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Bounce averaged trapped electron fluid equations for plasma turbulence

M. A. Beer and G. W. Hammett

Princeton University Plasma Physics Laboratory, Princeton, NJ 08543

Abstract

A novel set of nonlinear fluid equations for mirror-trapped electrons is developed which differs from conventional fluid equations in two main respects: 1) the trapped-fluid moments average over only two of three velocity space dimensions, retaining the full pitch angle dependence of the trapped electron dynamics, and 2) closure approximations include the effects of collisionless wave-particle resonances with the toroidal precession drift. Collisional pitch angle scattering is also included. By speeding up calculations by at least $\sqrt{m_i/m_e}$, these bounce averaged fluid equations make possible realistic nonlinear simulations of turbulent particle transport and electron heat transport in tokamaks and other magnetically confined plasmas.

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I. Introduction

Mirror-trapped particles often play an important role in long mean-free-path plasma dynamics, especially in magnetic confinement fusion devices and planetary magnetospheres. This paper presents a reduced nonlinear fluid-like description for mirror-trapped particles. These equations should be useful for describing nonlinear trapped particle dynamics in a wide range of plasma phenomena, but we will focus on tokamaks, where trapped electrons can be an important cause of turbulent transport.[1] Through wave-particle resonances, trapped electrons can destabilize the dissipative or collisionless trapped electron mode (TEM), and, as shown below, can double the growth rate of the ion temperature gradient (ITG) mode in some regimes. We include these kinetic resonances by using an extension of the method of Ref. 2 to take fluid moments of the bounce averaged drift kinetic equation of Ref. 3.

Although much progress has been made recently in nonlinear simulations of electrostatic core tokamak turbulence arising from the ITG instability, more realistic simulations require proper treatment of the trapped electron dynamics. To date, most simulations have focused on ion heat transport and have assumed adiabatic electrons, i.e. $\tilde{n}_e = n_0 e\tilde{\Phi}/T_e$, where $\tilde{n}_e$ is the fluctuating electron density and $\tilde{\Phi}$ is the fluctuating electrostatic potential. For realistic tokamak parameters, however, the *non*adiabatic electron response, which primarily comes from trapped electrons, is often important. To describe electron heat transport and particle transport in addition to ion heat transport, proper treatment of the nonadiabatic electron response is necessary. If the turbulence is electrostatic and the electrons are purely adiabatic, there is no net particle transport, since the $\mathbf{E} \times \mathbf{B}$ convection of the perturbed electron density is zero ($\mathbf{E} \times \mathbf{B} \cdot \nabla \tilde{n}_e = 0$).

In this paper, a sophisticated bounce averaged trapped electron fluid model is derived which retains the pitch angle dependence of the electron response, as opposed to more simplified models which assume all electrons are deeply trapped.[1] Retaining this pitch angle dependence is important for advanced tokamak configurations in the second stability regime or with reversed magnetic shear,[4] where a large fraction of the trapped electrons have favorable toroidal precession drift. This approach also allows use of a full pitch angle scattering operator for electron collisions, not a Krook-type algebraic approximation,[5] so these equations are continuously valid in the collisionless regime, where the electron response is driven by the toroidal precession resonance, in the dissipative regime, and also in the very collisional regime where the electrons become adiabatic. Since bounce averaging removes the fast parallel time scale, these trapped electron fluid equations are not numerically stiff. Coupled with the gyrofluid ion equations derived in Refs. 6–8, these equations can be used efficiently in high resolution three-dimensional (3D) toroidal simulations which simultaneously include trapped electron effects as well as the ITG drive. In addition, these equations enable calculation of the full transport matrix: electron and ion heat fluxes and particle fluxes.

II. Nonlinear Bounce Averaged Kinetic Equation

The electron dynamics are actually simpler than the ion dynamics in two respects, because $m_e \ll m_i$. First, the turbulent scales are on the order of the ion gyroradius, so $k_\perp \rho_e \ll 1$ and we can neglect finite Larmor radius effects for the electrons and use the drift kinetic equation. Second, the turbulent time scales (on the order of the ion transit frequency, $\omega_{ti} = v_{ti}/qR$, or the diamagnetic frequency, $\omega_* = k_\perp \rho_i v_{ti}/L_{ne}$) are long compared to the electron bounce frequency, $\omega \ll \omega_{be} = \sqrt{\epsilon} v_{te}/qR$. Thus we can average over the fast bounce motion so that the trapped electron dynamics are described by the nonlinear bounce averaged drift kinetic equation.[3] It is useful to rewrite this equation for $\langle f_e \rangle_b$, the bounce averaged distribution function, instead of the nonadiabatic piece h_e as in Ref. 3; the two are related by $f_e = F_e e\Phi/T_e + h_e$, where F_e is the Maxwellian equilibrium. At this point we normalize Φ to e/T_e. In addition, we use the field-aligned coordinate system given by the transformation Eq. (10) in Ref. 9, where x is the radial variable, y is perpendicular and mostly poloidal, and $z = qR\theta$ is the coordinate along the field line at fixed x and y. Ref. 8 gives details of the simplification of Eq. (31) of Ref. 3, which can be rewritten:

$$(\frac{d}{dt} + i\omega_{de})\langle f_e\rangle_b = \langle C\rangle_b(\langle f_e\rangle_b - F_e\langle\Phi\rangle_b) + iF_e(\omega_{de} - \omega_{*e}^T)\langle\Phi\rangle_b. \quad (1)$$

This equation is four dimensional (4D) (two velocity and two space dimensions), since the variation along the field line has been removed by bounce averaging and the rapid particle gyration frequency, $\omega_{ce} = eB/m_e c$, has been averaged over. Eq. (1) employs the usual two-scale expansion, where the fluctuation scales are much smaller than the equilibrium scales. The bounce average is defined by $\langle A\rangle_b = (\oint dz\, A/|v_{||}|)/(\oint dz/|v_{||}|)$, where the integration is along an orbit. To lowest order in ω/ω_{be}, the fast electron parallel motion causes h_e to be constant along a field line, which prescribes $f_e = \langle f_e\rangle_b - F_e\langle\Phi\rangle_b + F_e\Phi$ for trapped electrons and $f_e = F_e\Phi$ for passing electrons. The nonlinear term describing convection by the bounce averaged $\mathbf{E}\times\mathbf{B}$ drift has been absorbed in $d/dt = \partial/\partial t + \hat{\mathbf{b}}\times\langle\Phi\rangle_b\cdot\nabla$. The collision term is discussed below. The diamagnetic frequency is $\omega_{*e}^T = (k_y cT_e/eBL_{ne})[1 + \eta_e(v^2/2v_{te}^2 - 3/2)]$, where $\eta_e = L_{ne}/L_{Te}$, and the toroidal precession frequency ω_{de} is the bounce averaged ∇B and curvature drift frequency. Our derivation is correct for general magnetic geometry, but by expanding for large-aspect-ratio circular flux surfaces, the bounce average can be written in terms of elliptic integrals.[3] We combine the geometric and pitch angle dependence in the usual manner in G: $\omega_{de} = (k_y cT_e/eBR)(v^2/2v_{te}^2)G(\hat{s},\kappa)$. It is important to keep the pitch angle dependence of ω_{de} to describe the stabilization of the TEM in reversed shear configurations ($\hat{s} < 0$). The limiting values at $\kappa = 0$ and $\kappa = 1$ are independent of shear, but as $\hat{s}$ decreases, the precession drifts of barely trapped particles are reversed, so they cannot resonate with the TEM. We have recently emphasized that the Shafranov shift can be even more effective in reversing these drifts and stabilizing the TEM.[10]

It will be most convenient to use the velocity space variables v and κ, where v is the total velocity ($E = mv^2/2$) and κ is a pitch angle variable defined by $\kappa^2 = (1 - \mu B_{\min}/E)/2\epsilon_B$, where $\epsilon_B = (B_{\max} - B_{\min})/2B_{\max}$, $B_{\max}$ and $B_{\min}$ are the maximum and minimum values of the magnetic field on the flux surface, and $\mu = mv_\perp^2/2B$. Thus κ is the pitch angle at the outer midplane normalized to unity at the trapped-passing boundary (where $E = \mu B_{\max}$), and is a constant of the bounce motion. For deeply trapped electrons (with $E = \mu B_{\min}$), $\kappa = 0$, and the maximum κ for passing particles (where $\mu = 0$) is $1/\sqrt{2\epsilon_B}$. For trapped particles ($\kappa < 1$), the poloidal angle of the banana tip or turning point, θ_t, is related to κ by $\kappa = \sin(\theta_t/2)$. This can be seen by using $E = \mu B_t$ where B at the turning point is $B_t = B_0/(1 + \epsilon\cos\theta_t)$. Our pitch angle variable differs slightly from Ref. 3, but for trapped particles the difference is negligible since $v \approx v_\perp$. Writing $|v_{||}|$ in terms of v and κ: $|v_{||}| = v\sqrt{1-(1-2\epsilon_B\kappa^2)B/B_{\min}}$, the bounce time is $\tau_b(\kappa) = \oint dl/|v_{||}|$, and the bounce average becomes $\langle\Phi\rangle_b(x,y,\kappa) = (qR/v\tau_b)\int_{-\theta_t}^{\theta_t} d\theta\,\Phi(x,y,\theta)/\sqrt{1-(1-2\epsilon_B\kappa^2)B/B_{\min}}$.

Before taking moments of Eq. (1), it is instructive to calculate the total electron density, which we break into separate integrals over passing and trapped particles. Since the passing particles are adiabatic: $n_e = \int_p d^3v\Phi F_e + \int_t d^3v f_e = \int_p d^3v\Phi F_e + \int_t d^3v(\langle f_e\rangle_b - \langle\Phi\rangle_b F_e + \Phi F_e)$. Combining the adiabatic pieces for trapped and passing particles gives: $n_e = n_0\Phi + \int_t d^3v\,(\langle f_e\rangle_b - \langle\Phi\rangle_b F_e)$. The velocity space integral over trapped particles in v and κ variables is:

$$\int_t d^3v\langle f_e\rangle_b = \int_0^\infty 4\pi dv\,v^2\int_{\sin(\theta/2)}^1 2B\epsilon_B\kappa\langle f_e\rangle_b d\kappa/\sqrt{B_{\min}^2 - BB_{\min}(1-2\epsilon_B\kappa^2)}.$$

We introduce the following shorthand notation for the pitch angle part of this integration:

$$\langle A(\kappa)\rangle_\kappa = \int_{\sin(\theta/2)}^1 \frac{2B\epsilon_B\kappa A(\kappa)d\kappa}{\sqrt{B_{\min}^2 - BB_{\min}(1-2\epsilon_B\kappa^2)}}. \tag{2}$$

Averaging in pitch angle turns functions of κ into functions of θ, because of the θ dependence of the Jacobian and the turning points. The electron density in real space is just the κ average of the v-averaged $\langle f_e\rangle_b$. Defining a κ-dependent "density" by integrating only over v: $n_t(x,y,\kappa) = \int_0^\infty 4\pi dv\,v^2\langle f_e\rangle_b$, the total density in real space is:

$$n_e(x,y,z) = n_0\Phi + \langle n_t(x,y,\kappa)\rangle_\kappa - n_0\langle\langle\Phi\rangle_b(x,y,\kappa)\rangle_\kappa. \tag{3}$$

The κ average of $\langle\Phi\rangle_b$ in Eq. (3) is analogous to the polarization density in the ion real space density, and comes from the z-dependent part of the total electron distribution function.

III. Bounce Averaged Fluid Equations

The separable v and κ dependence of Eq. (1) and the pitch angle dependence of $\langle\Phi\rangle_b$ suggest a significantly different approach for deriving trapped electron fluid equations. Both the gyrokinetic and drift kinetic equations have already reduced the velocity space dimensions from three to two by gyroaveraging. For the ions, we take moments over $v_{||}$ and $v_\perp$ of the five-dimensional (5D) $f_i(x, y, z, v_{||}, v_\perp)$ to obtain 3D ion fluid equations.[8] For the electrons, we start with the 5D $f_e(x, y, z, v, \kappa)$ and bounce average, which removes the parallel coordinate. Then we only need to take moments over v of $\langle f_e\rangle_b(x, y, v, \kappa)$ to obtain 3D pitch angle dependent "fluid" equations for the electrons, which are functions of x, y, and κ. These moments can be thought of as the electron density, pressure, etc., of banana tips, since κ is directly related to the turning point by $\kappa = \sin(\theta_t/2)$. The resulting trapped electron fluid equations look similar to the 3D ion fluid equations derived in Ref. 8, with the parallel coordinate replaced by the pitch angle variable, κ. This has the advantage of retaining the full pitch angle dependence of the electron moments, the toroidal precession frequency ω_{de}, and the bounce averaged potential. When the real space electron density or pressure is needed, we perform the κ average in Eq. (2).

We derive trapped electron fluid equations by averaging Eq. (1) over v. Since only even powers of v appear in Eq. (1), we will only need even moments: $n_t(x, y, \kappa) = \frac{4\pi}{n_0}\int_0^\infty dv\, v^2\langle f_e\rangle_b$, $p_t(x, y, \kappa) = \frac{4\pi}{3n_0 v_{te}^2}\int_0^\infty dv\, v^4\langle f_e\rangle_b$, $r_t(x, y, \kappa) = \frac{4\pi}{15 n_0 v_{te}^4}\int_0^\infty dv\, v^6\langle f_e\rangle_b$, $t_t(x, y, \kappa) = \frac{4\pi}{105 n_0 v_{te}^6}\int_0^\infty dv\, v^8\langle f_e\rangle_b$, and $v_t(x, y, \kappa) = \frac{4\pi}{945 n_0 v_{te}^8}\int_0^\infty dv\, v^{10}\langle f_e\rangle_b$, which have been normalized to their Maxwellian values. The v^2 dependence of ω_{de} brings the next higher even moment into each dynamical equation, introducing the usual closure problem of the coupled moments hierarchy. Performing the v integration and redefining $\omega_{de} = Gk_\theta cT_e/eBR$ and $\omega_{*e} = k_\theta cT_e/eBL_{ne}$, we have:

$$\frac{dn_t}{dt} + \frac{3}{2}i\omega_{de}(p_t - \langle\Phi\rangle_b) + i\omega_{*e}\langle\Phi\rangle_b = \langle C\rangle_b(n_t - \langle\Phi\rangle_b), \tag{4}$$

$$\frac{dp_t}{dt} + \frac{5}{2}i\omega_{de}(r_t - \langle\Phi\rangle_b) + i(1+\eta_e)\omega_{*e}\langle\Phi\rangle_b = \langle C\rangle_b(p_t - \langle\Phi\rangle_b),$$

$$\frac{dr_t}{dt} + \frac{7}{2} i\omega_{de}(t_t - \langle\Phi\rangle_b) + i(1+2\eta_e)\omega_{*e}\langle\Phi\rangle_b = \langle C\rangle_b(r_t - \langle\Phi\rangle_b),$$
$$\frac{dt_t}{dt} + \frac{9}{2} i\omega_{de}(v_t - \langle\Phi\rangle_b) + i(1+3\eta_e)\omega_{*e}\langle\Phi\rangle_b = \langle C\rangle_b(t_t - \langle\Phi\rangle_b).$$

We require a closure approximation for the highest moment to model toroidal precession drift phase mixing, using an extension of the method of Ref. 2. For a 3-moment electron model (evolving n_t, p_t, and r_t) we choose: $t_t = -i\frac{|\omega_{de}|}{\omega_{de}}(\nu_a n_t + \nu_b p_t + \nu_c r_t)$, and in the 4-moment model (also evolving t_t), we choose: $v_t = -i\frac{|\omega_{de}|}{\omega_{de}}(\nu_a n_t + \nu_b p_t + \nu_c r_t + \nu_d t_t)$. As in Ref. 8, each closure coefficient has both a dissipative and non-dissipative piece, $\nu = \nu_r + i\nu_i|\omega_{de}|/\omega_{de}$, but now ω_{de} is pitch angle dependent. We choose these closure coefficients to closely match the collisionless bounce averaged kinetic response function, given by: $R_e = n_t(\kappa)/\langle\Phi\rangle_b(\kappa) = (4\pi/n_0)\int dv\, v^2 F_e(-\omega_{de} + \omega_{*e}^T)/(\omega - \omega_{de})$. This can be factored into the form: $R_e = R_{e0} + \frac{\omega_{*e}}{\omega_{de}} R_{e1} + \frac{\omega_{*e}\eta_e}{\omega_{de}} R_{e2}$. These integrals[11] become functions of $x_e = \omega/\omega_{de}$ and κ (through $\omega_{de}(\kappa)$): $R_{e0} = 1 + 2x_e - 2x_e^{3/2} Z(-\sqrt{x_e})$, $R_{e1} = -2[1 - \sqrt{x_e} Z(-\sqrt{x_e})]$, $R_{e2} = -[1 + 2x_e - 2x_e^{3/2} Z(-\sqrt{x_e})] + 3[1 - \sqrt{x_e} Z(-\sqrt{x_e})]$, where Z is the plasma dispersion function. The corresponding response functions from the 3-moment electron fluid equations are ($\sigma = \omega_{de}/|\omega_{de}|$):

$$R_{e0} = \frac{-12x_e^2 + 42i\sigma\nu_c x_e - 30x_e + 105(i\sigma\nu_b + i\sigma\nu_c - 1)}{8x_e^3 - 28i\sigma\nu_c x_e^2 - 70i\sigma\nu_b x_e - 105i\sigma\nu_a},$$
$$R_{e1} = \frac{8x_e^2 - 28i\sigma\nu_c x_e - 70i\sigma\nu_b + 12x_e - 42i\sigma\nu_c + 30}{8x_e^3 - 28i\sigma\nu_c x_e^2 - 70i\sigma\nu_b x_e - 105i\sigma\nu_a},$$
$$R_{e2} = \frac{12x_e - 42i\sigma\nu_c + 60}{8x_e^3 - 28i\sigma\nu_c x_e^2 - 70i\sigma\nu_b x_e - 105i\sigma\nu_a},$$

with similar expressions for four moments.[8] We use Powell's method[12] to find the closure coefficients by minimizing the error between the fluid and kinetic response functions, R_{e0}, R_{e1}, and R_{e2}, along the real x_e axis. The best fits are $\nu_a = (.290, -.071)$, $\nu_b = (-1.102, -.689)$, and $\nu_c = (.817, 1.774)$ for the 3-moment model, and $\nu_a = (-.038, .073)$, $\nu_b = (.657, -.060)$, $\nu_c = (-1.522, -1.085)$, and $\nu_d = (.905, 2.073)$ for the 4-moment model. The response function for the 3 and 4-moment models are shown in Fig. 1. Very good agreement is obtained for both models.

We now derive collision terms from the Lorentz collision operator: $C = (\nu_e(v)/2)\partial/\partial\xi[(1-\xi^2)\partial f_e/\partial\xi]$, where $\xi = v_{||}/v$. The energy dependent collision frequency is: $\nu_e(v) = (4\pi n_e e^4 \ln\Lambda/m_e^2 v^3)$ $(Z_{\text{eff}} + H_{ee}(v/v_{te}))$, where the Z_{eff} part accounts for electron-ion collisions (assuming $v \gg v_{ti}$) summed over ion species ($Z_{\text{eff}} = \sum_j Z_j^2 n_j/n_e$), and the $H_{ee}(x)$ part is from electron-electron collisions, where $H_{ee}(x) = \sqrt{2/\pi}\exp(-x^2/2)/x + [1 - 1/(x^2)]\text{erf}(x/\sqrt{2})$. This collision operator conserves particles and energy, but not momentum. The bounce average of this collision operator[13] enters Eq. (1), and in our variables, is:

$$\langle C\rangle_b = \frac{\nu_e}{8\epsilon_B^2|\kappa|\tau_b}\frac{\partial}{\partial\kappa}\left[(1-2\epsilon_B\kappa^2)\frac{\tau_b}{|\kappa|}\left\{\left\langle\frac{B_{\min}}{B}\right\rangle_b - 1 + 2\epsilon_B\kappa^2\right\}\frac{\partial}{\partial\kappa}\left(\langle f_e\rangle_b - F_e\langle\Phi\rangle_b\right)\right], \qquad (5)$$

To make f_e continuous at the trapped-passing boundary, we apply the boundary condition $\langle f_e\rangle_b = F_e\langle\Phi\rangle_b$ at $\kappa = 1$. The use of a full differential operator in κ for C instead of an algebraic Krook model[5] automatically incorporates the increasing importance of pitch angle scattering near the trapped-passing boundary. Thus, barely trapped electrons will scatter more quickly into the passing region than deeply trapped electrons. Once electrons are scattered to passing, they are free to move rapidly along the field line and phase mix, relaxing to an adiabatic response. Because we assume that passing electrons become adiabatic instantaneously (since $\omega_{be} \sim k_{||}v_{te}$ is large), there is some boundary layer near the trapped-passing boundary which we do not resolve. The width of this boundary layer is perhaps of $\mathcal{O}(\sqrt{\omega\nu_{\text{eff}}}/\omega_{be})$, and so is usually negligible for the moderate collisionalities we consider. Note that the bounce averaging procedure of Ref. 3, which we follow, uses the ordering $\omega \sim \omega_{*e} \sim \omega_{de} \sim \nu_{\text{eff}} \ll \omega_{be}$, and so can continuously handle the transition from what is usually called the "collisionless trapped electron" regime ($\nu_{\text{eff}} < \omega$, where the trapped electrons give a significant nonadiabatic response due to precession resonances) to what is usually called the "dissipative trapped electron" regime ($\omega < \nu_{\text{eff}} < \omega_{be}$, where collisions wipe out most of the trapped electrons and the nonadiabatic electron response $h_e \sim \mathcal{O}(\omega/\nu_{\text{eff}})$ is becoming small). This has been confirmed in comparisons of our calculations with fully kinetic calculations which do not bounce average,[14] which we will report in the future. However, the present ordering cannot handle very large collision frequencies, $\nu_{\text{eff}} \gg \omega_{be} \sim k_{\|} v_{te}$, where collisional drag on passing electrons begins to impede their parallel flow. This is the dominant drive for the classic resistive drift wave instability. For $\nu_{\text{eff}} \sim \omega_{be}$, the nonadiabatic response due to drag on passing electrons scales as $\nu_{\text{eff}}\omega/\omega_{be}^2$, and is still relatively small because $\omega \ll \omega_{be}$. Thus we believe that the present equations are appropriate for moderate to low collisionalities typical of most tokamaks, but they would need extension to include collisional drag on passing electrons, which may become important very close to the edge of some tokamaks.

The collision operator in Eq. (5) must be integrated over v to find the collision terms in the trapped electron fluid equations. The velocity dependence of ν_e should introduce coupling between different fluid moment equations, just as the velocity dependence of ω_{de} did. However, for the time being we will assume $\nu_e =$ constant when integrating over v, which leads to the simple form of the collision terms in Eqs. (4). A better approximation will be described in future work, which leads to weaker collision terms in the higher moment equations to model the $\nu_e \sim 1/v^3$ dependence.

We now describe how these electron moment equations are solved. The emphasis is on numerical solution, but analytic solution follows conceptually similar procedures. In our numerical simulations,[8,9,15] the ion gyrofluid moments are stored and evolved in (x, y, z) space. The electron moments are stored and evolved in (x, y, κ) space, and separate electron moments are independently evolved in each magnetic well along z. The bounce averaged $\langle\Phi\rangle_b(\kappa)$ is calculated from $\Phi(z)$ by numerically integrating along z, and is then used to advance the electron moments in time. The electron nonlinearities are evaluated pseudospectrally, as the ion nonlinearities, but in κ rather than in z. The electron collision terms are evaluated implicitly. Only the electron density needs to be evaluated in real space. To solve the gyrokinetic quasineutrality equation, the real space density, $n_e(z)$, is calculated by performing the κ averages of $n_t(\kappa)$ and $\langle\Phi\rangle_b$ as given by Eqs. (2) and (3). Then the quasineutrality equation is solved for Φ, and the cycle is repeated.

As in the adiabatic limit, special treatment is required for toroidally symmetric perturbations with $k_y = 0$, which have a component which is constant on flux surfaces. When $k_y \neq 0$, trapped electrons scattered onto passing orbits quickly become adiabatic, but this is not true if $k_y = 0$. When $k_y = 0$, $\omega_{de} = \omega_{*e} = 0$, so the bounce averaged kinetic equation reduces to $d\langle f_e\rangle_b/dt = \langle C\rangle_b(\langle f_e\rangle_b - F_e\langle\Phi\rangle_b)$. This equation applies to passing particles with $1 < \kappa < 1/\sqrt{2\epsilon_B}$ as well as trapped particles with $0 < \kappa < 1$. Thus the passing $k_y = 0$ electron moments interact only via collisions with trapped $k_y = 0$ moments, which in turn interact with trapped $k_y \neq 0$ moments only through the nonlinear term in $d/dt = \partial/\partial t + \hat{\mathbf{b}} \times \langle\Phi\rangle_b \cdot \nabla$. Conservative boundary conditions for $\langle C\rangle_b$ ensure that there is no flux across the $\kappa = 1/\sqrt{2\epsilon_B}$ boundary. The bounce average is generalized for $\kappa > 1$ to an orbit average with $\theta \to \pm\infty$ so that only the $k_y = 0$ component of Φ or f_e leads to a nonzero $\langle\Phi\rangle_b$ or $\langle f_e\rangle_b$, since Φ and f_e must vanish as $\theta \to \pm\infty$ for $k_y \neq 0$ but not for $k_y = 0$. The upper bounds on the κ integrals in Eqs. (2) and (3) are extended to $\kappa = 1/\sqrt{2\epsilon_B}$ for $k_y = 0$ modes. Note that in the final analysis there is no $k_y = 0$ electron response to a component $\bar{\Phi}$ of Φ which is constant on a flux surface, since $\langle\bar{\Phi}\rangle_b = \bar{\Phi}$ is independent of κ so $\langle C\rangle_b\langle\bar{\Phi}\rangle_b = 0$.

IV. Comparison with Kinetic Theory

To conclude, we demonstrate the accuracy of these trapped electron fluid equations by comparing fully nonlocal linear results with kinetic theory in the collisionless limit. The eigenfrequencies from the six moment toroidal gyrofluid equations[8] and the three moment trapped electron fluid equations are compared with fully kinetic calculations[14] in Fig. 2. These results are for a pure deuterium plasma with $\eta_i = \eta_e = 3$, $\hat{s} = 1$, $q = 1.5$, $L_{ne}/R = 1/3$, and $r/R = 1/6$, as in Fig. 1 of Ref. 14. The gyrofluid results with adiabatic electrons are also shown. The trapped electron response doubles the growth rates for these parameters, even though this is an ITG mode. Our trapped electron model also agrees quite well for the TEM. Initial nonlinear results using this model have been presented in Refs. 15 and 8. Quite recently, we have found that this model reproduces several interesting features of the transport in the core of supershots and Enhanced Reversed Shear discharges,[16] where the TEM dominates. These results will be reported in a future publication.

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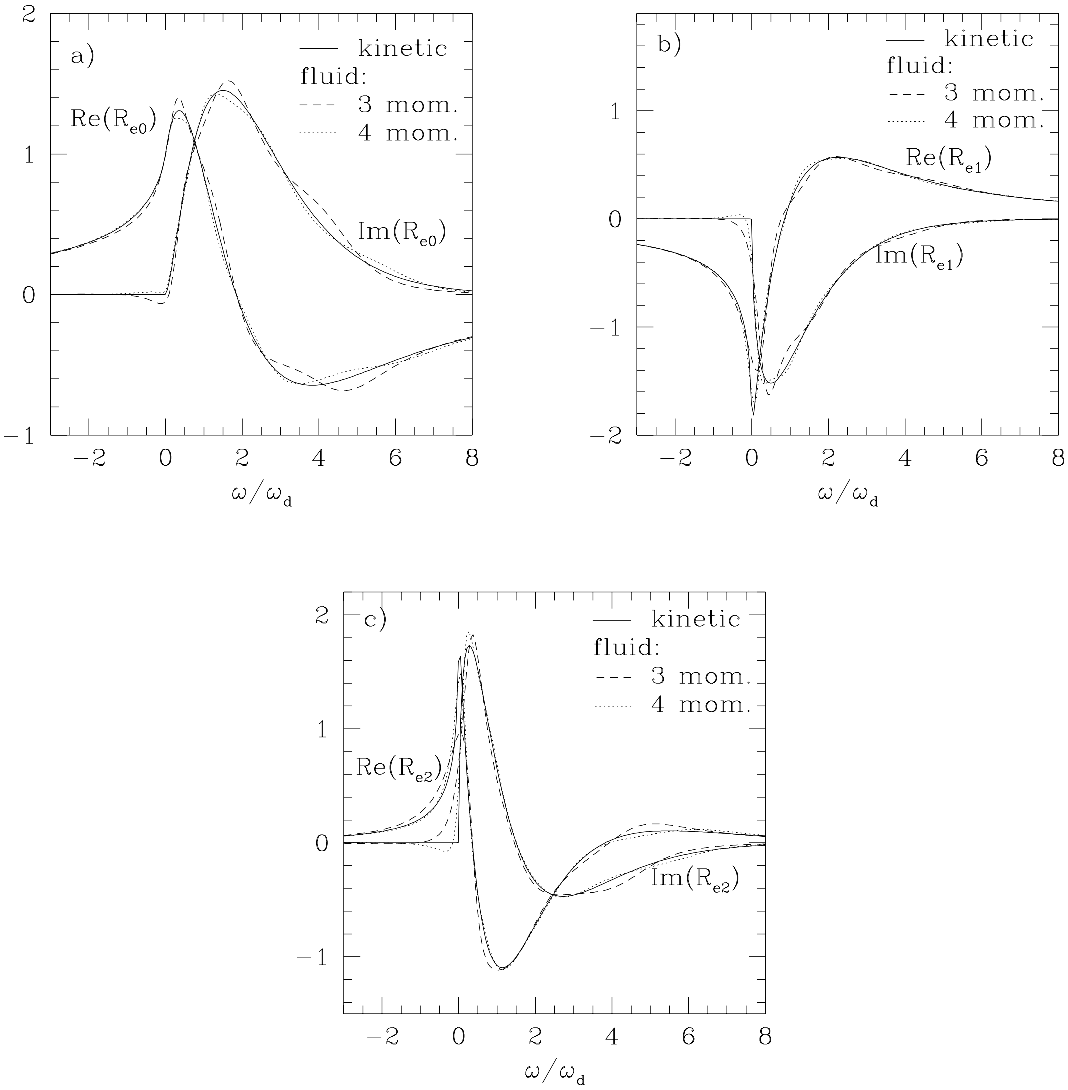
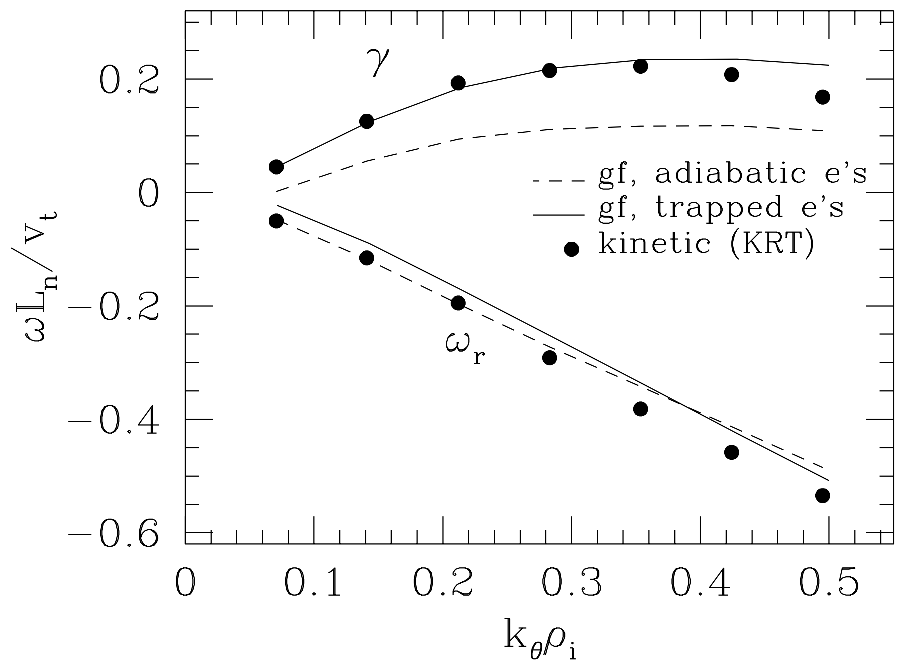

a)
kinetic
fluid:
3 mom.
4 mom.
Re(R_e0)
Im(R_e0)
ω/ω_d
b)
kinetic
fluid:
3 mom.
4 mom.
Re(R_e1)
Im(R_e1)
ω/ω_d
c)
kinetic
fluid:
3 mom.
4 mom.
Re(R_e2)
Im(R_e2)
ω/ω_d

Figure 1:



γ
ω_r
gf, adiabatic e's
gf, trapped e's
kinetic (KRT)
0.2
0
−0.2
−0.4
−0.6
0
0.1
0.2
0.3
0.4
0.5
$\omega L_n / v_t$
$k_\theta \rho_i$

Figure 2: