

Problem Set #2 (due September 27)

1. In class, we derived the Debye shielding by electrons of the potential produced by a test charge Q in a plasma. Calculate the resulting electron density and verify that the test charge is indeed shielded by showing that the total excess electron charge is $-Q$.
2. In the Debye shielding calculation done in class, we assumed the ion density remained uniform and used a Boltzmann equilibrium distribution for the electrons only. Extend the Debye shielding calculation to include a Boltzmann equilibrium distribution for the ions as well, where the ions have charge Z_i and density $n_{i0} = n_{e0}/Z_i$ (the plasma is neutral in the absence of the test charge). Allow the ion temperature T_i to be different than the electron temperature T_e (we will discover later in this class that the electron-ion temperature equilibration rate can be relatively slow). Which species provides the dominant shielding if $T_i \ll T_e$? [Something to think about: Suppose the test charge is *moving*. How do you expect the Debye shielding to change?]
3. Consider a homogeneous cold electron plasma, where the ions may be assumed to be an infinitely massive neutralizing background. Suppose that a small fraction of the electrons in the slab $x_1 < x < x_2$ are removed and then placed at $x_1 + L < x < x_2 + L$. Sketch the subsequent evolution of the electron density, the electric field, and the electron fluid velocity. Sketch the trajectory of a single electron.
4. Since much of the galaxy is filled with an electron-proton plasma, one might expect that the propagation of starlight reaching the earth could be affected. Calculate the dispersion relation for plane electromagnetic waves in this plasma, by modeling the plasma as a uniform cold electron fluid, with a uniform background of stationary ions. (This calculation will be related to the ω_{pe} plasma oscillation calculation we did in class, but you will need to include the fluctuating magnetic field also.) If the electron density were 1 cm^{-3} , what restriction is based on the observation of radio wave signals? What other effects might have to be included to obtain a realistic description of the propagation of light in the inter-stellar plasma?