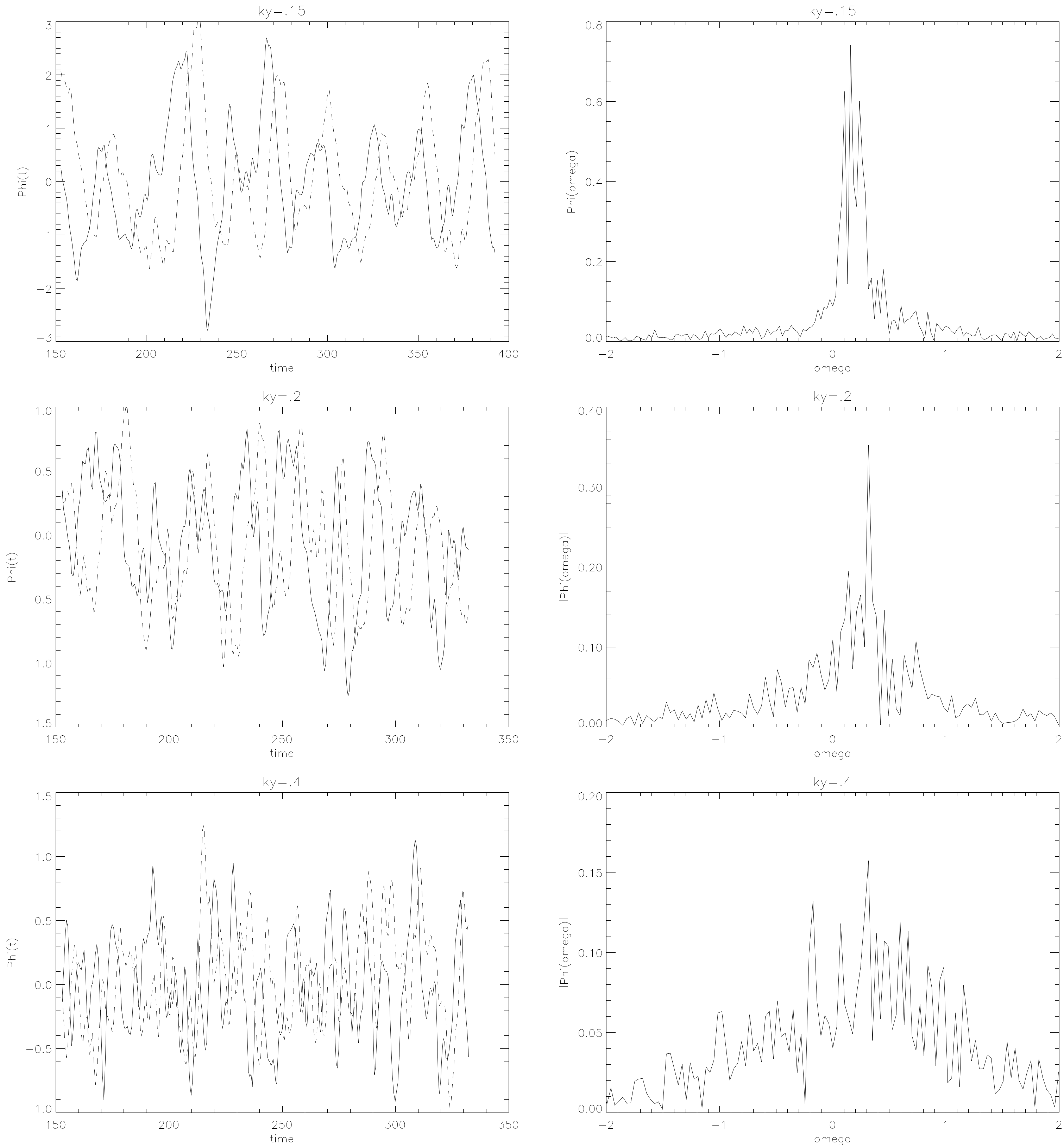
Conclusions

- Linear flow damping tests show good agreement between gyrokinetic and gyrofluid fast linear damping rates for all k_r
- Standard gyrofluid model does not model residual component accurately
- Correlation time of radial modes is on the order of damping time, but longer than damping time for low k_r
- Nonlinear effects appear to dominate evolution of residual flow component
- Nonlinear GyroKinetic Particle (GKP) vs. GyroFluid (GF) comparisons:
  - GF/GKP discrepancy is typically 2-3. Not understood at present. Good agreement at $\hat{s} = 0$ for NTP case.
  - Differences in linear radial mode dynamics do not appear to be the cause of GF/GKP discrepancy, except perhaps near marginal stability

Frequency Spectra for $k_y > 0$ Modes

Gyrofluid frequency spectra seem broader than GKP spectra, possible source of GF/GKP discrepancy?





ky=.15
Phi(t)
time
150
200
250
300
350
400
3
2
1
0
−1
−2
−3
ky=.15
|Phi(omega)|
omega
0.8
0.6
0.4
0.2
0.0
−2
−1
0
1
2
ky=.2
Phi(t)
time
1.0
0.5
0.0
−0.5
−1.0
−1.5
150
200
250
300
350
ky=.2
|Phi(omega)|
omega
0.40
0.30
0.20
0.10
0.00
ky=.4
Phi(t)
time
1.5
1.0
0.5
0.0
−0.5
−1.0
ky=.4
|Phi(omega)|
omega
0.20
0.15
0.10
0.05
0.00