AS554 Irreversible Processes in Plasmas
March 9, 1999

Problem Set #5 (due Mar. 23, 1999)

Question 1

Determine the electrical conductivity of a Lorentz plasma. [Take Z_i large so that only electron-ion collisions need to be considered. Write $f_e = f_M + f_1$ where f_M is a Maxwellian. Linearize the Fokker-Planck equation (including an external electric field) in E, assuming $f_1 = O(E)$. Solve the resulting equation.]

Question 2

Using the Rosenbluth potentials, evaluate the collisional flux $\mathbf{J}^{a/b}$ when f_b is a Maxwellian. Express in terms of the relaxation rates $\nu_s^{a/b}$, $\nu_\parallel^{a/b}$, and $\nu_\perp^{a/b}$ (see Plasma Formulary). Put the answer in a form similar to the one in the Formulary (under Fokker–Planck Equation) but note that the answer in older editions of the Formulary is wrong!
Useful formulas:

$$\mathbf{J}^{a/b} = 4\pi\nu_0^{a/b}v^3\left[-\frac{m_a}{m_b}(\nabla_\mathbf{v}\phi_b)f_a + (\nabla_\mathbf{v}\nabla_\mathbf{v}\psi_b)\cdot\nabla_\mathbf{v}f_a\right]$$

where

$$\nu_0^{a/b} = \frac{e_a^2 e_b^2 \log\Lambda n_b}{4\pi\epsilon_0^2 m_a^2 v^3},$$

and

$$\nabla_\mathbf{v}^2\phi_b = f_b, \qquad \nabla_\mathbf{v}^2\psi_b = \phi_b, \qquad \int f_b\, d^3\mathbf{v} = 1.$$

If f_b is spherically symmetric, ϕ_b and ψ_b are found by straightforward integration. In particular if f_b is a Mawellian with temperature T_b then

$$\frac{d\phi_b}{dv} = \frac{1}{4\pi v^2}(\mathrm{erf}(u) - u\,\mathrm{erf}'(u)) = \frac{1}{4\pi v^2}\frac{\nu_s^{a/b}}{(1+m_a/m_b)\nu_0^{a/b}}$$

$$\frac{d\psi_b}{dv} = -\frac{1}{16\pi}\left(\left(2-\frac{1}{u^2}\right)\mathrm{erf}(u) + \frac{\mathrm{erf}'(u)}{u}\right) = -\frac{1}{16\pi}\frac{\nu_\perp^{a/b}}{\nu_0^{a/b}}$$

$$\frac{d^2\psi_b}{dv^2} = -\frac{1}{8\pi v}\left(\frac{\mathrm{erf}(u)}{u^2} - \frac{\mathrm{erf}'(u)}{u}\right) = -\frac{1}{8\pi v}\frac{\nu_\parallel^{a/b}}{\nu_0^{a/b}}$$

$$\mathrm{erf}(u) = \frac{2}{\sqrt{\pi}}\int_0^u \exp(-x^2)\,dx, \qquad \mathrm{erf}'(u) = \frac{2}{\sqrt{\pi}}\exp(-u^2)$$

$$u = v/\sqrt{2T_b/m_b}$$