Background Information for GS2

I. NONLINEAR GYROKINETIC SIMULATIONS

A. Model equations and numerical techniques

GS2 is based on the electromagnetic nonlinear gyrokinetic equation [1–5]. This equation describes the evolution of fluctuations which satisfy

$$\frac{\tilde{h}}{F_0} \sim \frac{e\tilde{\phi}}{T} \sim \frac{\tilde{A}_\|}{B\rho} \sim \frac{\tilde{B}_\|}{B} \sim \frac{\omega}{\Omega} \sim \frac{\rho}{L} = \epsilon \ll 1\,, \quad k_\| L \sim k_\perp \rho \sim 1\,, \tag{1}$$

where $\tilde{h}$ is the nonadiabatic part of the perturbed distribution function, F_0 is the equilibrium distribution function, $\tilde{\phi}$ and $\tilde{A}_\|$ are the perturbed parts of the electrostatic and parallel vector potential, $\tilde{B}_\|$ is the perturbed parallel magnetic field, B is the equilibrium magnetic field, L is an equilibrium scale length (of density, temperature, or magnetic field), and $\Omega = eB/(mc)$ and $\rho = v_t/\Omega$ are the cyclotron frequency and thermal gyroradius of a given particle species with thermal velocity $v_t^2 = T/m$ and charge e. The simulations are performed in field-line-following coordinates using toroidal flux tubes [6–8]. In such coordinates, the nonlinear gyrokinetic equation may be written as

$$\left(\frac{d}{dt} + v_\| \mathbf{b}\cdot\nabla + i\omega_d\right)\tilde{h} = i\omega_*^T \tilde{\chi} - e\frac{\partial F_0}{\partial \epsilon}\frac{\partial \tilde{\chi}}{\partial t}\,. \tag{2}$$

Here, the distribution function $F_0 = F_0(\epsilon, \Psi)$ depends only on the energy $\epsilon = mv^2/2$ and the flux surface label Ψ, where Ψ is the equilibrium poloidal magnetic flux enclosed by the magnetic surface of interest. The total time derivative is given by $d_t = \partial_t + (c/B)\,[\tilde{\chi}, \cdot]$, where $[\cdot,\cdot]$ is the Poisson bracket. The perpendicular curvature and ∇B drifts are given by $\omega_d = \mathbf{k}_\perp \cdot \mathbf{B} \times \left(mv_\|^2\, \mathbf{b}\cdot\nabla\mathbf{b} + \mu\nabla B\right)/(Bm\Omega)$, where $\mu = mv_\perp^2/(2B)$ and the fields are represented by

$$\tilde{\chi} = J_0(\gamma)\left(\tilde{\phi} - \frac{v_\|}{c}\tilde{A}_\|\right) + \frac{J_1(\gamma)}{\gamma}\frac{mv_\perp^2}{e}\frac{\tilde{B}_\|}{B}\,. \tag{3}$$

Here, $\gamma = k_\perp v_\perp/\Omega$ and $\omega_*^T = n_0 c\,\partial_\Psi F_0$, where n_0 is the toroidal mode number of the perturbation. The self-consistent electromagnetic field fluctuations are computed from the gyrokinetic Poisson-Ampère equations,

$$\nabla_\perp^2 \tilde{\phi} = 4\pi \sum_s e \int d\mathbf{v}\, \left[e\tilde{\phi}\frac{\partial F_0}{\partial \epsilon} + J_0(\gamma)\tilde{h}\right]\,, \tag{4}$$

$$\nabla_\perp^2 \tilde{A}_\| = -\frac{4\pi}{c}\sum_s \int d\mathbf{v}\, e v_\| J_0(\gamma)\tilde{h}\,, \tag{5}$$

$$\frac{\tilde{B}_\|}{B} = -\frac{4\pi}{B^2}\sum_s \int d\mathbf{v}\, mv_\perp^2 \frac{J_1(\gamma)}{\gamma}\tilde{h}\,. \tag{6}$$

The Bessel functions J_0 and J_1 arise because Eqs. (4-6) are formulated in particle space $\mathbf{x}$, rather than in gyrocenter space $\mathbf{R}$. We retain the Debye-shielding term $\nabla_\perp^2\tilde{\phi}$ in Poisson's equation, since the electron Debye length λ_{De} can be comparable to ρ_e in laboratory fusion experiments. Of course, this term can be neglected when only ion-scale instabilities are studied.

GS2 is a nonlinear generalization of a widely used gyrokinetic stability code [9]. An operator splitting scheme is used, so that the linear terms [including Eqs. (4–6)] may be treated implicitly [9]. The nonlinear terms are evaluated with a dealiased pseudospectral algorithm in the plane perpendicular to the field line. A second-order Adams-Bashforth scheme is used to advance the nonlinear terms in time. Non-uniform coordinate meshes are used in velocity space to improve the resolution, particularly for the trapped-passing boundary. A small amount of upwind diffusion is typically used, only in the direction along the field line. In the absence of upwind diffusion, the algorithm is second-order accurate in space and time. Good parallel performance is achieved by employing multiple-domain decomposition in four of the five dimensions at all times.

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