Notes for Astrophysical Sciences 552: General Plasma Physics II

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April 26, 2001

Abstract

These are some notes to supplement my lectures, on the microinstabilities part of this course. The main reference for this course are Tang's lecture notes (available in the PPPL library), and references therein.

1 Iterative solution of the collisionless drift wave instability (with corrections)

There was an error in my iterative solution of the dispersion relation for the the collisionless drift wave instability. Start with the cold ion fluid response ($T_i = 0$ was assumed) derived in class:

$$\frac{n_{i1}}{n_0} = \frac{|e|\Phi}{T_e}\left[\frac{\omega_{*e}}{\omega} - b_s + \frac{k_\parallel^2 c_s^2}{\omega^2}\right]$$

where $b_s = k_\perp^2 \rho_s^2$, $\rho_s^2 = c_s^2/\Omega_{ci}^2$, and $c_s^2 = T_e/m_i$ (which differs from Tang's convention $c_{sTang}^2 = 2T_e/m_i$ by a factor of 2). The general electron response found from the linearized drift-kinetic equation is

$$\frac{n_{e1}}{n_0} = \frac{|e|\Phi}{T_e}\left[1 + \frac{\omega - \omega_{*e}}{|k_\parallel| v_{te}\sqrt{2}} Z(\zeta_e)\right].$$

where $\zeta_e = \omega/(|k_\parallel| v_{te}\sqrt{2})$, and the plasma dispersion function $Z(\zeta)$ is as defined in the NRL plasma formulary. In the standard drift-wave limit $\omega/k_\parallel \ll v_{te}$, we find an adiabiatic response (also called a Boltzmann response) plus a next order correction:

$$\frac{n_{e1}}{n_0} = \frac{|e|\Phi}{T_e}\left[1 + i\sqrt{\pi}\frac{\omega - \omega_{*e}}{|k_\parallel| v_{te}\sqrt{2}}\right].$$

Assuming quasineutrality, these ion and electron responses can be combined to give the dispersion relation

$$1 + b_s = \frac{\omega_{*e}}{\omega} + \frac{k_\parallel^2 c_s^2}{\omega^2} + i\sqrt{\pi}\frac{\omega_{*e} - \omega}{|k_\parallel| v_{te}\sqrt{2}}$$

or

$$\omega = \frac{1}{1+b_s}\left[\omega_{*e} + \frac{k_\parallel^2 c_s^2}{\omega} + i\sqrt{\pi}\frac{\omega(\omega_{*e}-\omega)}{|k_\parallel| v_{te}\sqrt{2}}\right]$$

[In class, I accidently replaced $\omega(\omega_{*e}-\omega))$ in the last term with $\omega_{*e}(\omega_{*e}-\omega))$, which led to another factor of $1/(1+b_s)$ missing from the final growth rate formula. Also, I may have missed the $\sqrt{2}$ factor in this equation.] In the usual drift-wave limit $v_{ti} \ll \omega/k_\parallel \ll v_{te}$, to lowest order we have $\omega = \omega_{*e}/(1+b_s)$. Substituting this value of ω into small terms on the right hand side, we obtain an improved estimate for ω:

$$\omega = \frac{\omega_{*e}}{1+b_s} + \frac{k_\parallel^2 c_s^2}{\omega_{*e}} + i\sqrt{\pi}\frac{\omega_{*e}^2 b_s}{(1+b_s)^3 |k_\parallel| v_{te}\sqrt{2}}$$

If we consider the $k_x = 0$ limit so $b_s = k_y^2\rho_s^2$, and plot the growth rate vs. k_y, we find that the growth rate $\gamma \propto k_y^4$ for $k_y\rho_s \ll 1$, and the growth rate $\gamma \propto 1/k_y^2$ for $k_y\rho_s \gg 1$ (with this error fixed, this is now consistent with my experience of the usual behavior at high $k_\perp$). It is because the peak growth rate occurs roughly in the vicinity of $k_y\rho_s \sim 1$ (i.e., on a relatively small ion gyroradius scale) that these are called "microinstabilities".