AS551 General Plasma Physics I
November 23, 2004

Homework #9, due Wednesday Dec. 1

1. Generalized sound waves. Extend Goldston and Rutherford's calculation of the sound wave dispersion relation Eq. 16.23 to the case of ions with arbitrary charge Z_i. You can assume quasineutrality†, $Z_i n_i = n_e$ in the derivation. For a $T_i = T_e$ plasma, what condition is required so that the sound wave is much faster than the thermal ion velocity? (Ion acoustic waves in a high Z_i plasma can be important in various situations, such as in some inertial fusion problems problems, where the outer layers of the pellet may use a high Z_i material.)

2. Goldston & Rutherford Problem 16.2.

3. Goldston & Rutherford Problem 16.3.

† What is meant by "quasineutrality" is that rather than solve the full Poisson equation, G&R Eq. 16.19, one can just set the LHS of the Poisson equation to zero and require that the RHS vanish, so that $Z_i n_{i1} = n_{e1}$. (The equilibrium is usually assumed to be exactly neutral $Z_i n_{i0} = n_{e0}$.) Since $n_{e1} \propto \phi_1$ (in this case), the requirement of quasineutrality will determine the electrostatic potential once the ion density is known. This is a very useful and widely used approximation, but is sometimes confusing to those new to plasma physics. The quasineutral assumption does not mean that the electrostatic potential ϕ is zero. Instead, it means that even tiny charge imbalances can generate very large electric fields (to which the electrons would very quickly respond), so that in the end the electric field is determined primarily by the requirement that the electron and ion density be nearly equal to each other ("quasineutral"). Another way to say this is that one can compare the LHS of G&R Eq. 16.19 with the electron response on the RHS:

$$\epsilon_0 \nabla \cdot \vec{E}/(e n_e 1) = \epsilon_0 k^2 \phi_1/(n_{e0} e^2 \phi_1/T_e) = k^2 \lambda_{De}^2$$

which is small for waves that are much longer wavelength than the Debye length λ_{De}, so if one is focussing on long wavelength modes, one can just drop the LHS of the equation and assume quasineutrality. (The quasineutral approximation is often used in other cases even where the electron response is more general than a Bolzmann distribution, but there are cases where quasineutrality can't be assumed. Quasinuetrality is assumed also in MHD, so we will return to this topic later.) One way to gain insight into quasineutrality is to solve problems with the full Poisson equation but multiply the LHS by a coefficient ϵ, and investigate what happens in the $\epsilon \to 0$ limit (which is the quasineutral limit). But this is easy to do for the ion acoustic wave in MKS units, since the LHS is already multiplied by ϵ_0, so one can see that $\epsilon_0 \to 0$ corresponds to $\lambda_D \to 0$, and the dispersion relation in G&R Eq. 16.23 simplifies to $\omega \propto k$. Assuming quasineutrality from the start often simplifies long calculations in other problems and so is often used for long wavelength (compared to the Debye length) phenomena like MHD and drift waves and sound waves. But it can't always be used, particularly for higher-frequency electromagnetic waves where the displacement current in Ampere's law is important. (Since Ampere's law insures that the Poisson equation remains true as long as it is initially true, there is a consistency condition between approximations on the displacement current and the LHS of the Poisson Eq.)