AS551 General Plasma Physics 1
Dec. 12., 2003

Problem Set # 10 (due Friday Jan 16, 2004)

G&R refers to Goldston and Rutherford's textbook.

1. G&R problem 11.4. (describe process of approaching thermal equilibrium)

2. G&R problem 12.1. (solutions to diffusion equation)

3. Collisional transport in air. Work out a rough estimate of the collisional diffusion coefficient for air (this is often called "molecular diffusion"), using the collision frequency, mean free path, and thermal speed of molecules under typical atmospheric conditions. Compare this to an estimate of the diffusion coefficient due to turbulent air motions observed outside on a typical afternoon (assuming the turbulence corresponds to a random walk process with $\Delta x \sim 10m$ and $\Delta t \sim \Delta x / v_{turb}$ with $v_{turb} \sim 10km/hour$. You are in charge of homeland security in a certain city. Using this estimate of the turbulent diffusion coefficient, roughly how long would it take for an aerosol released in the air to be spread over an area of 1 square kilometer (after being spread over such a large volume, the toxicity is assumed to be reduced to acceptable levels...).

4. Fusion random walk estimates. Particle diffusion coefficients roughly of order $D = 1m^2/s$ are typical of tokamaks. Fluctuations are observed in the plasma with a time scale of order $\Delta t \approx 10^{-5}s$. Assuming this is the step time of some random walk process, what is the step size Δx needed to explain the observed D? Assuming this is due to randomly fluctuating $E \times B$ drifts, so that $v_{E\times B} \approx \Delta x/\Delta t$, use this to estimate the ratio $v_{E\times B}/v_{ti}$ (where v_{ti} is the thermal ion speed) for a typical 10 keV fusion plasma.

5. Steady-state diffusion with a source. Particle transport in a cylinder of plasma of radius a is modelled with the equation

$$\frac{\partial n}{\partial t} = D\nabla^2 n + S$$

where the particle diffusion coefficient D and the particle fuelling rate S are constants independent of time or space. Find the steady-state solution $n(r)$ to this equation in cylindrical geometry with the boundary condition that $n(r = a) = 0$ (where plasma particles are assumed to be lost to the wall). Define the integrated particle density by $N = \int dr 2\pi r n$, and define the average particle confinement time by $\tau_p = N/(S\pi a^2)$ (or in other words, the total fuelling rate must be $S\pi a^2 = N/\tau_p$ to maintain the plasma against diffusive losses). Show how τ_p can be expressed in terms of D and a.

5. G&R problem 12.6. (Effect of ions with charge Z on transport coefficients) (Approximate answers using scalings from random-walk arguments are sufficient here.)

6. G&R problem 23.4. Kinetic ion acoustic wave.

7. G&R problem 23.5. Another version of a two-stream instability.