Investigate Langevin nu_eff, including Krommes trick.

```
> restart; kernelopts(version); interface(version);
```

Maple 7.00, IBM INTEL LINUX, May 28 2001 Build ID 96223

Maple Worksheet Interface, Maple 7.00, IBM INTEL LINUX, May 28 2001 Build ID 96223

```
> # We are working with the Langevin equation of the following form.
  # Note that the conjugate(f) convention is being used, to make it
  similar to
  # the practice in the DIA/EDQNM/RMC.
  #
  # Note that the damping rate is denoted by eta in the paper, but
  by nu here.

  diff(psi(t),t) = - nu * psi + conjugate(f);
```

$$\frac{\partial}{\partial t}\psi(t)=-\nu\,\psi+\bar{f}$$

```
> # Here is C0/(nu_eff+conjugate(nu_eff)) :

  e1 := (nu_f
  +conjugate(nu_f))/(conjugate(nu_f)-nu)*(1/(nu+conjugate(nu_eff))/(
  nu+conjugate(nu))/(nu+nu_f)

  -1/(conjugate(nu_f)+conjugate(nu_eff))/(nu_f+conjugate(nu_f))/(con
  jugate(nu_f)+conjugate(nu)) );
```

$$e1 := \frac{(nu\_f+\overline{nu\_f})\left(\frac{1}{(\nu+\overline{nu\_eff})\,(\nu+\bar{\nu})\,(\nu+nu\_f)}-\frac{1}{(\overline{nu\_f}+\overline{nu\_eff})\,(nu\_f+\overline{nu\_f})\,(\overline{nu\_f}+\bar{\nu})}\right)}{\overline{nu\_f}-\nu}$$

```
> # simplify(%); # too complex
> # Here is C0:

  c0 := (nu_f
  +conjugate(nu_f))/(conjugate(nu_f)-nu)*(1/(nu+conjugate(nu))/(nu+n
  u_f)

  -1/(nu_f+conjugate(nu_f))/(conjugate(nu_f)+conjugate(nu)) );
```

$$c0 := \frac{(nu\_f+\overline{nu\_f})\left(\frac{1}{(\nu+\bar{\nu})\,(\nu+nu\_f)}-\frac{1}{(nu\_f+\overline{nu\_f})\,(\overline{nu\_f}+\bar{\nu})}\right)}{\overline{nu\_f}-\nu}$$

```
> # simplify(e1/c0);  # too long
> # My own calculation (which apparently has an error in it)
  # gives 1/(nu_eff+conjugate(nu_eff)) as:

  e10 := ( ( nu_f + conjugate(nu) ) * ( nu_f + conjugate(nu_f) +
  conjugate(nu_eff) )
```

```
        +( nu + conjugate(nu_f) ) * ( nu   + conjugate(nu)   +
conjugate(nu_eff) )
        + nu * nu_f - conjugate(nu) * conjugate(nu_f) )
      / (nu + conjugate(nu) + nu_f + conjugate(nu_f) ) / (nu +
conjugate(nu_eff)) / (nu_f + conjugate(nu_eff))
;
```

$$e10 := \frac{(nu\_f+\overline{\nu})\,(nu\_f+\overline{nu\_f}+\overline{nu\_eff})+(\nu+\overline{nu\_f})\,(\nu+\overline{\nu}+\overline{nu\_eff})+\nu\; nu\_f-\overline{\nu}\;\overline{nu\_f}}{(\nu+\overline{\nu}+nu\_f+\overline{nu\_f})\,(\nu+\overline{nu\_eff})\,(nu\_f+\overline{nu\_eff})}$$

```
> e10 - e1/c0;  # If e10 is right, then this should be zero.
```

$$\frac{(nu\_f+\overline{\nu})\,(nu\_f+\overline{nu\_f}+\overline{nu\_eff})+(\nu+\overline{nu\_f})\,(\nu+\overline{\nu}+\overline{nu\_eff})+\nu\; nu\_f-\overline{\nu}\;\overline{nu\_f}}{(\nu+\overline{\nu}+nu\_f+\overline{nu\_f})\,(\nu+\overline{nu\_eff})\,(nu\_f+\overline{nu\_eff})}$$

$$-\frac{\dfrac{1}{(\nu+\overline{nu\_eff})\,(\nu+\overline{\nu})\,(\nu+nu\_f)}-\dfrac{1}{(nu\_f+\overline{nu\_eff})\,(nu\_f+\overline{nu\_f})\,(nu\_f+\overline{\nu})}}{\dfrac{1}{(\nu+\overline{\nu})\,(\nu+nu\_f)}-\dfrac{1}{(nu\_f+\overline{nu\_f})\,(nu\_f+\overline{\nu})}}$$

```
> # simplify(%);  # This calculation takes enormously long...
> e10*c0;
```

$$((nu\_f+\overline{\nu})\,(nu\_f+\overline{nu\_f}+\overline{nu\_eff})+(\nu+\overline{nu\_f})\,(\nu+\overline{\nu}+\overline{nu\_eff})+\nu\; nu\_f-\overline{\nu}\;\overline{nu\_f})\,(nu\_f+\overline{nu\_f})$$
$$\left(\frac{1}{(\nu+\overline{\nu})\,(\nu+nu\_f)}-\frac{1}{(nu\_f+\overline{nu\_f})\,(nu\_f+\overline{\nu})}\right)\Big/\;((\nu+\overline{\nu}+nu\_f+\overline{nu\_f})\,(\nu+\overline{nu\_eff})$$
$$(nu\_f+\overline{nu\_eff})\,(\overline{nu\_f}-\nu))$$

```
> # After trying lots of things, I eventually decided to try to
guide Maple in repeating the
  # calculation I did by hand, by having it do polynomial division
(there is a "divide" command).
  # However, in the process of putting it in a "normal" form
(polynomial numerator / polynomial denominator)
  # I discovered that this automatically simplifies it, eliminating
the apparent singularity!

  e12 := normal(e1/c0);
```

$$e12 := (\nu^2+\overline{nu\_eff}\,\nu+\nu\; nu\_f+\nu\,\overline{\nu}+\nu\,\overline{nu\_f}+\overline{nu\_eff}\; nu\_f+\overline{nu\_eff}\,\overline{\nu}+nu\_f\,\overline{\nu}+\overline{nu\_eff}\;\overline{nu\_f}$$
$$+\overline{nu\_f}\; nu\_f+\overline{\nu}\;\overline{nu\_f}+\overline{nu\_f}^{\,2})\Big/\;((\nu+\overline{\nu}+nu\_f+\overline{nu\_f})\,(nu\_f+\overline{nu\_eff})\,(\nu+\overline{nu\_eff}))$$

```
> # Check the real limit:
  e14 := subs(nu=g, nu_f=g_f, nu_eff=g_eff, e12);
```

$$e14 := ($$
$$g^2+\overline{g\_eff}\,g+g\,g\_f+g\,\overline{g}+g\,\overline{g\_f}+\overline{g\_eff}\,g\_f+\overline{g\_eff}\,\overline{g}+g\_f\,\overline{g}+\overline{g\_eff}\,\overline{g\_f}+\overline{g\_f}\,g\_f+\overline{g}\,\overline{g\_f}+\overline{g\_f}^{\,2})$$
$$\Big/\;((g+\overline{g}+g\_f+\overline{g\_f})\,(\overline{g\_f}+\overline{g\_eff})\,(g+\overline{g\_eff}))$$

```
> e16 := simplify(evalc(%));
```

$$e16 := \frac{g + g\_f + g\_eff}{(g + g\_eff)\,(g\_f + g\_eff)}$$

```
> e17 := numer(%)*2*g_eff-denom(%);
```

$$e17 := 2\,(g + g\_f + g\_eff)\,g\_eff - (g + g\_eff)\,(g\_f + g\_eff)$$

```
> e18 := factor(e17);
```

$$e18 := g\_eff\,g + g\_eff\,g\_f + g\_eff^2 - g\,g\_f$$

```
> # This is indeed equivalent to  g_eff = g*g_f/(g+g_f_g_eff)

> e19 := solve(e18=0, g_eff);
```

$$e19 := -\frac{1}{2}\,g - \frac{1}{2}\,g\_f + \frac{1}{2}\sqrt{g^2 + 6\,g\,g\_f + g\_f^2},\ -\frac{1}{2}\,g - \frac{1}{2}\,g\_f - \frac{1}{2}\sqrt{g^2 + 6\,g\,g\_f + g\_f^2}$$

```
> # Now return to the general case with complex coefficients:

  e20 := numer(e12)*(nu_eff+conjugate(nu_eff))-denom(e12);
```

$$e20 := (\nu^2 + \overline{nu\_eff}\,\nu + \nu\,nu\_f + \nu\,\overline{\nu} + \nu\,\overline{nu\_f} + \overline{nu\_eff}\,nu\_f + \overline{nu\_eff}\,\overline{\nu} + nu\_f\,\overline{\nu} + \overline{nu\_eff}\,\overline{nu\_f} + \overline{nu\_f}\,nu\_f + \overline{\nu}\,\overline{nu\_f} + \overline{nu\_f}^2)\,(nu\_eff + \overline{nu\_eff}) - (\nu + \overline{\nu} + nu\_f + \overline{nu\_f})\,(\overline{nu\_f} + \overline{nu\_eff})\,(\nu + \overline{nu\_eff})$$

```
> simplify(%);
```

$$nu\_f\,\overline{\nu}\,nu\_eff + \nu\,\overline{nu\_f}\,nu\_eff + \nu^2\,nu\_eff + \overline{nu\_f}^2\,nu\_eff - \nu^2\,\overline{nu\_f} - \overline{nu\_f}^2\,\nu + \overline{nu\_eff}\,\overline{nu\_f}\,nu\_eff - \overline{nu\_f}\,nu\_f\,\nu + \overline{nu\_eff}\,\overline{\nu}\,nu\_eff + \overline{\nu}\,\overline{nu\_f}\,nu\_eff + \overline{nu\_f}\,nu\_f\,nu\_eff - \nu\,\overline{nu\_f}\,\overline{nu\_eff} + \overline{nu\_eff}\,nu\_f\,nu\_eff + \nu\,\overline{\nu}\,nu\_eff + \nu\,nu\_f\,nu\_eff + \overline{nu\_eff}\,\nu\,nu\_eff + nu\_f\,\overline{\nu}\,\overline{nu\_eff} - \overline{\nu}\,\overline{nu\_f}\,\nu$$

```
> expand(e20);
```

$$nu\_f\,\overline{\nu}\,nu\_eff + \nu\,\overline{nu\_f}\,nu\_eff + \nu^2\,nu\_eff + \overline{nu\_f}^2\,nu\_eff - \nu^2\,\overline{nu\_f} - \overline{nu\_f}^2\,\nu + \overline{nu\_eff}\,\overline{nu\_f}\,nu\_eff - \overline{nu\_f}\,nu\_f\,\nu + \overline{nu\_eff}\,\overline{\nu}\,nu\_eff + \overline{\nu}\,\overline{nu\_f}\,nu\_eff + \overline{nu\_f}\,nu\_f\,nu\_eff - \nu\,\overline{nu\_f}\,\overline{nu\_eff} + \overline{nu\_eff}\,nu\_f\,nu\_eff + \nu\,\overline{\nu}\,nu\_eff + \nu\,nu\_f\,nu\_eff + \overline{nu\_eff}\,\nu\,nu\_eff + nu\_f\,\overline{\nu}\,\overline{nu\_eff} - \overline{\nu}\,\overline{nu\_f}\,\nu$$

```
> factor(e20);
```

$$nu\_f\,\overline{\nu}\,nu\_eff + \nu\,\overline{nu\_f}\,nu\_eff + \nu^2\,nu\_eff + \overline{nu\_f}^2\,nu\_eff - \nu^2\,\overline{nu\_f} - \overline{nu\_f}^2\,\nu + \overline{nu\_eff}\,\overline{nu\_f}\,nu\_eff - \overline{nu\_f}\,nu\_f\,\nu + \overline{nu\_eff}\,\overline{\nu}\,nu\_eff + \overline{\nu}\,\overline{nu\_f}\,nu\_eff + \overline{nu\_f}\,nu\_f\,nu\_eff - \nu\,\overline{nu\_f}\,\overline{nu\_eff} + \overline{nu\_eff}\,nu\_f\,nu\_eff + \nu\,\overline{\nu}\,nu\_eff + \nu\,nu\_f\,nu\_eff + \overline{nu\_eff}\,\nu\,nu\_eff + nu\_f\,\overline{\nu}\,\overline{nu\_eff} - \overline{\nu}\,\overline{nu\_f}\,\nu$$

```
> factor(numer(e12));
```

$$\nu^2 + \overline{nu\_eff}\,\nu + \nu\,nu\_f + \nu\,\overline{\nu} + \nu\,\overline{nu\_f} + \overline{nu\_eff}\,nu\_f + \overline{nu\_eff}\,\overline{\nu} + nu\_f\,\overline{\nu} + \overline{nu\_eff}\,\overline{nu\_f} + \overline{nu\_f}\,nu\_f + \overline{\nu}\,\overline{nu\_f} + \overline{nu\_f}^2$$

```
> simplify(%);
```

$$\nu^2 + \overline{nu\_eff}\,\nu + \nu\,nu\_f + \nu\,\overline{\nu} + \nu\,\overline{nu\_f} + \overline{nu\_eff}\,nu\_f + \overline{nu\_eff}\,\overline{\nu} + nu\_f\,\overline{\nu} + \overline{nu\_eff}\,\overline{nu\_f} + \overline{nu\_f}\,nu\_f + \overline{\nu}\,\overline{nu\_f} + \overline{nu\_f}^2$$

```
> # Because both nu_eff and conjugate(nu_eff) appear in the
  equations, it isn't straightforward
  # to solve for nu_eff.  Denote conjugate(nu_eff) as a special
```

```
symbol, and then solve for
# nu_eff:

e30 := subs(conjugate(nu_eff)=nu_effcc, e20);
```

$$e30 := (\nu^2 + \mathit{nu\_effcc}\,\nu + \nu\,\mathit{nu\_f} + \nu\,\bar{\nu} + \nu\,\overline{\mathit{nu\_f}} + \mathit{nu\_effcc}\,\mathit{nu\_f} + \mathit{nu\_effcc}\,\bar{\nu} + \mathit{nu\_f}\,\bar{\nu} + \mathit{nu\_effcc}\,\overline{\mathit{nu\_f}} + \overline{\mathit{nu\_f}}\,\mathit{nu\_f} + \bar{\nu}\,\overline{\mathit{nu\_f}} + \overline{\mathit{nu\_f}}^2)\,(\mathit{nu\_eff} + \mathit{nu\_effcc}) - (\nu + \bar{\nu} + \mathit{nu\_f} + \overline{\mathit{nu\_f}})\,(\overline{\mathit{nu\_f}} + \mathit{nu\_effcc})\,(\nu + \mathit{nu\_effcc})$$

```
> e31 := solve(e30=0, nu_eff);
```

$$e31 := -(-\overline{\mathit{nu\_f}}^2\,\nu + \mathit{nu\_f}\,\bar{\nu}\,\mathit{nu\_effcc} - \nu\,\overline{\mathit{nu\_f}}\,\mathit{nu\_effcc} - \bar{\nu}\,\overline{\mathit{nu\_f}}\,\nu - \nu^2\,\overline{\mathit{nu\_f}} - \overline{\mathit{nu\_f}}\,\mathit{nu\_f}\,\nu) \Big/ (\nu^2 + \mathit{nu\_effcc}\,\nu + \nu\,\mathit{nu\_f} + \nu\,\bar{\nu} + \nu\,\overline{\mathit{nu\_f}} + \mathit{nu\_effcc}\,\mathit{nu\_f} + \mathit{nu\_effcc}\,\bar{\nu} + \mathit{nu\_f}\,\bar{\nu} + \mathit{nu\_effcc}\,\overline{\mathit{nu\_f}} + \overline{\mathit{nu\_f}}\,\mathit{nu\_f} + \bar{\nu}\,\overline{\mathit{nu\_f}} + \overline{\mathit{nu\_f}}^2)$$

```
> ##############################################################
# e32 is our main answer for nu_eff as a function of nu and nu_eff
# (it is a recursive definition, since conjugate(nu_eff) appears
on the RHS,
# but it can probably be solved by just iterating a few times...)

e32 := subs(nu_effcc=conjugate(nu_eff), e31);
```

$$e32 := -(-\overline{\mathit{nu\_f}}^2\,\nu + \mathit{nu\_f}\,\bar{\nu}\,\overline{\mathit{nu\_eff}} - \nu\,\overline{\mathit{nu\_f}}\,\overline{\mathit{nu\_eff}} - \bar{\nu}\,\overline{\mathit{nu\_f}}\,\nu - \nu^2\,\overline{\mathit{nu\_f}} - \overline{\mathit{nu\_f}}\,\mathit{nu\_f}\,\nu) \Big/ (\nu^2 + \overline{\mathit{nu\_eff}}\,\nu + \nu\,\mathit{nu\_f} + \nu\,\bar{\nu} + \nu\,\overline{\mathit{nu\_f}} + \overline{\mathit{nu\_eff}}\,\mathit{nu\_f} + \overline{\mathit{nu\_eff}}\,\bar{\nu} + \mathit{nu\_f}\,\bar{\nu} + \overline{\mathit{nu\_eff}}\,\overline{\mathit{nu\_f}} + \overline{\mathit{nu\_f}}\,\mathit{nu\_f} + \bar{\nu}\,\overline{\mathit{nu\_f}} + \overline{\mathit{nu\_f}}^2)$$

```
>
simplify(%);
```

$$-(-\overline{\mathit{nu\_f}}^2\,\nu + \mathit{nu\_f}\,\bar{\nu}\,\overline{\mathit{nu\_eff}} - \nu\,\overline{\mathit{nu\_f}}\,\overline{\mathit{nu\_eff}} - \bar{\nu}\,\overline{\mathit{nu\_f}}\,\nu - \nu^2\,\overline{\mathit{nu\_f}} - \overline{\mathit{nu\_f}}\,\mathit{nu\_f}\,\nu) \Big/ (\nu^2 + \overline{\mathit{nu\_eff}}\,\nu + \nu\,\mathit{nu\_f} + \nu\,\bar{\nu} + \nu\,\overline{\mathit{nu\_f}} + \overline{\mathit{nu\_eff}}\,\mathit{nu\_f} + \overline{\mathit{nu\_eff}}\,\bar{\nu} + \mathit{nu\_f}\,\bar{\nu} + \overline{\mathit{nu\_eff}}\,\overline{\mathit{nu\_f}} + \overline{\mathit{nu\_f}}\,\mathit{nu\_f} + \bar{\nu}\,\overline{\mathit{nu\_f}} + \overline{\mathit{nu\_f}}^2)$$

```
> factor(denom(e32));
```

$$\nu^2 + \overline{\mathit{nu\_eff}}\,\nu + \nu\,\mathit{nu\_f} + \nu\,\bar{\nu} + \nu\,\overline{\mathit{nu\_f}} + \overline{\mathit{nu\_eff}}\,\mathit{nu\_f} + \overline{\mathit{nu\_eff}}\,\bar{\nu} + \mathit{nu\_f}\,\bar{\nu} + \overline{\mathit{nu\_eff}}\,\overline{\mathit{nu\_f}} + \overline{\mathit{nu\_f}}\,\mathit{nu\_f} + \bar{\nu}\,\overline{\mathit{nu\_f}} + \overline{\mathit{nu\_f}}^2$$

```
> # check e32 in the real limit:
simplify(evalc(e32));
```

$$\frac{\nu\,\mathit{nu\_f}}{\nu + \mathit{nu\_eff} + \mathit{nu\_f}}$$

```
> factor(numer(e32));
```

$$\overline{\mathit{nu\_f}}^2\,\nu - \mathit{nu\_f}\,\bar{\nu}\,\overline{\mathit{nu\_eff}} + \nu\,\overline{\mathit{nu\_f}}\,\overline{\mathit{nu\_eff}} + \bar{\nu}\,\overline{\mathit{nu\_f}}\,\nu + \nu^2\,\overline{\mathit{nu\_f}} + \overline{\mathit{nu\_f}}\,\mathit{nu\_f}\,\nu$$

```
> simplify(%);
```

$$\overline{nu\_f}^{2}\, \nu - nu\_f\, \overline{\nu}\, \overline{nu\_eff} + \nu\, \overline{nu\_f}\, \overline{nu\_eff} + \overline{\nu}\, \overline{nu\_f}\, \nu + \nu^{2}\, \overline{nu\_f} + \overline{nu\_f}\, nu\_f\, \nu$$

```
> denom(e32);
```

$$\nu^{2} + \overline{nu\_eff}\, \nu + \nu\, nu\_f + \nu\, \overline{\nu} + \nu\, \overline{nu\_f} + \overline{nu\_eff}\, nu\_f + \overline{nu\_eff}\, \overline{\nu} + nu\_f\, \overline{\nu} + \overline{nu\_eff}\, \overline{nu\_f} + \overline{nu\_f}\, nu\_f$$
$$+ \overline{\nu}\, \overline{nu\_f} + \overline{nu\_f}^{2}$$

```
> simplify(%);
```

$$\nu^{2} + \overline{nu\_eff}\, \nu + \nu\, nu\_f + \nu\, \overline{\nu} + \nu\, \overline{nu\_f} + \overline{nu\_eff}\, nu\_f + \overline{nu\_eff}\, \overline{\nu} + nu\_f\, \overline{\nu} + \overline{nu\_eff}\, \overline{nu\_f} + \overline{nu\_f}\, nu\_f$$
$$+ \overline{\nu}\, \overline{nu\_f} + \overline{nu\_f}^{2}$$

```
> factor(%);
```

$$\nu^{2} + \overline{nu\_eff}\, \nu + \nu\, nu\_f + \nu\, \overline{\nu} + \nu\, \overline{nu\_f} + \overline{nu\_eff}\, nu\_f + \overline{nu\_eff}\, \overline{\nu} + nu\_f\, \overline{\nu} + \overline{nu\_eff}\, \overline{nu\_f} + \overline{nu\_f}\, nu\_f$$
$$+ \overline{\nu}\, \overline{nu\_f} + \overline{nu\_f}^{2}$$

```
> coeff(%,conjugate(nu_eff));
```

$$\nu + \overline{\nu} + nu\_f + \overline{nu\_f}$$

```
> e40 := subs(nu=g+I*w, nu_f= g_f+I*w_f, nu_eff=g_eff+I*w+eff, e32);
```

$$e40 := -(-\overline{(g\_f + I\, w\_f)}^{2}\, (g + I\, w) + (g\_f + I\, w\_f)\, \overline{(g + I\, w)}\, \overline{(g\_eff + I\, w + eff)}$$
$$- (g + I\, w)\, \overline{(g\_f + I\, w\_f)}\, \overline{(g\_eff + I\, w + eff)} - \overline{(g + I\, w)}\, \overline{(g\_f + I\, w\_f)}\, (g + I\, w) - (g + I\, w)^{2}\, \overline{(g\_f + I\, w\_f)}$$
$$- \overline{(g\_f + I\, w\_f)}\, (g\_f + I\, w\_f)\, (g + I\, w)) \Big/ ((g + I\, w)^{2} + \overline{(g\_eff + I\, w + eff)}\, (g + I\, w)$$
$$+ (g + I\, w)\, (g\_f + I\, w\_f) + (g + I\, w)\, \overline{(g + I\, w)} + (g + I\, w)\, \overline{(g\_f + I\, w\_f)}$$
$$+ \overline{(g\_eff + I\, w + eff)}\, (g\_f + I\, w\_f) + \overline{(g\_eff + I\, w + eff)}\, \overline{(g + I\, w)} + (g\_f + I\, w\_f)\, \overline{(g + I\, w)}$$
$$+ \overline{(g\_eff + I\, w + eff)}\, \overline{(g\_f + I\, w\_f)} + \overline{(g\_f + I\, w\_f)}\, (g\_f + I\, w\_f) + \overline{(g + I\, w)}\, \overline{(g\_f + I\, w\_f)}$$
$$+ \overline{(g\_f + I\, w\_f)}^{2})$$

```
> evalc(%);
```

$$(-(-g\_f^{2} + w\_f^{2})\, g + 2\, g\_f\, w\_f\, w - (g\, g\_f + w\_f\, w)\, (g\_eff + eff) - 2\, (w\_f\, g - g\_f\, w)\, w$$
$$- (-g\, g\_f - w\_f\, w)\, (g\_eff + eff) - (-g\, g\_f + w\_f\, w)\, g + (g\_f\, w + w\_f\, g)\, w - (-g^{2} + w^{2})\, g\_f$$
$$+ 2\, g\, w\, w\_f - (-g\_f^{2} - w\_f^{2})\, g)\, (2\, g^{2} + 2\, (g\_eff + eff)\, g + 4\, g\, g\_f + 2\, (g\_eff + eff)\, g\_f + 2\, g\_f^{2}) \Big/ ($$
$$(2\, g^{2} + 2\, (g\_eff + eff)\, g + 4\, g\, g\_f + 2\, (g\_eff + eff)\, g\_f + 2\, g\_f^{2})^{2} + (-2\, g\_f\, w - 2\, g\_f\, w\_f)^{2}) + ($$
$$-2\, g\_f\, w\_f\, g - (-g\_f^{2} + w\_f^{2})\, w - 2\, (w\_f\, g - g\_f\, w)\, (g\_eff + eff) + (g\, g\_f + w\_f\, w)\, w$$
$$+ (-g\, g\_f - w\_f\, w)\, w - (g\_f\, w + w\_f\, g)\, g - (-g\, g\_f + w\_f\, w)\, w + 2\, g\, w\, g\_f + (-g^{2} + w^{2})\, w\_f$$
$$- (-g\_f^{2} - w\_f^{2})\, w)\, (-2\, g\_f\, w - 2\, g\_f\, w\_f) \Big/ ($$
$$(2\, g^{2} + 2\, (g\_eff + eff)\, g + 4\, g\, g\_f + 2\, (g\_eff + eff)\, g\_f + 2\, g\_f^{2})^{2} + (-2\, g\_f\, w - 2\, g\_f\, w\_f)^{2}) + I\, (($$
$$-2\, g\_f\, w\_f\, g - (-g\_f^{2} + w\_f^{2})\, w - 2\, (w\_f\, g - g\_f\, w)\, (g\_eff + eff) + (g\, g\_f + w\_f\, w)\, w$$
$$+ (-g\, g\_f - w\_f\, w)\, w - (g\_f\, w + w\_f\, g)\, g - (-g\, g\_f + w\_f\, w)\, w + 2\, g\, w\, g\_f + (-g^{2} + w^{2})\, w\_f$$
$$- (-g\_f^{2} - w\_f^{2})\, w)\, (2\, g^{2} + 2\, (g\_eff + eff)\, g + 4\, g\, g\_f + 2\, (g\_eff + eff)\, g\_f + 2\, g\_f^{2}) \Big/ ($$

$$\left(2 g^2+2 (g\_eff+eff) g+4 g\, g\_f+2 (g\_eff+eff) g\_f+2 g\_f^2\right)^2+\left(-2 g\_f\, w-2 g\_f\, w\_f\right)^2\Big)-\Big($$
$$-(-g\_f^2+w\_f^2) g+2 g\_f\, w\_f\, w-(g\, g\_f+w\_f\, w)(g\_eff+eff)-2 (w\_f\, g-g\_f\, w) w$$
$$-(-g\, g\_f-w\_f\, w)(g\_eff+eff)-(-g\, g\_f+w\_f\, w) g+(g\_f\, w+w\_f\, g) w-(-g^2+w^2) g\_f$$
$$+2 g\, w\, w\_f-(-g\_f^2-w\_f^2) g\Big)(-2 g\_f\, w-2 g\_f\, w\_f) \Big/ \Big($$
$$\left(2 g^2+2 (g\_eff+eff) g+4 g\, g\_f+2 (g\_eff+eff) g\_f+2 g\_f^2\right)^2+\left(-2 g\_f\, w-2 g\_f\, w\_f\right)^2\Big)\Big)$$

```
> simplify(%);
```

$$-(-2 g\_f\, w\_f\, w\, g\_eff\, g-2 g\_f\, w\_f\, w\, g\, eff-g\_f\, w\_f^2\, g\, g\_eff-g\_f^2\, w\_f^2\, g-2 g^2\, g\_eff\, g\_f^2$$
$$-3 g^3 g\_f^2-3 g^2 g\_f^3-g\_f\, g^4-g\, g\_f^4-2 w\_f\, w\, g\, g\_f^2-g\_f\, w^2\, g\, eff-g\_f\, w^2\, g\_eff\, g$$
$$-2 g\_f\, w\_f\, w\, g^2-g\_f\, w\_f^2\, g\, eff+I\, w\_f\, g^2\, eff^2+I\, w\_f\, g^2\, g\_eff^2-2 I\, g\_f\, w\, eff\, g^2-I\, g\_f\, w\, eff^2\, g$$
$$-2 I\, g\_f\, w\, g\_eff\, g\, eff+4 I\, g\_f\, w\_f\, g^2\, g\_eff+4 I\, g\_f\, w\_f\, g^2\, eff+2 I\, g\_f^2\, w\_f\, g\, g\_eff$$
$$+2 I\, g\_f^2\, w\_f\, g\, eff+2 I\, w\_f\, g\, g\_eff\, g\_f\, eff-4 I\, w\, g\_f^2\, g\_eff\, g-2 I\, w\, g\_f^3\, eff-4 I\, w\, g\_f^3\, g$$
$$-4 I\, w\, g\_f^2\, g^2+2 I\, g\_f^2\, w\_f\, g^2+3 I\, g\_f\, w\_f\, g^3-2 I\, w\, g\_f^3\, g\_eff-4 I\, w\, g\_f^2\, g\, eff$$
$$+2 I\, w\_f\, g^2\, g\_eff\, eff-2 I\, g\_f\, w\, g\_eff\, g^2-I\, g\_f\, w\, g\_eff^2\, g-I\, g\_f^2\, w\, w\_f^2-2 I\, g\_f^2\, w\_f\, w^2$$
$$-I\, g^3\, w\, g\_f-I\, g\_f^2\, w\, eff^2-I\, g\_f^2\, w\, g\_eff^2+2 I\, w\_f\, g^3\, eff+2 I\, w\_f\, g^3\, g\_eff+I\, w\_f\, g\, g\_eff^2\, g\_f$$
$$+I\, w\_f\, g\, eff^2\, g\_f+I\, w\_f\, g^4-2 I\, g\_f^2\, w\, g\_eff\, eff-I\, w\, g\_f^4-g\_f\, w\_f^2\, g^2-g\_f\, w^2\, g^2-g\_f\, g^3\, eff$$
$$-g\_f\, g^3\, g\_eff-w^2\, g\, g\_f^2-I\, g\_f^2\, w^3-2 g^2\, g\_f^2\, eff-g\, eff\, g\_f^3-g\_eff\, g\, g\_f^3) \Big/ (4 g\_eff\, g\, g\_f\, eff$$
$$+2 g^3\, g\_eff+2 g^3\, eff+4 g^3\, g\_f+6 g^2\, g\_f^2+g\_eff^2\, g^2+g^2\, eff^2+6 g^2\, g\_eff\, g\_f+g^4+g\_f^4$$
$$+g\_f^2\, w^2+g\_f^2\, w\_f^2+4 g\, g\_f^3+g\_eff^2\, g\_f^2+2 g\_eff\, g\_f^3+g\_f^2\, eff^2+2 g\_f^3\, eff+2 g\_f^2\, w\, w\_f$$
$$+2 g\, eff^2\, g\_f+6 g\_eff\, g\, g\_f^2+2 g\_eff^2\, g\, g\_f+2 g\_eff\, g^2\, eff+2 g\_eff\, g\_f^2\, eff+6 g\, eff\, g\_f^2$$
$$+6 g^2\, g\_f\, eff)$$

```
> numer(%);
```

$$I\, w\, g\_f^4-I\, w\_f\, g^4+2 g\_f\, w\_f\, w\, g\_eff\, g+2 g\_f\, w\_f\, w\, g\, eff+g\_f\, w\_f^2\, g\, g\_eff+g\_f^2\, w\_f^2\, g$$
$$+2 g^2\, g\_eff\, g\_f^2+3 g^3\, g\_f^2+3 g^2\, g\_f^3+g\_f\, g^4+g\, g\_f^4+2 w\_f\, w\, g\, g\_f^2+g\_f\, w^2\, g\, eff$$
$$+g\_f\, w^2\, g\_eff\, g+2 g\_f\, w\_f\, w\, g^2+g\_f\, w\_f^2\, g\, eff+I\, g\_f^2\, w\, w\_f^2+I\, g^3\, w\, g\_f+I\, g\_f^2\, w\, eff^2$$
$$+I\, g\_f^2\, w\, g\_eff^2+I\, g\_f\, w\, eff^2\, g+I\, g\_f\, w\, g\_eff^2\, g+I\, g\_f^2\, w^3+2 I\, g\_f^2\, w\_f\, w^2-I\, w\_f\, g^2\, eff^2$$
$$-2 I\, w\_f\, g^3\, eff-I\, w\_f\, g^2\, g\_eff^2-2 I\, w\_f\, g^3\, g\_eff+2 I\, w\, g\_f^3\, eff+2 I\, w\, g\_f^3\, g\_eff+4 I\, w\, g\_f^3\, g$$
$$+4 I\, w\, g\_f^2\, g^2-2 I\, g\_f^2\, w\_f\, g^2-3 I\, g\_f\, w\_f\, g^3+4 I\, w\, g\_f^2\, g\, eff-2 I\, w\_f\, g^2\, g\_eff\, eff$$
$$-I\, w\_f\, g\, g\_eff^2\, g\_f-I\, w\_f\, g\, eff^2\, g\_f+2 I\, g\_f\, w\, g\_eff\, g^2+2 I\, g\_f^2\, w\, g\_eff\, eff+2 I\, g\_f\, w\, eff\, g^2$$
$$+2 I\, g\_f\, w\, g\_eff\, g\, eff-4 I\, g\_f\, w\_f\, g^2\, g\_eff-4 I\, g\_f\, w\_f\, g^2\, eff-2 I\, g\_f^2\, w\_f\, g\, g\_eff$$
$$-2 I\, g\_f^2\, w\_f\, g\, eff-2 I\, w\_f\, g\, g\_eff\, g\_f\, eff+4 I\, w\, g\_f^2\, g\_eff\, g+g\_f\, w\_f^2\, g^2+g\_f\, w^2\, g^2$$
$$+g\_f\, g^3\, eff+g\_f\, g^3\, g\_eff+w^2\, g\, g\_f^2+2 g^2\, g\_f^2\, eff+g\, eff\, g\_f^3+g\_eff\, g\, g\_f^3$$

```
> denom(%%);
```

$$4 g\_eff\, g\, g\_f\, eff+2 g^3\, g\_eff+2 g^3\, eff+4 g^3\, g\_f+6 g^2\, g\_f^2+g\_eff^2\, g^2+g^2\, eff^2+6 g^2\, g\_eff\, g\_f$$

$$+g^4+g\_f^4+g\_f^2\,w^2+g\_f^2\,w\_f^2+4\,g\,g\_f^3+g\_eff^2\,g\_f^2+2\,g\_eff\,g\_f^3+g\_f^2\,eff^2+2\,g\_f^3\,eff$$
$$+2\,g\_f^2\,w\,w\_f+2\,g\,eff^2\,g\_f+6\,g\_eff\,g\,g\_f^2+2\,g\_eff^2\,g\,g\_f+2\,g\_eff\,g^2\,eff+2\,g\_eff\,g\_f^2\,eff$$
$$+6\,g\,eff\,g\_f^2+6\,g^2\,g\_f\,eff$$

```
> factor(%);
```

$$4\,g\_eff\,g\,g\_f\,eff+2\,g^3\,g\_eff+2\,g^3\,eff+4\,g^3\,g\_f+6\,g^2\,g\_f^2+g\_eff^2\,g^2+g^2\,eff^2+6\,g^2\,g\_eff\,g\_f$$
$$+g^4+g\_f^4+g\_f^2\,w^2+g\_f^2\,w\_f^2+4\,g\,g\_f^3+g\_eff^2\,g\_f^2+2\,g\_eff\,g\_f^3+g\_f^2\,eff^2+2\,g\_f^3\,eff$$
$$+2\,g\_f^2\,w\,w\_f+2\,g\,eff^2\,g\_f+6\,g\_eff\,g\,g\_f^2+2\,g\_eff^2\,g\,g\_f+2\,g\_eff\,g^2\,eff+2\,g\_eff\,g\_f^2\,eff$$
$$+6\,g\,eff\,g\_f^2+6\,g^2\,g\_f\,eff$$

```
>
# I couldn't simplify e32 by computer, but I could by hand.  My
claim is that nu_eff is given by:

e40 := ( nu*conjugate(nu_f)*(nu+nu_f +
conjugate(nu)+conjugate(nu_f))
       + conjugate(nu_eff) * (nu * conjugate(nu_f) -
nu_f*conjugate(nu)) )
     / ( (nu +conjugate(nu_eff)) * (nu + nu_f + conjugate(nu) +
conjugate(nu_f))
    + (conjugate(nu_f)+conjugate(nu))*(nu_f+conjugate(nu_f)) ) ;
```

$$e40 := \frac{\nu\,\overline{nu\_f}\,(\nu+\overline{\nu}+nu\_f+\overline{nu\_f})+\overline{nu\_eff}\,(\nu\,\overline{nu\_f}-nu\_f\,\overline{\nu})}{(\nu+\overline{nu\_eff})\,(\nu+\overline{\nu}+nu\_f+\overline{nu\_f})+(\overline{nu\_f}+\overline{\nu})\,(nu\_f+\overline{nu\_f})}$$

```
> # Verify that this is true:

normal(e40-e32);
```

$$0$$

```
> # From my Red RMC treatment of the Langevin equation, I found that
nu_eff should be given by:
e50 := nu - conjugate(theta)*(nu_eff
+conjugate(nu_eff))*(nu+conjugate(nu))
       / (theta + conjugate(theta))/(conjugate(nu_eff) +
conjugate(nu_f));
```

$$e50 := \nu-\frac{\overline{\theta}\,(nu\_eff+\overline{nu\_eff})\,(\nu+\overline{\nu})}{(\theta+\overline{\theta})\,(\overline{nu\_f}+\overline{nu\_eff})}$$

```
> e51 := subs(conjugate(nu_eff)=nu_effcc, e50);
```

$$e51 := \nu-\frac{\overline{\theta}\,(nu\_eff+nu\_effcc)\,(\nu+\overline{\nu})}{(\theta+\overline{\theta})\,(\overline{nu\_f}+nu\_effcc)}$$

```
> e52 := nu_eff -e51;
```

$$e52 := nu\_eff-\nu+\frac{\overline{\theta}\,(nu\_eff+nu\_effcc)\,(\nu+\overline{\nu})}{(\theta+\overline{\theta})\,(\overline{nu\_f}+nu\_effcc)}$$

```
> e53 := solve(e52=0, nu_eff);
```

$$e53 := -\frac{-\nu\,\theta\,\overline{nu\_f} - \nu\,\theta\,nu\_effcc - \nu\,\overline{\theta}\,\overline{nu\_f} + \overline{\theta}\,nu\_effcc\,\overline{\nu}}{\theta\,\overline{nu\_f} + \theta\,nu\_effcc + \overline{\theta}\,\overline{nu\_f} + \overline{\theta}\,nu\_effcc + \overline{\theta}\,\nu + \overline{\theta}\,\overline{\nu}}$$

```
> e54 := subs(theta=1/(nu+nu_f), nu_effcc=conjugate(nu_eff), e53);
```

$$e54 := -\frac{-\frac{\nu\,\overline{nu\_f}}{\nu+nu\_f} - \frac{\nu\,\overline{nu\_eff}}{\nu+nu\_f} - \nu\,\overline{\left(\frac{1}{\nu+nu\_f}\right)}\,\overline{nu\_f} + \overline{\left(\frac{1}{\nu+nu\_f}\right)}\,\overline{nu\_eff}\,\overline{\nu}}{\frac{\overline{nu\_f}}{\nu+nu\_f} + \frac{\overline{nu\_eff}}{\nu+nu\_f} + \overline{\left(\frac{1}{\nu+nu\_f}\right)}\,\overline{nu\_f} + \overline{\left(\frac{1}{\nu+nu\_f}\right)}\,\overline{nu\_eff} + \overline{\left(\frac{1}{\nu+nu\_f}\right)}\,\nu + \overline{\left(\frac{1}{\nu+nu\_f}\right)}\,\overline{\nu}}$$

```
> e55 := normal(e54);
```

$$e55 := -\big(-\nu\,\overline{nu\_f}\,\overline{(\nu+nu\_f)} - \overline{nu\_eff}\,\nu\,\overline{(\nu+nu\_f)} - \nu^2\,\overline{nu\_f} - \overline{nu\_f}\,nu\_f\,\nu + \overline{nu\_eff}\,\overline{\nu}\,\nu + nu\_f\,\overline{\nu}\,\overline{nu\_eff}\big) \Big/ \big(\overline{nu\_f}\,\overline{(\nu+nu\_f)} + \overline{nu\_eff}\,\overline{(\nu+nu\_f)} + \nu\,\overline{nu\_f} + \overline{nu\_f}\,nu\_f + \overline{nu\_eff}\,\nu + \overline{nu\_eff}\,nu\_f + \nu^2 + \nu\,nu\_f + \nu\,\overline{\nu} + nu\_f\,\overline{\nu}\big)$$

```
> numer(e55)-numer(e32);
```

$$\nu\,\overline{nu\_f}\,\overline{(\nu+nu\_f)} + \overline{nu\_eff}\,\nu\,\overline{(\nu+nu\_f)} - \overline{nu\_eff}\,\overline{\nu}\,\nu - \overline{nu\_f}^{\,2}\,\nu - \nu\,\overline{nu\_f}\,\overline{nu\_eff} - \overline{\nu}\,\overline{nu\_f}\,\nu$$

```
> # Maple doesn't realize
  conjugate(nu+nu_f)=conjugate(nu)+conjugate(nu_f) until you force
  # it to expand:
  expand(%);
```

$$0$$

```
> expand(denom(e55)-denom(e32));
```

$$0$$

```
> ################################################################
  # We can use e32 recursively to deterine nu_eff in most cases.
  But in cases where we want
  # a direct solution, we will need to solve some simultaneous
  equations:

  e60 := subs(nu_eff=g_eff+I*w_eff,nu=g+I*w,nu_f=g_f+I*w_f,e20);
```

$$e60 := \big((g+I\,w)^2 + \overline{(g\_eff+I\,w\_eff)}\,(g+I\,w) + (g+I\,w)\,(g\_f+I\,w\_f) + (g+I\,w)\,\overline{(g+I\,w)}$$
$$+ (g+I\,w)\,\overline{(g\_f+I\,w\_f)} + \overline{(g\_eff+I\,w\_eff)}\,(g\_f+I\,w\_f) + \overline{(g\_eff+I\,w\_eff)}\,\overline{(g+I\,w)}$$
$$+ (g\_f+I\,w\_f)\,\overline{(g+I\,w)} + \overline{(g\_eff+I\,w\_eff)}\,\overline{(g\_f+I\,w\_f)} + \overline{(g\_f+I\,w\_f)}\,(g\_f+I\,w\_f)$$
$$+ \overline{(g+I\,w)}\,\overline{(g\_f+I\,w\_f)} + \overline{(g\_f+I\,w\_f)}^{\,2}\big)\,\big(g\_eff+I\,w\_eff+\overline{(g\_eff+I\,w\_eff)}\big) -$$
$$\big(g+I\,w+\overline{(g+I\,w)}+g\_f+I\,w\_f+\overline{(g\_f+I\,w\_f)}\big)\,\big(\overline{(g\_f+I\,w\_f)}+\overline{(g\_eff+I\,w\_eff)}\big)$$
$$\big(g+I\,w+\overline{(g\_eff+I\,w\_eff)}\big)$$

```
> e61 := evalc(%);
```

$$e61 := 2\,(2\,g^2 + 2\,g\_eff\,g + 4\,g\,g\_f + 2\,g\_eff\,g\_f + 2\,g\_f^2)\,g\_eff$$
$$+ (-2\,g - 2\,g\_f)\,(g\_f+g\_eff)\,(g+g\_eff) - (-2\,g-2\,g\_f)\,(-w\_f-w\_eff)\,(w-w\_eff) + I\,($$
$$2\,(2\,g\,w - 2\,w\_eff\,g - 2\,w\_eff\,g\_f - 2\,g\_f\,w\_f)\,g\_eff + (-2\,g-2\,g\_f)\,(-w\_f-w\_eff)\,(g+g\_eff)$$

$$+(-2\,g-2\,g\_f)\,(g\_f+g\_eff)\,(w-w\_eff))$$

```
> e62 := evalc(Re(e61));
```

$$e62 := 4\,g\_eff\,g\,g\_f-2\,w\_eff\,g\_f\,w+2\,g\_f\,w\_f\,w\_eff-2\,g\_f\,w\_f\,w-2\,w\_eff\,g\,w+2\,g\,w\_f\,w\_eff$$
$$-2\,g\,w\_f\,w+2\,w\_eff^2\,g\_f+2\,w\_eff^2\,g-2\,g\_f^2\,g+2\,g\_eff\,g^2+2\,g\_eff^2\,g+2\,g\_eff^2\,g\_f$$
$$+2\,g\_eff\,g\_f^2-2\,g^2\,g\_f$$

```
> e63 := evalc(Im(e61));
```

$$e63 := 2\,g\_eff\,g\,w-2\,g\_eff\,g\_f\,w\_f+2\,g^2\,w\_f+2\,g\,w\_f\,g\_eff+2\,w\_eff\,g^2+2\,g\_f\,w\_f\,g$$
$$+4\,w\_eff\,g\_f\,g-2\,g\,g\_f\,w-2\,g\_f^2\,w+2\,g\_f^2\,w\_eff-2\,g\_eff\,g\_f\,w$$

```
> # Brute strength solution doesn't work (it's a 4th order
  polynomial):
  e65 := solve({e62=0, e63=0}, {g_eff,w_eff});
```

$$e65 := \{g\_eff=\mathrm{RootOf}((4\,g^3\,g\_f+g\_f^4+g^4+g^2\,w\_f^2+g\_f^2\,w^2+4\,g\_f^3\,g+2\,g^2\,w\,w\_f$$
$$-2\,g\_f\,w\_f^2\,g-4\,g\,w\,g\_f\,w\_f-2\,g\,w^2\,g\_f+2\,g\_f^2\,w\_f\,w+6\,g^2\,g\_f^2+g\_f^2\,w\_f^2+g^2\,w^2)\,\_Z^2$$
$$-g^3\,g\_f-2\,g^2\,g\_f^2-g\_f\,w\_f^2\,g-g\,w^2\,g\_f-g\_f^3\,g-2\,g\,w\,g\_f\,w\_f+(4\,g^3\,g\_f+g\_f^4+g^4$$
$$+g^2\,w\_f^2+g\_f^2\,w^2+4\,g\_f^3\,g+2\,g^2\,w\,w\_f-2\,g\_f\,w\_f^2\,g-4\,g\,w\,g\_f\,w\_f-2\,g\,w^2\,g\_f$$
$$+2\,g\_f^2\,w\_f\,w+6\,g^2\,g\_f^2+g\_f^2\,w\_f^2+g^2\,w^2)\,\_Z,\ label=\_L1)\,(g+g\_f),\ w\_eff=-(\mathrm{RootOf}(($$
$$4\,g^3\,g\_f+g\_f^4+g^4+g^2\,w\_f^2+g\_f^2\,w^2+4\,g\_f^3\,g+2\,g^2\,w\,w\_f-2\,g\_f\,w\_f^2\,g-4\,g\,w\,g\_f\,w\_f$$
$$-2\,g\,w^2\,g\_f+2\,g\_f^2\,w\_f\,w+6\,g^2\,g\_f^2+g\_f^2\,w\_f^2+g^2\,w^2)\,\_Z^2-g^3\,g\_f-2\,g^2\,g\_f^2-g\_f\,w\_f^2\,g$$
$$-g\,w^2\,g\_f-g\_f^3\,g-2\,g\,w\,g\_f\,w\_f+(4\,g^3\,g\_f+g\_f^4+g^4+g^2\,w\_f^2+g\_f^2\,w^2+4\,g\_f^3\,g$$
$$+2\,g^2\,w\,w\_f-2\,g\_f\,w\_f^2\,g-4\,g\,w\,g\_f\,w\_f-2\,g\,w^2\,g\_f+2\,g\_f^2\,w\_f\,w+6\,g^2\,g\_f^2+g\_f^2\,w\_f^2$$
$$+g^2\,w^2)\,\_Z,\ label=\_L1)\,g\,w+w\_f\,g+g\,w\_f\,\mathrm{RootOf}((4\,g^3\,g\_f+g\_f^4+g^4+g^2\,w\_f^2+g\_f^2\,w^2$$
$$+4\,g\_f^3\,g+2\,g^2\,w\,w\_f-2\,g\_f\,w\_f^2\,g-4\,g\,w\,g\_f\,w\_f-2\,g\,w^2\,g\_f+2\,g\_f^2\,w\_f\,w+6\,g^2\,g\_f^2$$
$$+g\_f^2\,w\_f^2+g^2\,w^2)\,\_Z^2-g^3\,g\_f-2\,g^2\,g\_f^2-g\_f\,w\_f^2\,g-g\,w^2\,g\_f-g\_f^3\,g-2\,g\,w\,g\_f\,w\_f+($$
$$4\,g^3\,g\_f+g\_f^4+g^4+g^2\,w\_f^2+g\_f^2\,w^2+4\,g\_f^3\,g+2\,g^2\,w\,w\_f-2\,g\_f\,w\_f^2\,g-4\,g\,w\,g\_f\,w\_f$$
$$-2\,g\,w^2\,g\_f+2\,g\_f^2\,w\_f\,w+6\,g^2\,g\_f^2+g\_f^2\,w\_f^2+g^2\,w^2)\,\_Z,\ label=\_L1)-\mathrm{RootOf}((4\,g^3\,g\_f$$
$$+g\_f^4+g^4+g^2\,w\_f^2+g\_f^2\,w^2+4\,g\_f^3\,g+2\,g^2\,w\,w\_f-2\,g\_f\,w\_f^2\,g-4\,g\,w\,g\_f\,w\_f-2\,g\,w^2\,g\_f$$
$$+2\,g\_f^2\,w\_f\,w+6\,g^2\,g\_f^2+g\_f^2\,w\_f^2+g^2\,w^2)\,\_Z^2-g^3\,g\_f-2\,g^2\,g\_f^2-g\_f\,w\_f^2\,g-g\,w^2\,g\_f$$
$$-g\_f^3\,g-2\,g\,w\,g\_f\,w\_f+(4\,g^3\,g\_f+g\_f^4+g^4+g^2\,w\_f^2+g\_f^2\,w^2+4\,g\_f^3\,g+2\,g^2\,w\,w\_f$$
$$-2\,g\_f\,w\_f^2\,g-4\,g\,w\,g\_f\,w\_f-2\,g\,w^2\,g\_f+2\,g\_f^2\,w\_f\,w+6\,g^2\,g\_f^2+g\_f^2\,w\_f^2+g^2\,w^2)\,\_Z,$$
$$label=\_L1)\,g\_f\,w\_f-g\_f\,w-\mathrm{RootOf}((4\,g^3\,g\_f+g\_f^4+g^4+g^2\,w\_f^2+g\_f^2\,w^2+4\,g\_f^3\,g$$
$$+2\,g^2\,w\,w\_f-2\,g\_f\,w\_f^2\,g-4\,g\,w\,g\_f\,w\_f-2\,g\,w^2\,g\_f+2\,g\_f^2\,w\_f\,w+6\,g^2\,g\_f^2+g\_f^2\,w\_f^2$$
$$+g^2\,w^2)\,\_Z^2-g^3\,g\_f-2\,g^2\,g\_f^2-g\_f\,w\_f^2\,g-g\,w^2\,g\_f-g\_f^3\,g-2\,g\,w\,g\_f\,w\_f+(4\,g^3\,g\_f$$
$$+g\_f^4+g^4+g^2\,w\_f^2+g\_f^2\,w^2+4\,g\_f^3\,g+2\,g^2\,w\,w\_f-2\,g\_f\,w\_f^2\,g-4\,g\,w\,g\_f\,w\_f-2\,g\,w^2\,g\_f$$
$$+2\,g\_f^2\,w\_f\,w+6\,g^2\,g\_f^2+g\_f^2\,w\_f^2+g^2\,w^2)\,\_Z,\ label=\_L1)\,g\_f\,w)\,/\,(g+g\_f)\}$$

```
> e66 := solve(e62=0,g_eff);
```

$$e66 := -\frac{1}{2}g - \frac{1}{2}g\_f + \frac{1}{2}\sqrt{6\,g\,g\_f + g\_f^2 + g^2 + 4\,w\_eff\,w - 4\,w\_f\,w\_eff + 4\,w\,w\_f - 4\,w\_eff^2},$$
$$-\frac{1}{2}g - \frac{1}{2}g\_f - \frac{1}{2}\sqrt{6\,g\,g\_f + g\_f^2 + g^2 + 4\,w\_eff\,w - 4\,w\_f\,w\_eff + 4\,w\,w\_f - 4\,w\_eff^2}$$

```
> e67 := solve(e63=0,w_eff);
```

$$e67 := -\frac{g\_eff\,g\,w - g\_eff\,g\_f\,w\_f + g^2\,w\_f + g\,w\_f\,g\_eff - g\_f^2\,w + g\_f\,w\_f\,g - g\_eff\,g\_f\,w - g\,g\_f\,w}{2\,g\,g\_f + g\_f^2 + g^2}$$

```
> e68 := solve(e62=0,w_eff);
```

$$e68 := \frac{1}{2}w - \frac{1}{2}w\_f + \frac{1}{2}\sqrt{w^2 + 2\,w\,w\_f + w\_f^2 - 4\,g\_eff\,g + 4\,g\,g\_f - 4\,g\_eff^2 - 4\,g\_eff\,g\_f},$$
$$\frac{1}{2}w - \frac{1}{2}w\_f - \frac{1}{2}\sqrt{w^2 + 2\,w\,w\_f + w\_f^2 - 4\,g\_eff\,g + 4\,g\,g\_f - 4\,g\_eff^2 - 4\,g\_eff\,g\_f}$$

```
> e69 := solve(e63=0,g_eff);
```

$$e69 := -\frac{w\_eff\,g^2 + g\_f\,w\_f\,g + g^2\,w\_f - g\,g\_f\,w - g\_f^2\,w + g\_f^2\,w\_eff + 2\,w\_eff\,g\_f\,g}{g\,w - g\_f\,w\_f + w\_f\,g - g\_f\,w}$$

```
> # Surprisingly, The imaginary part of the equation e61 (in e63) is linear in w_eff or g_eff, and
  # so can be solved more easily.  It seems more natural to let this determine w_eff, since in the
  # real limit w=w_f=0, w_eff should be zero also, while g_eff should still involve a quadratic.
  # Thus use e67 to determine w_eff, and substitute into e66 to determine g_eff:

  e70 := subs(w_eff=e67, e62);
```

$$e70 := 4\,g\_eff\,g\,g\_f + 2\,(g\_eff\,g\,w - g\_eff\,g\_f\,w\_f + g^2\,w\_f + g\,w\_f\,g\_eff - g\_f^2\,w + g\_f\,w\_f\,g - g\_eff\,g\_f\,w - g\,g\_f\,w)\,g\_f\,w \Big/ (2\,g\,g\_f + g\_f^2 + g^2) - 2\,g\_f\,w\_f\,(g\_eff\,g\,w - g\_eff\,g\_f\,w\_f + g^2\,w\_f + g\,w\_f\,g\_eff - g\_f^2\,w + g\_f\,w\_f\,g - g\_eff\,g\_f\,w - g\,g\_f\,w) \Big/ (2\,g\,g\_f + g\_f^2 + g^2) - 2\,g\_f\,w\_f\,w + 2\,(g\_eff\,g\,w - g\_eff\,g\_f\,w\_f + g^2\,w\_f + g\,w\_f\,g\_eff - g\_f^2\,w + g\_f\,w\_f\,g - g\_eff\,g\_f\,w - g\,g\_f\,w)\,g\,w \Big/ (2\,g\,g\_f + g\_f^2 + g^2) - 2\,g\,w\_f\,(g\_eff\,g\,w - g\_eff\,g\_f\,w\_f + g^2\,w\_f + g\,w\_f\,g\_eff - g\_f^2\,w + g\_f\,w\_f\,g - g\_eff\,g\_f\,w - g\,g\_f\,w) \Big/ (2\,g\,g\_f + g\_f^2 + g^2) - 2\,g\,w\_f\,w + 2\,(g\_eff\,g\,w - g\_eff\,g\_f\,w\_f + g^2\,w\_f + g\,w\_f\,g\_eff - g\_f^2\,w + g\_f\,w\_f\,g - g\_eff\,g\_f\,w - g\,g\_f\,w)^2\,g\_f$$

$$\Big/ (2\, g\, g_f + g_f^2 + g^2)^2 + 2$$

$$(g_{\mathit{eff}}\, g\, w - g_{\mathit{eff}}\, g_f\, w_f + g^2\, w_f + g\, w_f\, g_{\mathit{eff}} - g_f^2\, w + g_f\, w_f\, g - g_{\mathit{eff}}\, g_f\, w - g\, g_f\, w)^2\, g$$

$$\Big/ (2\, g\, g_f + g_f^2 + g^2)^2 - 2\, g_f^2\, g + 2\, g_{\mathit{eff}}\, g^2 + 2\, g_{\mathit{eff}}^2\, g + 2\, g_{\mathit{eff}}^2\, g_f + 2\, g_{\mathit{eff}}\, g_f^2$$

$$- 2\, g^2\, g_f$$

```
> simplify(%);
```

$$2\, (-g_f\, w_f^2\, g^3 - 2\, g_f^2\, w_f^2\, g^2 - 2\, g^2\, g_f^2\, w^2 - g\, g_f^3\, w^2 - w_f^2\, g_f^3\, g - w^2\, g^3\, g_f - g^5\, g_f$$
$$- 6\, g^3\, g_f^3 - 4\, g^4\, g_f^2 - g\, g_f^5 - 4\, g^2\, g_f^4 - 4\, g_f^2\, w_f\, g^2\, w - 2\, g_f^3\, w_f\, g\, w - 2\, g^3\, g_f\, w\, w_f$$
$$+ g^3\, w_f^2\, g_{\mathit{eff}} + g^2\, w_f^2\, g_{\mathit{eff}}^2 + g_f^3\, w^2\, g_{\mathit{eff}} + g_{\mathit{eff}}^2\, g_f^2\, w^2 - 2\, g_{\mathit{eff}}\, g_f^2\, w_f\, g\, w$$
$$- 2\, g_{\mathit{eff}}^2\, g_f\, w_f^2\, g + 2\, g_{\mathit{eff}}\, g_f^3\, w_f\, w - g_{\mathit{eff}}\, g_f^2\, w_f^2\, g + 2\, g_{\mathit{eff}}^2\, g_f^2\, w_f\, w$$
$$- 2\, g_{\mathit{eff}}\, g^2\, w\, g_f\, w_f - 2\, g_{\mathit{eff}}^2\, g\, w^2\, g_f - g_{\mathit{eff}}\, g^2\, w^2\, g_f - g_{\mathit{eff}}\, g_f\, w_f^2\, g^2$$
$$- 4\, g_{\mathit{eff}}^2\, g\, w\, g_f\, w_f + 2\, g_{\mathit{eff}}\, g^3\, w\, w_f + 2\, g_{\mathit{eff}}^2\, g^2\, w\, w_f - g_{\mathit{eff}}\, g\, w^2\, g_f^2 + g_{\mathit{eff}}^2\, g^2\, w^2$$
$$+ g_{\mathit{eff}}^2\, g_f^2\, w_f^2 + g_{\mathit{eff}}^2\, g_f^4 + g_f^3\, w_f^2\, g_{\mathit{eff}} + g_{\mathit{eff}}\, g_f^5 + g^5\, g_{\mathit{eff}} + 4\, g\, g_{\mathit{eff}}^2\, g_f^3$$
$$+ g^4\, g_{\mathit{eff}}^2 + 5\, g^4\, g_{\mathit{eff}}\, g_f + g^3\, w^2\, g_{\mathit{eff}} + 4\, g^3\, g_{\mathit{eff}}^2\, g_f + 10\, g^3\, g_{\mathit{eff}}\, g_f^2 + 10\, g^2\, g_{\mathit{eff}}\, g_f^3$$
$$+ 6\, g^2\, g_{\mathit{eff}}^2\, g_f^2 + 5\, g\, g_f^4\, g_{\mathit{eff}}) \Big/ ((g + g_f)\, (2\, g\, g_f + g_f^2 + g^2))$$

```
> e71 := solve(%=0, g_eff);
```

$$e71 := \frac{1}{2} (-4\, g^3\, g_f - g_f^4 - g^4 - g^2\, w_f^2 - g_f^2\, w^2 - 4\, g_f^3\, g - 2\, g^2\, w\, w_f + 2\, g_f\, w_f^2\, g$$
$$+ 4\, g\, w\, g_f\, w_f + 2\, g\, w^2\, g_f - 2\, g_f^2\, w_f\, w - 6\, g^2\, g_f^2 - g_f^2\, w_f^2 - g^2\, w^2 + \mathrm{sqrt}(g_f^4\, w_f^4$$
$$+ w_f^4\, g^4 - 2\, w_f^4\, g^2\, g_f^2 + 116\, g_f^5\, g^3 + 12\, g_f^7\, g + 52\, g^6\, g_f^2 + 12\, g^7\, g_f + 116\, g^5\, g_f^3$$
$$+ 150\, g^4\, g_f^4 + 2\, g_f^6\, w^2 + 2\, g_f^6\, w_f^2 + 2\, g^6\, w_f^2 + 12\, g_f^5\, w_f^2\, g + 28\, g^4\, w\, w_f\, g_f^2$$
$$+ 24\, g^5\, w\, w_f\, g_f + 12\, g^5\, w^2\, g_f + 14\, g_f^2\, w_f^2\, g^4 + 14\, g_f^4\, w_f^2\, g^2 - 2\, g^2\, w^4\, g_f^2 + 8\, g^3\, w^2\, g_f^3$$
$$+ 14\, g^4\, w^2\, g_f^2 + 14\, g^2\, w^2\, g_f^4 + 12\, g_f^5\, w^2\, g + 12\, g^5\, w_f^2\, g_f - 8\, g_f^2\, w_f^3\, g^2\, w$$
$$- 8\, g^2\, w^3\, g_f^2\, w_f + 16\, g_f^3\, w_f\, w\, g^3 + 24\, g_f^5\, w_f\, w\, g - 12\, g_f^2\, w_f^2\, g^2\, w^2 + 28\, g_f^4\, w_f\, w\, g^2$$
$$+ 4\, g_f^4\, w_f^3\, w + 4\, g^4\, w_f^3\, w + 6\, g^4\, w_f^2\, w^2 + 4\, g_f^4\, w^3\, w_f + 6\, g_f^4\, w^2\, w_f^2 + 4\, g^4\, w^3\, w_f$$
$$+ 2\, g^6\, w^2 + g_f^4\, w^4 + g^4\, w^4 + 4\, g^6\, w\, w_f + 4\, g_f^6\, w_f\, w + g_f^8 + g^8 + 52\, g_f^6\, g^2 + 8\, g_f^3\, w_f^2\, g^3)$$
$$)\, (g + g_f) \Big/ (4\, g^3\, g_f + g_f^4 + g^4 + g^2\, w_f^2 + g_f^2\, w^2 + 4\, g_f^3\, g + 2\, g^2\, w\, w_f - 2\, g_f\, w_f^2\, g$$
$$- 4\, g\, w\, g_f\, w_f - 2\, g\, w^2\, g_f + 2\, g_f^2\, w_f\, w + 6\, g^2\, g_f^2 + g_f^2\, w_f^2 + g^2\, w^2),\ \frac{1}{2} (-4\, g^3\, g_f - g_f^4$$
$$- g^4 - g^2\, w_f^2 - g_f^2\, w^2 - 4\, g_f^3\, g - 2\, g^2\, w\, w_f + 2\, g_f\, w_f^2\, g + 4\, g\, w\, g_f\, w_f + 2\, g\, w^2\, g_f$$
$$- 2\, g_f^2\, w_f\, w - 6\, g^2\, g_f^2 - g_f^2\, w_f^2 - g^2\, w^2 - \mathrm{sqrt}(g_f^4\, w_f^4 + w_f^4\, g^4 - 2\, w_f^4\, g^2\, g_f^2$$
$$+ 116\, g_f^5\, g^3 + 12\, g_f^7\, g + 52\, g^6\, g_f^2 + 12\, g^7\, g_f + 116\, g^5\, g_f^3 + 150\, g^4\, g_f^4 + 2\, g_f^6\, w^2$$
$$+ 2\, g_f^6\, w_f^2 + 2\, g^6\, w_f^2 + 12\, g_f^5\, w_f^2\, g + 28\, g^4\, w\, w_f\, g_f^2 + 24\, g^5\, w\, w_f\, g_f + 12\, g^5\, w^2\, g_f$$

$$+14\, g_f^2\, w_f^2\, g^4 + 14\, g_f^4\, w_f^2\, g^2 - 2\, g^2\, w^4\, g_f^2 + 8\, g^3\, w^2\, g_f^3 + 14\, g^4\, w^2\, g_f^2 + 14\, g^2\, w^2\, g_f^4$$
$$+12\, g_f^5\, w^2\, g + 12\, g^5\, w_f^2\, g_f - 8\, g_f^2\, w_f^3\, g^2\, w - 8\, g^2\, w^3\, g_f^2\, w_f + 16\, g_f^3\, w_f\, w\, g^3$$
$$+24\, g_f^5\, w_f\, w\, g - 12\, g_f^2\, w_f^2\, g^2\, w^2 + 28\, g_f^4\, w_f\, w\, g^2 + 4\, g_f^4\, w_f^3\, w + 4\, g^4\, w_f^3\, w$$
$$+6\, g^4\, w_f^2\, w^2 + 4\, g_f^4\, w^3\, w_f + 6\, g_f^4\, w^2\, w_f^2 + 4\, g^4\, w^3\, w_f + 2\, g^6\, w^2 + g_f^4\, w^4 + g^4\, w^4$$
$$+4\, g^6\, w\, w_f + 4\, g_f^6\, w_f\, w + g_f^8 + g^8 + 52\, g_f^6\, g^2 + 8\, g_f^3\, w_f^2\, g^3))\, (g + g_f) \Big/ (4\, g^3\, g_f$$
$$+ g_f^4 + g^4 + g^2\, w_f^2 + g_f^2\, w^2 + 4\, g_f^3\, g + 2\, g^2\, w\, w_f - 2\, g_f\, w_f^2\, g - 4\, g\, w\, g_f\, w_f - 2\, g\, w^2\, g_f$$
$$+ 2\, g_f^2\, w_f\, w + 6\, g^2\, g_f^2 + g_f^2\, w_f^2 + g^2\, w^2)$$

```
> e72 := simplify(%[1]);
```

$$e72 := -\frac{1}{2} (4\, g^3\, g_f + g_f^4 + g^4 + g^2\, w_f^2 + g_f^2\, w^2 + 4\, g_f^3\, g + 2\, g^2\, w\, w_f - 2\, g_f\, w_f^2\, g$$
$$- 4\, g\, w\, g_f\, w_f - 2\, g\, w^2\, g_f + 2\, g_f^2\, w_f\, w + 6\, g^2\, g_f^2 + g_f^2\, w_f^2 + g^2\, w^2 - \mathrm{sqrt}((g + g_f)^2\, ($$
$$g_f^2\, w_f^4 + w_f^4\, g^2 - 2\, g_f\, w_f^4\, g + 8\, g_f\, w_f^2\, g^3 - 4\, g_f^2\, w_f^2\, g^2 - 4\, g^2\, g_f^2\, w^2 + 8\, g\, g_f^3\, w^2$$
$$+ 8\, w_f^2\, g_f^3\, g + 8\, w^2\, g^3\, g_f + 10\, g^5\, g_f + 44\, g^3\, g_f^3 + 31\, g^4\, g_f^2 + 10\, g\, g_f^5 + 31\, g^2\, g_f^4$$
$$- 8\, g_f^2\, w_f\, g^2\, w + 16\, g_f^3\, w_f\, g\, w + 16\, g^3\, g_f\, w\, w_f + 2\, g^4\, w^2 + 2\, g_f^4\, w^2 + g_f^6 + g^6$$
$$+ 4\, g^4\, w\, w_f + 6\, g^2\, w^2\, w_f^2 + 4\, g^2\, w\, w_f^3 + 4\, g^2\, w_f\, w^3 - 2\, g\, g_f\, w^4 + 2\, g^4\, w_f^2$$
$$- 12\, g\, g_f\, w^2\, w_f^2 - 8\, g\, g_f\, w\, w_f^3 - 8\, g\, g_f\, w_f\, w^3 + 4\, g_f^2\, w_f^3\, w + 4\, w^3\, g_f^2\, w_f$$
$$+ 6\, g_f^2\, w_f^2\, w^2 + 4\, g_f^4\, w_f\, w + 2\, g_f^4\, w_f^2 + w^4\, g_f^2 + g^2\, w^4)))\, (g + g_f) \Big/ (4\, g^3\, g_f + g_f^4$$
$$+ g^4 + g^2\, w_f^2 + g_f^2\, w^2 + 4\, g_f^3\, g + 2\, g^2\, w\, w_f - 2\, g_f\, w_f^2\, g - 4\, g\, w\, g_f\, w_f - 2\, g\, w^2\, g_f$$
$$+ 2\, g_f^2\, w_f\, w + 6\, g^2\, g_f^2 + g_f^2\, w_f^2 + g^2\, w^2)$$

```
>
# Define a procedure to evaluate nueff:

func_nueff := proc(nu,nu_f)
      local g, w, g_f, w_f , g_eff, w_eff;
      g   := Re(nu)   ; w   := Im(nu);
      g_f := Re(nu_f) ; w_f := Im(nu_f);
g_eff :=
-1/2*(g^4+g_f^2*w^2+w^2*g^2+6*g^2*g_f^2+4*g^3*g_f+4*g*g_f^3-2*g*g_
f*w^2+g_f^4-4*g*w*w_f*g_f+w_f^2*g_f^2+g^2*w_f^2+2*g^2*w*w_f-2*w_f^
2*g_f*g+2*g_f^2*w*w_f-sqrt((g+g_f)^2*(g^6-2*w_f^4*g_f*g+w^4*g^2+w^
4*g_f^2+2*w_f^2*g_f^4+4*g_f^4*w*w_f+4*g_f^2*w^3*w_f+6*w^2*w_f^2*g_
f^2+4*w*w_f^3*g_f^2-12*g*g_f*w_f^2*w^2-8*g*g_f*w_f*w^3-8*g*g_f*w*w
_f^3+2*g^4*w_f^2+4*g^4*w*w_f+6*g^2*w_f^2*w^2+4*g^2*w_f*w^3+4*g^2*w
*w_f^3+2*g^4*w^2+2*g_f^4*w^2+g^2*w_f^4+w_f^4*g_f^2+g_f^6-2*w^4*g*g
_f-4*g_f^2*g^2*w^2+8*g*g_f^3*w^2+8*g^3*g_f*w^2+31*g^2*g_f^4+44*g^3
*g_f^3+8*w_f^2*g_f*g^3-4*w_f^2*g_f^2*g^2+8*w_f^2*g_f^3*g+31*g^4*g_
f^2-8*w_f*g_f^2*g^2*w+16*w_f*g_f^3*g*w+16*g^3*g_f*w*w_f+10*g^5*g_f
+10*g*g_f^5)))*(g+g_f)/(g^4+g_f^2*w^2+w^2*g^2+6*g^2*g_f^2+4*g^3*g_
f+4*g*g_f^3-2*g*g_f*w^2+g_f^4-4*g*w*w_f*g_f+w_f^2*g_f^2+g^2*w_f^2+
2*g^2*w*w_f-2*w_f^2*g_f*g+2*g_f^2*w*w_f)           ;
```

```
w_eff :=
-(g*w*g_eff-g_eff*w_f*g_f+g^2*w_f+g*w_f*g_eff+w_f*g_f*g-g*g_f*w-g_f^2*w-g_f*w*g_eff)/(g^2+g_f^2+2*g*g_f) ;
g+I*w_eff;
end ;
```

func_nueff := **proc**(ν, *nu_f*)
local *g*, *w*, *g_f*, *w_f*, *g_eff*, *w_eff*;
$g := \Re(\nu)$;
$w := \Im(\nu)$;
$g\_f := \Re(nu\_f)$;
$w\_f := \Im(nu\_f)$;
g_eff := −1 / 2*(g^4 + g_f^2*w^2 + w^2*g^2 + 6*g^2*g_f^2 + 4*g^3*g_f + 4*g*g_f^3
− 2*g*g_f*w^2 + g_f^4 − 4*g*w*w_f*g_f + w_f^2*g_f^2 + g^2*w_f^2 + 2*g^2*w*w_f
− 2*w_f^2*g_f*g + 2*g_f^2*w*w_f − sqrt((g + g_f)^2*(−8*g*g_f*w_f*w^3
− 8*g*g_f*w*w_f^3 − 8*w_f*g_f^2*g^2*w + 16*w_f*g_f^3*g*w + 16*g^3*g_f*w*w_f
+ 2*g^4*w_f^2 − 12*g*g_f*w_f^2*w^2 + g^6 + g_f^6 + w_f^4*g_f^2 + g^2*w_f^4
+ w^4*g_f^2 + w^4*g^2 + 31*g^2*g_f^4 + 2*g_f^4*w^2 + 2*g^4*w^2 + 2*w_f^2*g_f^4
+ 10*g*g_f^5 + 10*g^5*g_f + 31*g^4*g_f^2 + 44*g^3*g_f^3 − 2*w^4*g*g_f
+ 4*g^2*w*w_f^3 + 4*g^2*w_f*w^3 + 6*g^2*w_f^2*w^2 + 4*g^4*w*w_f
+ 4*w*w_f^3*g_f^2 + 6*w^2*w_f^2*g_f^2 + 4*g_f^2*w^3*w_f + 4*g_f^4*w*w_f
− 2*w_f^4*g_f*g + 8*w_f^2*g_f^3*g − 4*w_f^2*g_f^2*g^2 + 8*w_f^2*g_f*g^3
+ 8*g^3*g_f*w^2 + 8*g*g_f^3*w^2 − 4*g_f^2*g^2*w^2)))*(g + g_f) / (g^4 + g_f^2*w^2
+ w^2*g^2 + 6*g^2*g_f^2 + 4*g^3*g_f + 4*g*g_f^3 − 2*g*g_f*w^2 + g_f^4
− 4*g*w*w_f*g_f + w_f^2*g_f^2 + g^2*w_f^2 + 2*g^2*w*w_f − 2*w_f^2*g_f*g
+ 2*g_f^2*w*w_f);
w_eff := −(g*w*g_eff − g_eff*w_f*g_f + g^2*w_f + g*w_f*g_eff + w_f*g_f*g − g*g_f*w
− g_f^2*w − g_f*w*g_eff) / (g^2 + g_f^2 + 2*g*g_f);
g + I*w_eff
end proc

```
> func_nueff(1.0,1.0+16*I);
```

$1.0 - 8.000000000\, I$

```
>
>
```