Shaping Effects on Gyrokinetic Plasma Turbulence

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Abstract

The effects of flux surface shape and other plasma parameters on the gyrokinetic stability and transport of tokamak plasmas has been studied. Preliminary results indicate that increasing the triangularity and triangularity gradient at a finite elongation has a stabilizing influence at high α ($\approx -R_0 q^2 \partial\beta/\partial\rho$, related to the second derivative of the Shafranov shift). This high degree of shaping, in effect, opens up access to a type of 2nd-microstability regime at high α , though it can have a destabilizing influence at low α . This behavior is qualitatively reminiscent of the effects of triangularity on MHD ballooning stability in previous studies[1]. In addition, results from the application of reduced-order models to these gyrokinetic results, employing trial eigenfunctions and/or sub-grid turbulence models to enhance the efficiency of the calculations, are also reported. In the near future, we will compare these results with data from JET and explore their implications for proposed highly shaped tokamaks such as FIRE.

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Drift waves are commonly held responsible for anomalous transport & the resulting anomalously high heat loss in tokamak configurations.

- Turbulence in plasmas ultimately reduces the efficiency of a fusion reactor by allowing heat to escape from the containment device faster because of increased transport processes.
- Using simulations to understand plasma turbulence and transport could help to improve the performance of tokamaks by helping to design a device with optimal confinement properties.
- **In these studies, the gyrokinetic code GS2 was used to examine the effects of local flux shape (particularly elongation, triangularity, and pressure gradient) on plasma turbulence and transport.**

The Miller local equilibrium model[1] was used to obtain a realistic treatment of plasma shape.

- Nine parameters are required to fully describe the local equilibrium: κ (elongation), δ (triangularity), s (global magnetic shear), α (pressure gradient) , A, q, $\partial_r R_0$, $\partial_r \kappa$, $\partial_r \delta$
- The shape of a flux surface is specified using a standard formula for D-shaped plasmas:

$$R_s = R_0(r) + r \cos\{\theta + [\sin^{-1}\delta(r)]\sin\theta\}$$

$$Z_s = \kappa(r)\, r \sin\theta$$

- **The parameters can be individually varied, thus allowing for systematic studies of the effects of each upon stability and transport for shaped flux surfaces.**

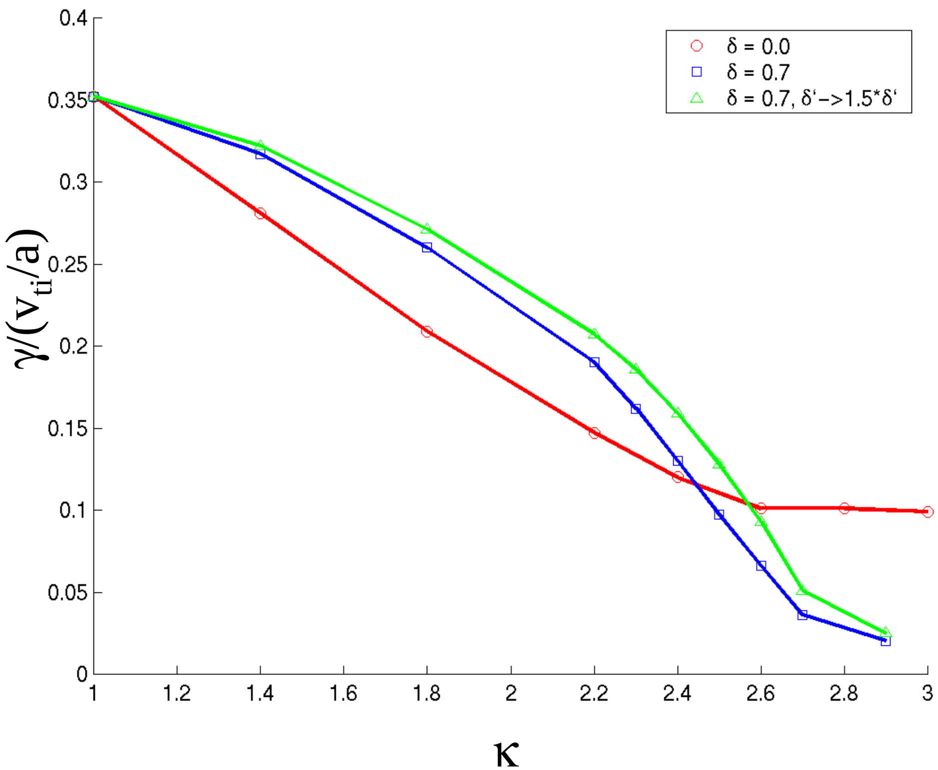
Gyrokinetic simulations of ITG-driven turbulence for a range of equilibrium flux surface shapes were done using the GS2 code[2].

- GS2 contains a nearly full, parallel implementation of the 5D Frieman and Chen nonlinear gyrokinetic equation in the flux tube limit.
- It treats electrons and an arbitrary number of ion species on an equal footing and includes trapped particles, electromagnetic perturbations, and a pitch-angle scattering collision operator with a momentum conserving term.
- The following reported growth rates at any given point in parameter space were maximized over a scan of $k_y\rho_i$ ranging from 0.1 to 1.0.

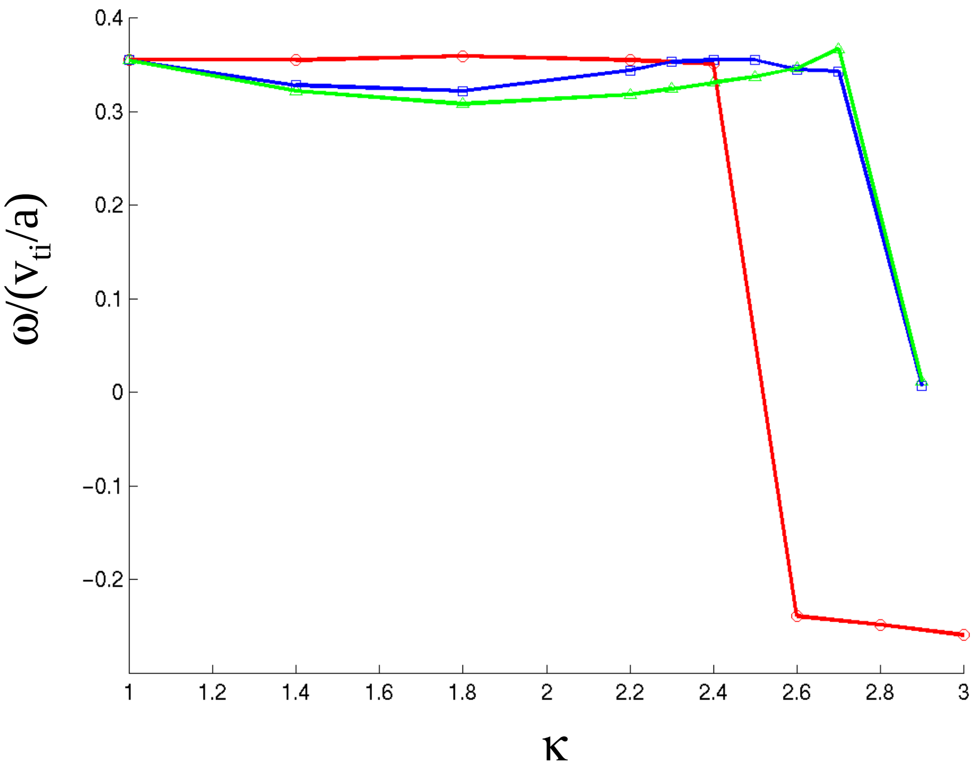
Scans over κ show a general improvement at high elongation, particularly when coupled with high triangularity.

$$\delta(\kappa) \approx 3/4*(0.416/0.66)(\kappa-1), \quad \partial_r \delta \approx \delta/r, \quad \partial_r \kappa \approx (\kappa-1)/r$$

Linear growth rate vs. κ, fixed α



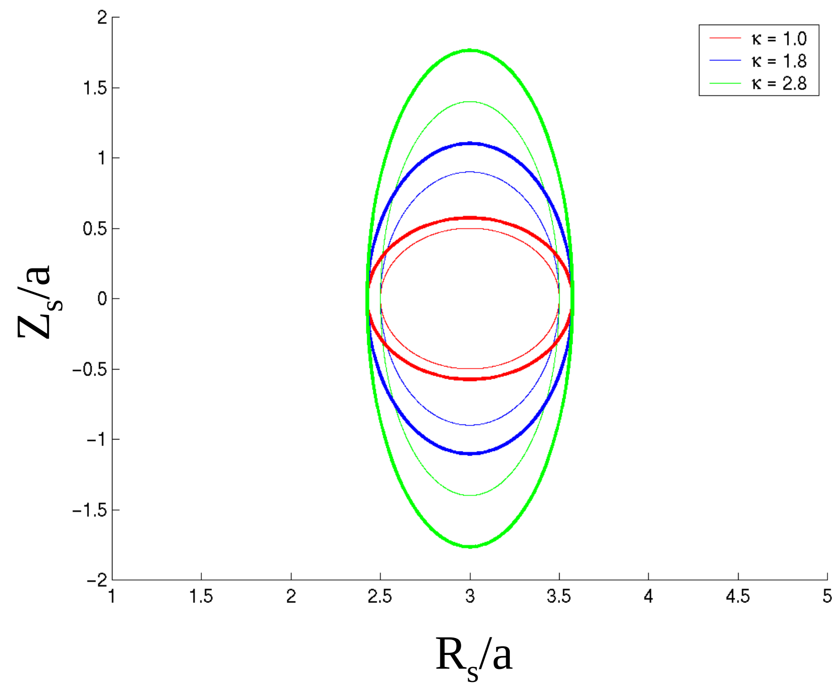
Real frequency vs. κ, fixed α



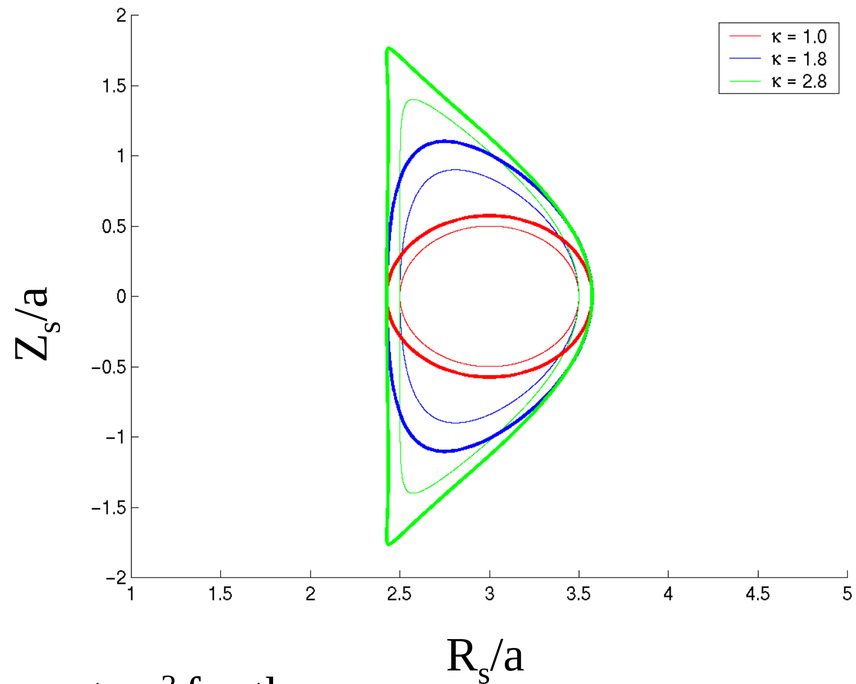
Also, a higher triangularity gradient is more stabilizing.

Plots of neighboring flux surfaces for the κ scan show the effects of κ and δ on the equilibrium.

Flux surface shapes for $\delta = 0.0$



Flux surface shapes for δ (κ)



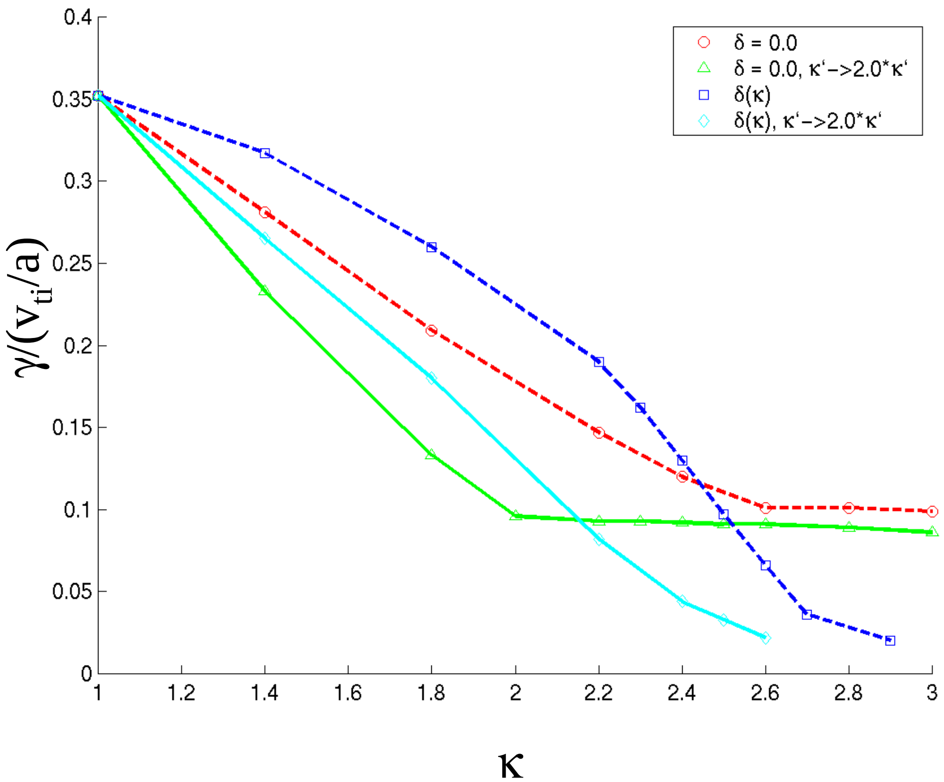
Standard local parameters[3] for the κ scan:

$r/a = 0.5$	**$q = 2.0$**	**$a/L_T = 3.0$**
$R/a = 3.0$	**$s = 1.0$**	**$a/L_n = 1.0$**
$\partial_r R_0 = -0.0$	**$\alpha = 0.0$**	**$T_i/T_e = 1.0$**
$\nu_{ei} = 0.0$	**$k_y \rho_i = 0.4$**	

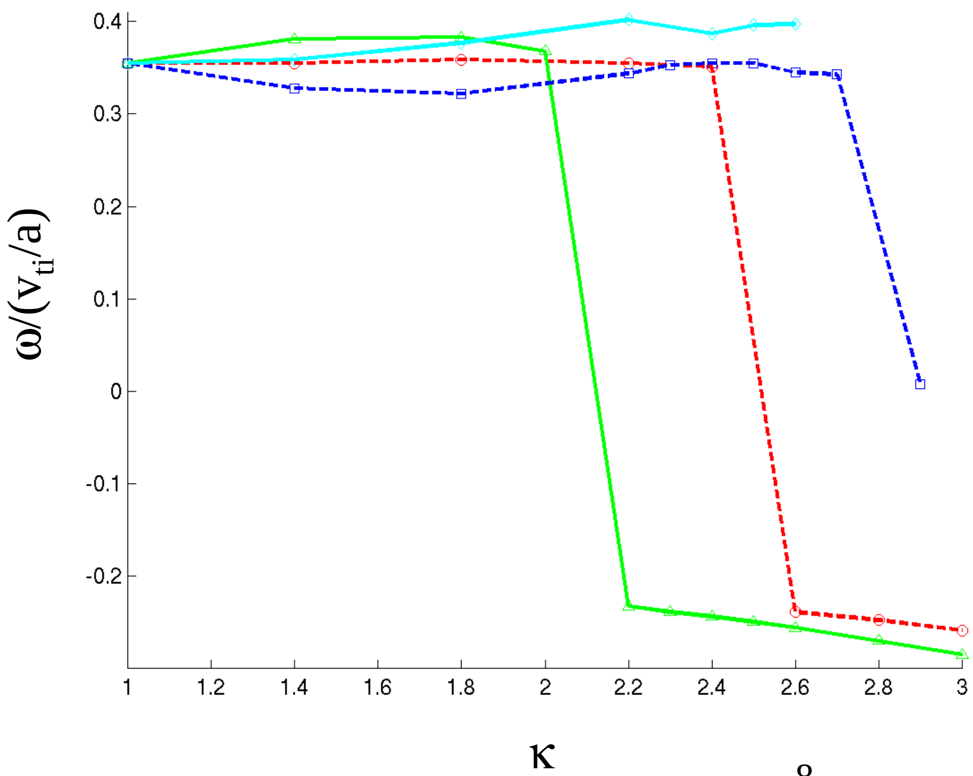
It was also observed that a high elongation gradient is generally more stabilizing.

$\delta(\kappa) \approx 3/4*(0.416/0.66)(\kappa-1)$, $\partial_r \delta \approx \delta/r$, $\partial_r \kappa \approx (\kappa-1)/r$

Linear growth rate vs. κ, fixed α



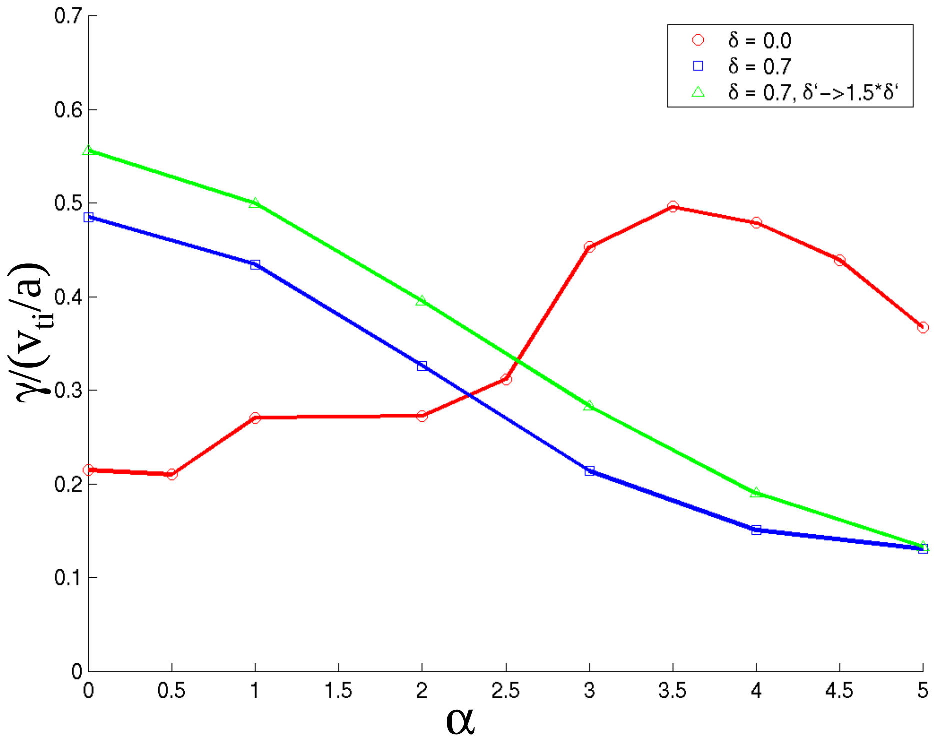
Real frequency vs. κ, fixed α



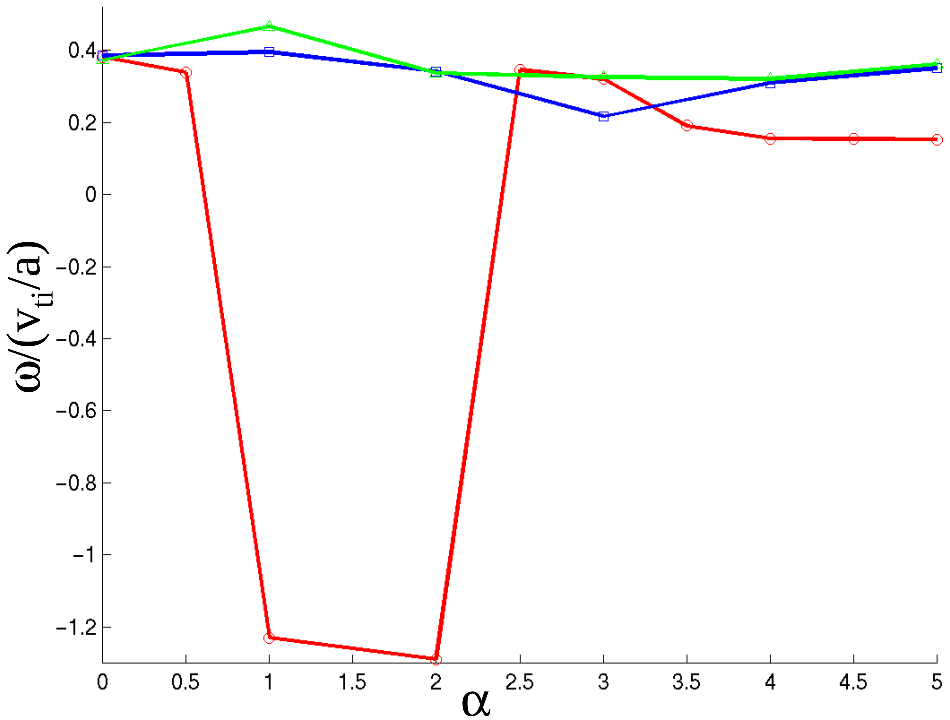
Scans over α at fixed κ show that increasing δ has a stabilizing influence at high α, yet a destabilizing influence at low α.

s=2.85

Linear growth rate vs. α, fixed κ



Real frequency vs. α, fixed κ

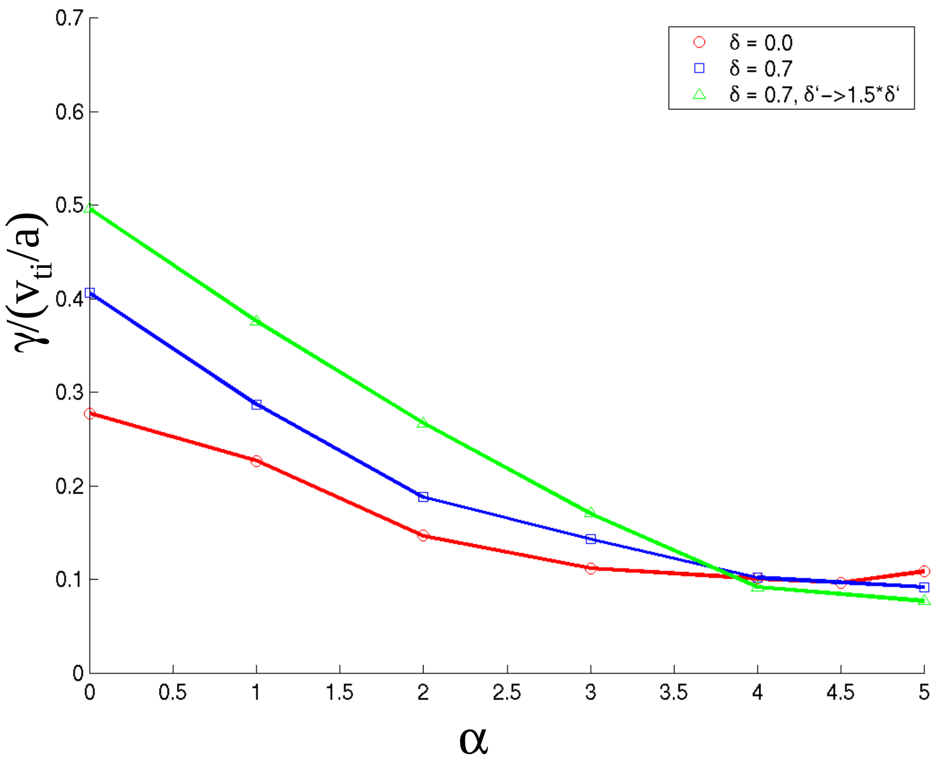


The 2nd-microstability regime at high α and high δ is qualitatively similar to previous studies of the effects of δ on MHD ballooning stability[1].

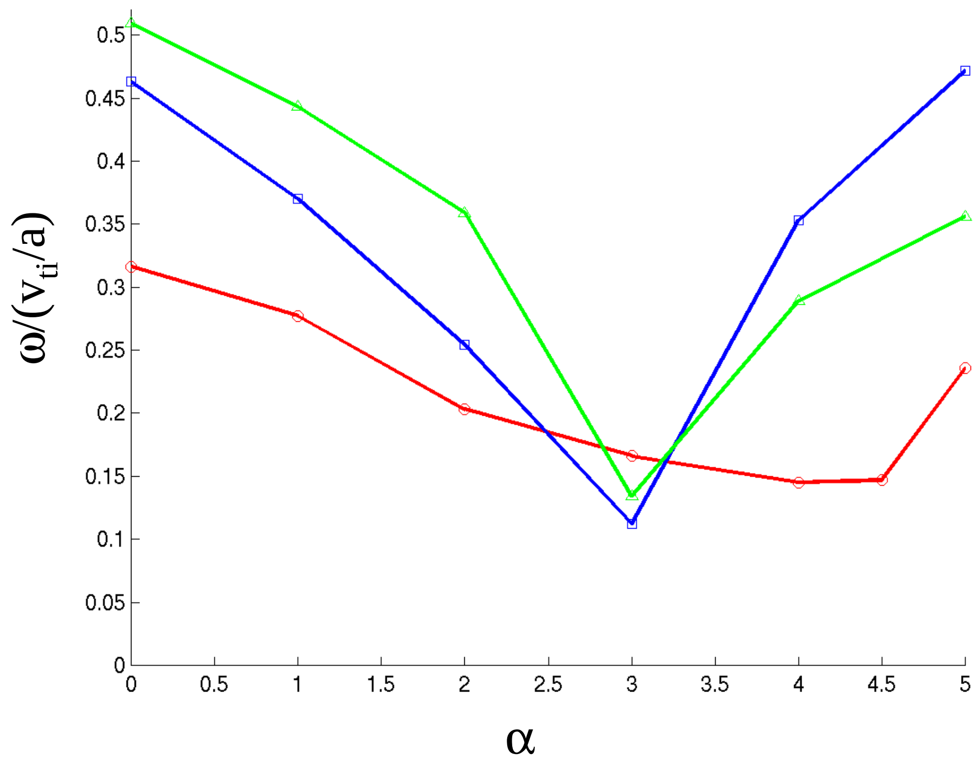
However, at lower shear, δ does not have an effect on the stability at high α.

s=1.00

Linear growth rate vs. α, fixed κ



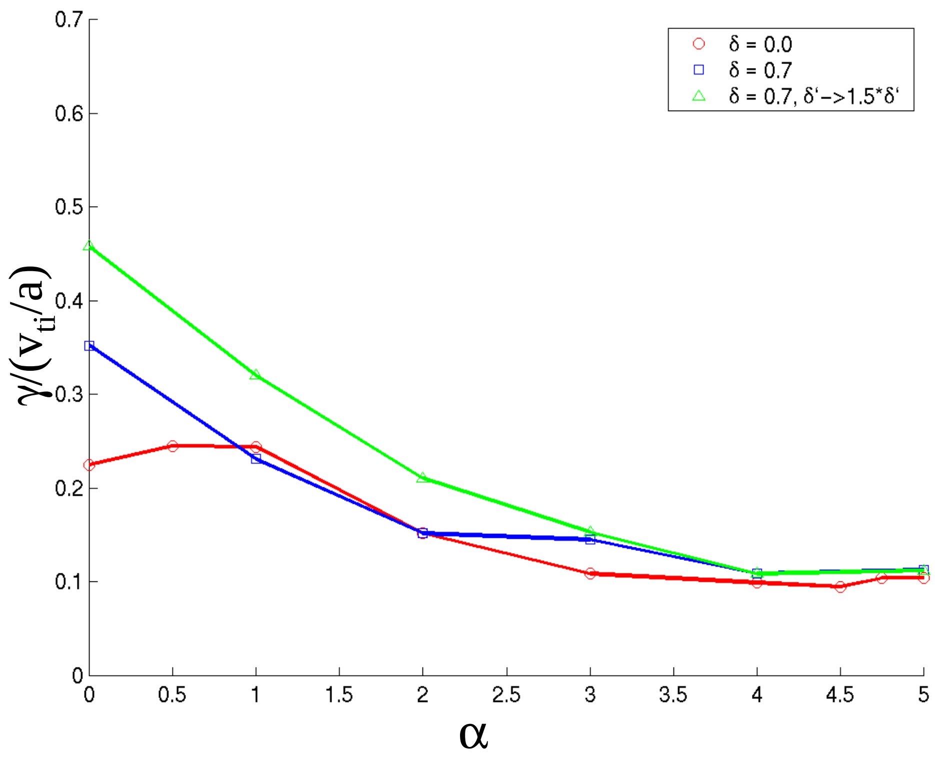
Real frequency vs. α, fixed κ



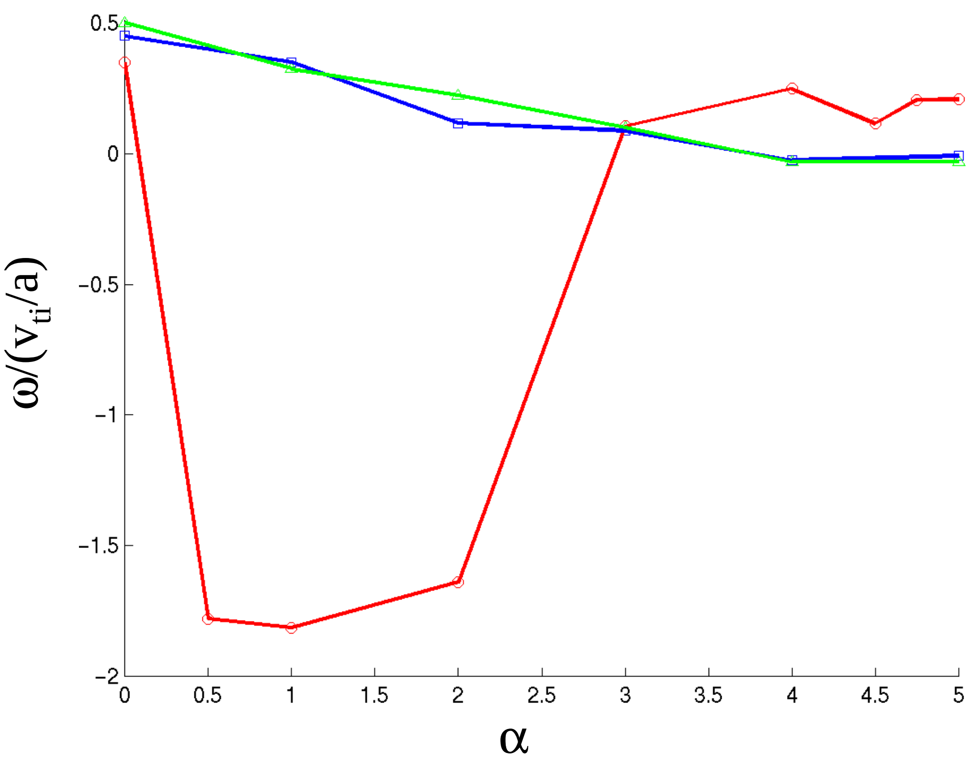
Though, the destabilizing effects of δ & $\partial_r \delta$ at fixed κ at low α are qualitatively similar to that at high shear.

s=0.00

Linear growth rate vs. α, fixed κ

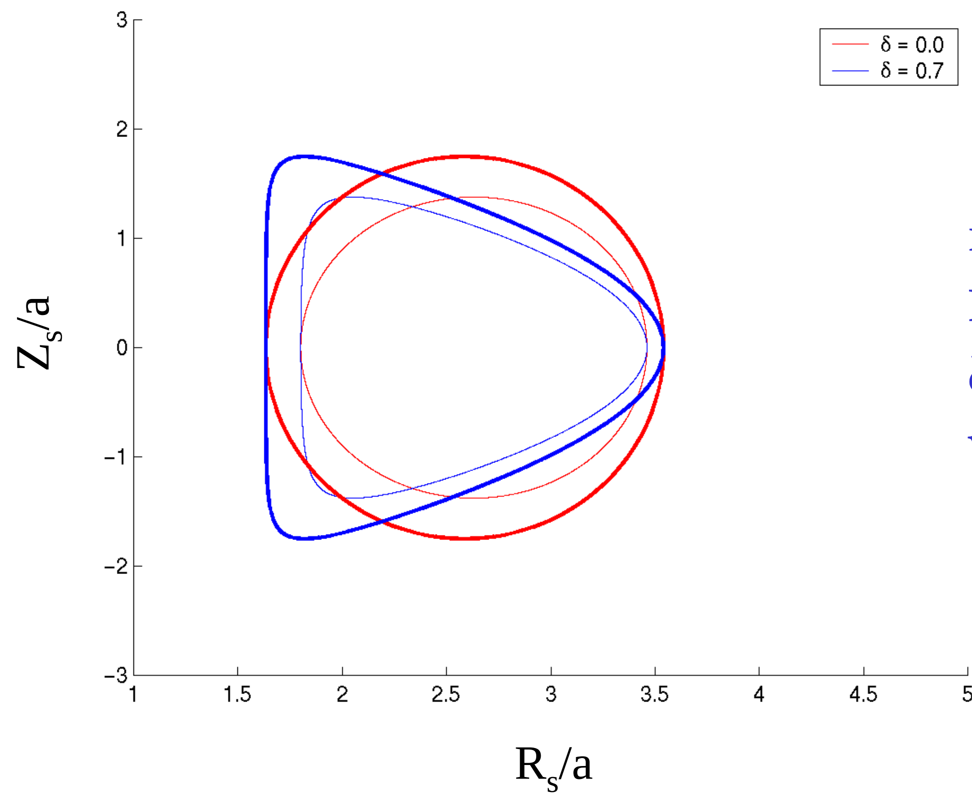


Real frequency vs. α, fixed κ



Plots of neighboring flux surfaces for the α scan show the effects of δ & $\partial_r \delta$ and on the equilibrium.

Flux surface shape



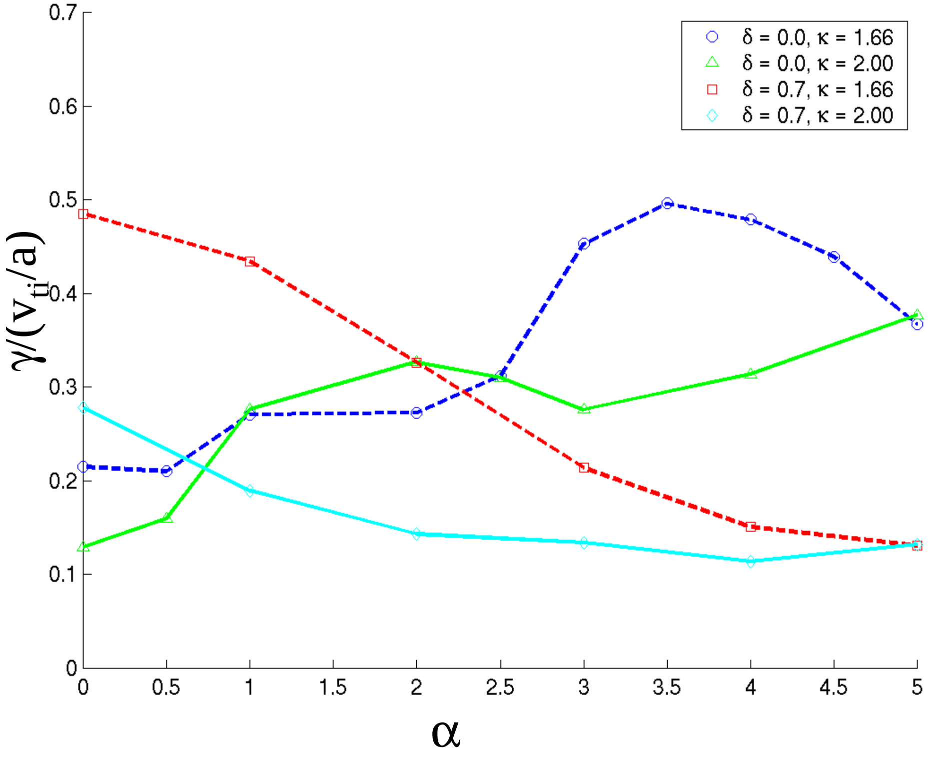
Standard local parameters[1] for the α scan:

$r/a = 0.83$	$q = 3.03$	$a/L_T = 3.0$
$R/a = 2.631$	$\kappa = 1.66$	$a/L_n = 1.0$
$\partial_r R_0 = -0.354$	$\partial_\rho \kappa = 1.4$	$T_i/T_e = 1.0$
$\nu_{ei} = 0.0$		

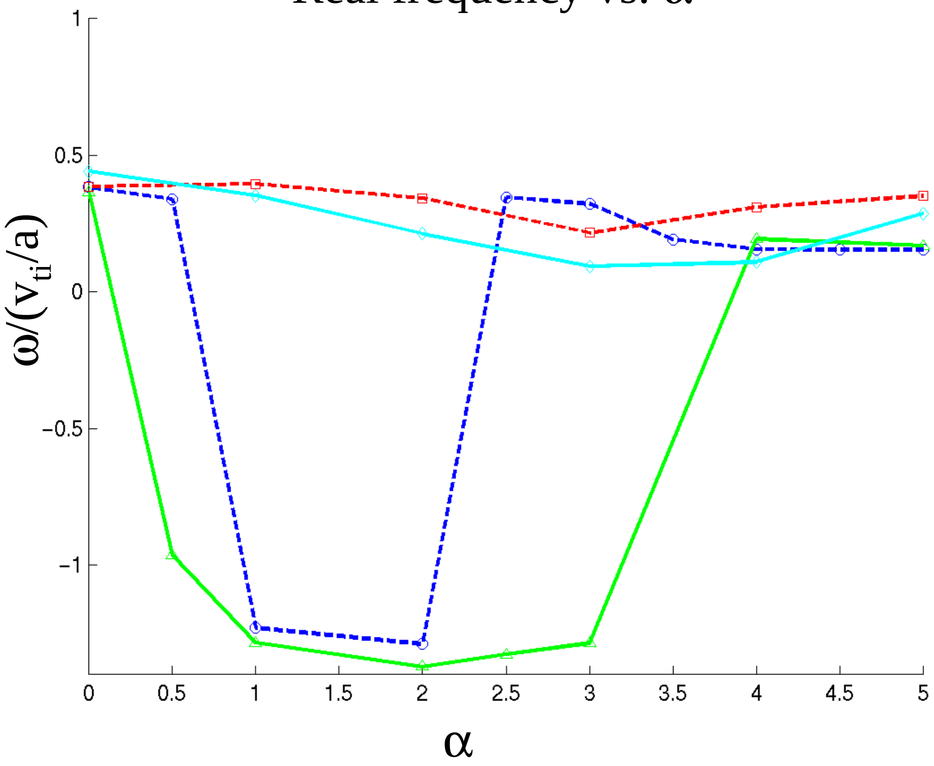
High elongation has a stabilizing effect at both low and high α.

s=2.85

Linear growth rate vs. α



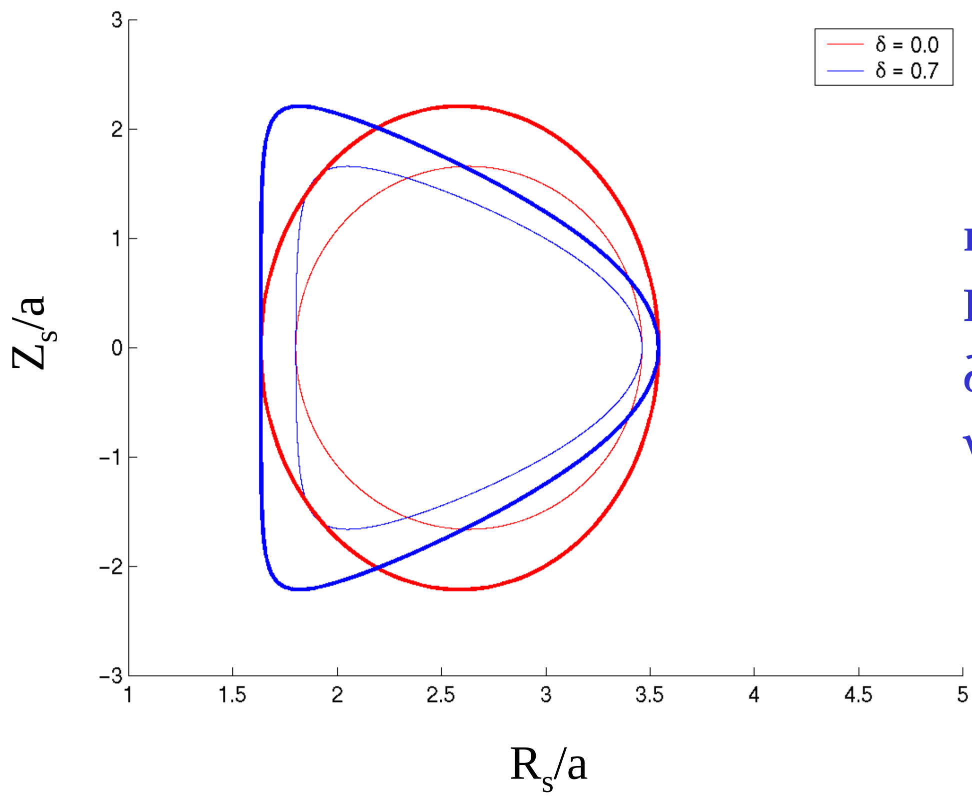
Real frequency vs. α



This result is qualitatively consistent with the previous κ scan at fixed α.

Plots of neighboring flux surfaces for the case of higher elongation and elongation gradient

Flux surface shape



Standard local parameters[1] for the α scan:

r/a	**= 0.83**	**q**	**= 3.03**	**a/L_T**	**= 3.0**
R/a	**= 2.631**	**κ**	**= 2.0**	**a/L_n**	**= 1.0**
$\partial_r R_0$	**= -0.354**	**$\partial_\rho \kappa$**	**= 2.556**	**T_i/T_e**	**= 1.0**
ν_{ei}	**= 0.0**				

A local gyrokinetic code employing a trial function model[4] has been developed to enhance the efficiency of the microstability calculations.

- In general, the equilibrium parameters (particularly the curvature drift, ∇B drift, $k_\perp$, and $k_\parallel$) depend on the ballooning mode extended angle θ.
- To reduce these equations to one dimension yet still retain the effects of the geometry, weighted averages of these parameters over a Gaussian trial function are computed as follows: $<F> = \int d\theta\, F \exp(-\theta^2/(4\sigma^2)) / \int d\theta \exp(-\theta^2/(4\sigma^2))$

  where σ is the trial function width
- With this model, the gyrokinetic equation reduces to:

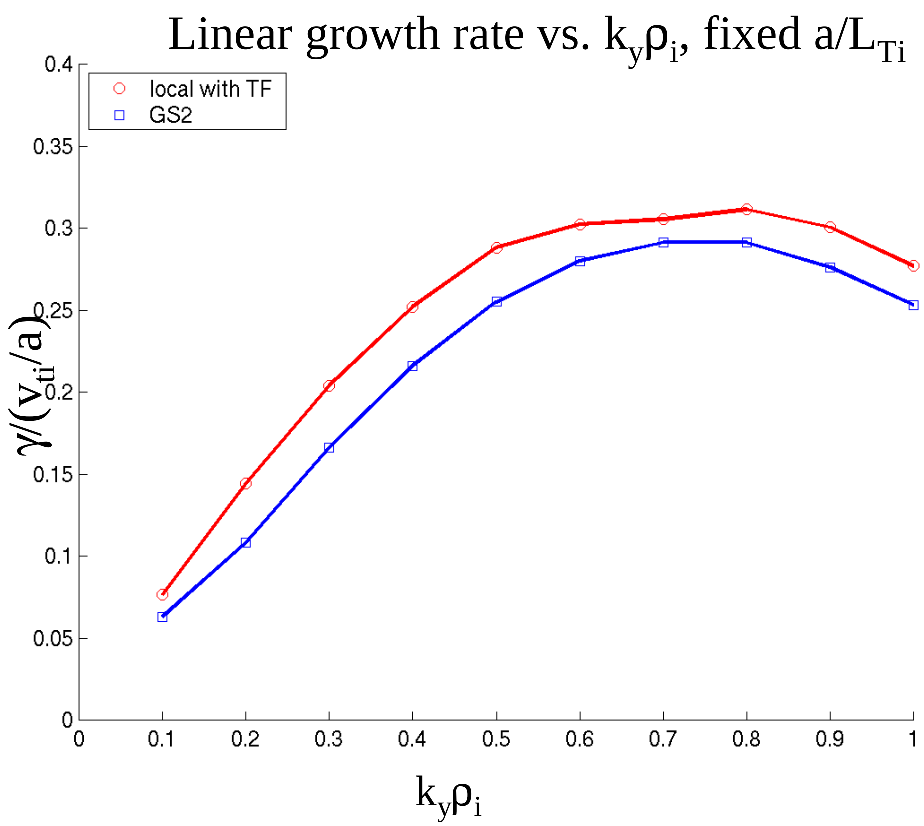
$$\frac{\partial f}{\partial t} = i(\omega_{*T} - \sqrt{<k_\parallel^2>}v_\parallel - <\omega_\kappa> v_\parallel^2 - \frac{1}{2}<\omega_{\nabla B}> v_\perp^2) F_o J_o(\sqrt{<k_\perp^2>} v_\perp / \Omega) Ze\phi / T_o$$
$$- i(\sqrt{<k_\parallel^2>}v_\parallel + <\omega_\kappa> v_\parallel^2 + \frac{1}{2}<\omega_{\nabla B}> v_\perp^2) f$$

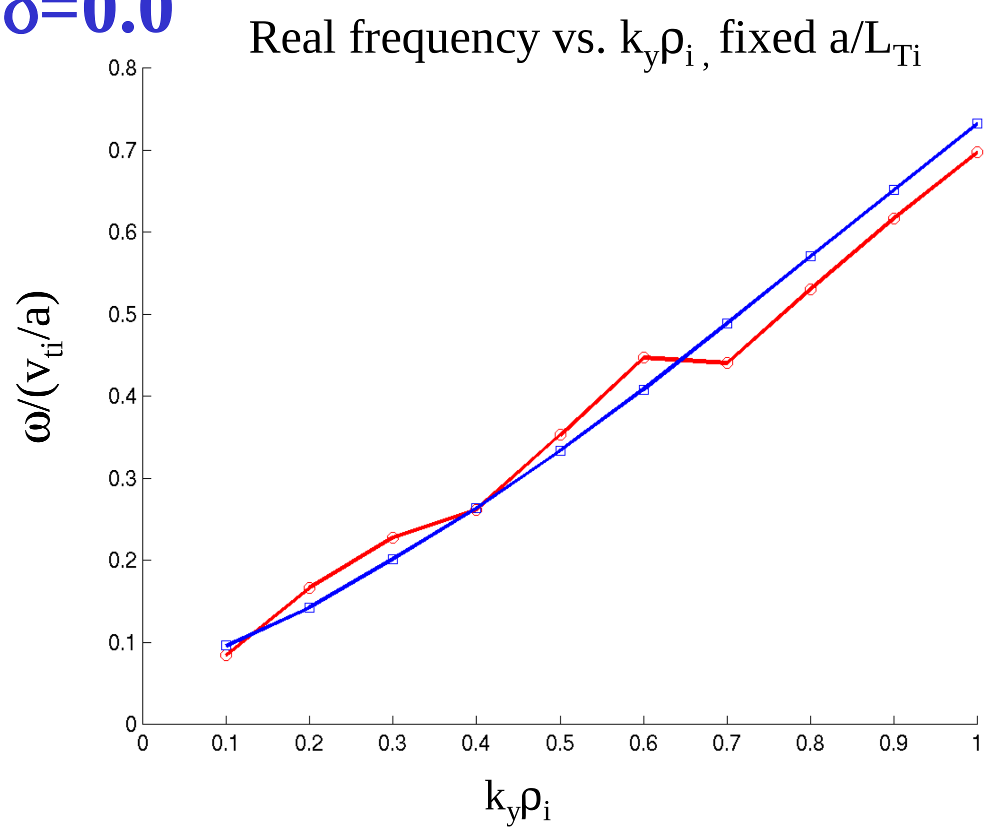
The local code solves the linearized gyrokinetic equation for ions (assuming adiabatic electrons) in the collisionless, electrostatic limit.

- **For a given gyrokinetic calculation, the weighted averages are computed for a range of trial function widths σ and the gyrokinetic eqn is solved for each σ. The largest growth rate over this σ scan is then reported as the worst-case result.**
- Since trapped particle effects are currently not accounted for in the model, the standard case parameters are set to minimize these effects (i.e. r/a = 0.001, q = 10.0, s = 0.5).
- Overall, the trial function model allows for time efficient studies of the microstability of plasma shaping effects. **Such a code might then be more practical than a full geometry code for use in a transport code**, where a large number of iterations are required to accurately describe the heat transfer coefficient.

Benchmarks of the local trial function code with GS2 for scans over $k_y\rho_i$ were successful.

δ=0.0





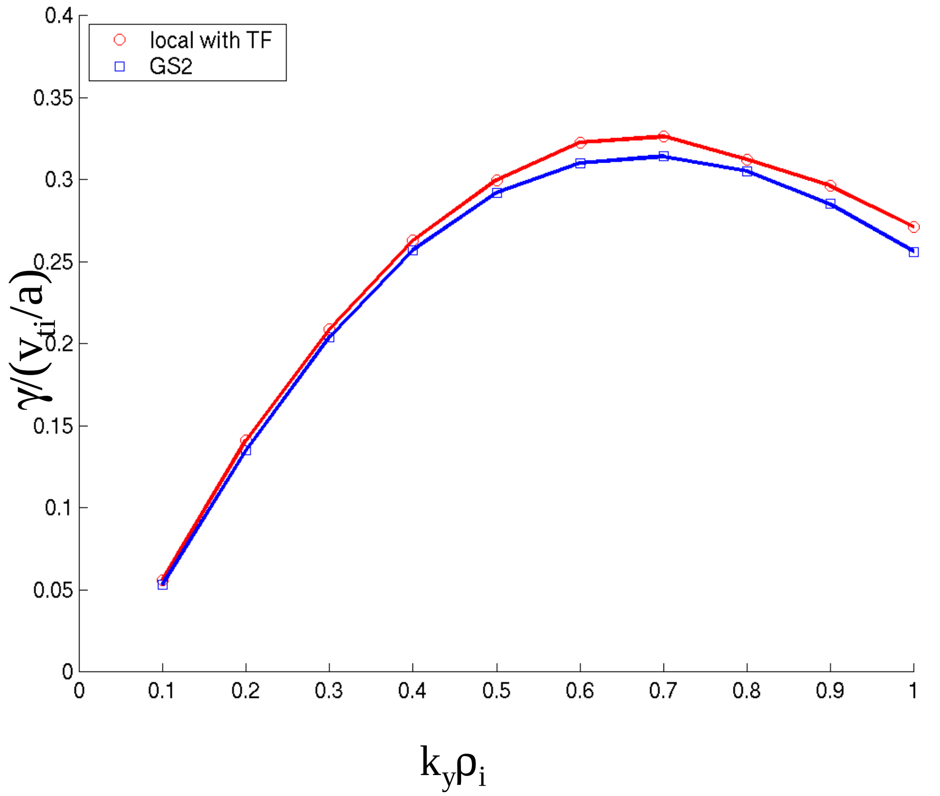
Standard local parameters:

r/a = 0.001	**q = 10.0**	**κ = 1.66**	**a/L_{ni} = 1.0**
R/a = 2.631	**s = 0.5**	**$\partial_\rho\kappa$ = 1162**	**a/L_{Ti} = 3.0**
$\partial_r R_0$ = -0.354	**α = 0.0**	**ν_{ei} = 0.0**	**T_i/T_e = 1.0**

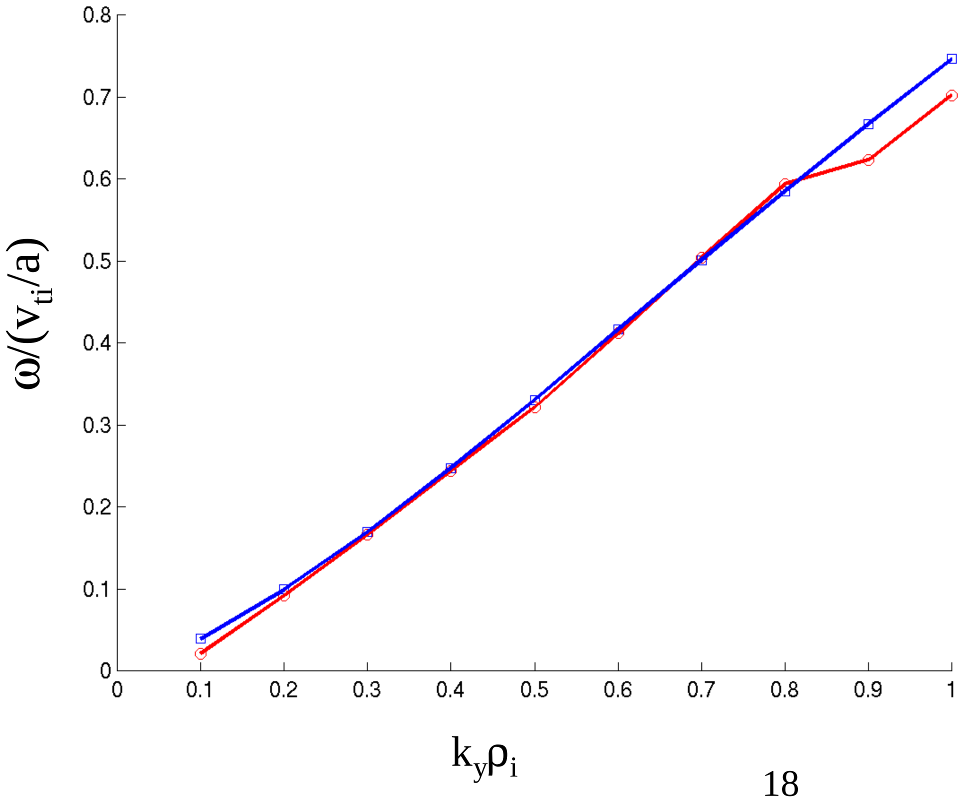
The local trial function code matches GS2 even for plasmas with high triangularity.

$\delta=0.7 \quad \partial_r \delta \approx \delta/r$

Linear growth rate vs. $k_y\rho_i$, fixed a/L_{Ti}

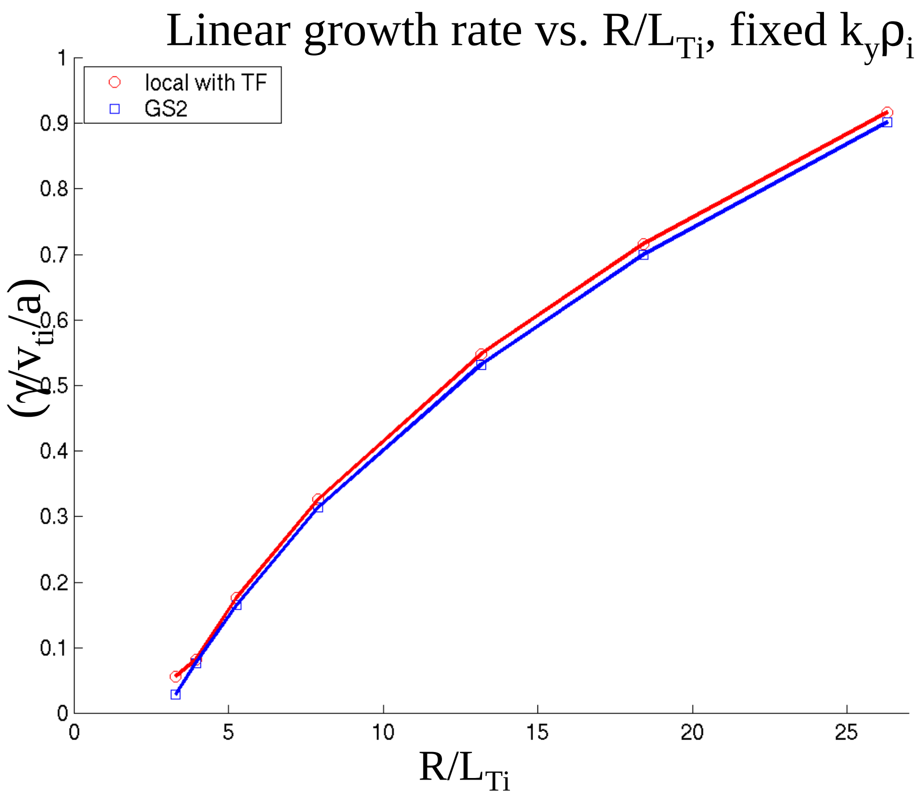


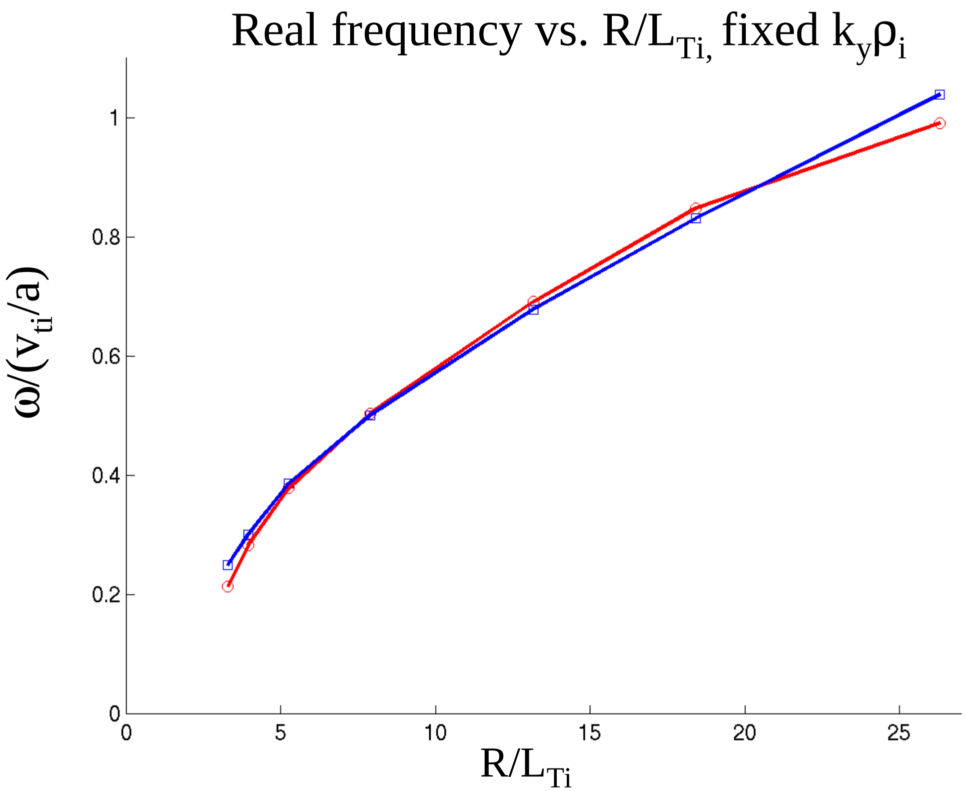
Real frequency vs. $k_y\rho_i$, fixed a/L_{Ti}



Benchmarks of the local trial function code with GS2 for scans over R/L_{Ti} were also successful.

$\delta=0.7 \quad \partial_r \delta \approx \delta/r \quad k_y\rho_i = 0.7$



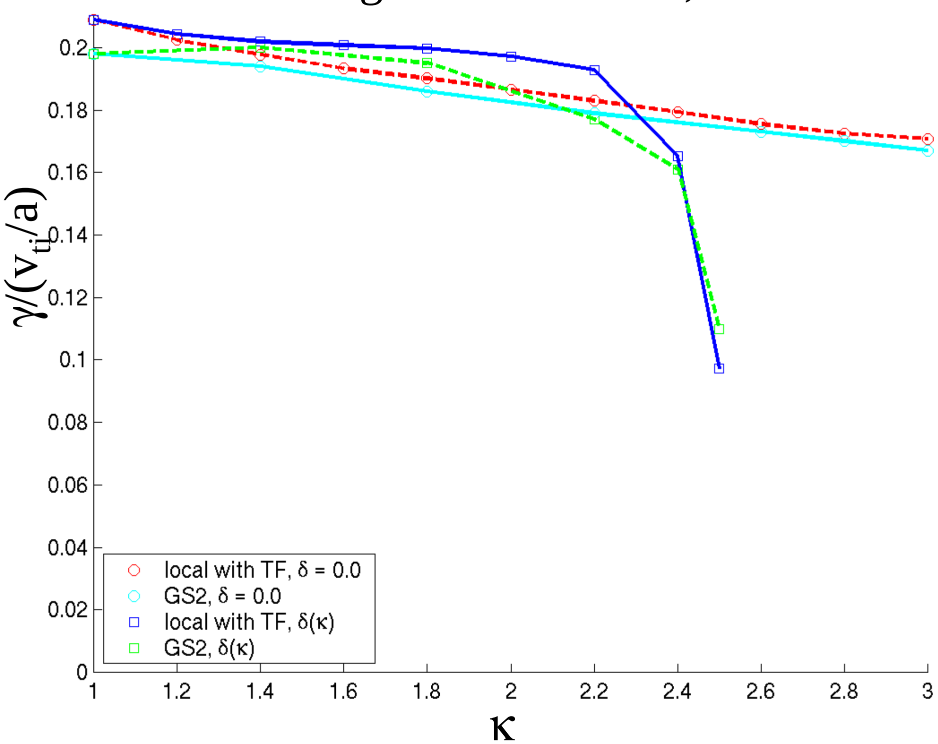


The ability of the local trial function code to track the growth rate & the critical temperature gradient will be studied more extensively in the future over a wider range of parameters.

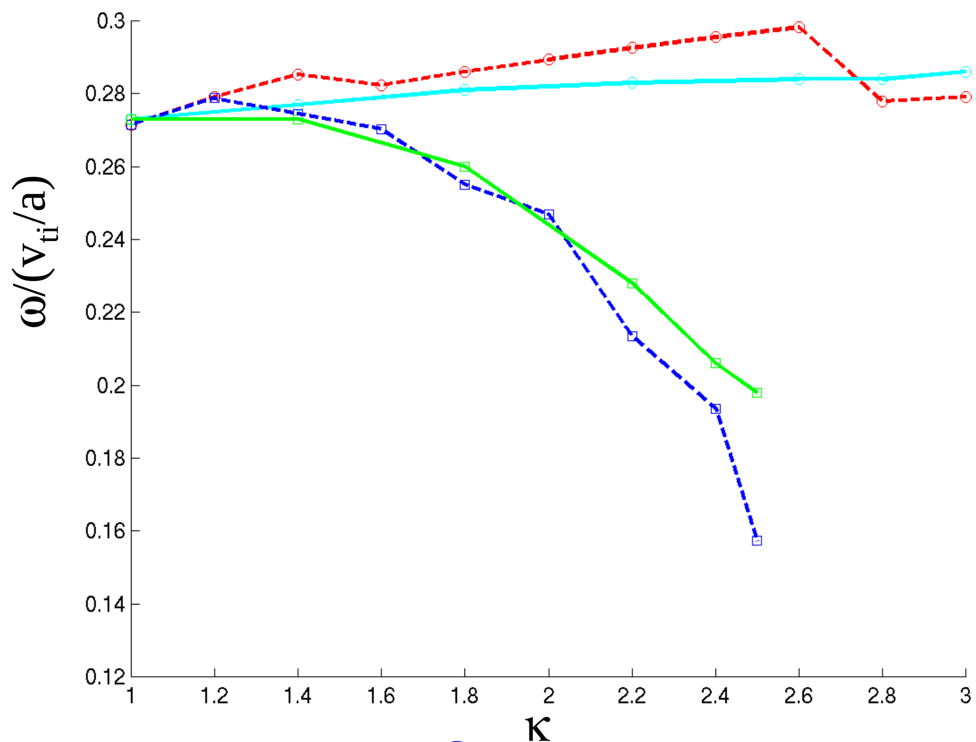
The qualitative trend of improved stability with high δ at high κ was also observed with the trial function code.

$\delta(\kappa) \approx (0.416/0.66)(\kappa\text{-}1)$, $\partial_r \delta \approx \delta/r$, $\partial_r \kappa \approx (\kappa\text{-}1)/r$

Linear growth rate vs. κ, fixed α



Real frequency vs. κ, fixed α



Though, high κ has little effect in the absence of δ for these parameters .

Conclusions

- The GS2 code has been used to study the effects of flux surface shape and other plasma parameters on the gyrokinetic stability and transport of tokamak plasmas.
- A trial function-based linearized local code has been developed and successfully benchmarked with the GS2 code in the collisionless, electrostatic, single species limit for a range of shaped flux surface equilibria.
- In general, the most stabilizing influences are seen with 1) high kappa and kappa gradient with high triangularity at a fixed α, and 2)high triangularity at high α at a finite elongation.

Future Work

- Extend the local trial function code to model trapped particle effects and to include collisional, electromagnetic, and multi-gyrokinetic species effects.
- Incorporate the trial function code in a transport code.
- Compare these results with experimental data and explore the implications for FIRE.

References

[1] R. L. Miller, et al, Phys. Plasmas **5**, 973 (1998).

[2] M. Kotschenreuther, G. Rewoldt, and W. M. Tang, Comput. Phys. Commun. **88**, 128 (1995).

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[4] R. E. Waltz, et al, Phys. Plasmas **4**, 2482 (1997).

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