Turbulence-Driven Zonal Flow Dynamics in Gyrofluid Simulations

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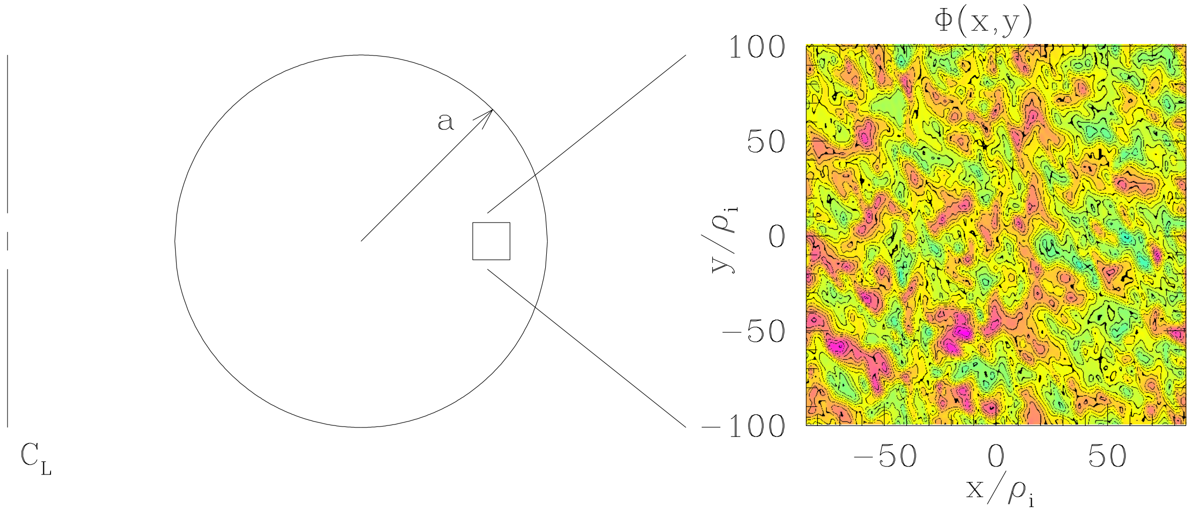
Outline

- Background and Motivation
  - importance of sheared zonal $\mathbf{E} \times \mathbf{B}$ flows in regulating turbulence
  - residual undamped component of zonal flows emphasized by [Rosenbluth & Hinton, PRL (1998)], not retained in previous gyrofluid simulations. Hypothesized to be the cause of gyrokinetic/gyrofluid disagreement
- Derivation of new gyrofluid closures which attempt to retain residual component
  - reasonable comparisons with linear collisionless gyrokinetic simulations for moderate k_r, not as good for lower or higher k_r
- Nonlinear tests of the importance of the residual undamped component
  - away from marginal stability, nonlinear effects appear to keep the residual flow component from growing indefinitely
  - near marginal stability residual component can completely quench turbulence

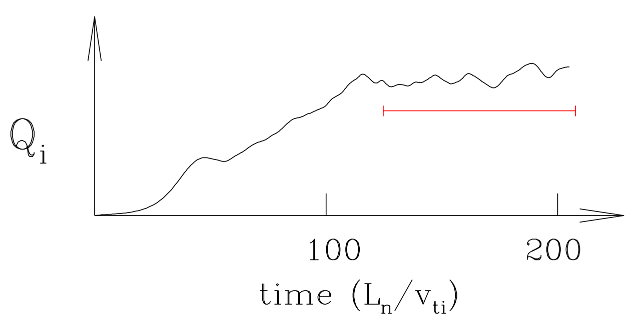
Flux Tube Simulation Model

Simulate small perpendicular cross section. Exploit separation between equilibrium scales $\sim a$, and fluctuations $\sim 10\rho_i \ll a$.



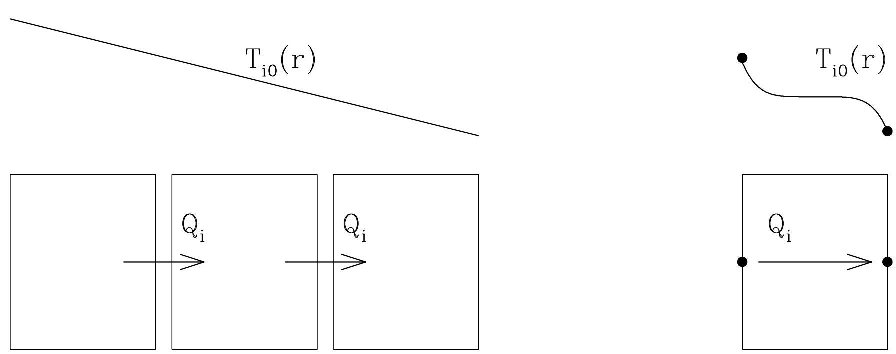
Equilibrium parameters used as inputs: q, $\hat{s}$, L_n, L_T, r/R, T_i/T_e,...

Gradients $\nabla n_0, \nabla T_0$ drive instabilities which evolve into turbulence.



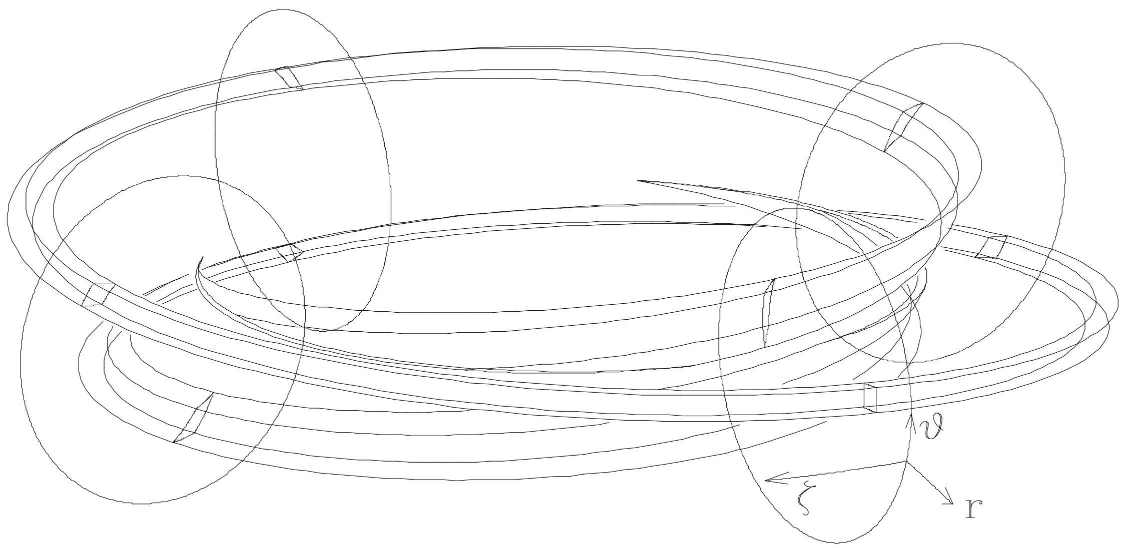
$$Q_i = \langle \tilde{v}_{Er} \tilde{p}_i \rangle \equiv -n_0 \chi_i^{tot} \nabla T_{i0}$$

Radial periodicity inhibits flattening of equilibrium.



Realistic 3D Toroidal Geometry is Used

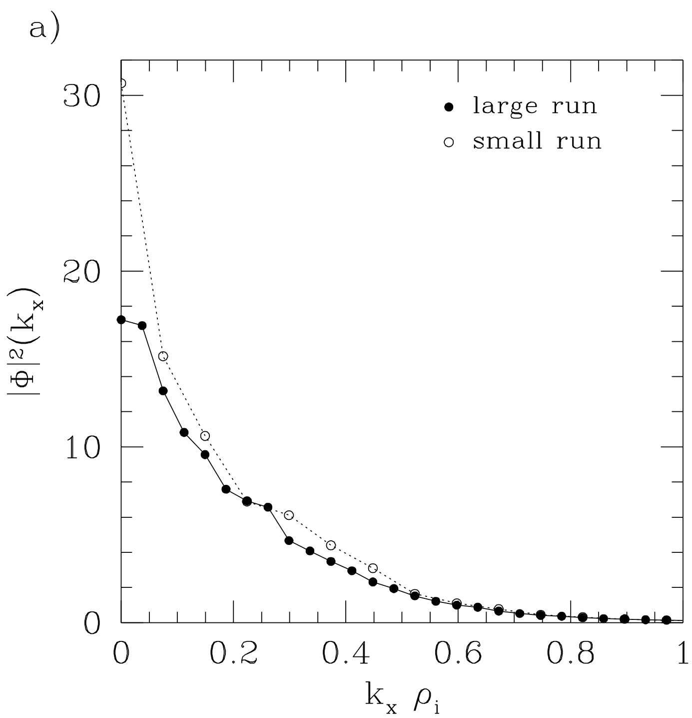
Flux tube simulation domain aligned with sheared field lines.



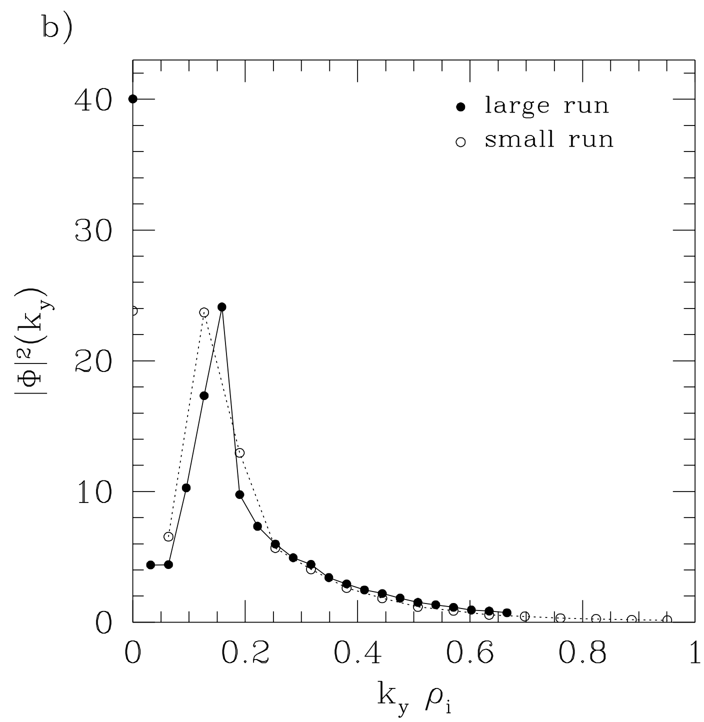
Equivalent to simulating a fraction $(1/n_0)$ of a toroidal annulus, with a coarse grid in toroidal mode number $n \in \{0, n_0, 2n_0, 3n_0, \ldots\}$

Fluctuation Spectra Resemble BES

Toroidal gyrofluid simulations reproduce long-wavelength peak measured by BES [Fonck, *et al.*, PRL (1993)]. Growth rate peaks at higher k_θ: nonlinear downshift to long wavelengths.



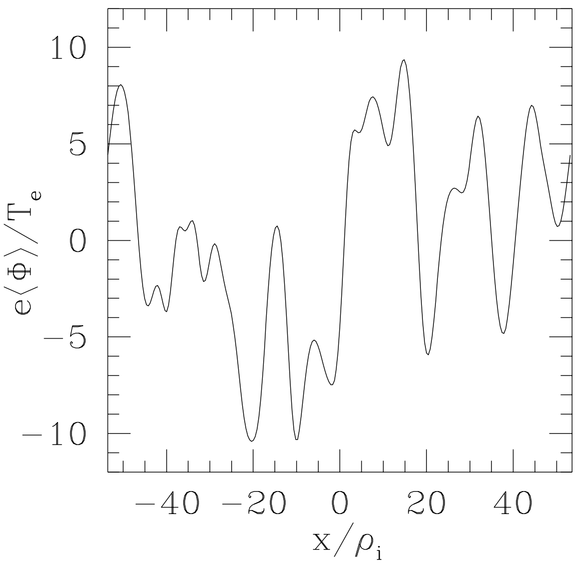
radial

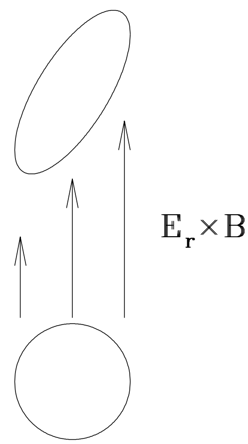


poloidal

Also seen in Gyrokinetic particle simulations [Parker, *et al.*, PRL (1993)].

Sheared $\mathbf{E} \times \mathbf{B}$ Zonal Flows Play an Important Role in the Turbulent Dynamics





- Zonal flows are constant on a flux surface, but vary radially. Also called radial modes [Waltz, *et al.* (1994)]. Potential leads to radially sheared perpendicular $\mathbf{E} \times \mathbf{B}$ flow.

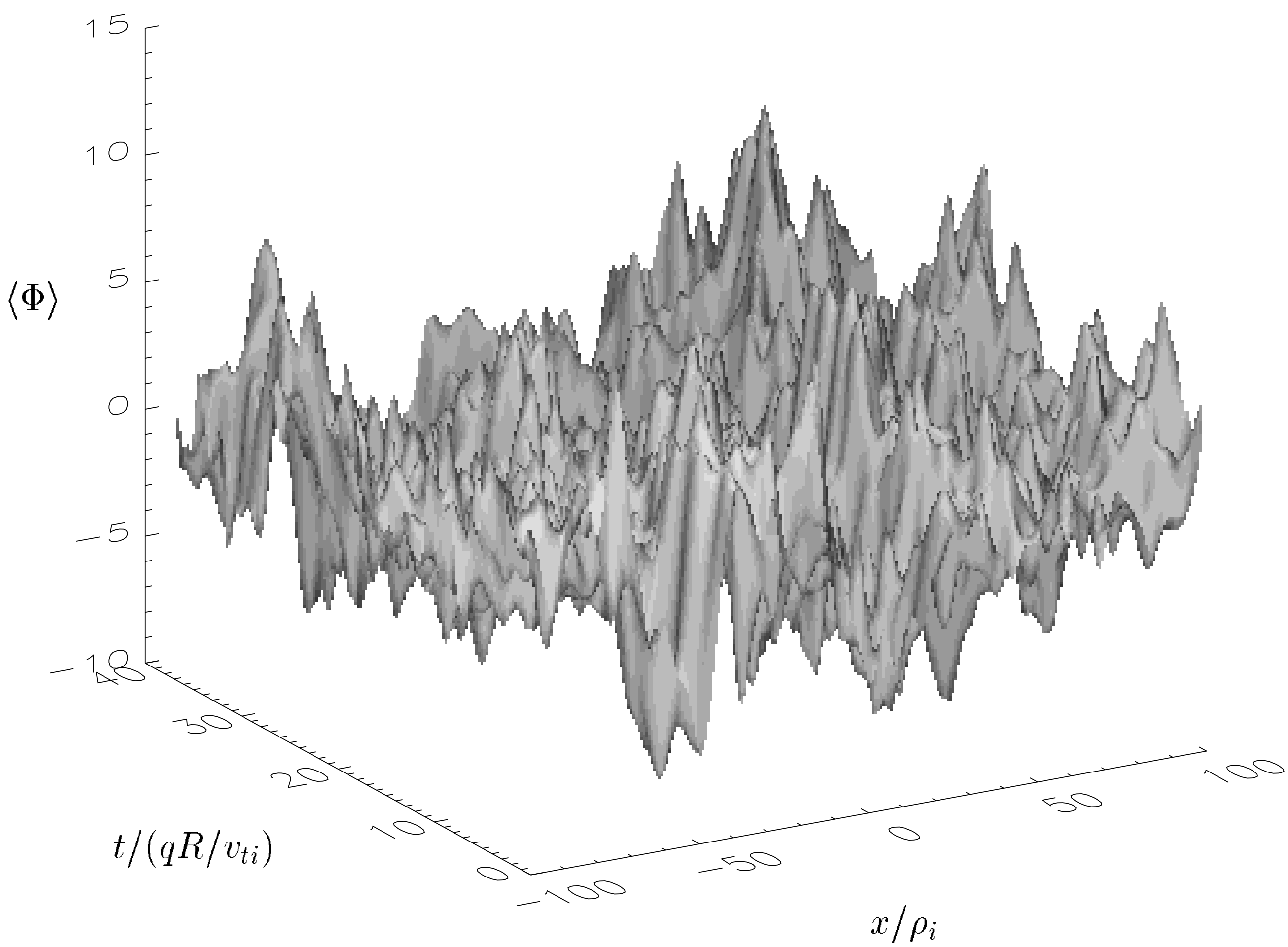
- Have small radial scales $\sim$ turbulent scales, not equilibrium

- Flows are nonlinearly generated by the turbulence [Hasegawa & Wakatani (1987)], [Carreras, *et al.* (1991)], [Diamond & Kim, (1991)]

- Sheared flows stretch turbulent eddies, decreasing radial correlation and regulating the turbulence [Biglari, Diamond, Terry, (1990)]

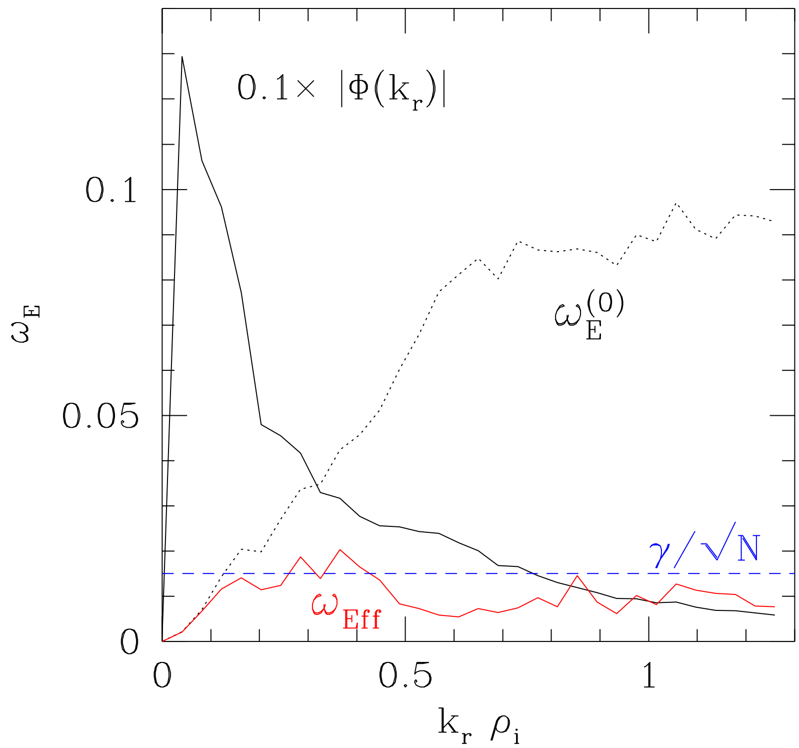
Zonal Flows Fluctuate on Turbulent Time and Space Scales

Time history of the flux surface averaged potential, $\langle \Phi(r,t) \rangle$, from the saturated phase of a nonlinear run for DIIID #81499 parameters at $\rho = 0.5$: $\hat{s} = .776$, $q = 1.4$, $\eta_i = 3.11$, $\epsilon_n = 0.45$, $T_i = T_e$.



Time Averaged Flow Spectrum

Spectrum of saturated flux surface averaged potential $|\Phi(k_r)|$ obtained by Fourier Transforming in r and averaging in t.



Shearing rate peaks at high k_r: $\gamma_{\text{shear}} = k_r^2|\Phi(k_r)|$

While highest k_r shearing rates are large, they have small correlation times and thus small effect on turbulence (Hahm, Diamond).

Maximum $\gamma_{\text{lin}} \approx 0.1$

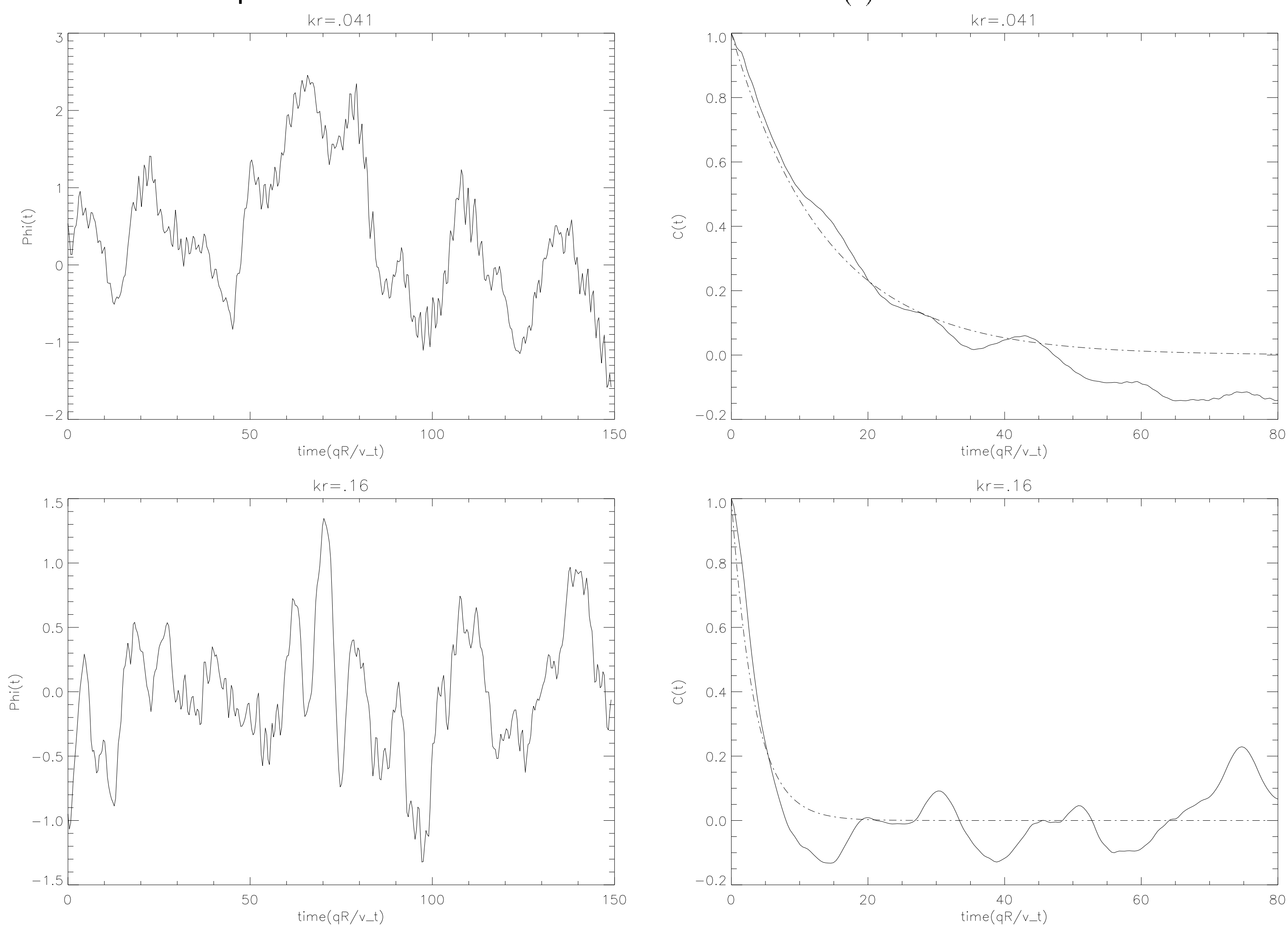
Using simulation zonal flow spectrum and using simulation zonal flow time history to calculate $\tau_{\text{corr}}(k_r)$, we estimate the time dependent effective shearing rate ω_{Eff} [Hahm] and find close to $\gamma_{\text{lin}}/\sqrt{N}$

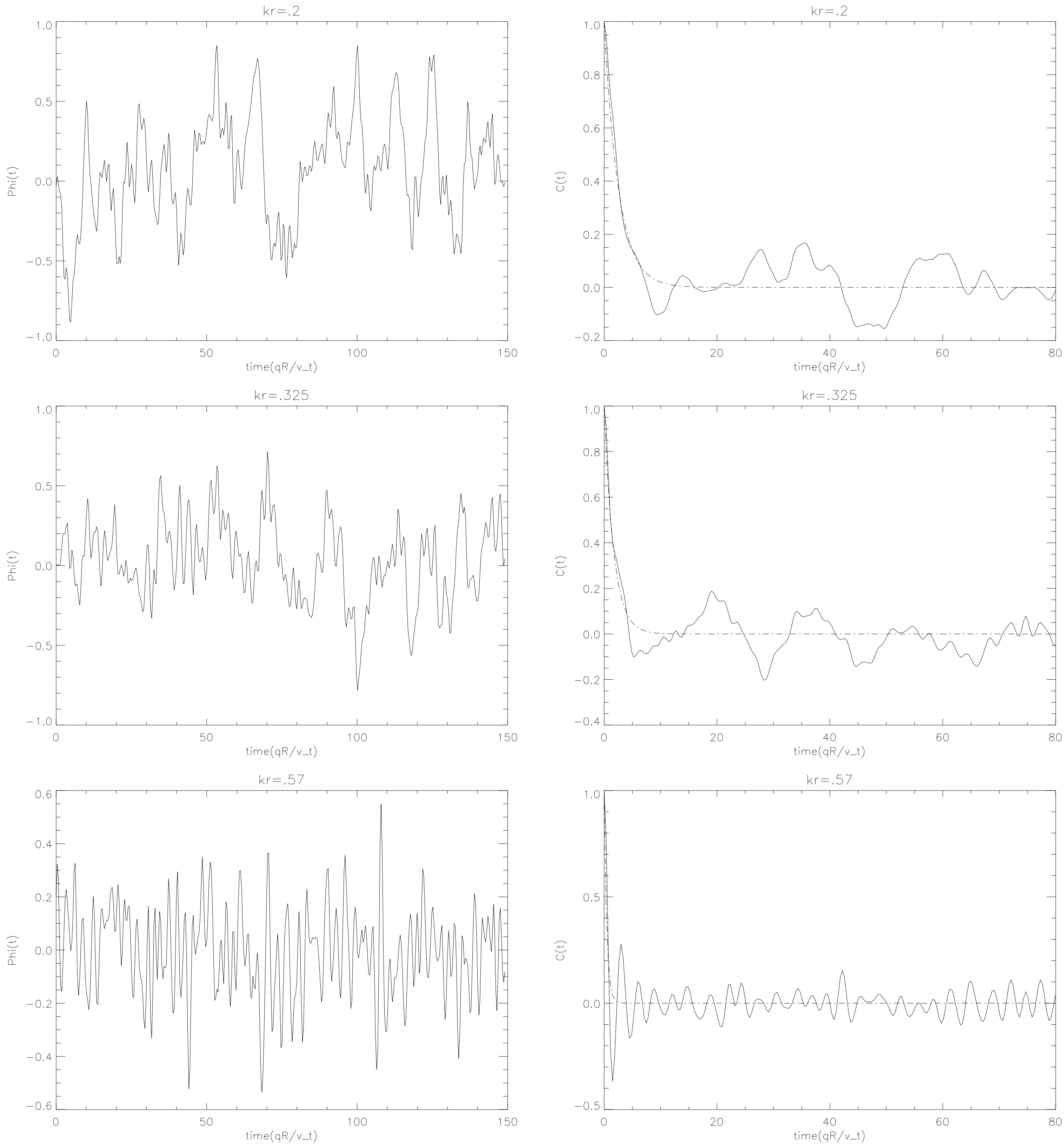
Flow Correlation Functions

After transforming in r, the correlation function can be obtained from the time series $\Phi(k_r, t)$:

$$C(t) = \int dt\, e^{-i\omega t} \Phi^* \Phi(\omega)$$

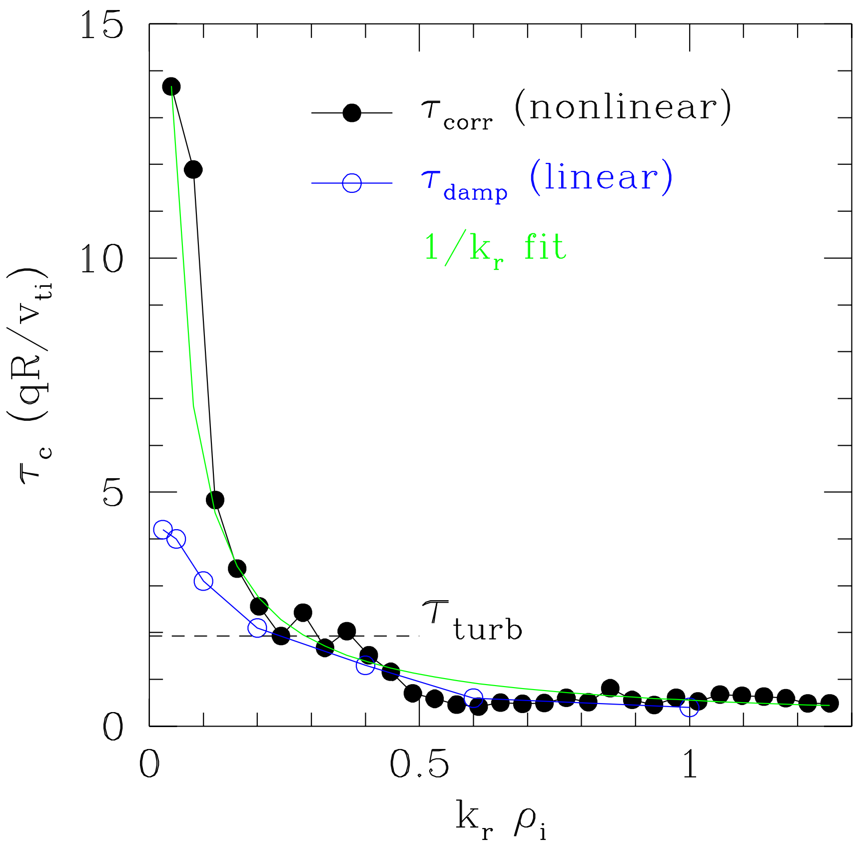
A least squares fit to the numerical data of the form $C(t) = e^{-t/\tau_c}$ is also shown




kr=.2
Phi(t)
time(qR/v_t)
kr=.2
C(t)
time(qR/v_t)
kr=.325
Phi(t)
time(qR/v_t)
kr=.325
C(t)
time(qR/v_t)
kr=.57
Phi(t)
time(qR/v_t)
kr=.57
C(t)
time(qR/v_t)

Simulation $\tau_{\rm corr}$ Similar to Expt

$\tau_{\rm corr}$ vs. k_r similar to measurements by Coda [APS 1997]:



$\tau_c \approx \tau_{\rm damp}$ except for small k_r, where $\tau_c > \tau_{\rm damp}$.

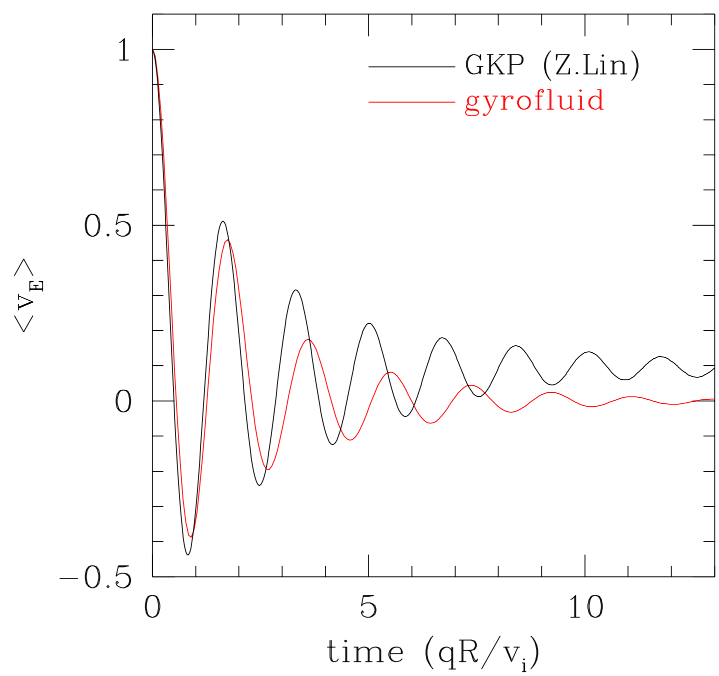
$\tau_c > \tau_{\rm damp}$ implies that finite spectral width of nonlinear source $S(\omega)$ is dominating τ_c:

$$\frac{\partial \Phi}{\partial t} + i\omega_r \Phi = -\nu \Phi + S \quad \Rightarrow \quad |\Phi^2(\omega)| = \frac{|S^2(\omega)|}{(\omega-\omega_r)^2+\nu^2}$$

$\Rightarrow$ NL Source is not white at low k_r.

Importance of Linear Zonal Flow Damping

- Two phases: fast collisionless damping & slow collisional damping. Depends on initial flow conditions
- In [Beer, Ph.D. Thesis (1995)] showed that our gyrofluid equations accurately model the fast linear collisionless damping for $t < qR/v_{ti}\sqrt{\epsilon}$. Argued that long time linear flow dynamics are not important, nonlinear effects will dominate long term nonlinear flow evolution.



- [Rosenbluth & Hinton, PRL (1998)] emphasized a linearly undamped flow component. This "residual" flow damped by collisional effects. Argued that *nonlinearly*, residual component should grow in time $\sim \sqrt{t}$ in collisionless limit. Modeled nonlinear drive term as a white noise source.
- Since our original gyrofluid eqns underestimate residual component, if residual component is important nonlinearly, gyrofluid simulations would underestimate $\mathbf{E} \times \mathbf{B}$ flow levels and overpredict χ_i.

Toroidal Gyrofluid Equations for Ion Species

[Beer & Hammett, PoP 3, 4046 (1996)]

For ions, evolve moments of nonlinear electrostatic toroidal gyrokinetic eqn. $(n, u_\parallel, T_\parallel, T_\perp, q_\parallel, q_\perp)$: **[Frieman&Chen, Lee, Dubin, Krommes, Hahm]**

$$\frac{\partial f}{\partial t} + (v_\parallel \hat{\mathbf{b}} + \bar{\mathbf{v}}_E + \mathbf{v}_d) \cdot \nabla f + \left(\frac{e}{m} \bar{E}_\parallel - \mu \nabla_\parallel B + v_\parallel (\hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}}) \cdot \bar{\mathbf{v}}_E \right) \frac{\partial f}{\partial v_\parallel} = C(f)$$

$$\begin{aligned}
\frac{\partial n}{\partial t} &+ \bar{\mathbf{v}}_E \cdot \nabla n + B \nabla_\parallel \frac{u_\parallel}{B} - \left(1 + \frac{\eta_\perp}{2} \hat{\nabla}_\perp^2 \right) i\omega_* \bar{\Phi} \\
&+ \left(2 + \frac{3}{2} \hat{\nabla}_\perp^2 - \hat{\hat{\nabla}}_\perp^2 \right) i\omega_d \bar{\Phi} + i\omega_d (p_\parallel + p_\perp) = 0 \\
\frac{\partial u_\parallel}{\partial t} &+ \bar{\mathbf{v}}_E \cdot \nabla u_\parallel + B \nabla_\parallel \frac{p_\parallel}{B} + \nabla_\parallel \bar{\Phi} + \left(p_\perp + \frac{1}{2} \hat{\nabla}_\perp^2 \bar{\Phi} \right) \frac{1}{B} \nabla_\parallel B \\
&+ i\omega_d (q_\parallel + q_\perp + 4u_\parallel) = 0 \\
&\vdots \\
\frac{\partial q_\parallel}{\partial t} &+ \bar{\mathbf{v}}_E \cdot \nabla q_\parallel + (3 + \beta_\parallel) \nabla_\parallel T_\parallel + \sqrt{2} D_\parallel |k_\parallel| q_\parallel \\
&+ i\omega_d (-3q_\parallel - 3q_\perp + 6u_\parallel) + i|\omega_d| (\nu_5 u_\parallel + \nu_6 q_\parallel + \nu_7 q_\perp) = -\nu_{ii} q_\parallel
\end{aligned}$$

- each moment equation has $\mathbf{E} \times \mathbf{B}$ nonlinear term
- toroidal terms: $i\omega_d \equiv (cT/eB^3) \mathbf{B} \times \nabla B \cdot \nabla$
- H&P type parallel and toroidal closures: $|k_\parallel|$, $|\omega_d|$
- trapped ion CGL terms, ion-ion collisions (ν_{ii})
- FLR closures, $\hat{\nabla}_\perp$, $\hat{\hat{\nabla}}_\perp$

Physics of the Undamped Flow Component

Since $\mathbf{v}_E \cdot \nabla F_0 = 0$ for the zonal flows, they obey a simplified collisionless electrostatic toroidal gyrokinetic equation:

$$\frac{\partial f}{\partial t} + v_\parallel \nabla_\parallel f + i\omega_d f + i(eF_0/T)\omega_d \Phi + v_\parallel (eF_0/T)\nabla_\parallel \Phi = 0,$$

where $i\omega_d = \mathbf{v}_d \cdot \nabla = i(k_r \rho_i / v_t R)(v_\parallel^2 + v_\perp^2/2)\sin\theta$.

Rosenbluth and Hinton found a general equilibrium solution:

$$f = -(e\Phi/T)F_0 + h(E,\mu)e^{-ik_r\rho_i(qB_0 v_\parallel/\epsilon B v_t)},$$

where $h(E,\mu)$ is arbitrary but satisfies $\partial h/\partial l = 0$. The $v_\parallel$ in the exponential keeps f non-Maxwellian.

In this equilibrium, parallel variations in f balance the velocity dependent cross field drifts.

Expanding for small banana width $k_r \rho_i q/\epsilon \ll 1$:

$$f = -(e\Phi/T)F_0 + h(E,\mu)[1 - ik_r\rho_i \frac{qB_0 v_\parallel}{\epsilon B v_t}],$$

we see that moments of f will be supported by radial gradients of higher moments, e.g. $u_\parallel$ is driven by $k_r p_\parallel$, analogous to Pfirsch-Schlüter flow:

$$n_0 u_\parallel = \int d^3v\, v_\parallel f = -ik_r\rho_i \frac{qB_0}{\epsilon B v_t} \int d^3v\, v_\parallel^2 h(E,\mu)$$

New Closures for Zonal Flows Which Retain Residual Component

If we choose $h(E,\mu)$ to be a perturbed Maxwellian with no n perturbation:

$$f = -(e\Phi/T)F_0 + F_0\left(\frac{mv^2}{2T_0} - \frac{3}{2}\right)\frac{\delta T}{T_0}\left[1 - ik_r\rho_i\frac{qB_0 v_\parallel}{\epsilon B v_t}\right],$$

we can integrate this and find equilibrium $q_\parallel$ and $q_\perp$ moments:

$$q_\parallel^{(0)} = 3ik_r\rho_i\frac{qB_0}{\epsilon B}\delta T \quad \text{and} \quad q_\perp^{(0)} = ik_r\rho_i\frac{qB_0}{\epsilon B}\delta T.$$

Generalizing to non-isotropic h leads to:

$$q_\parallel^{(0)} = 3ik_r\rho_i\frac{qB_0}{\epsilon B}T_\parallel \quad \text{and} \quad q_\perp^{(0)} = ik_r\rho_i\frac{qB_0}{\epsilon B}T_\perp.$$

Our old parallel closures damped $q_\parallel$ and $q_\perp$ to zero, but now we replace:

$$\begin{array}{ll} \sqrt{2}D_\parallel|k_\parallel|q_\parallel \rightarrow \sqrt{2}D_\parallel|k_\parallel|(q_\parallel - q_\parallel^{(0)}) & \text{in the q}_\parallel\text{ eqn} \\ \sqrt{2}D_\perp|k_\parallel|q_\perp \rightarrow \sqrt{2}D_\perp|k_\parallel|(q_\perp - q_\perp^{(0)}) & \text{in the q}_\perp\text{ eqn} \end{array}$$

We also have to modify the toroidal closures in the $p_\parallel$ and $p_\perp$ eqns to support this equilibrium. We have not found a completely satisfactory way to do this. Two possibilities are:

closure (a): $\nu_1 = \nu_2 = \nu_3 = \nu_4 = 0$, or

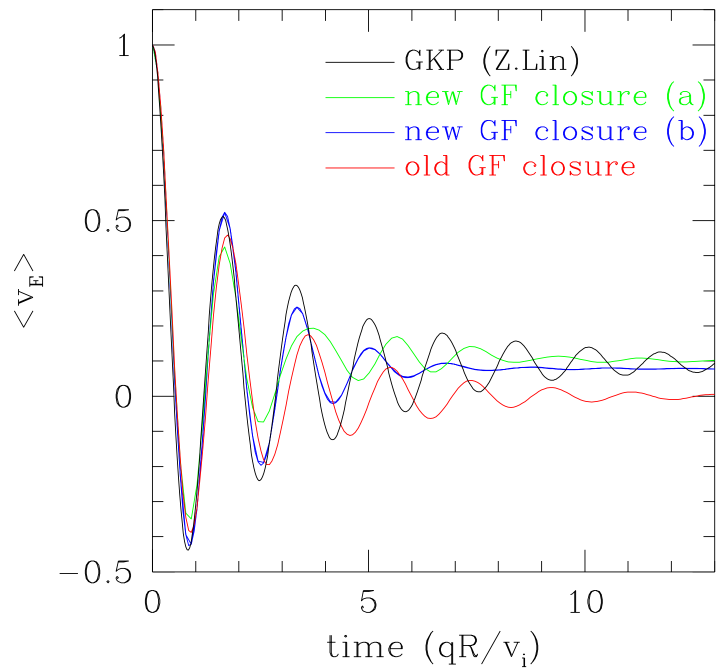
closure (b): $\nu_1 = (0,-3)$, $\nu_2 = (0,1)$, $\nu_3 = (0,0)$, $\nu_4 = (0,-3/2)$, and $q_\parallel^{(0)} = q_\perp^{(0)} = 0$, which makes less physical sense but doesn't do too poorly.

Both with ν_5-ν_{10}=0.

Because our flux-tube code is spectral, we can modify these evolution eqns for the zonal flows without changing the $k_\theta \neq 0$ components.

Comparison of Gyrokinetic and Gyrofluid Flow Damping With New Closures

New closures agree reasonably well with gyrokinetic results on amplitude of residual component for $k_r \rho_i = 0.2$:



This is for DIII-D 81499 parameters, $\epsilon = .18$, $q = 1.4$.

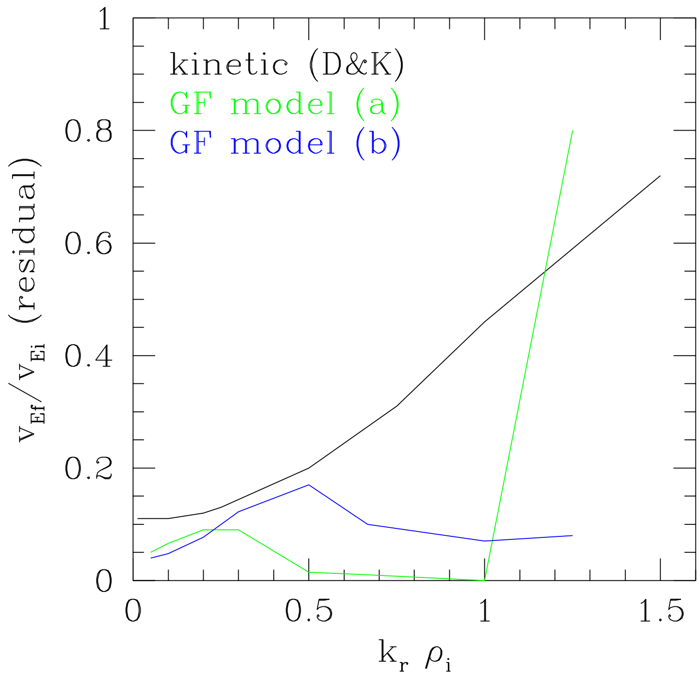
Reasonable agreement with Rosenbluth-Hinton formula:

$$\frac{v_{Ef}}{v_{Ei}} = \frac{c\sqrt{\epsilon}/q^2}{1 + c\sqrt{\epsilon}/q^2}$$

where $c = 0.625$, which predicts $v_{Ef}/v_{Ei} = 0.12$.

Comparison of Gyrokinetic and Gyrofluid Residual Component vs. k_r

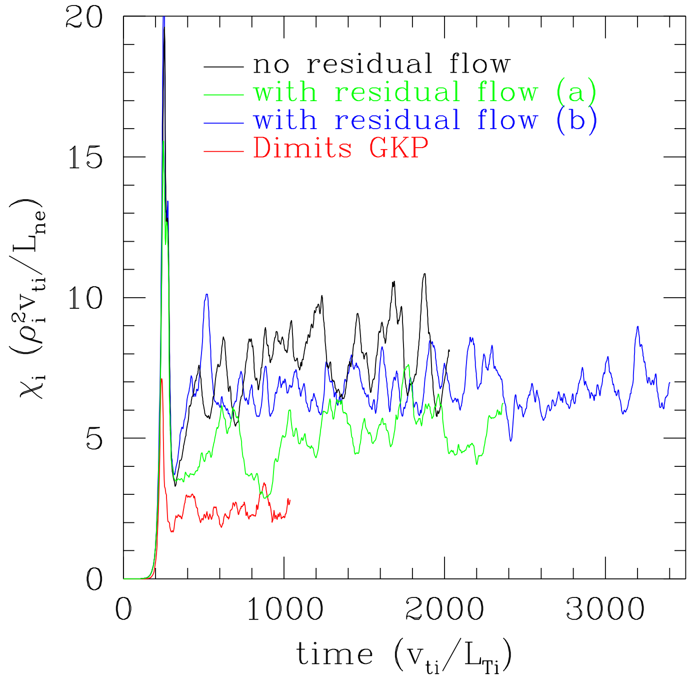
New closures don't do so well for other k_r's:



Closure (a) does slightly better at low k_r, which seems to be more important.

Nonlinear Tests of Importance of Residual Flow

For parameters from DIII-D shot 81499 (the Cyclone base case, with $R/L_{Ti} = 6.9$), we repeat nonlinear runs with the new closures (a) and (b), both including undamped components of the zonal flow.

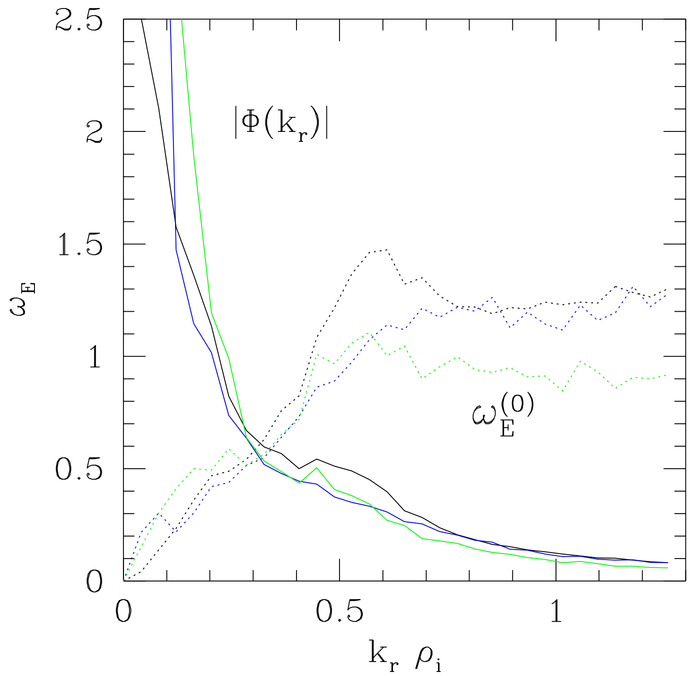


With residual flows, flux drops by up to about 35%, for this case

Nonlinear effects (*e.g.* turbulent viscosity) keep linearly undamped residual components from growing indefinitely

Time Averaged Zonal Flow Spectra with New Closures

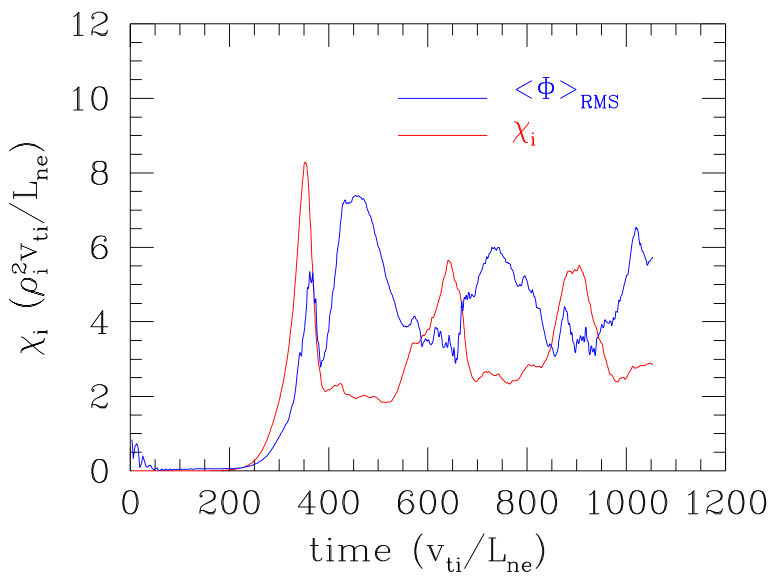
Low k_r zonal flows are larger with new closures



Since low k_r residual component is too small for our new closures, might expect more of an effect as we improve model further

Zonal Flows Can Cause Bursting Near Marginal Stability

Nearer marginal stability ($R/L_{Ti} = 4$), with the old closures we find intermittent behavior:

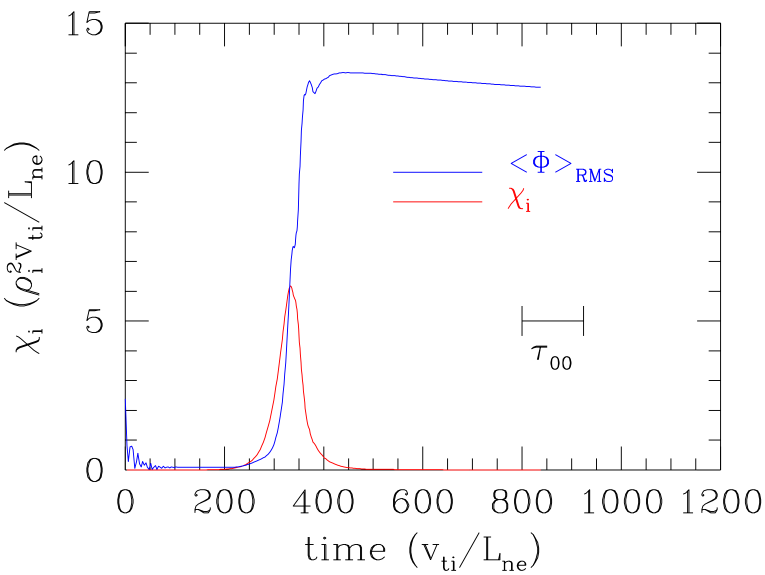


Turbulence (χ_i) drives zonal flows ($\langle\Phi\rangle_{RMS}$) which then damp turbulence. Flows then slowly damp and turbulence grows again.

Bursting is on 1ms time-scale, similar to Mazzucato's fluctuation measurements in RS, which are likely near marginal stability

Zero-flux State Near Marginal Stability With Undamped Residual Flow Component

Repeating this marginal stability case ($R/L_{Ti} = 4$) with the new closures, we find that the turbulence drives one burst of flow which is now undamped. Leads to nonlinear upshift in critical gradient (DIMITS, SHERWOOD 1998)



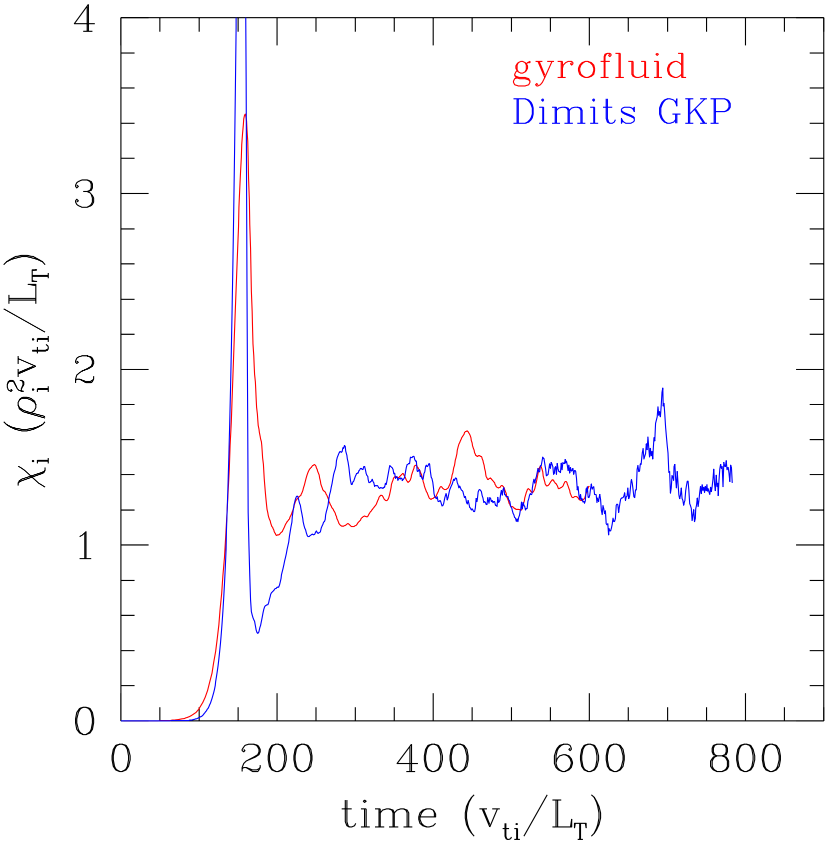
This is in the collisionless limit. A realistic amount of collisions would damp the zonal flows on a time scale $\tau_{00} = \epsilon/1.5\nu_{ii}$ (ROSENBLUTH, APS 1997) and would likely lead to bursty behavior or a turbulent steady state

Possibly an artifact of initial conditions. We could initialize arbitrarily large flow and get zero flux for any R/L_{Ti}

Gyrofluid/Gyrokinetic Flux-tube Simulation Comparisons: NTP test case with $\hat{s} = 0$

For the NTP test case parameters with $\hat{s} = 0$, GF and GKP agree

Parameters taken from TFTR L-mode:
$\hat{s} = 1.5$, $q = 2.4$, $\eta_i = 4$, $\epsilon_n = 0.4$, $\epsilon = 0.2$, $T_i = T_e$



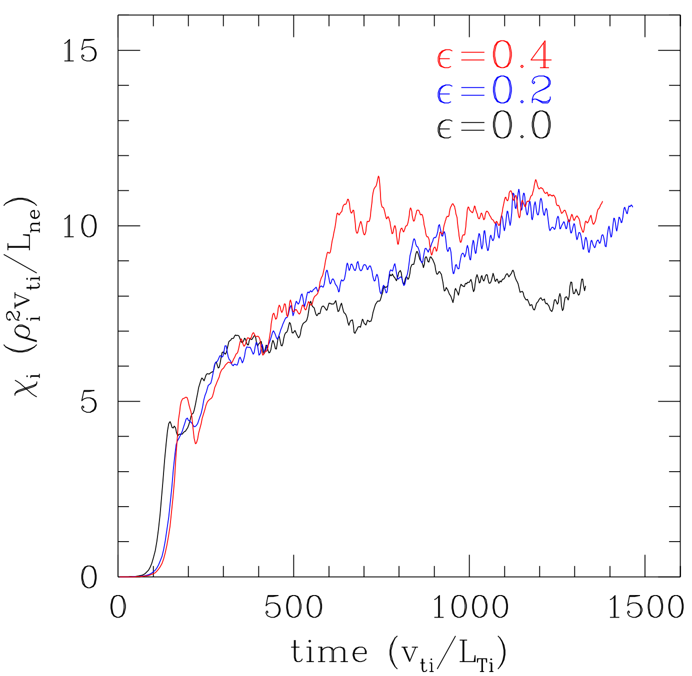
Gyrofluid/Gyrokinetic Comparisons:
ϵ scan to test Residual Flow Effects

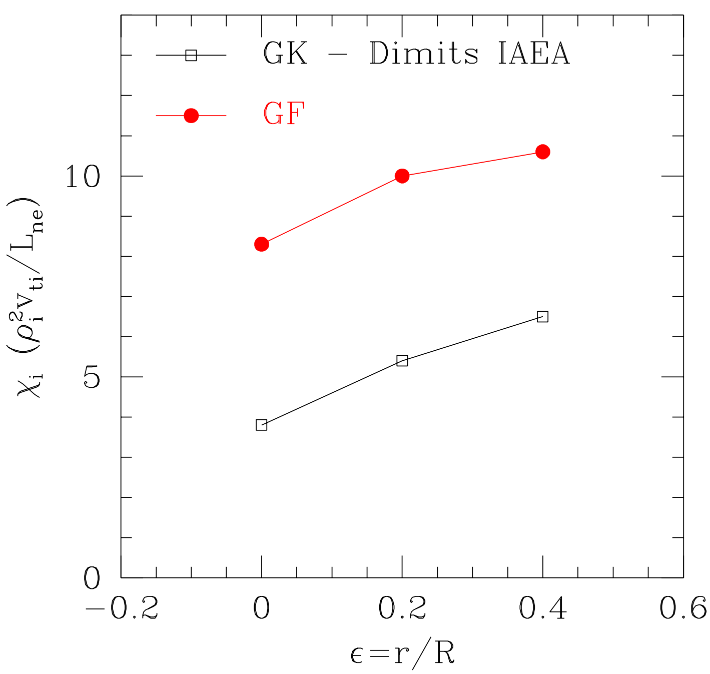
Amount of residual flow after an initial flow perturbation has damped away is controlled by $\epsilon = r/R$, as given by Rosenbluth & Hinton and verified by Dimits ($c = 0.625$):

$$\frac{\mathbf{v}_{Ef}}{\mathbf{v}_{Ei}} = \frac{c\sqrt{\epsilon}/q^2}{1 + c\sqrt{\epsilon}/q^2}$$

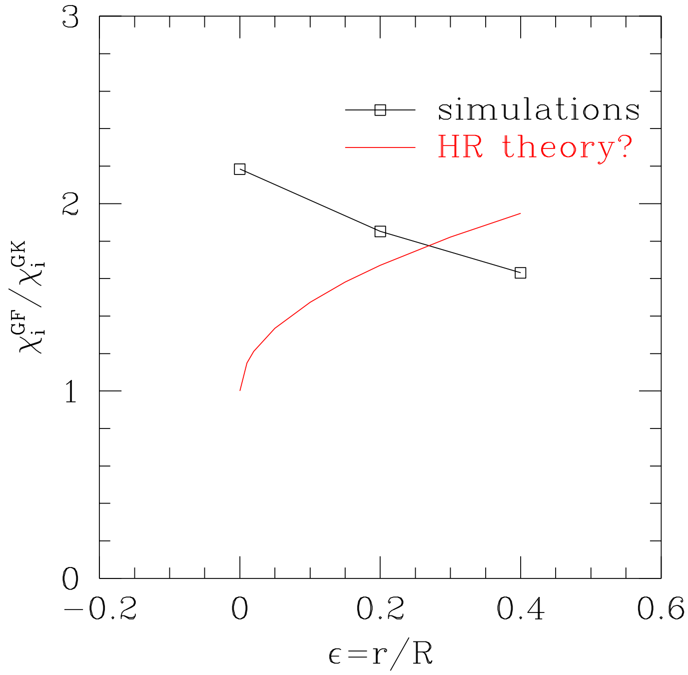
Residual flow component can be turned off by taking $\epsilon \to 0$.

Dimits reported an ϵ scan for the NTP test case parameters in his IAEA (1994) paper which we repeated with GF simulations.





Gyrofluid/Gyrokinetic Comparisons: ϵ scan to test Residual Flow Effects

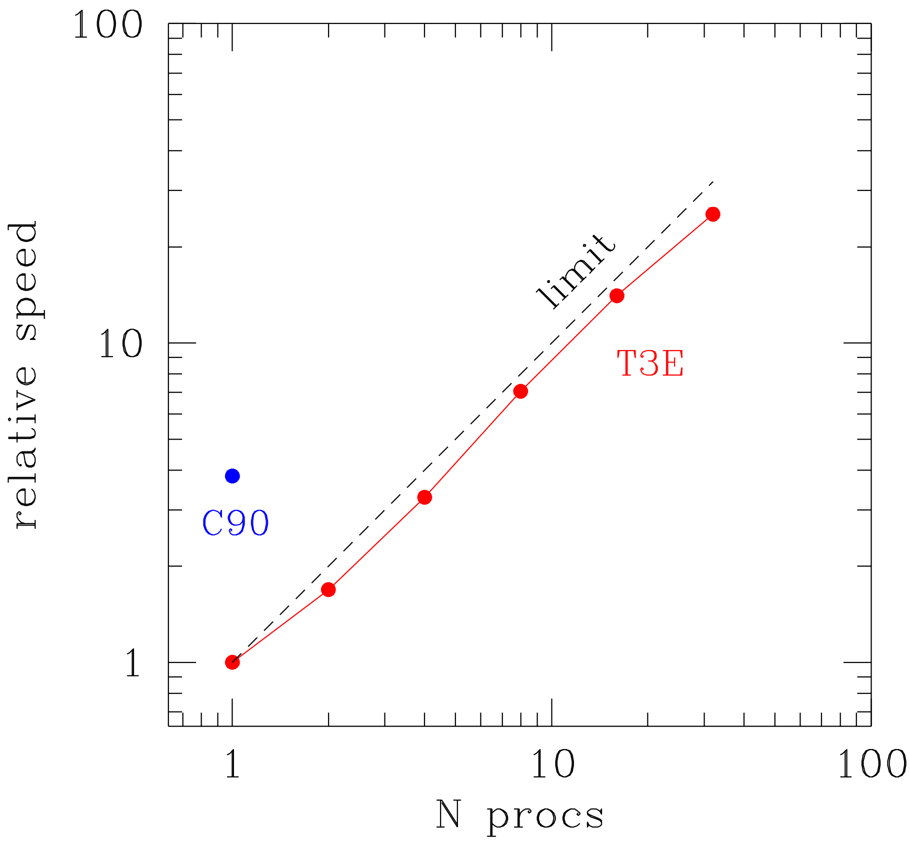


Ratio χ_{GF}/χ_{GK} does not change as residual flows are turned off.

Evidence (?) that residual flows are not dominant source of GF vs. GKP discrepancy, and that turbulent viscosity is keeping residual components from growing to large amplitudes

Parallel Version of Gyrofluid Code Performs Well on T3E

Thanks to Dorland and Liu, and NTTP for T3E time.



Conclusions

- New gyrofluid closures derived which retain linearly undamped residual zonal flow components
- Agreement at low k_r is not great, further closure modifications being investigated
- Nonlinear comparisons show that including residual component has up to 35% effect for Cyclone DIII-D base case
- Might expect larger effect as low k_r behavior is improved
- If undamped flow effect is important, a small amount of collisions may increase χ_i, but χ_i may depend weakly on ν_{ii}.
- Near marginal stability system can bifurcate into all flow, zero flux state
  - collisional flow damping will be important here
  - intermittent or bursty behavior seen with some flow damping
- In strong turbulence regimes nonlinear effects appear to saturate residual flow component, (turbulent viscosity keeps residual components from growing indefinitely)
- Nonlinear GyroKinetic Particle (GKP) vs. GyroFluid (GF) comparisons:
  - GF/GKP discrepancy is typically 2-3.
  - Differences in linear zonal flow dynamics may account for some of the GF/GKP discrepancy, especially near marginal stability
  - Adding additional physics (e.g. TE's, collisions) may move system farther from marginal stability and improve agreement
- Future work
  - Investigate collisionality and IC dependence of flux near marginal stability
  - Perhaps move to more flexible frequency dependent closures (Mattor)