Thoughts on Continuum Algorithms.

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at PMP "1st Annual Summer Frontier Center"

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① Gyrofluids : 6 velocity moments
GK : ~400 velocities

} χ_i within a factor of x 2.

Can one reduce # of v's in GK code?

Can one use 20 – 100 ~~[illegible]~~ v's ?

Adjustable to check convergence...

② "Recurrence problem"

(Present codes have some numerical dissipation to cope: ~~[illegible]~~
slight upwinding, slight off-center implicit, collisions...)

⇒ Hyper collision operator ⇒ ② eliminate recurrence
can reduce numerical dissipation elsewhere
① reduce # of v's.

Coping with well-known "Recurrence problem"

in this "Continuum code" ("Vlasov code", finite-difference "Eulerian")

(related, in a way, to noise in a Lagrangian PIC (Particle-in-cell) code or to Landau damping)

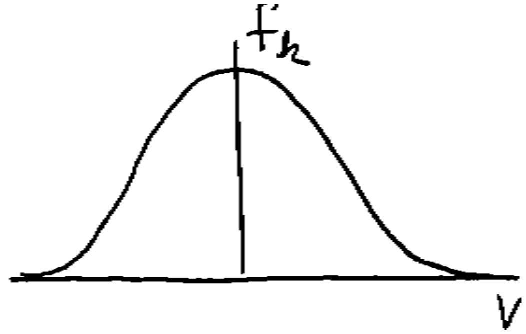
Consider simple 1-D free streaming:

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial z} = 0$$

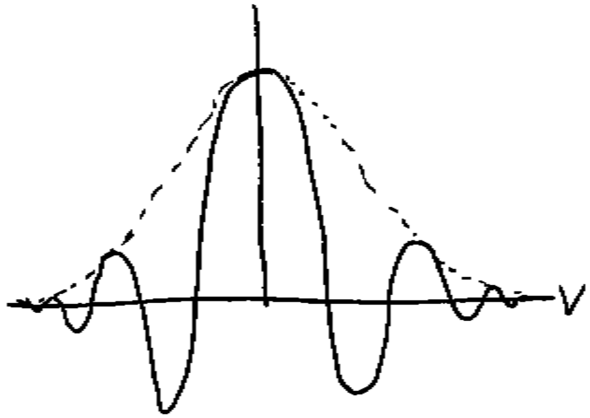
$$\frac{\partial f_k}{\partial t} = -ikv\, f_k$$

$$f_k(t) = e^{-ikvt} \underbrace{e^{-\frac{v^2}{2v_t^2}}}_{f_k(v,\,t=0)}$$

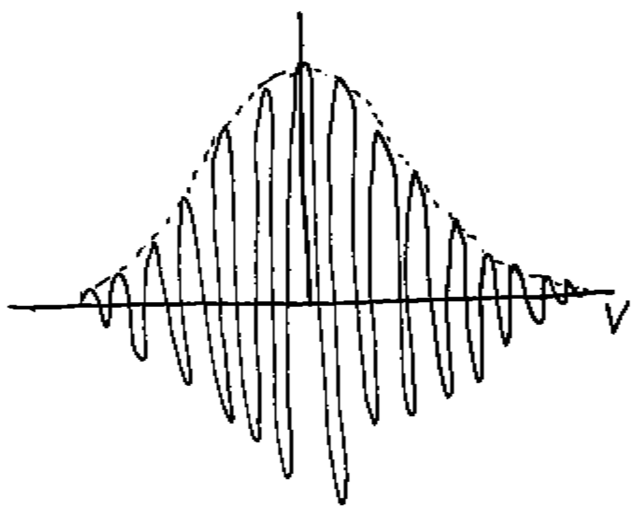
$t=0$



$t=t_1$



$t=4t_1$



Need high velocity resolution!

Physically: small v scales damped by collisions.

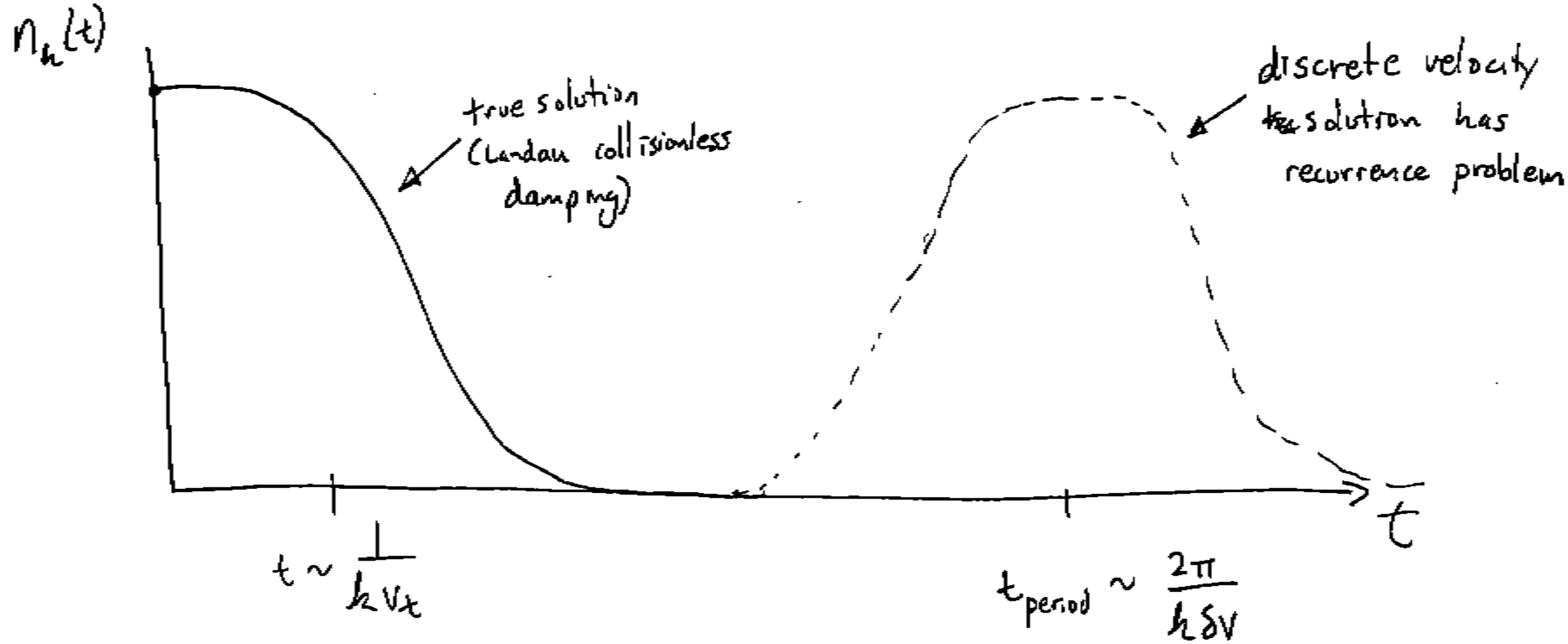
$$n(t) = \int dv\, f_k \propto e^{-k^2 v_t^2 t^2/2}$$

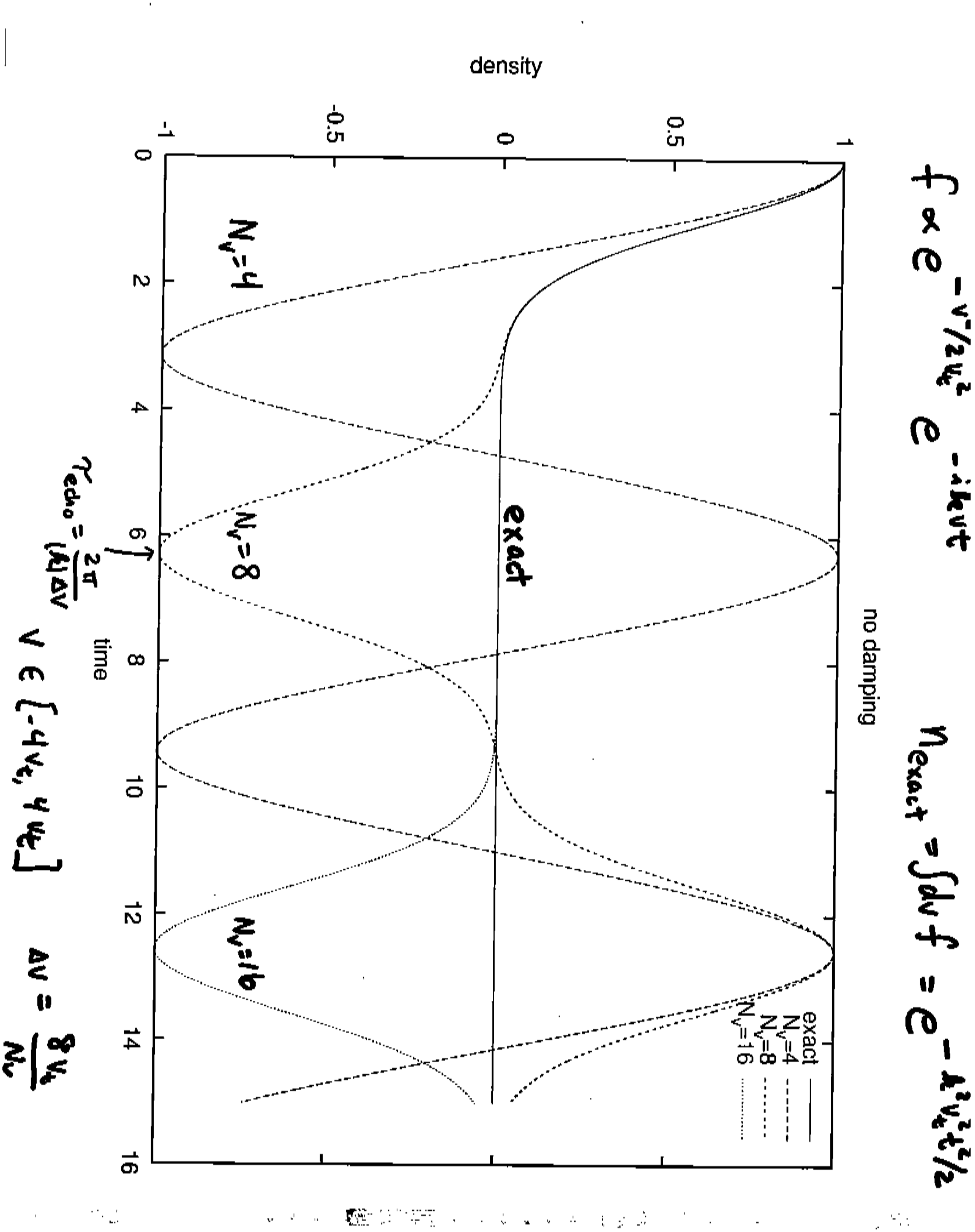
But if a discrete velocity grid is used:

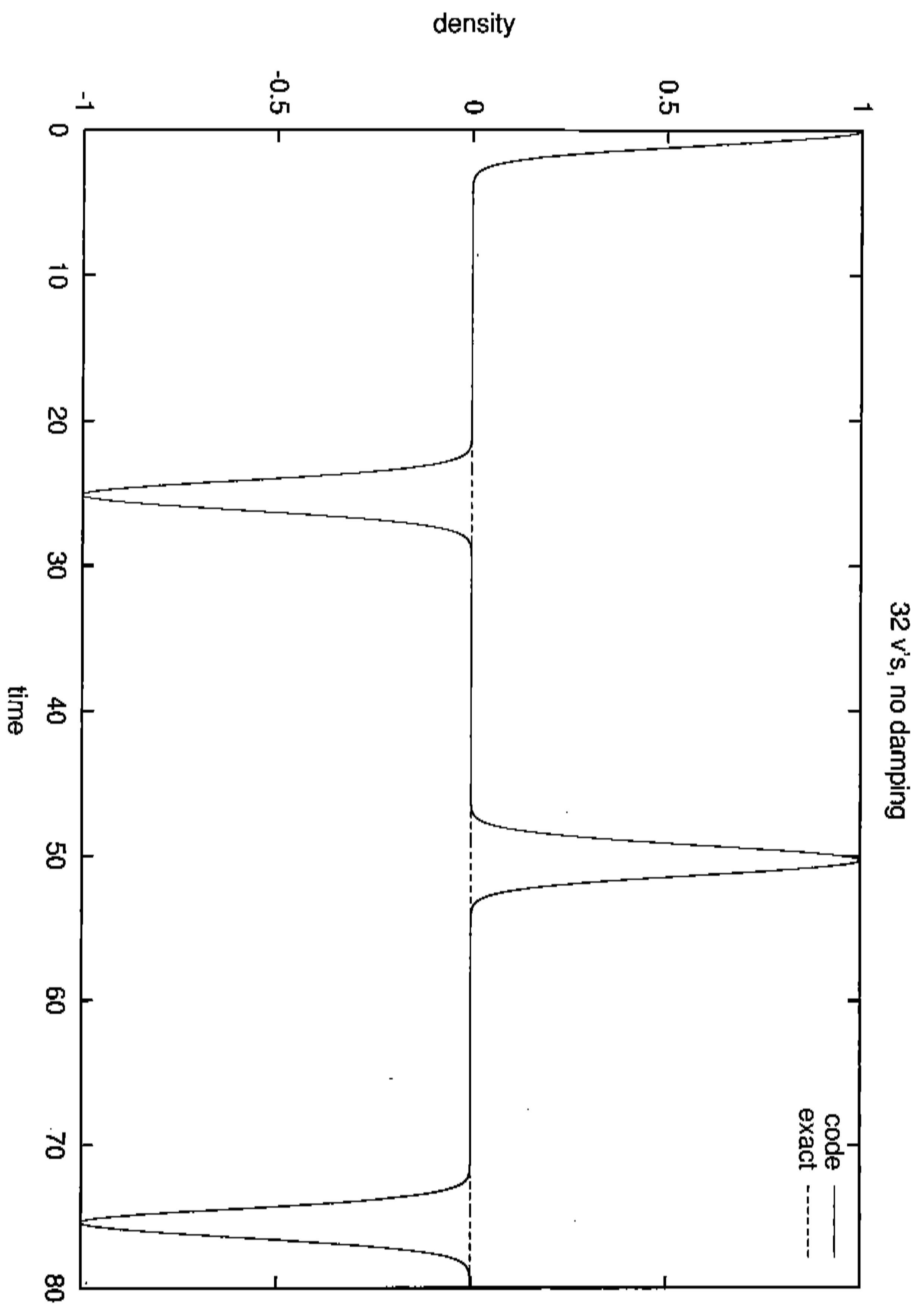
$$n_k(t) = \sum_j \delta v \underbrace{e^{-ikv_j t}} e^{-\frac{v^2}{2v_t^2}}$$

Phases decohere temporarily

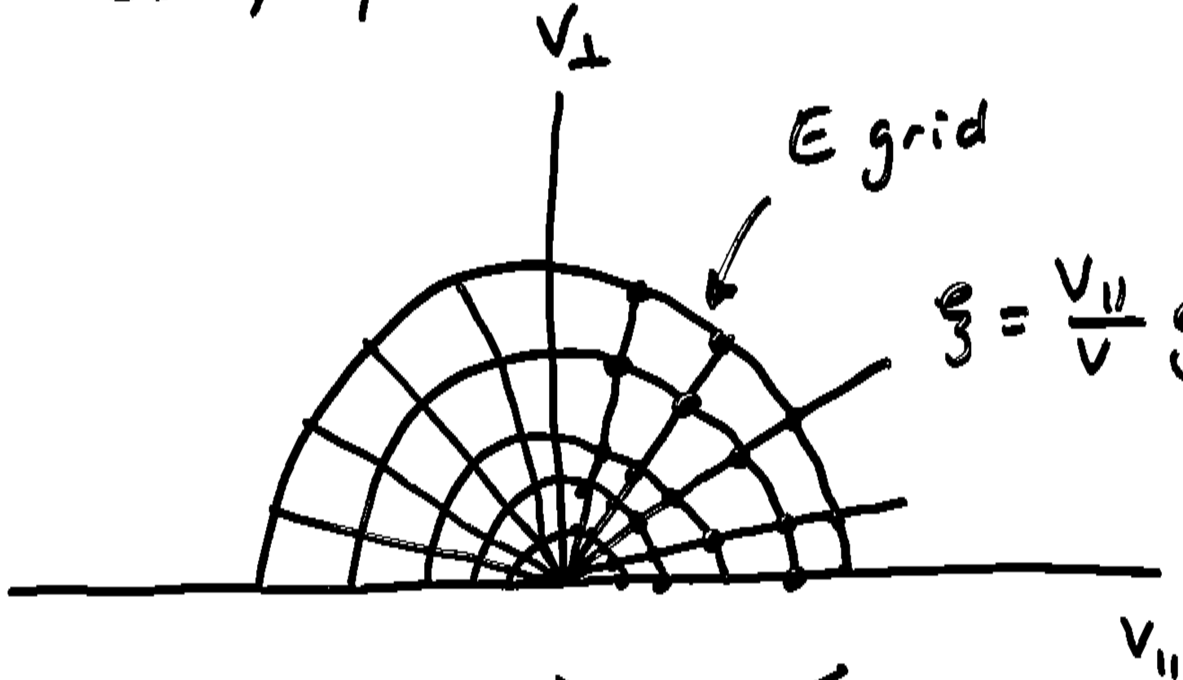
Periodic $t = \frac{2\pi}{k\delta v}$




no damping
density
time
exact
N_v=4
N_v=8
N_v=16
$f \propto e^{-v^2/2v_t^2} e^{-ikvt}$
$n_{exact} = \int dv\, f = e^{-k^2 v_t^2 t^2/2}$
$\tau_{echo} = \frac{2\pi}{k \Delta v}$
$v \in [-4v_t, 4v_t]$
$\Delta v = \frac{8 v_t}{N_v}$


32 v's, no damping
density
time
code
exact
-1
-0.5
0
0.5
1
0
10
20
30
40
50
60
70
80

GS2, Gyro



Set of $v_\parallel$'s not on a uniform grid.

$$n \approx \sum_j \delta v_{\parallel j} \underbrace{e^{-ik_\parallel v_j t}}_{\text{hard to get phases to coherently add}} f_j$$

hard to get phases to coherently add

$\Rightarrow$ get $\frac{1}{N_{v_\parallel}}$ "noise"

Smooth out small velocity scales, w/ std. collision operator:

$$\frac{\partial f}{\partial t} + \text{standard gyrokinetic terms} = \nu_2 v_t^2 \frac{\partial^2 f}{\partial v_\parallel^2}$$ (+ momentum & energy conserving terms)

Or **Hyper-collision operator:**

$$= -\nu_4 v_t^4 \frac{\partial^4 f}{\partial v_\parallel^4}$$

Advantages: less sensitive to choice of ν_4

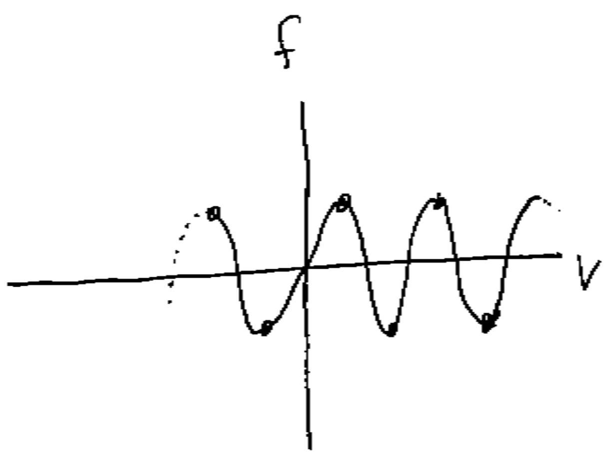
Conserves lowest 4 moments $\int dv_\parallel \, v_\parallel^{0-3} f$

Minimum Hyper-Damping rate requirement:
(confirmed numerically)

Need

Damping rate at barely resolved (Nyquist limit) scale $>$ Phase-mixing rate at which information should transfer to unresolved scales.

$$\nu_4 V_t^4 \frac{16}{(\Delta v)^4} > \underbrace{k \, \Delta V}_{\text{Frequency difference between adjacent grid points.}}$$



$$f_j \propto e^{i k v_j t}$$

Free Streaming + Hyper Collisions

$$\frac{\partial f}{\partial t} + v\frac{\partial f}{\partial z} = (-1)^n \nu_{2n} V_t^{2n} \frac{\partial^{2n} f}{\partial v^{2n}}$$

Fourier trans. in space:

$$\frac{\partial f}{\partial t} = -ikvf + (-1)^n \nu_{2n} V_t^{2n} \frac{\partial^{2n} f}{\partial v^{2n}}$$

w/ init. condition $f(v, t=0) = f_0(v)$

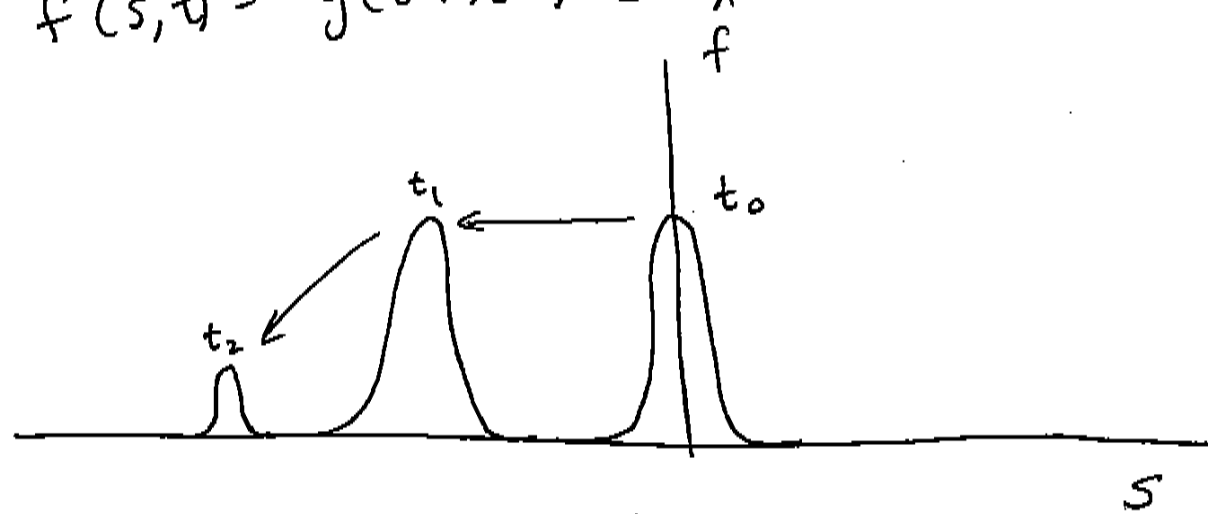
Fourier trans. in velocity:

$$f(v,t) = \frac{1}{\sqrt{2\pi}} \int ds\, e^{isv} \hat{f}(s,t)$$

$$\frac{\partial \hat{f}}{\partial t} = k\frac{\partial \hat{f}}{\partial s} - \nu_{2n} V_t^{2n} s^{2n} \hat{f}$$

Exact Soln. (Parker + ~~Mattor~~ Carditti ?)

$$\hat{f}(s,t) = g(s+kt)\, e^{\frac{\nu}{k} V_t^{2n} \frac{s^{2n+1}}{2n+1}}$$



Want to damp $\hat{f}$ before it reaches

Nyquist limit $|S_{max}| = \frac{\pi}{\delta v}$

& "reflects" + gives an "aliased echo" error.

Minimum damping (to keep echo error $< 10^{-3}$):

$$\boxed{\nu_{2n} > 1.2\ (2n+1)\left(\frac{\delta v}{2 v_t}\right)^{2n+1} |k|\, v_t}$$

Landau-Fluid models:

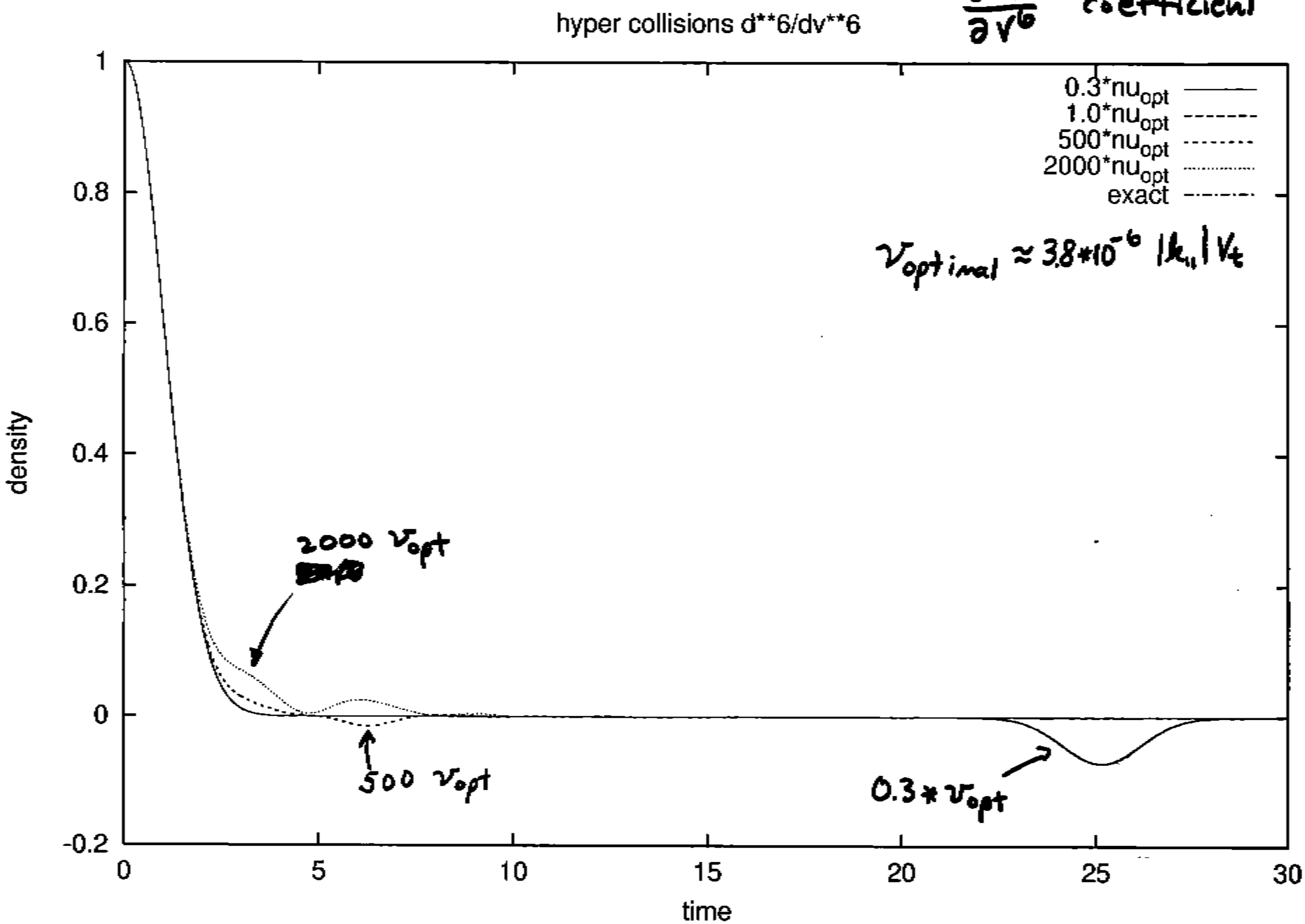
Apply damping $\propto |k|\, v_t$ in the highest Fluid moments ($\approx H_3$ Hermite polynomial $\approx$ hyper collisions with $2n = 4$).

$$\boxed{\nu_6 \gtrsim 4 * 10^{-6}\ |k|\, v_t \quad \text{at} \quad \delta v = \frac{v_t}{4}}$$

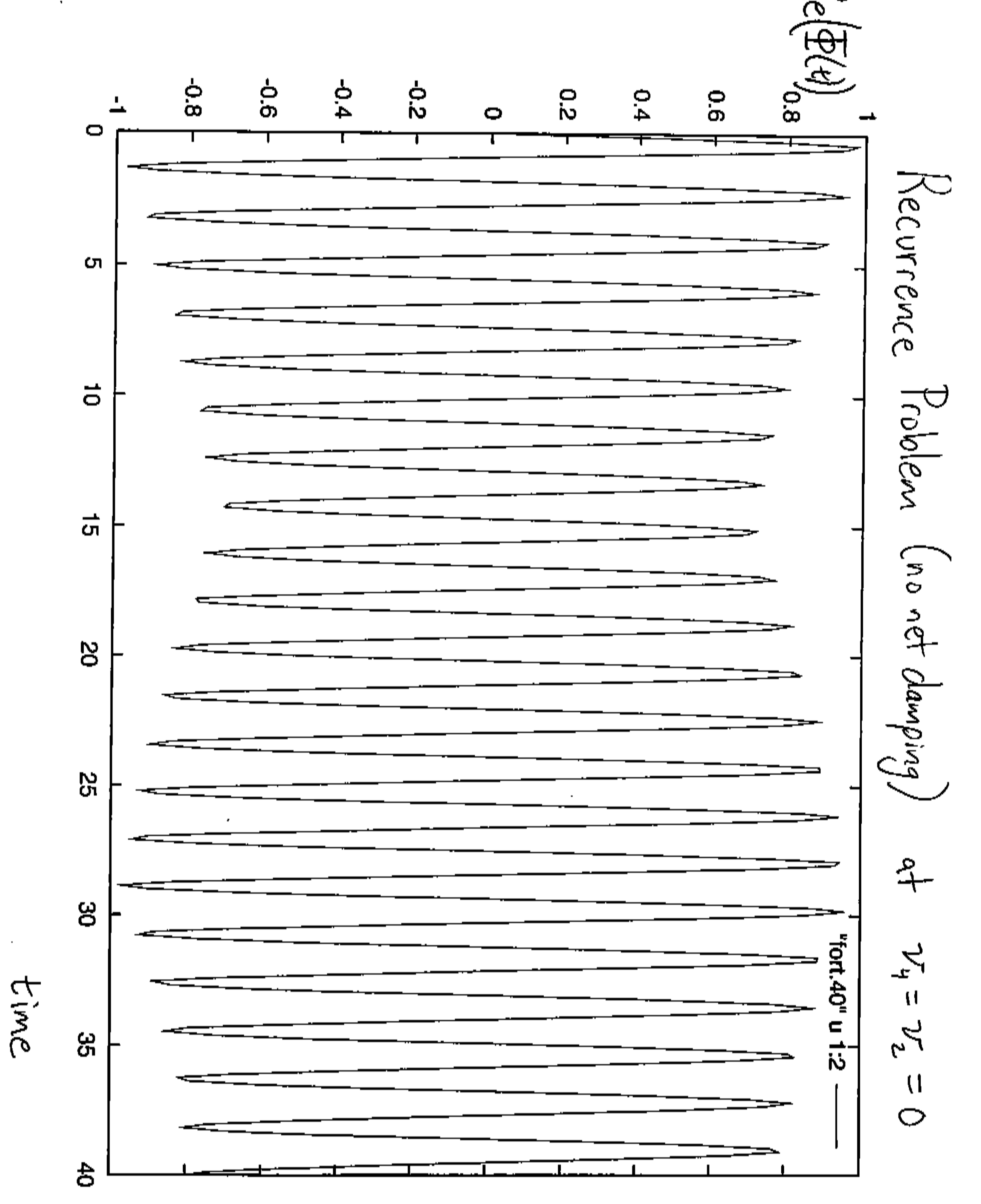
Has very little effect on $\delta v \sim v_t$ scales.

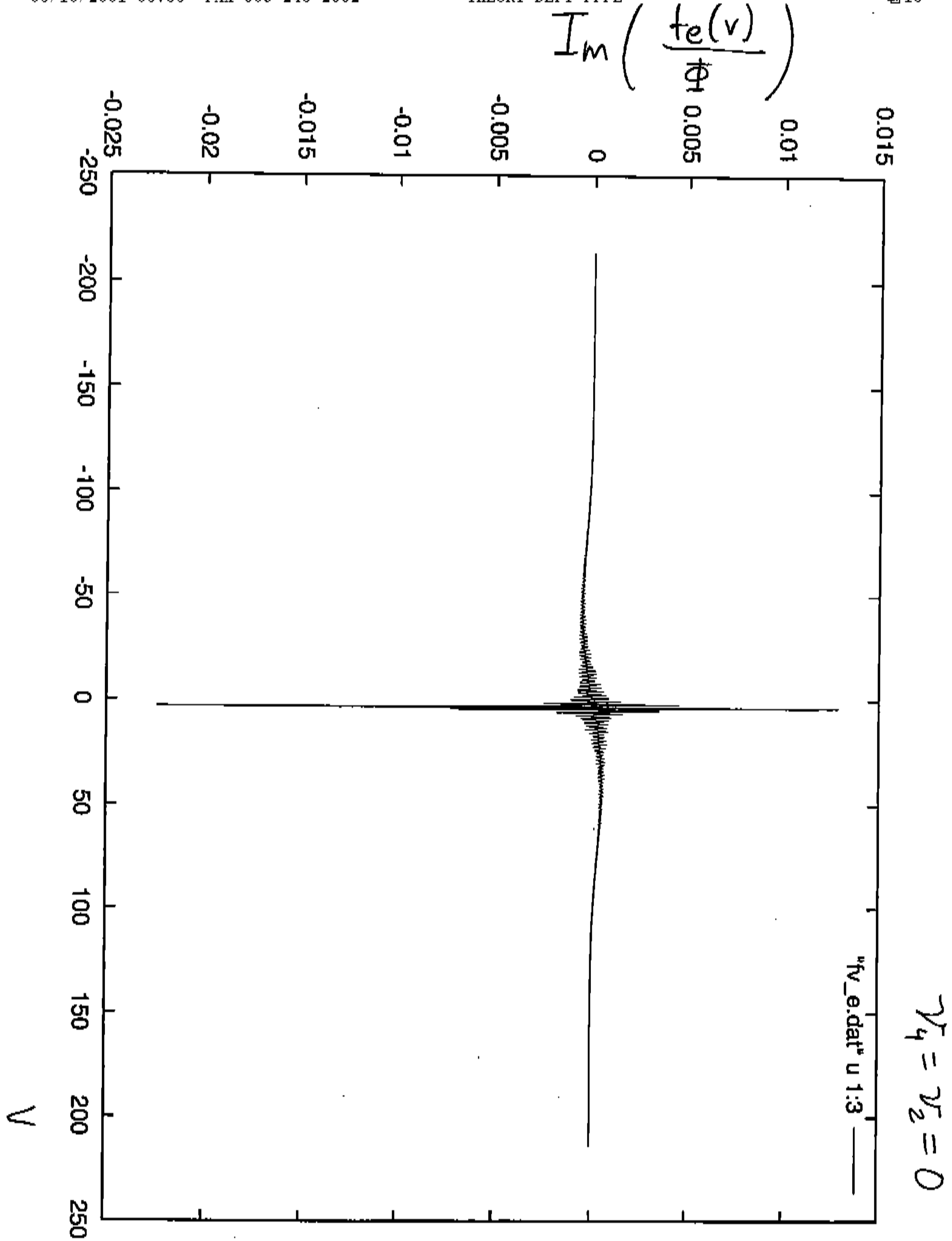
Use $k = k_{max}$ & apply to all modes $\Rightarrow$ avoid FFT's.

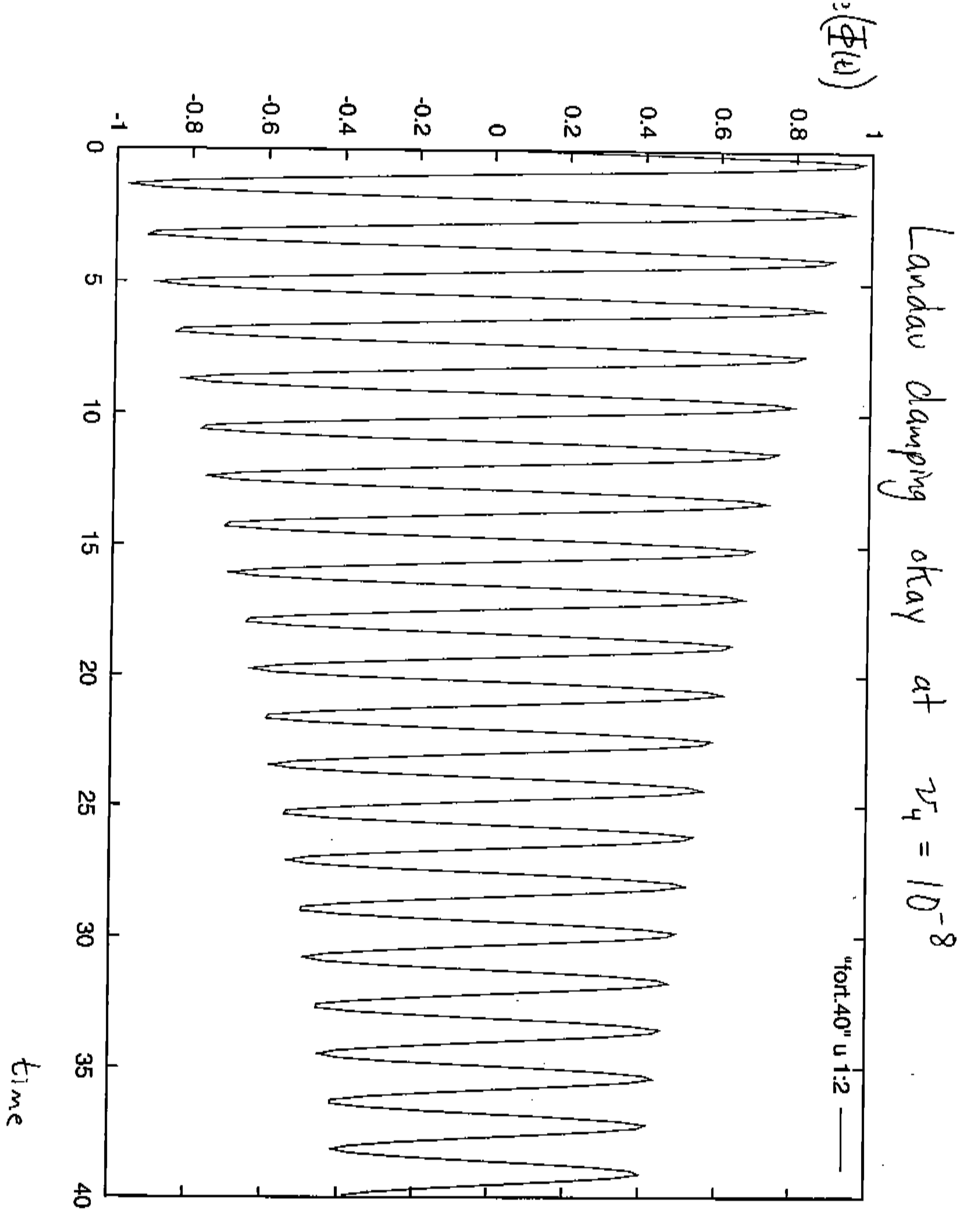
Results insensitive to factor $\sim 10^3$ variation in $\frac{\partial^6 f}{\partial v^6}$ coefficient

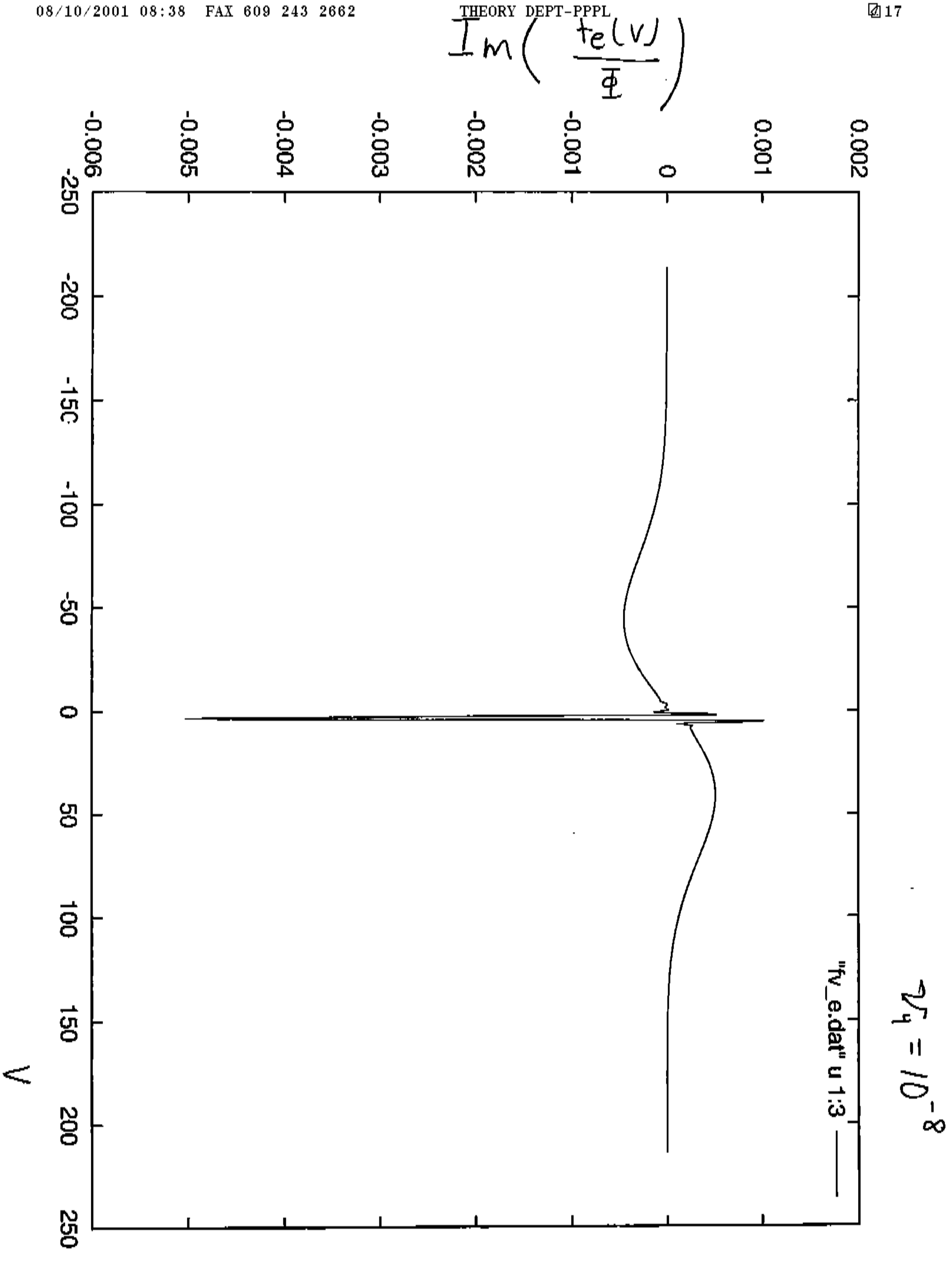


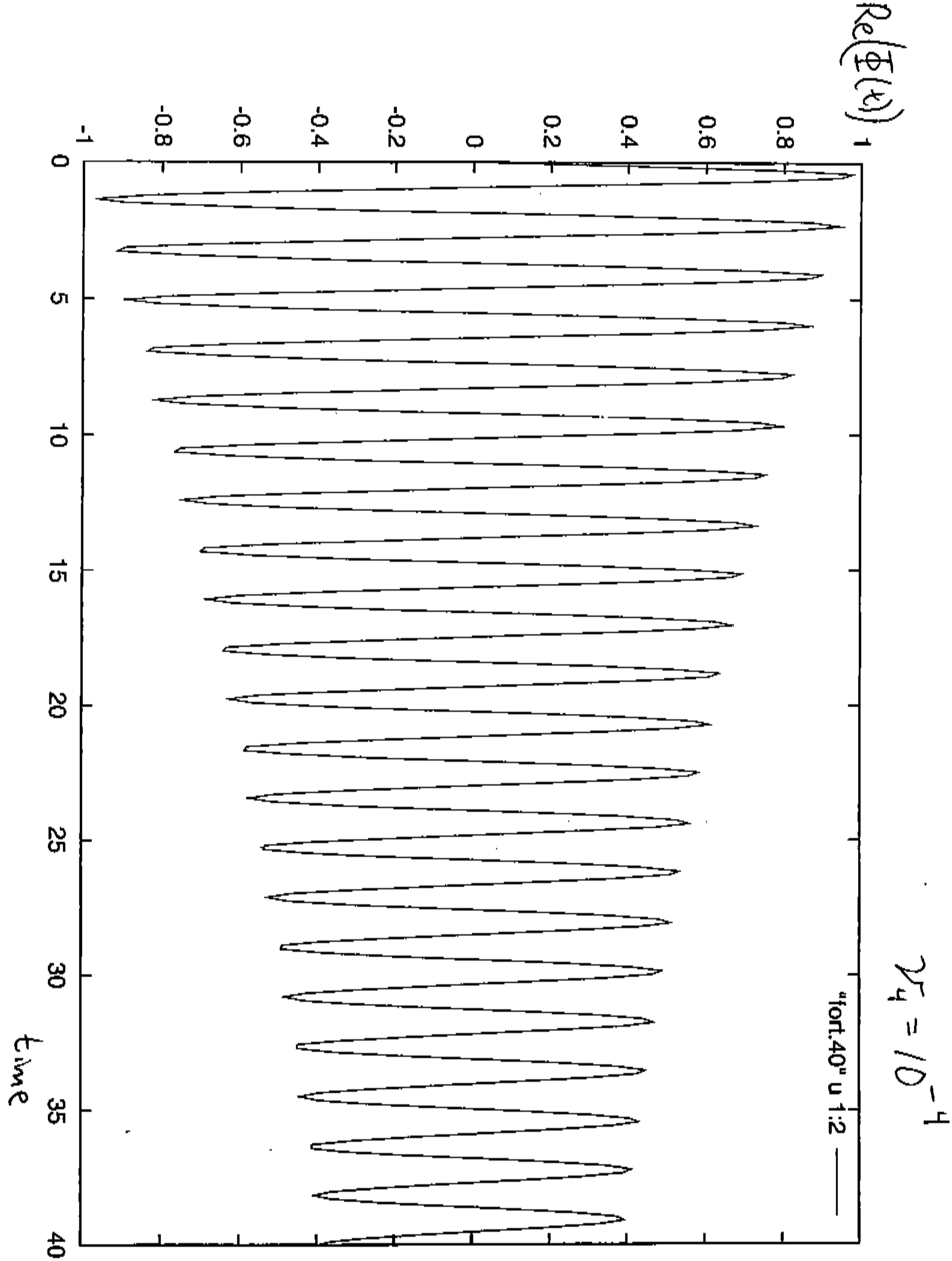
$N_v = 32 \quad \frac{\Delta v}{V_t} \approx \frac{1}{4}$

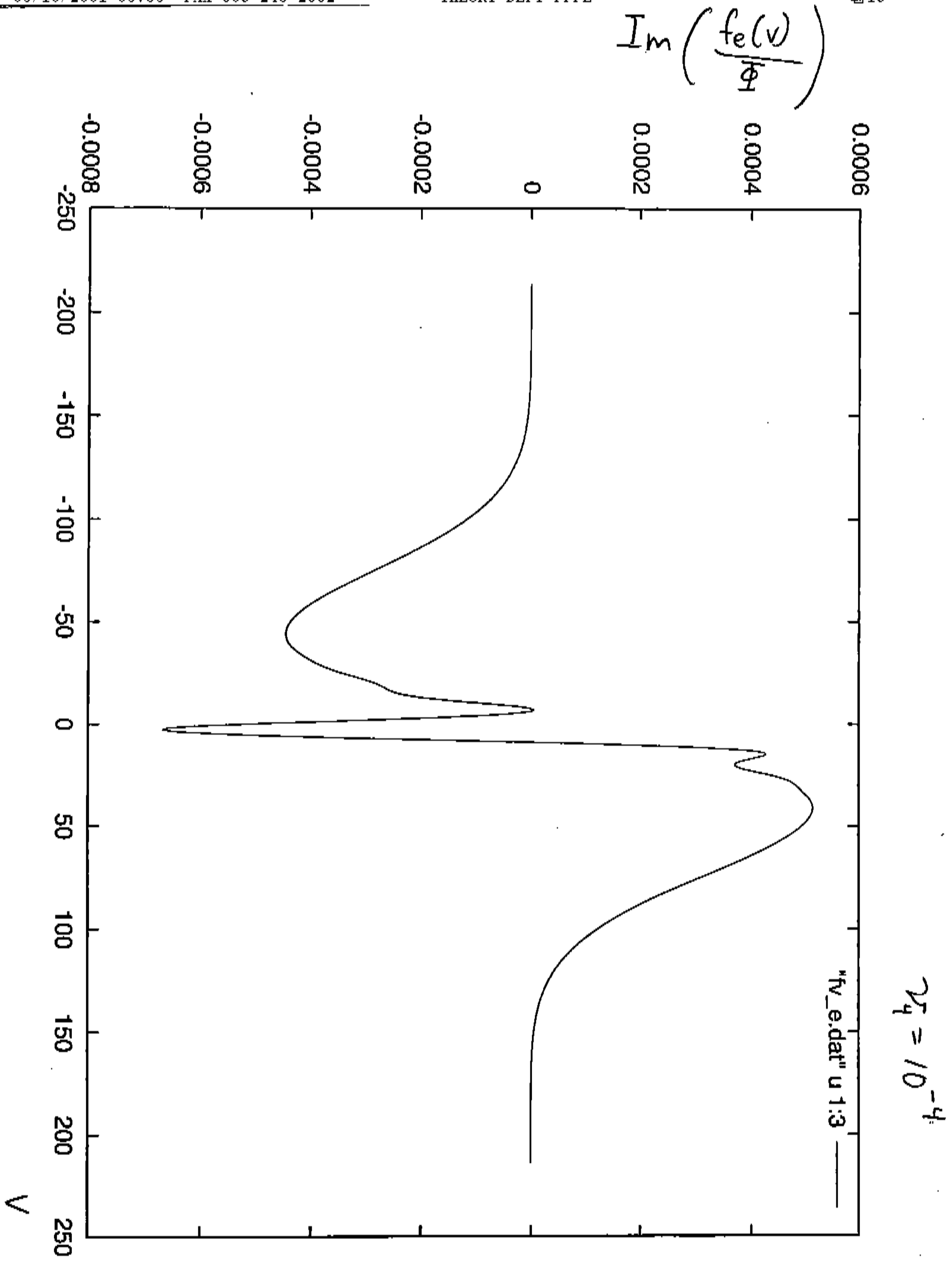

Recurrence Problem (no net damping) at $\nu_4 = \nu_2 = 0$
$e(\Phi(t))$
"fort.40" u 1:2
time
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
0
5
10
15
20
25
30
35
40

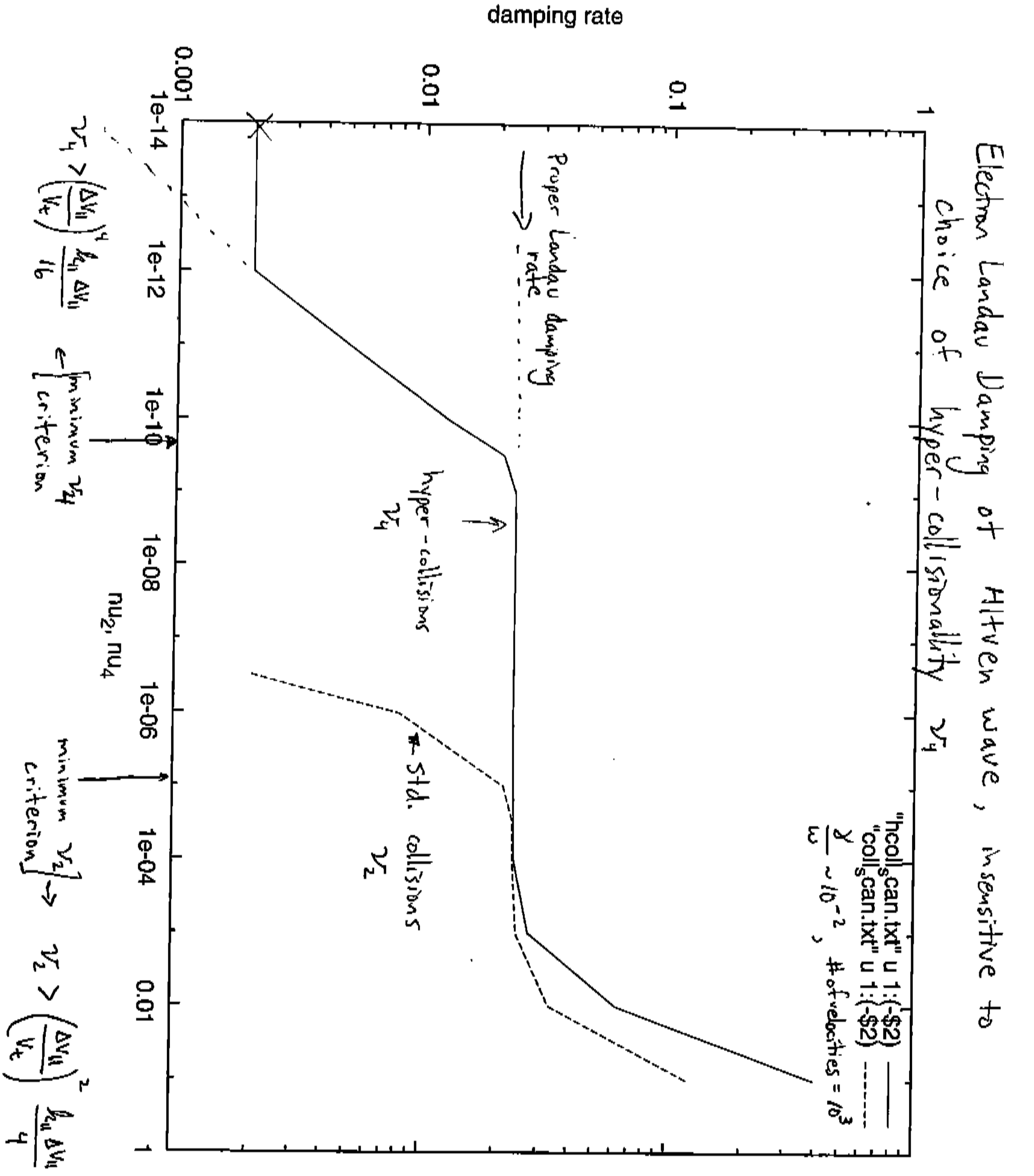

Im(fe(v) / Φ)
-0.025
-0.02
-0.015
-0.01
-0.005
0
0.005
0.01
0.015
-250
-200
-150
-100
-50
0
50
100
150
200
250
"fv_e.dat" u 1:3
ν4 = ν2 = 0
V


Landau damping okay at $\nu_4 = 10^{-8}$
$\Phi(t)$
time
"fort.40" u 1:2
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20
25
30
35
40


Im(fe(v) / Φ)
"fv_e.dat" u 1:3
νq = 10-8
V
-0.006
-0.005
-0.004
-0.003
-0.002
-0.001
0
0.001
0.002
-250
-200
-150
-100
-50
0
50
100
150
200
250


Re(Φ(t))
ν4 = 10^-4
"fort.40" u 1:2
time
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
0
5
10
15
20
25
30
35
40


Im(fe(v) / Φ)
0.0006
0.0004
0.0002
0
-0.0002
-0.0004
-0.0006
-0.0008
-250
-200
-150
-100
-50
0
50
100
150
200
250
V
"fv_e.dat" u 1:3
$\nu_f = 10^{-4}$


Electron Landau Damping of Alfven wave, insensitive to choice of hyper-collisionality ν_4
damping rate
1
0.1
0.01
0.001
1e-14
1e-12
1e-10
1e-08
1e-06
1e-04
0.01
1
nu_2, nu_4
"hcoll_scan.txt" u 1:(-$2)
"coll_scan.txt" u 1:(-$2)
$\frac{\gamma}{\omega} \sim 10^{-2}$, # of velocities = 10^3
Proper Landau damping rate
hyper-collisions ν_4
std. collisions ν_2
minimum ν_4 criterion
minimum ν_2 criterion
$\nu_4 > \left(\frac{\Delta v_\parallel}{v_t}\right)^4 \frac{k_\parallel \Delta v_\parallel}{16}$
$\nu_2 > \left(\frac{\Delta v_\parallel}{v_t}\right)^2 \frac{k_\parallel \Delta v_\parallel}{4}$

$\frac{c}{\omega_{pe}}$ in Jenko/Scott Formulation

1-D electromagnetic

$$\frac{\partial f}{\partial t} + v_\parallel \frac{\partial f}{\partial z} = \left(- \frac{\partial \Phi}{\partial z} - \frac{1}{c} \frac{\partial A_\parallel}{\partial t} \right) \frac{q v_\parallel}{T} F_M$$

Jenko & Scott

$$f = g - \frac{1}{c} A_\parallel \frac{q v_\parallel}{T} F_M$$

$$\frac{\partial g}{\partial t} + v_\parallel \frac{\partial g}{\partial z} = - \frac{\partial \Phi}{\partial z} \frac{q v_\parallel}{T} F_M + v_\parallel^2 \frac{\partial A_\parallel}{\partial z} \frac{q}{cT} F_M$$

Modified Ampere's Law:

$$\left[k_\perp^2 + \frac{1}{c^2} \underbrace{\sum_s \frac{4\pi q^2 \int d^3v\, F_M v_\parallel^2}{T}}_{\text{Analytically} = \omega_{pe}^2} \right] A_\parallel = \frac{4\pi}{c} \sum_s q \int d^3v\, g\, v_\parallel$$

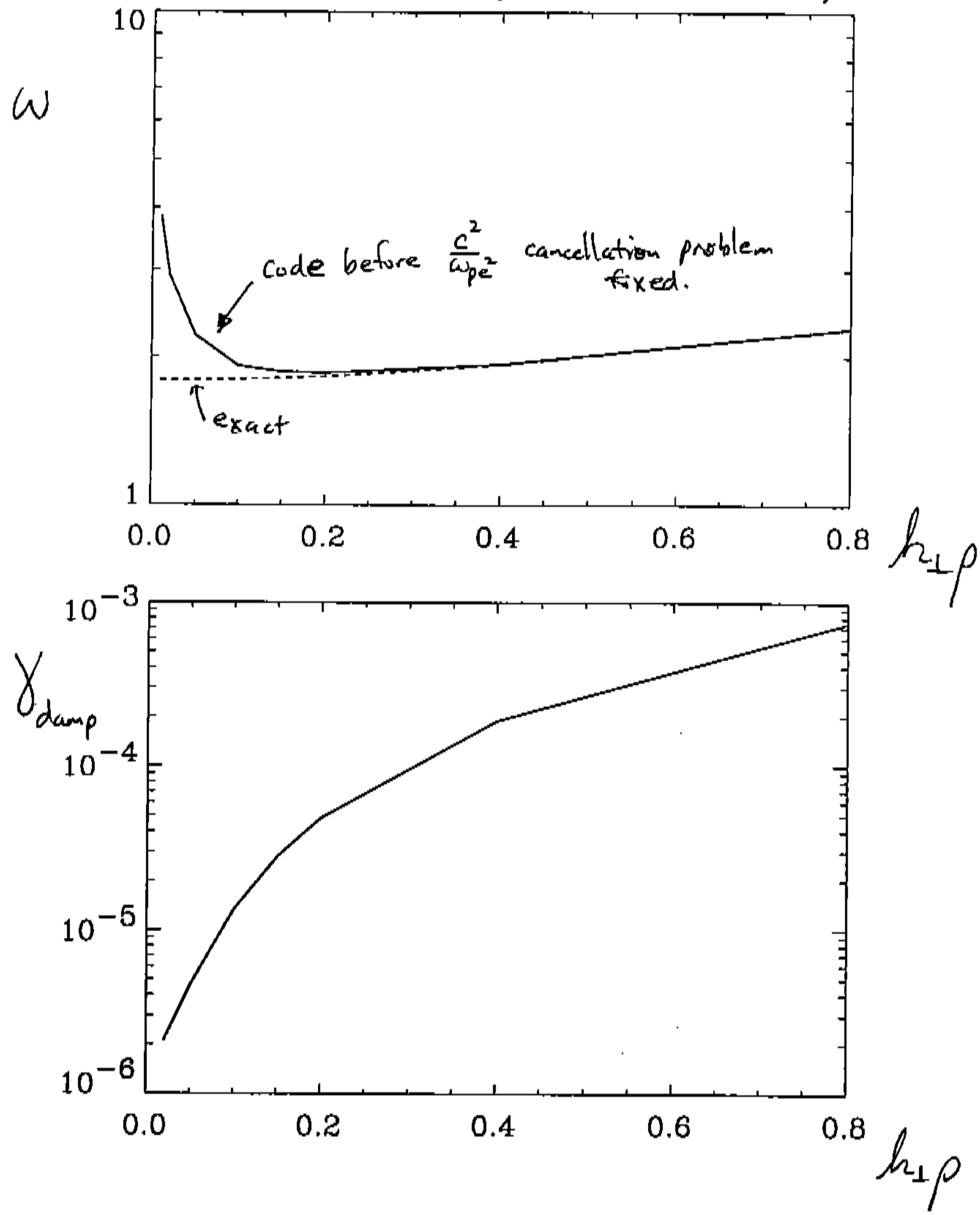
Need to do these integrals on equal footing to get cancellations right.

Fixed low $k_\perp$ problem Jenko pointed out.

Kinetic shear Alfvén waves: MHD limit

$\beta_e = 0.1$

Fully kinetic model (Jenko & Scott 1999)



Conclusions:

* Kotschenreuther's implicit (& fast) algorithm for linear terms recovers plasma dispersion Z function

  if $\frac{\omega \Delta t}{2} \ll 1$ & $\frac{k \Delta z}{2} \ll 1$

  i.e. Courant condition for fastest particles $v_{max} \Delta t < \Delta z$ can be violated & still keep accuracy for lower phase velocity waves.

* Hyper-collisions has little effect on long v scales, eliminates recurrence problems at small v scales, & reduces # of v grid points needed.

* Future work: Alternative noise filters (or projection operators) to further reduce # of v points needed?