Equilibrium $\mathbf{E} \times \mathbf{B}$ Flows in Nonlinear Gyrofluid Flux-Tube Simulations

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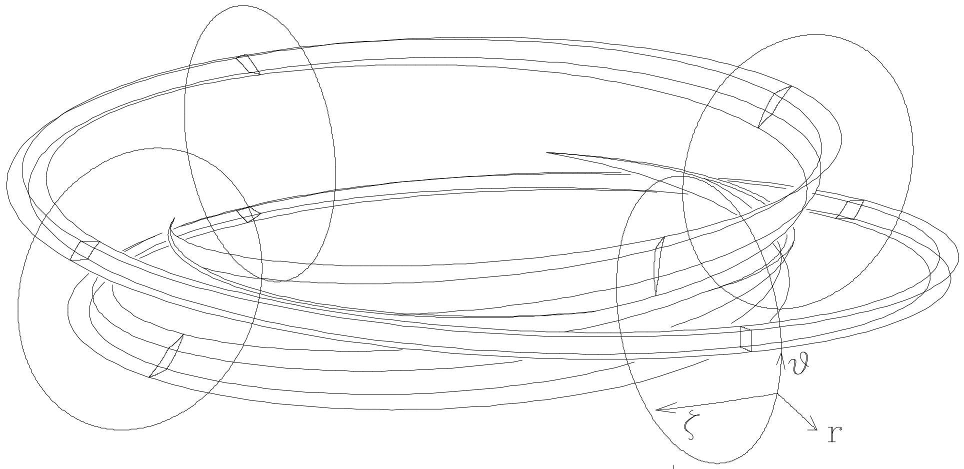
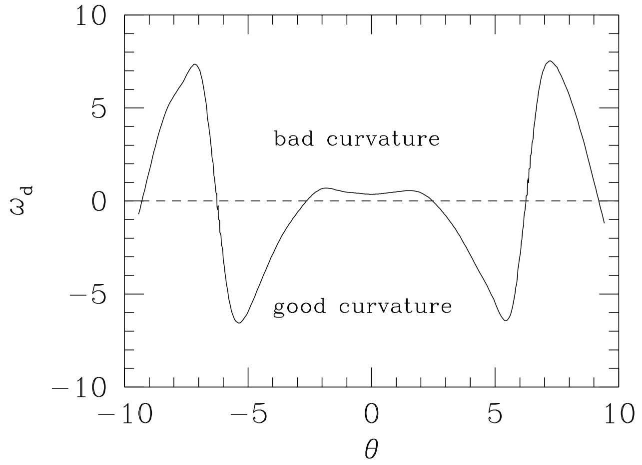
Outline

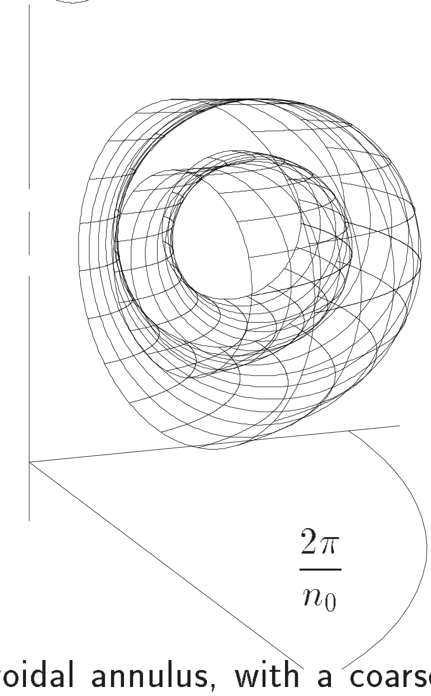
1. Equilibrium $\mathbf{E} \times \mathbf{B}$ flows in flux-tube simulations
   - Introduce a coordinate transformation which makes the flux-tube domain move with the equilibrium $\mathbf{E} \times \mathbf{B}$ flow
   - Consistent with radially periodic boundary conditions
   - Nonlinear results are consistent with conventional picture: turbulence is suppressed when the shearing rate $\Omega_E \approx \gamma_{\max}$
2. Review of results from gyrofluid closures which retain residual component
   - reasonable comparisons with linear collisionless gyrokinetic simulations for moderate k_r, not as good for lower or higher k_r
   - away from marginal stability, nonlinear effects appear to keep the residual flow component from growing indefinitely, but it can reduce χ_i by up to 40%
   - near marginal stability residual component can completely quench turbulence. Initial conditions and collisional damping of the flows can be important.

Flux-Tube Simulation Geometry

Flux tube simulation domain aligned with sheared field lines.







Equivalent to simulating a fraction $(1/n_0)$ of a toroidal annulus, with a coarse grid in toroidal mode number $n \in \{0, n_0, 2n_0, 3n_0, \ldots\}$

Variations of $\omega_d \propto \mathbf{B} \times \nabla B$ along field line contain effects of good and bad curvature

Incorporating Sheared $\mathbf{E} \times \mathbf{B}$ Flow in Nonlinear Simulations

In each fluid equation, $\mathbf{E} \times \mathbf{B}$ convection gives rise to three terms:

$$\frac{\partial n}{\partial t} + \mathbf{v}_E^{(0)} \cdot \nabla n + \mathbf{v}_E \cdot \nabla n_0 + \mathbf{v}_E \cdot \nabla n + \cdots = 0, \tag{1}$$

In our flux-tube simulations, we evolve $n(\alpha, \psi, \theta, t)$, where α and ψ are coordinates perpendicular to the field, $\mathbf{B} = \nabla\alpha \times \nabla\psi$, ψ is the poloidal flux, and θ measures distance along the field line. In these coordinates Eq. (1) becomes:

$$\frac{\partial n}{\partial t} - c\frac{\partial \phi^{(0)}}{\partial \psi}\frac{\partial n}{\partial \alpha} + c\frac{\partial \phi}{\partial \alpha}\frac{\partial n_0}{\partial \psi} + c\left(\frac{\partial \phi}{\partial \alpha}\frac{\partial n}{\partial \psi} - \frac{\partial \phi}{\partial \psi}\frac{\partial n}{\partial \alpha}\right) + \cdots = 0.$$

For linearly sheared $\mathbf{E}^{(0)} \times \mathbf{B}$ flow: $\frac{\partial \phi^{(0)}}{\partial \psi} = \frac{\partial^2 \phi^{(0)}}{\partial \psi^2}(\psi - \psi_0)$.

We introduce the transformation $\alpha' = \alpha - c\frac{\partial^2 \phi^{(0)}}{\partial \psi^2}(\psi - \psi_0)t$, so the new coordinates shear in the poloidal direction with the equilibrium flow. This introduces the $\mathbf{v}_E^{(0)} \cdot \nabla n$ term, consistent with smooth statistically periodic boundary conditions across the ψ domain.

Now radial derivatives become time dependent:

$$\frac{\partial}{\partial \psi} = \frac{\partial}{\partial \psi'} - c\frac{\partial^2 \phi^{(0)}}{\partial \psi^2} t \frac{\partial}{\partial \alpha}.$$

In more standard ballooning notation, this means that the θ_0's shift:

$$\theta_0' = \theta_0 + \Omega_E t / \hat{s},$$

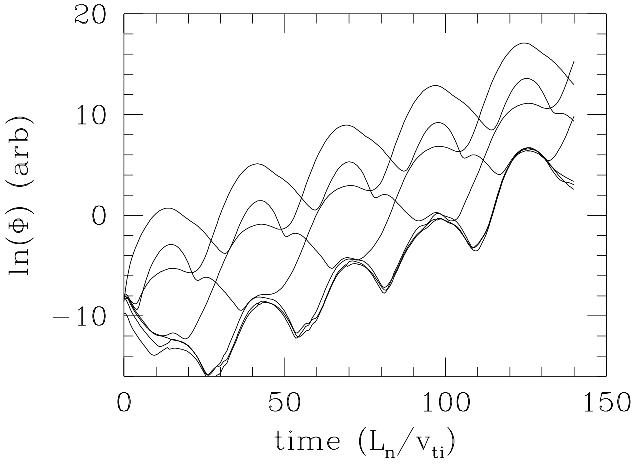
where $k_r = k_\theta \hat{s} \theta_0$, and

$$\Omega_E = \frac{e}{T_i}\frac{\partial^2 \phi^{(0)}}{\partial \rho^2}.$$

Note that Ω_E is a flux function. The simulation takes any geometric effects which may increase the shearing on the outboard midlplane into account automatically.

Linear Behavior with Sheared $\mathbf{E} \times \mathbf{B}$ Flow

In this representation the addition of flow shear introduces oscillations on top of exponential growth, and a reduced effective growth rate. $\Phi(t)$ for seven $\theta_0' s$ in the presence of $\mathbf{E} \times \mathbf{B}$ shear:



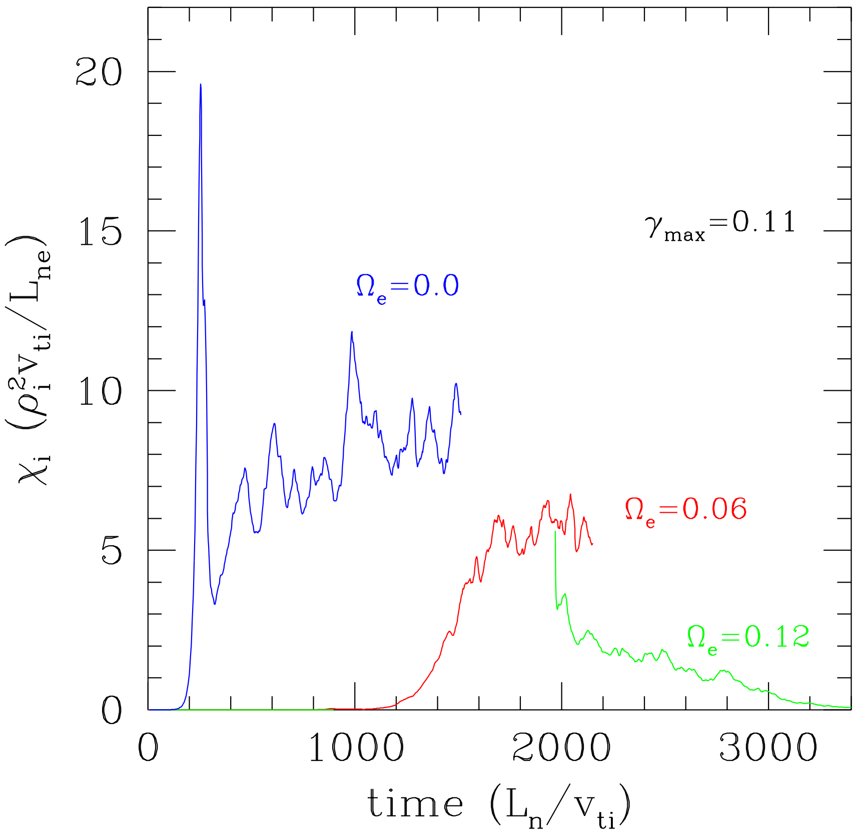
When a mode is sheared to sufficiently high θ_0, we reintroduce it at negative θ_0, to resolve the physically relevant part of k-space. Different θ_0's are linearly coupled through the boundary condition along the field line [**Beer, Cowley, & Hammett, PoP 2, 2687 (1995)**], making the linear calculation with flow shear 2D.

The net growth now depends on the θ_0 averaged growth rate, as a mode is convected poloidally. This approach and our results are similar to [**Waltz, Dewar, & Garbet, PoP 5, 1784 (1998)**]. Parallel flow shear not included.

Nonlinear Results with Sheared $\mathbf{E} \times \mathbf{B}$ Flow

Nonlinear results are consistent with [**Waltz, Dewar, & Garbet, PoP 5, 1784 (1998)**]: turbulence is suppressed when $\Omega_E \approx \gamma_{\max}$.

This is for DIII-D 81499 parameters.

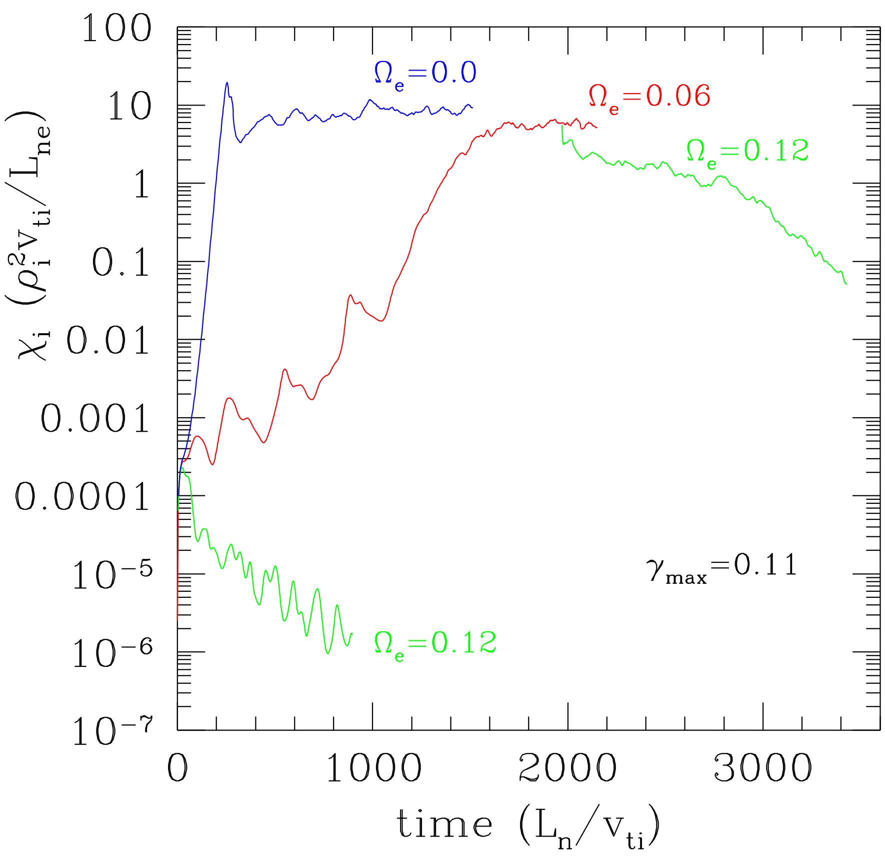


The case with $\Omega_E = 0.06$ is linearly only very weakly unstable, yet the saturated flux is not reduced much from the case with no sheared $\mathbf{E} \times \mathbf{B}$ flow.

Nonlinear Effects Tend to Sustain Turbulence when $\Omega_E = \gamma_{\max}$

When $\Omega_E = \gamma_{\max}$, for these parameters there is no linear instability.

However, if the simulation is started at finite amplitude, the turbulence decays less rapidly, due to nonlinear effects. Once the amplitude drops sufficiently, the nonlinear sustainment weakens and the turbulence decays more rapidly.



Physics of the Undamped Flow Component

Since $\mathbf{v}_E \cdot \nabla F_0 = 0$ for the zonal flows, they obey a simplified collisionless electrostatic toroidal gyrokinetic equation:

$$\frac{\partial f}{\partial t} + v_{\|}\nabla_{\|} f + i\omega_d f + i(eF_0/T)\omega_d \Phi + v_{\|}(eF_0/T)\nabla_{\|}\Phi = 0,$$

where $i\omega_d = \mathbf{v}_d \cdot \nabla = i(k_r \rho_i / v_t R)(v_{\|}^2 + v_{\perp}^2/2)\sin\theta$.

Rosenbluth and Hinton found a general equilibrium solution:

$$f = -(e\Phi/T)F_0 + h(E,\mu)e^{-ik_r\rho_i(qB_0 v_{\|}/\epsilon B v_t)},$$

where $h(E,\mu)$ is arbitrary but satisfies $\partial h/\partial l = 0$. The $v_{\|}$ in the exponential keeps f non-Maxwellian.

In this equilibrium, parallel variations in f balance the velocity dependent cross field drifts.

Expanding for small banana width $k_r \rho_i q/\epsilon \ll 1$:

$$f = -(e\Phi/T)F_0 + h(E,\mu)[1 - ik_r\rho_i \frac{qB_0 v_{\|}}{\epsilon B v_t}],$$

we see that moments of f will be supported by radial gradients of higher moments, e.g. $u_{\|}$ is driven by $k_r p_{\|}$, analogous to Pfirsch-Schlüter flow:

$$n_0 u_{\|} = \int d^3v\, v_{\|} f = -ik_r\rho_i \frac{qB_0}{\epsilon B v_t} \int d^3v\, v_{\|}^2 h(E,\mu)$$

Closures for Zonal Flows Which Retain Residual Component

If we choose $h(E, \mu)$ to be a perturbed Maxwellian with no n perturbation:

$$f = -(e\Phi/T)F_0 + F_0 \left(\frac{mv^2}{2T_0} - \frac{3}{2}\right)\frac{\delta T}{T_0}\left[1 - ik_r\rho_i\frac{qB_0 v_\parallel}{\epsilon B v_t}\right],$$

we can integrate this and find equilibrium $q_\parallel$ and $q_\perp$ moments:

$$q_\parallel^{(0)} = 3ik_r\rho_i\frac{qB_0}{\epsilon B}\delta T \qquad \text{and} \qquad q_\perp^{(0)} = ik_r\rho_i\frac{qB_0}{\epsilon B}\delta T.$$

Generalizing to non-isotropic h leads to:

$$q_\parallel^{(0)} = 3ik_r\rho_i\frac{qB_0}{\epsilon B}T_\parallel \qquad \text{and} \qquad q_\perp^{(0)} = ik_r\rho_i\frac{qB_0}{\epsilon B}T_\perp.$$

Our old parallel closures damped $q_\parallel$ and $q_\perp$ to zero, but now we replace:

$$\begin{array}{ll} \sqrt{2}D_\parallel|k_\parallel|q_\parallel \rightarrow \sqrt{2}D_\parallel|k_\parallel|(q_\parallel - q_\parallel^{(0)}) & \text{in the } \mathrm{q}_\parallel \text{ eqn} \\ \sqrt{2}D_\perp|k_\parallel|q_\perp \rightarrow \sqrt{2}D_\perp|k_\parallel|(q_\perp - q_\perp^{(0)}) & \text{in the } \mathrm{q}_\perp \text{ eqn} \end{array}$$

We also have to modify the toroidal closures in the $p_\parallel$ and $p_\perp$ eqns to support this equilibrium. We have not found a completely satisfactory way to do this. Two possibilities are:

closure (a): $\nu_1 = \nu_2 = \nu_3 = \nu_4 = 0$, or

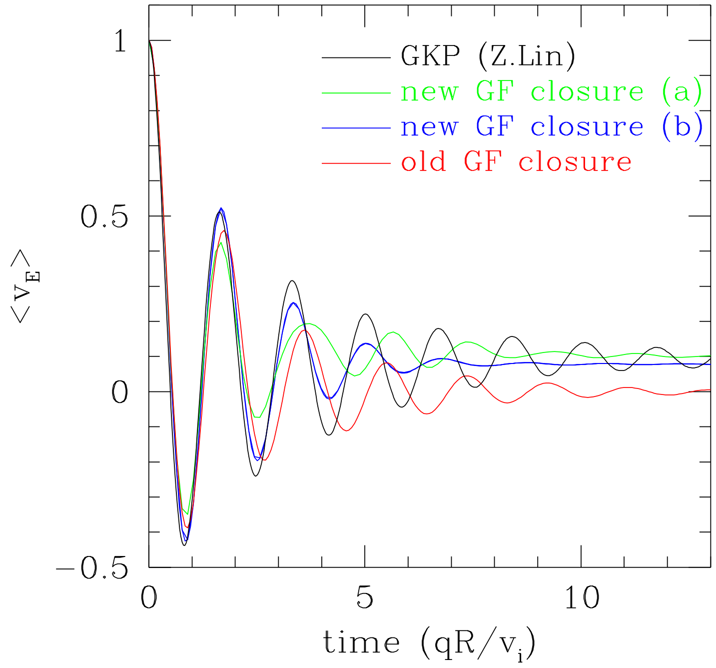
closure (b): $\nu_1 = (0, -3)$, $\nu_2 = (0, 1)$, $\nu_3 = (0, 0)$, $\nu_4 = (0, -3/2)$, and $q_\parallel^{(0)} = q_\perp^{(0)} = 0$, which makes less physical sense but doesn't do too poorly.

Both with ν_5-ν_{10}=0.

Because our flux-tube code is spectral, we can modify these evolution eqns for the zonal flows without changing the $k_\theta \neq 0$ components.

Comparison of Gyrokinetic and Gyrofluid Flow Damping With New Closures

New closures agree reasonably well with gyrokinetic results on amplitude of residual component for $k_r \rho_i = 0.2$:



This is for DIII-D 81499 parameters, $\epsilon = .18$, $q = 1.4$.

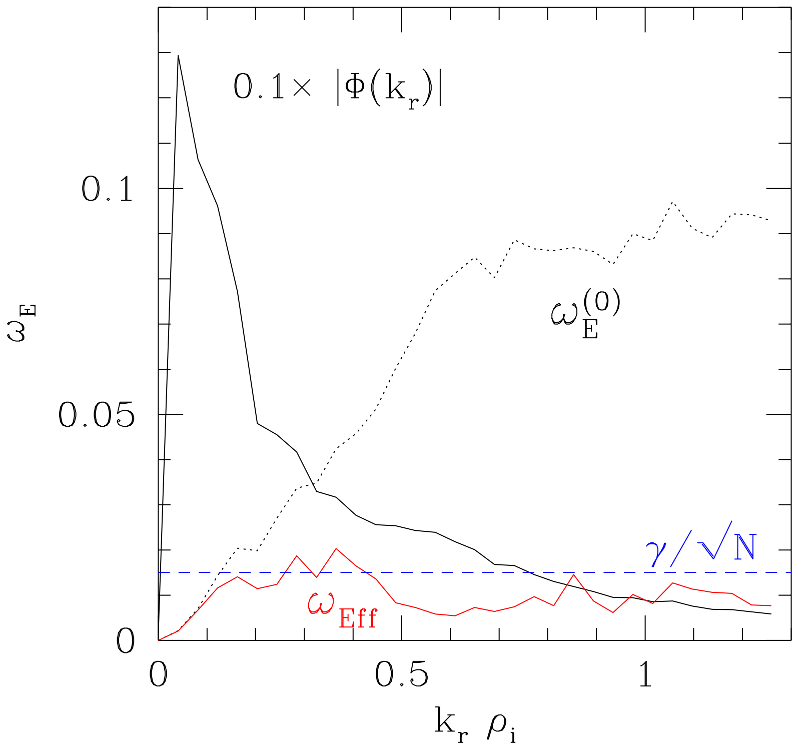
Reasonable agreement with Rosenbluth-Hinton formula:

$$\frac{v_{Ef}}{v_{Ei}} = \frac{c\sqrt{\epsilon}/q^2}{1 + c\sqrt{\epsilon}/q^2}$$

where $c = 0.625$, which predicts $v_{Ef}/v_{Ei} = 0.12$.

Time Averaged Flow Spectrum

Spectrum of saturated flux surface averaged potential $|\Phi(k_r)|$ obtained by Fourier Transforming in r and averaging in t.



Shearing rate peaks at high k_r: $\gamma_{\text{shear}} = k_r^2|\Phi(k_r)|$

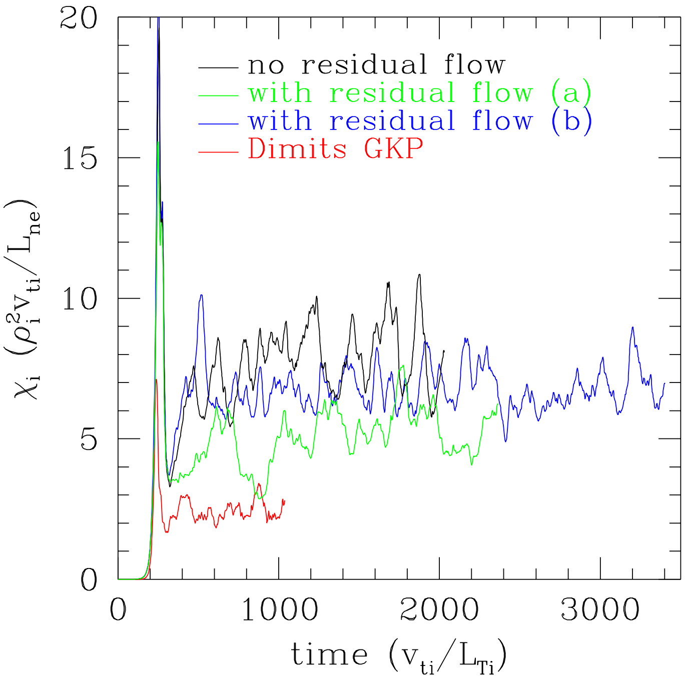
While highest k_r shearing rates are large, they have small correlation times and thus small effect on turbulence (Hahm, Diamond).

Maximum $\gamma_{\text{lin}} \approx 0.1$

Using simulation zonal flow spectrum and using simulation zonal flow time history to calculate $\tau_{\text{corr}}(k_r)$, we estimate the time dependent effective shearing rate ω_{Eff} [Hahm] and find close to $\gamma_{\text{lin}}/\sqrt{N}$

Nonlinear Tests of Importance of Residual Flow

For parameters from DIII-D shot 81499 (the Cyclone base case, with $R/L_{Ti} = 6.9$), we repeat nonlinear runs with the new closures (a) and (b), both including undamped components of the zonal flow.

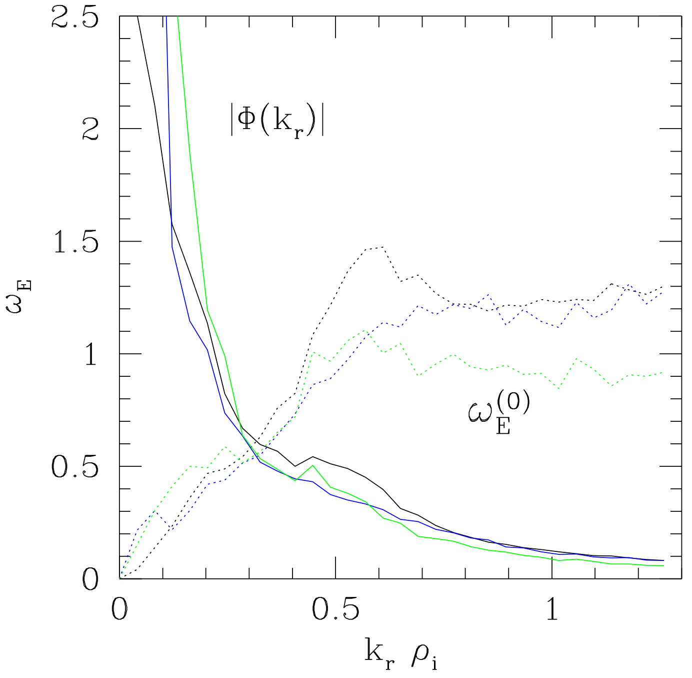


With residual flows, flux drops by up to about 35%, for this case

Nonlinear effects ($e.g.$ turbulent viscosity) keep linearly undamped residual components from growing indefinitely

Time Averaged Zonal Flow Spectra with New Closures

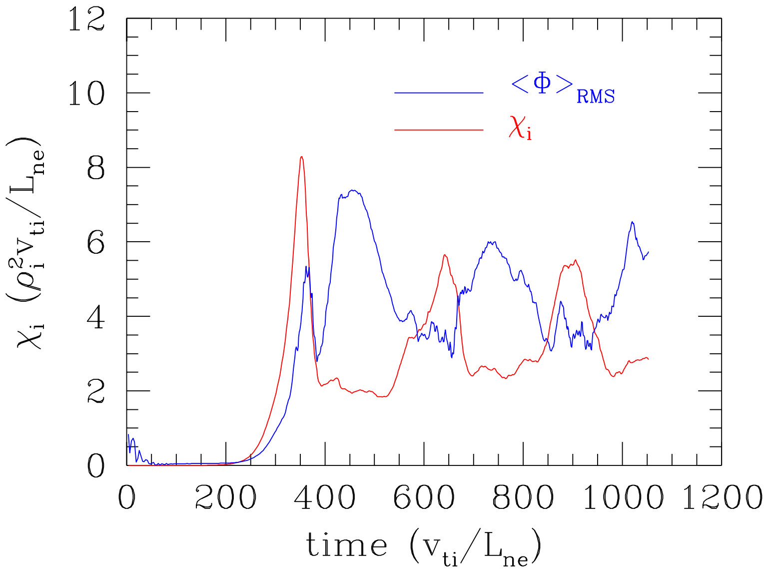
Low k_r zonal flows are larger with new closures



Since low k_r residual component is too small for our new closures, might expect more of an effect as we improve model further

Zonal Flows Can Cause Bursting Near Marginal Stability

Nearer marginal stability ($R/L_{Ti} = 4$), with the old closures we find intermittent behavior: [**Beer & Hammett, Varenna, 1998**]

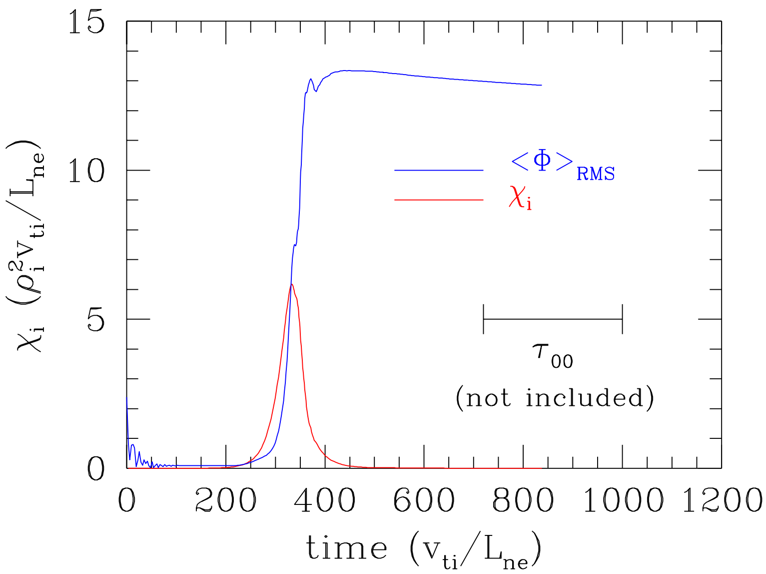


Turbulence (χ_i) drives zonal flows ($\langle\Phi\rangle_{RMS}$) which then damp turbulence. Flows then slowly damp and turbulence grows again.

Bursting is on 1ms time-scale, similar to Mazzucato's fluctuation measurements in RS, which are likely near marginal stability

Zero-flux State Near Marginal Stability With Undamped Residual Flow Component

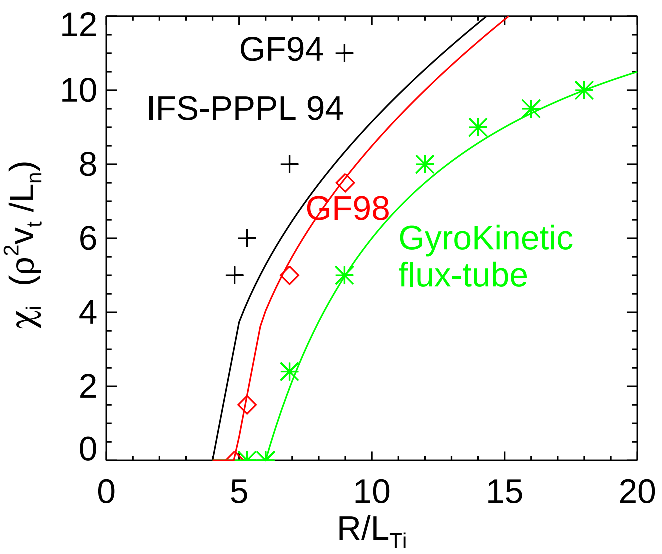
Repeating this marginal stability case ($R/L_{Ti} = 4$) with the new closures, we find that the turbulence drives one burst of flow which is now undamped. Leads to nonlinear upshift in critical gradient (Dimits, Sherwood 1998)

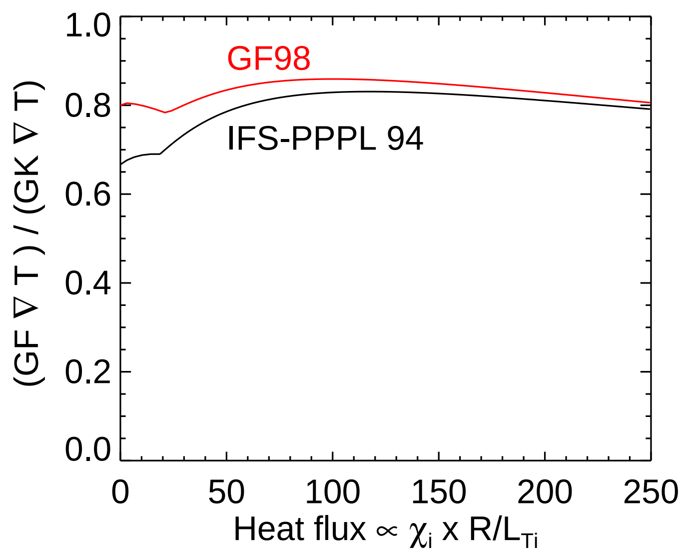


This is in the collisionless limit. A realistic amount of collisions would damp the zonal flows on a time scale $\tau_{00} = 1.5\epsilon/\nu_{ii}$ (Rosenbluth, APS 1997) and would likely lead to bursty behavior or a turbulent steady state

Possibly an artifact of initial conditions. We could initialize arbitrarily large flow and get zero flux for any R/L_{Ti}

Gyrofluid/gyrokinetic (GF/GK) simulation differences correspond to 20-30% differences in the predicted temperature gradient.





•Dimits (LLNL): good convergence in his gyrokinetic particle simulations

•New neoclassical gyrofluid closure significantly improves GF/GK comparison

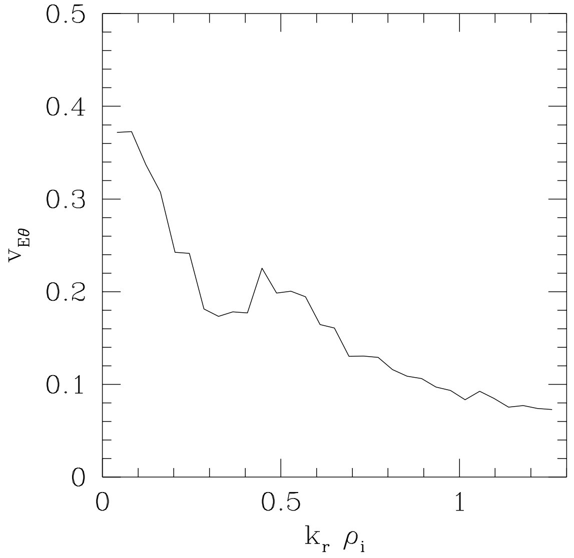
•Turning this plot around, for a fixed amount of heat flux, $\propto \chi \nabla T$, the temperature gradient predicted by the original gyrofluid-based IFS-PPPL model is 20-33% low. But $P_{\rm fusion} \propto T^2$, and so may increase by $\times 2$ or more.

•Nonlinear upshift in critical gradient may depend on: Rosenbluth-Hinton undamped zonal flows ↑ with elongation (W. Dorland), ↓ with weak collisions, ↓ with non-adiabatic electrons [may limit inverse cascade that drives zonal flows (Diamond, Liang, Terry-Horton, Waltz, ...) and ↑ turbulent viscosity].

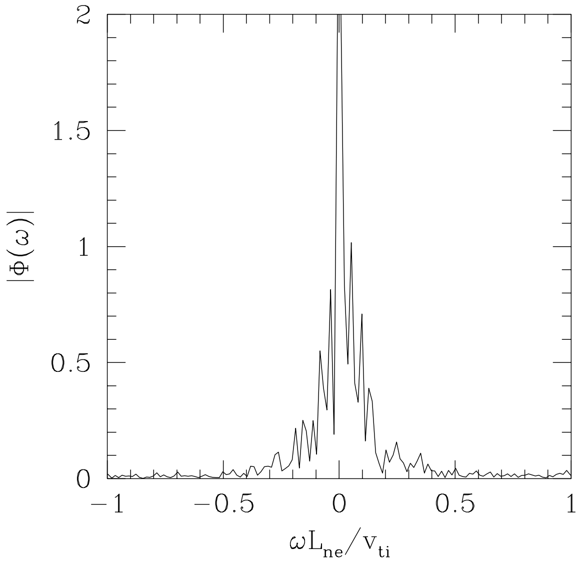
Experimental Signatures of Zonal Flows

$\mathbf{v}_{E\theta}(k_r)$ broad, with width $\sim k_r\rho_i$. $\mathbf{v}_{E\theta}$ is in units of $v_{ti}\rho_i/L_{ne}$.

For this shot, DIII-D #81499, $\rho_i/L_{ne} = .37\mathrm{cm}/76\mathrm{cm} = 1/205$, and $v_{ti} = 3.4 \times 10^7\mathrm{cm/s}$, so $\mathbf{v}_{E\theta} = 0.3$ corresponds to $500\mathrm{m/s}$.



Frequency spectrum for $k_r\rho_i = 0.08$:



$v_{ti}/L_{ne} = 4.5 \times 10^5\mathrm{s}^{-1}$, so $\omega L_{ne}/v_{ti} = 0.1$ corresponds to $\omega/2\pi = 7\,\mathrm{kHz}$

Higher k_r's have higher frequencies.

Conclusions

1. Equilibrium scale $\mathbf{E} \times \mathbf{B}$ flows added to nonlinear simulations
   - Results are consistent with previous experience and expectations from experiments
   - Now can compare with a much wider range of experiments self-consistently
   - Important for NSTX, JET, DIII-D and most other machines
   - Fully general geometric treatment
2. Gyrofluid simulations with linearly undamped residual zonal flow component
   - Agreement at low k_r is not great, further closure modifications being investigated
   - Nonlinear comparisons show that including residual component has 30%-40% effect for Cyclone DIII-D base case
   - Might expect larger effect as low k_r behavior is improved
   - Near marginal stability system can bifurcate into all flow, zero flux state
   - When undamped flow effect is important, a small amount of collisions can increase χ_i.