Kinetic Effects on Small Scale Plasma Turbulence & Magnetorotational Instabilities in Accretion Flows

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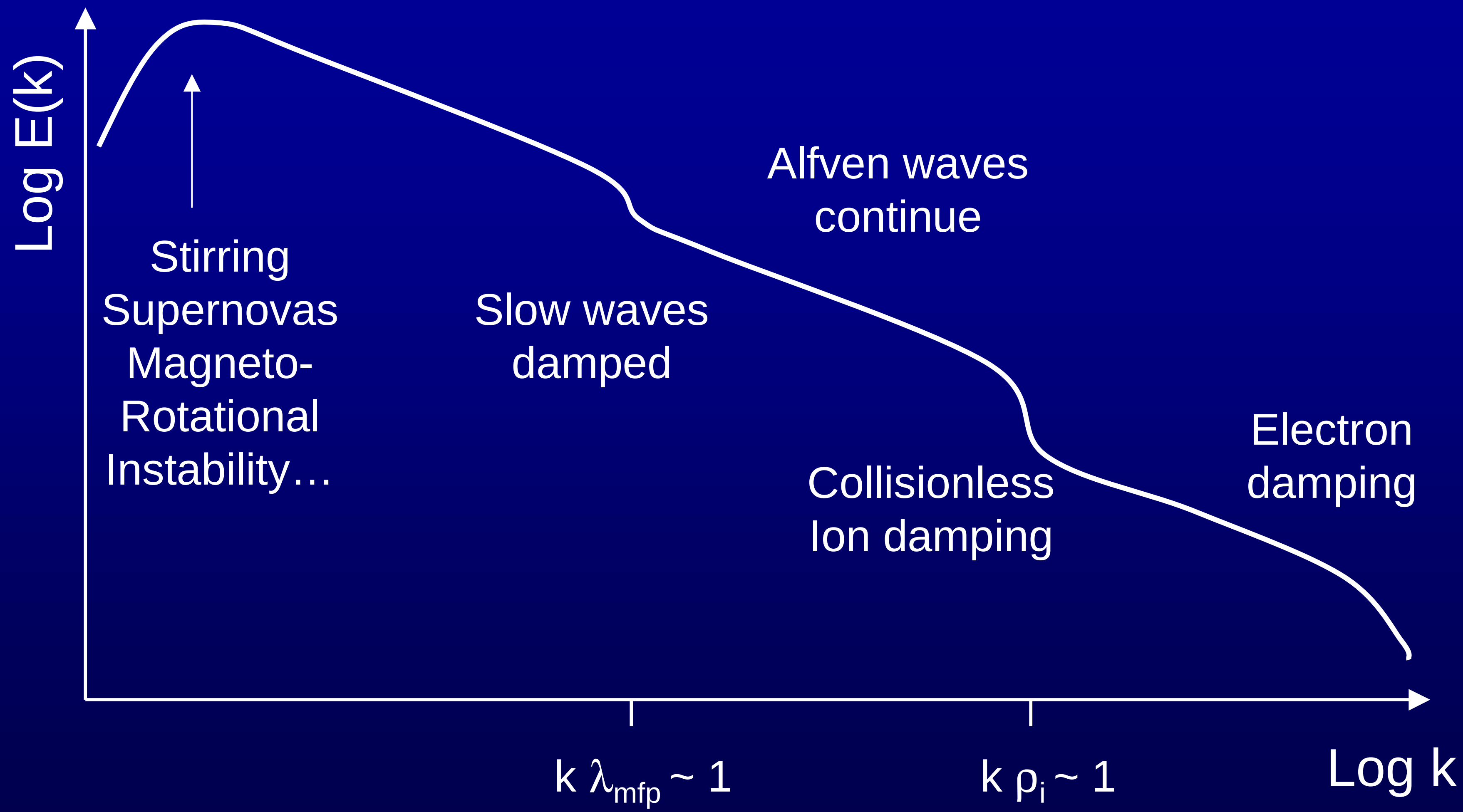
LMS Durham Astro Symposium Aug 6, 2002

Idealized Problem: What happens to tail of Alfven wave turbulent cascade: e vs i heating?



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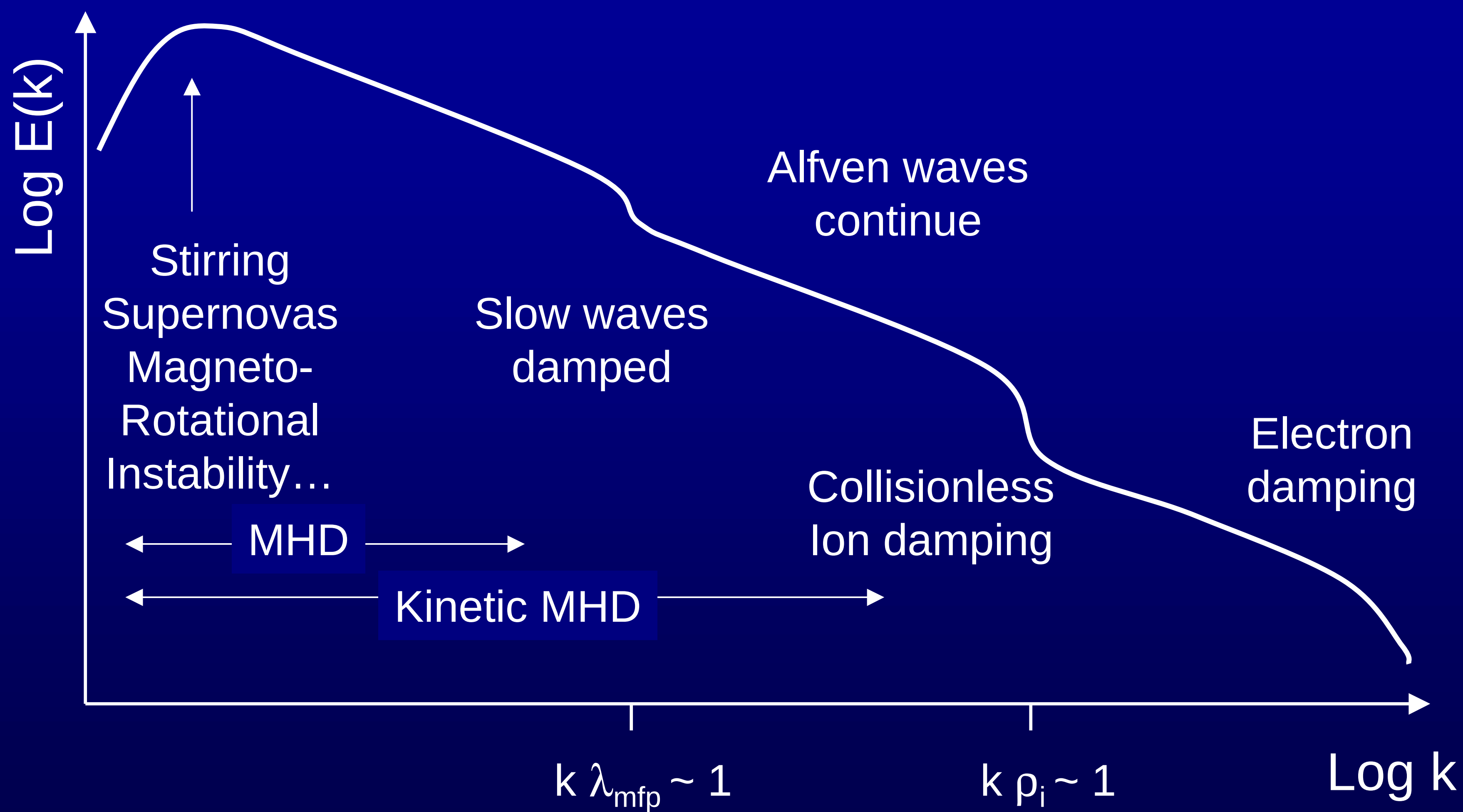

Kulsrud's formulation of kinetic MHD: anisotropic $P_\perp$ & $P_{||}$, determined by solving drift kinetic equation for distribution function. ($\omega/\Omega_{ci} \sim k_\perp \rho_i << 1$)

(Varenna 62, Handbook Plasma Physics 83)

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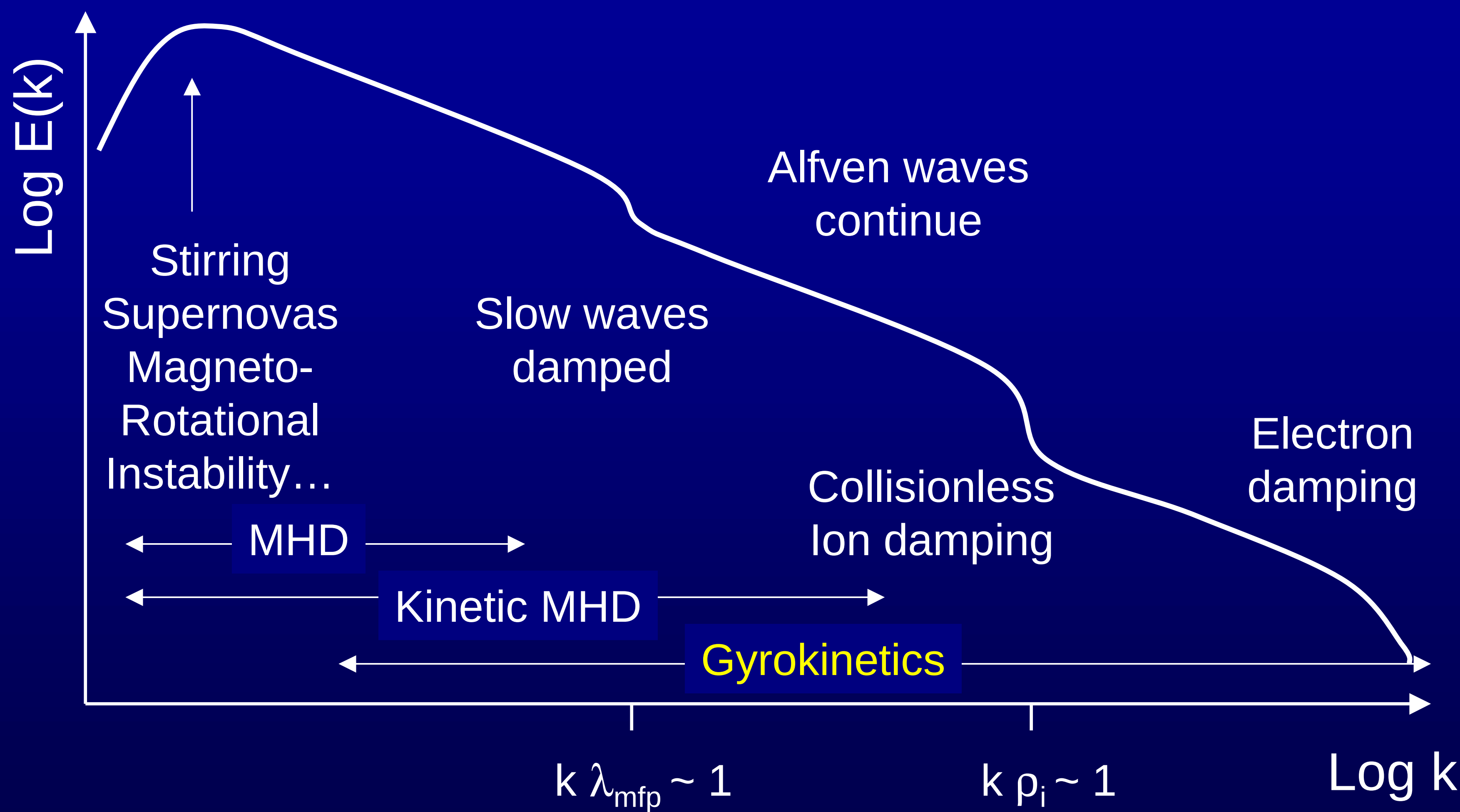


Answer requires more than MHD: collisionless kinetics, finite gyroradius. This is the regime of nonlinear gyrokinetic equations and codes developed in fusion energy research in 1980’s and 1990’s.

Overview

- **MHD Turbulence in Astrophysical Plasmas**
  - MHD turb. $\Rightarrow$ Gyrokinetic turb. on small scales
- **Astrophysical Applications**
  - Turbulence in the Interstellar Medium
  - Black Hole Accretion
  - Solar Corona & Wind
- **Gyrokinetic Simulations Needed & In Progress**

MHD Turbulence in Astrophysical Plasmas

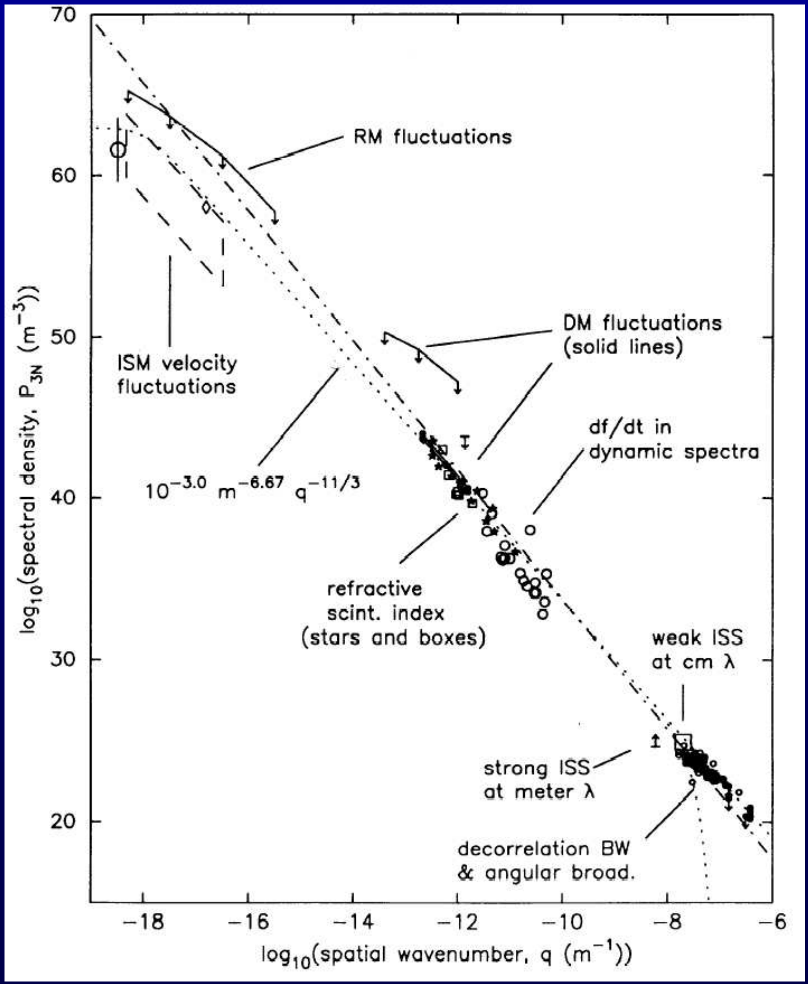
- Believed to play a central role in star formation, the transport of angular momentum in accretion disks, scintillation of interstellar media ...
- Typically $\beta \sim 1$ rather than $\beta \ll 1$ (fusion regime)
- Turbulence usually Driven or Generated by MHD Instability
  - $\rho_i/L \sim 10^{-10} \ll 1$ (L ~ system size)
  - dominant ~ ρ_i scale turbulence due to cascade of energy from larger scales, not ITG or other ~ ρ_i scale instability

Turbulence in the Interstellar Medium

Power Spectrum Of Electron Density Fluctuations

Consistent with Kolmogorov

Armstrong et al. 1995



Wavenumber (m^{-1})

Power law over ~ 12 orders of Magnitude ~5.4 Grand Pianos!

Density fluctuations change the index of refraction of the plasma & thus modify the propagation of radio waves: “Interstellar scintillation/scattering”

Incompressible MHD Turbulence

- View as nonlinear interactions btw. oppositely directed Alfven waves (e.g., Kraichnan 1965)

$$\omega = |k_{\|}| V_A$$

- Consider weak turbulence where nonlinear time >> linear time (e.g., Shebalin et al. 1983)

Resonance Conditions

$$\vec{k}_1 + \vec{k}_2 = \vec{k}_3$$

$$\omega_1 + \omega_2 = \omega_3$$

$$\Rightarrow k_{\|1} + k_{\|2} = k_{\|3} \quad \& \quad k_{\|1} - k_{\|2} = k_{\|3}$$

$k_{\|}$ cannot increase (true for 4-waves as well)

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Turbulence is Anisotropic: Energy Cascades Perpendicular to Local Magnetic Field

Strong MHD Turbulence
(Goldreich & Sridhar 1995)

- Perpendicular cascade becomes more & more nonlinear

- Hypothesize "critical balance": linear time ~ nonlinear time

$$\boxed{k_{\|} V_A \sim k_{\perp} V_{\perp}}$$

L = Outer Scale of Turbulence

- Anisotropic Kolmogorov Cascade

$$\boxed{k_{\|} \sim k_{\perp}^{2/3} L^{-1/3}}$$

$$\boxed{P(k_{\perp}) \propto k_{\perp}^{-5/3}}$$

more & more anisotropic on small scales

What Happens on Small Scales?

- At $\mathbf{k_\perp \rho_i \sim 1}$, MHD cascade has

$$\omega/\Omega_i \sim k_{\|}/k_\perp \sim (\rho_i/L)^{1/3} \sim 10^{-3} << 1$$

$$\delta B_\perp/B \sim \delta B_{\|}/B \sim (\rho_i/L)^{1/3} \sim 10^{-3} << 1$$

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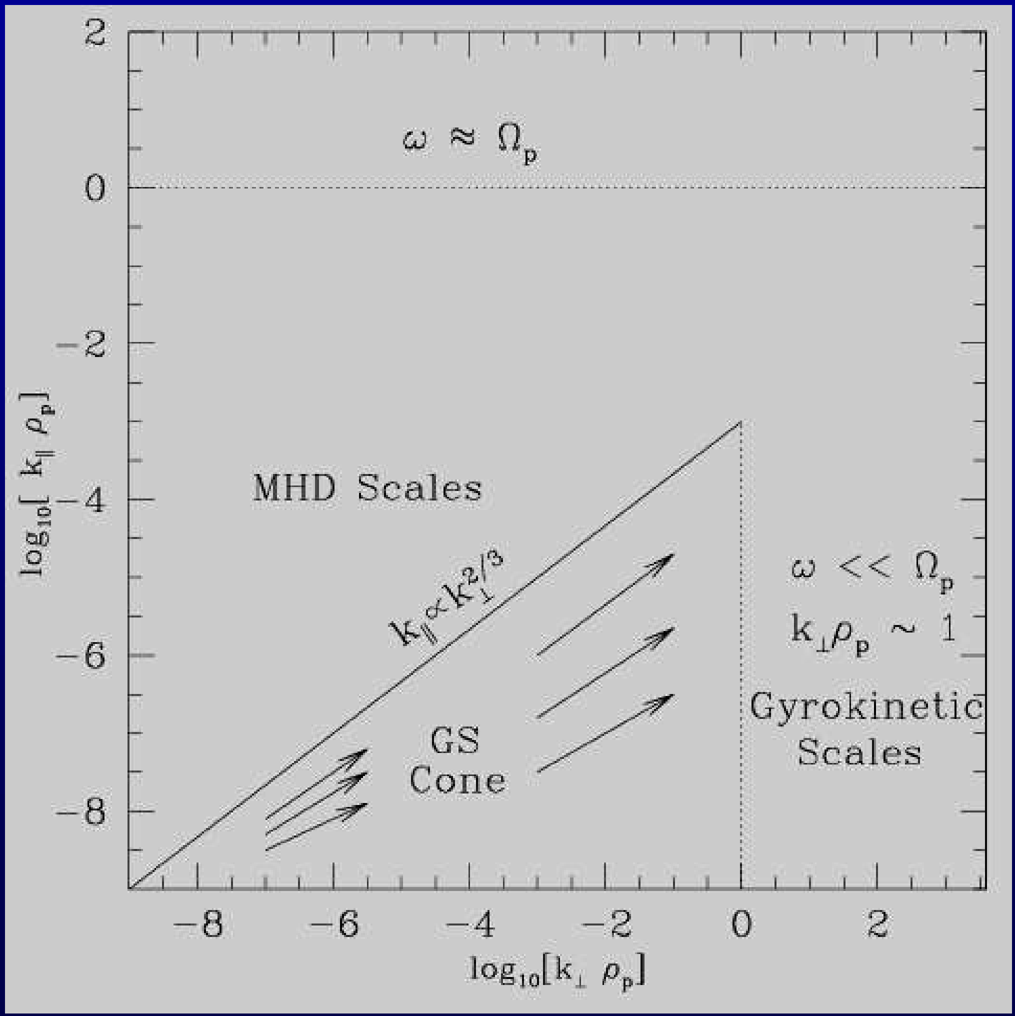
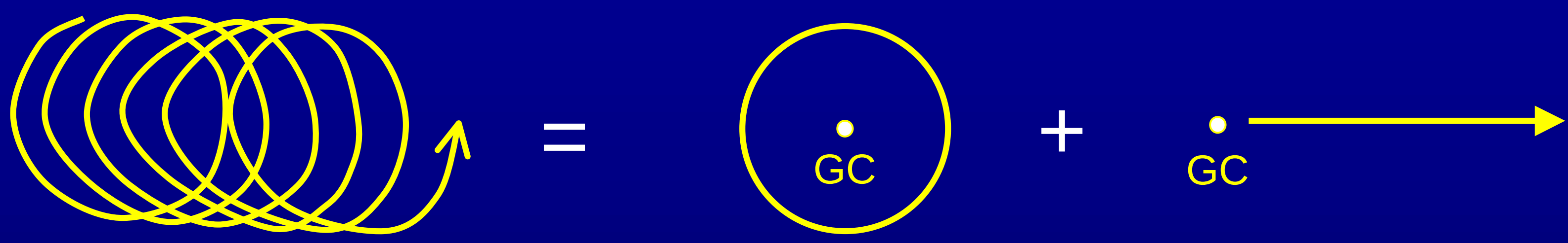
MHD Turbulence has become
Gyrokinetic Turbulence

MHD ⇒ Gyrokinetics



What are gyrokinetic equations?

- Average of full Vlasov Eq. over fast particle gyromotion



- Big advantage: eliminates fast ω_{pe} and gyrofrequencies.
- Gyrokinetic ordering:

$$\boldsymbol{\omega/\Omega_i \sim k_{||}/ k_\perp \sim (\rho_i/L) \sim \delta f/F_0 \sim V_{ExB}/v_t \sim \delta B/B_0 << 1}$$

(small gyrofrequency, parallel wavenumber, gyroradius, and perturbed particle distribution function, ExB drift, and perturbed magnetic field)

- No assumption on $k_\perp\rho_i$, β, ν_{ii}/ω, $(V_{ExB}\bullet\nabla)/\omega$

(wave numbers relative to gyroradius, plasma/magnetic pressure, collisionality, nonlinear frequency shifts)

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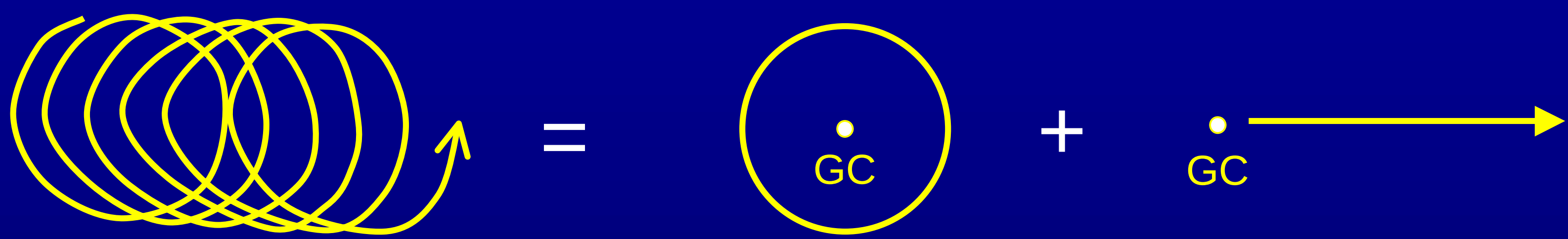

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- Caveat: Fast wave (sound wave at β>>1) ordered out:

$$\omega / \Omega_i \sim k_\perp c_s / \Omega_i \sim k_\perp\rho_i \sim 1$$

(but see Hong Qin, circa late 1990's).

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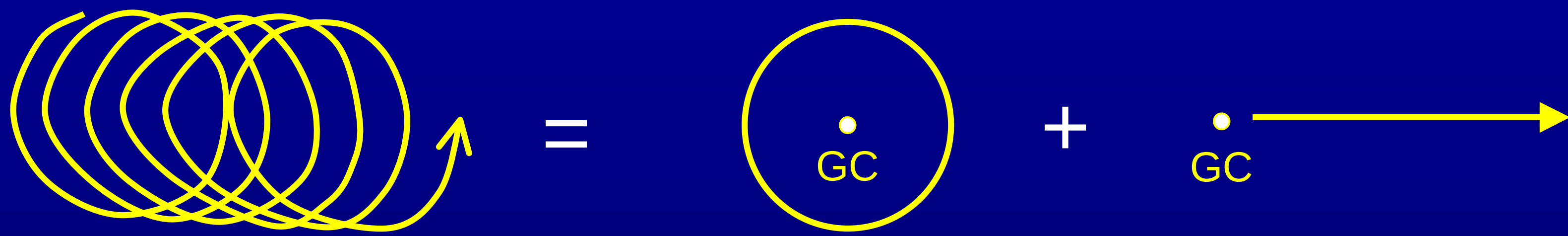

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(wave numbers relative to gyroradius, plasma/magnetic pressure, collisionality, nonlinear frequency shifts)

- But how is it nonlinear?

How can gyrokinetics be nonlinear?

- If all of these quantities are small:

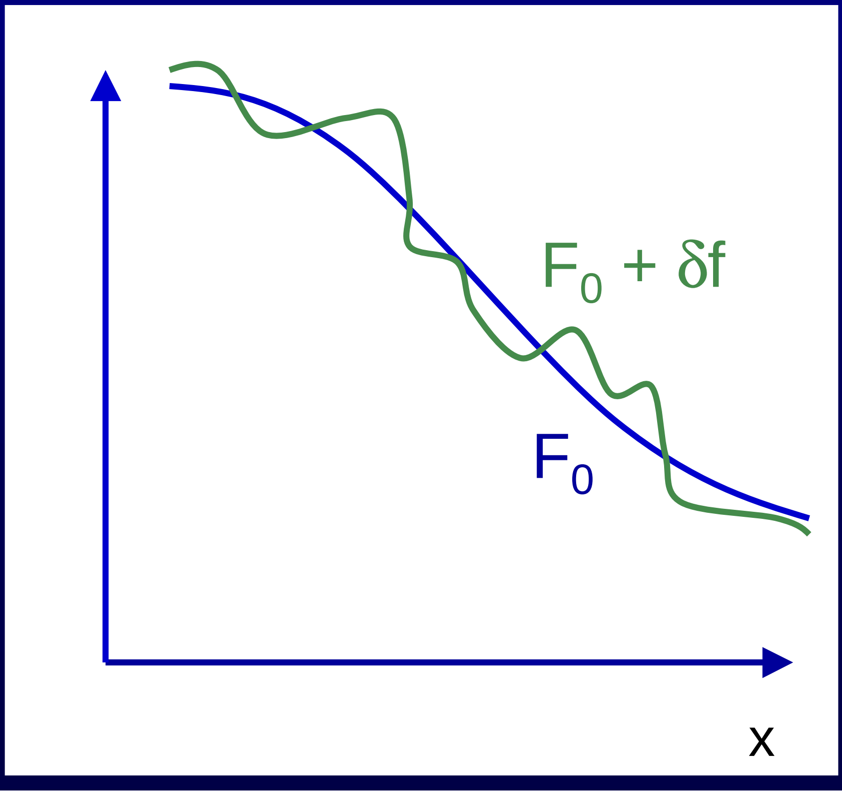
$$\omega/\Omega_i \sim k_{||}/\, k_\perp \sim (\rho_i/L) \sim \delta f/F_0 \sim V_{ExB}/v_t \sim \delta B/B_0 << 1$$

- No assumption on $k_\perp\rho_i$, β, $\nu_{ii}/\,\Omega_i$, $(V_{ExB}\bullet\nabla)/\omega$

Although $\delta f << F_0$
Nonlinear since:

$$\nabla\delta f \sim \nabla F_0$$
$$k_\perp\, \delta f \sim F_0/L$$

$$(k_\perp\, \rho_i)\, \delta f\, /\, F_0 \sim (\rho_i\, /\, L)$$



Gyrokinetic Equations Summary

- Gyro-averaged, non-adiabatic part of the perturbed distribution function, **$h=h_s(\vec{x},\varepsilon,\mu,t)$** determined by gyrokinetic Eq. (in deceptively compact form):

$$\frac{\partial h}{\partial t}+\left(\mathrm{v}_{\parallel}\hat{b}+\vec{\mathrm{v}}_d\right)\bullet\nabla h+\underbrace{\hat{b}\times\nabla\chi\bullet\nabla\left(h+F_0\right)}_{\substack{\text{Generalization of Nonlinear}\\ \text{ExB Drift incl. Magnetic fluctuations…}}}+q\frac{\partial F_0}{\partial\varepsilon}\frac{\partial\chi}{\partial t}=C(h)$$

Plus gyroaveraged Maxwell's Eqs. to get fields:

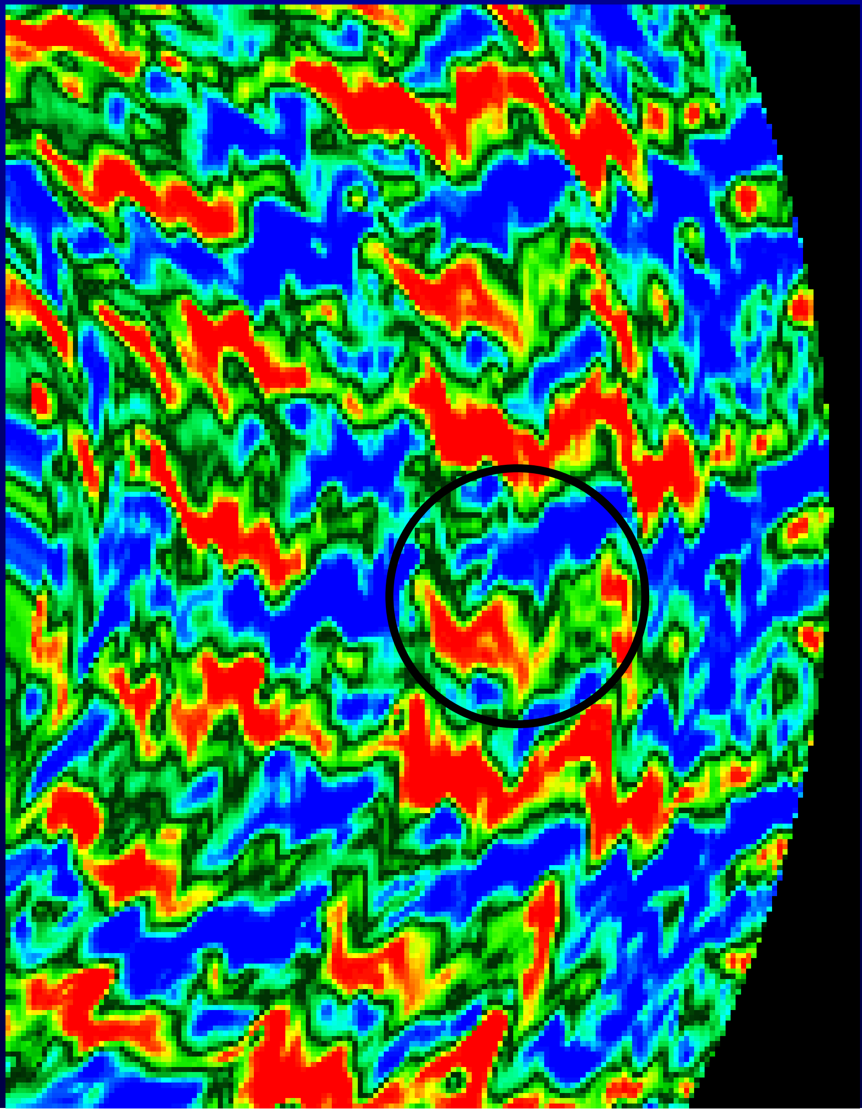
$$\boxed{\chi=J_0(k_\perp\rho)\left(\Phi-\frac{\mathrm{v}_\parallel}{c}A_\parallel\right)+\frac{J_1(k_\perp\rho)}{k_\perp\rho}\frac{m\mathrm{v}_\perp^2}{\mathrm{q}}\frac{\delta B_\parallel}{B_0}}$$

Bessel Functions represent averaging around particle gyro-orbit

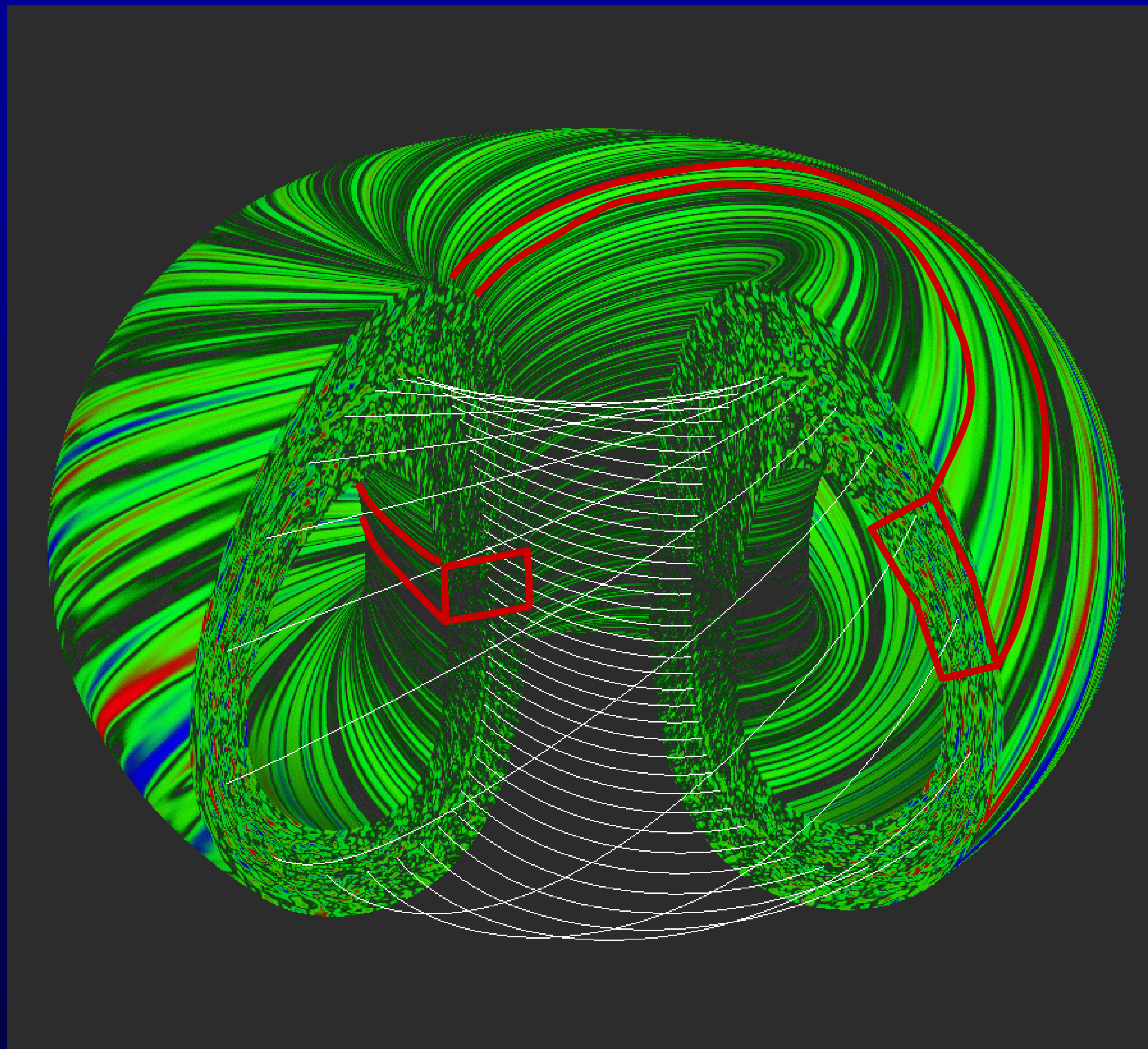
- Easy to evaluate in pseudo-spectra code. Fast multipoint Padé approx. in other codes.

$$\boxed{\chi = J_0(k_\perp \rho)\Phi}$$

$$\boxed{J_0(k_\perp \rho) = \oint d\theta\, e^{k_\perp \rho \cos(\theta)}}$$



Example of Gyrokinetic Calculation of Turbulence in Fusion Device



Gyrokinetic Numerical Methods

- Some gyrokinetic codes: explicit particle-in-cell algorithms

- GS2 code (linear: Kotschenreuther, nonlinear: Dorland):
  - pseudo-spectral in x,y (perp to B_0)
  - implicit finite-difference parallel to B_0
    useful for fast parallel electron and wave dynamics
  - grid in Energy and pitch angle ($V_{||}/V$), Gaussian integration

- Moderate resolution run:
  - x*y*z = 50*50*100, Energy*($V_{||}/V$)= 12*20, 5 eddy times
  - => ~3.5 hours on 340 proc. IBM SP2

- High resolution runs for Alfven cascade soon…

Status of gyrokinetic theory & codes

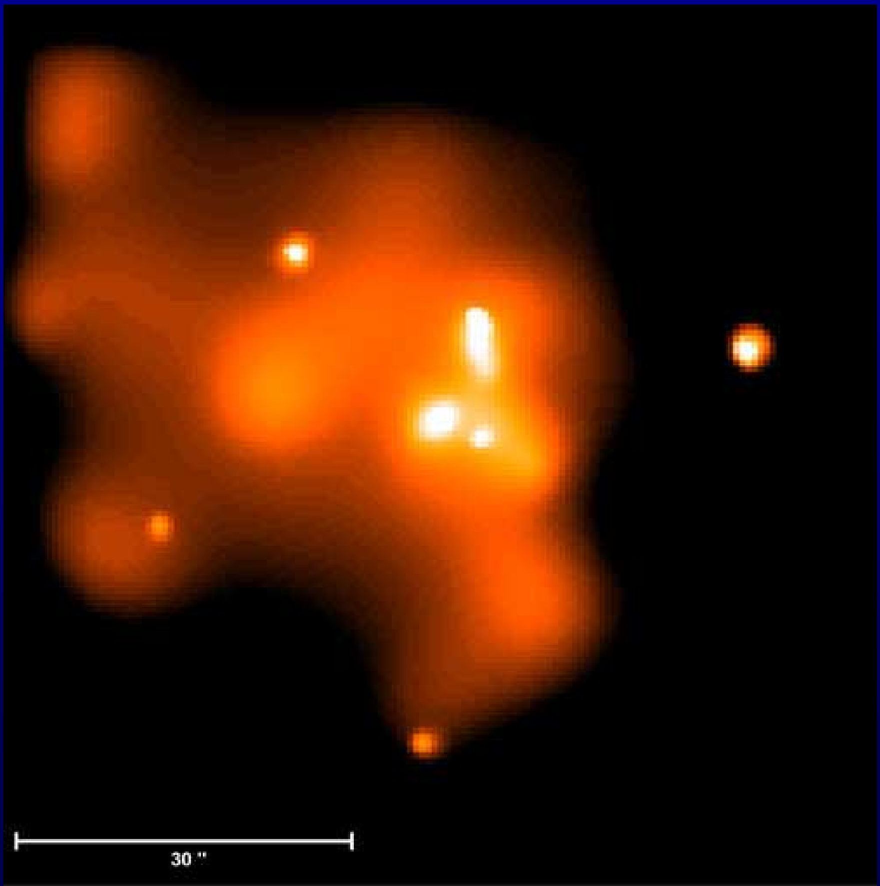
- Nonlinear gyrokinetics invented by Ed Frieman & Liu Chen (1982), studied & refined by W.W. Lee, Dubin, Krommes, Hahm, Brizard, Qin (1980's-1990's), …
- 3D nonlinear gyrokinetic codes 1990's. DOE Fusion grand challenge project, DOE SciDAC project.
- Early codes with fixed magnetic field ($\beta << 1$ in early fusion devices), turbulence and transport from ExB with $E = - \nabla \Phi$
- Dorland & Kotschenreuther GS2 code: first code to handle full magnetic fluctuations at arbitrary β, important for more efficient fusion devices (and astrophysics!)
  (Some algorithms have problems with $\beta > m_e/m_i$, $v_{te} > V_{Alfven}$)

Physics on Gyrokinetic Scales is Astrophysically Relevant When …

- Fluctuations on scales ~ ρ_i are observable
  - e.g., interstellar medium, solar wind
- System is Collisionless on its Dynamical Timescale
  - electron & ion energetics depend on heating by turbulence
  - e.g., solar wind, solar flares, plasmas around compact objects such as black holes and neutron stars

Black Hole Accretion

Center of Milky Way in X-rays (*Chandra*)

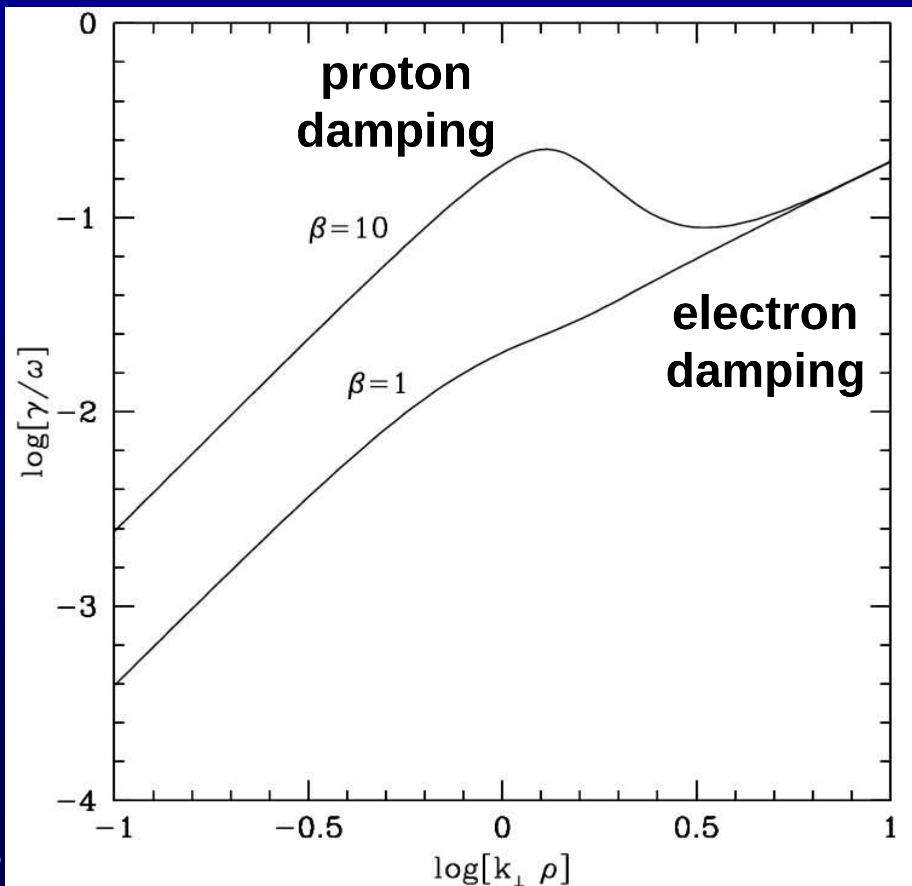


- 3×10^6 solar mass black hole
- $L \sim 10^{36}$ ergs s^{-1}
- Leading model for accretion onto the BH posits a **two-temperature collisionless plasma**

$$T_p \sim 100 \text{ MeV} >> T_e \sim 1 \text{ MeV}$$
$$n \sim 10^9 \text{ cm}^{-3} \quad B \sim 10^3 \text{ Gauss}$$

- **All observables (luminosity & spectrum) determined by amount of electron heating**

Collisionless Damping on ~ ρ_i scales



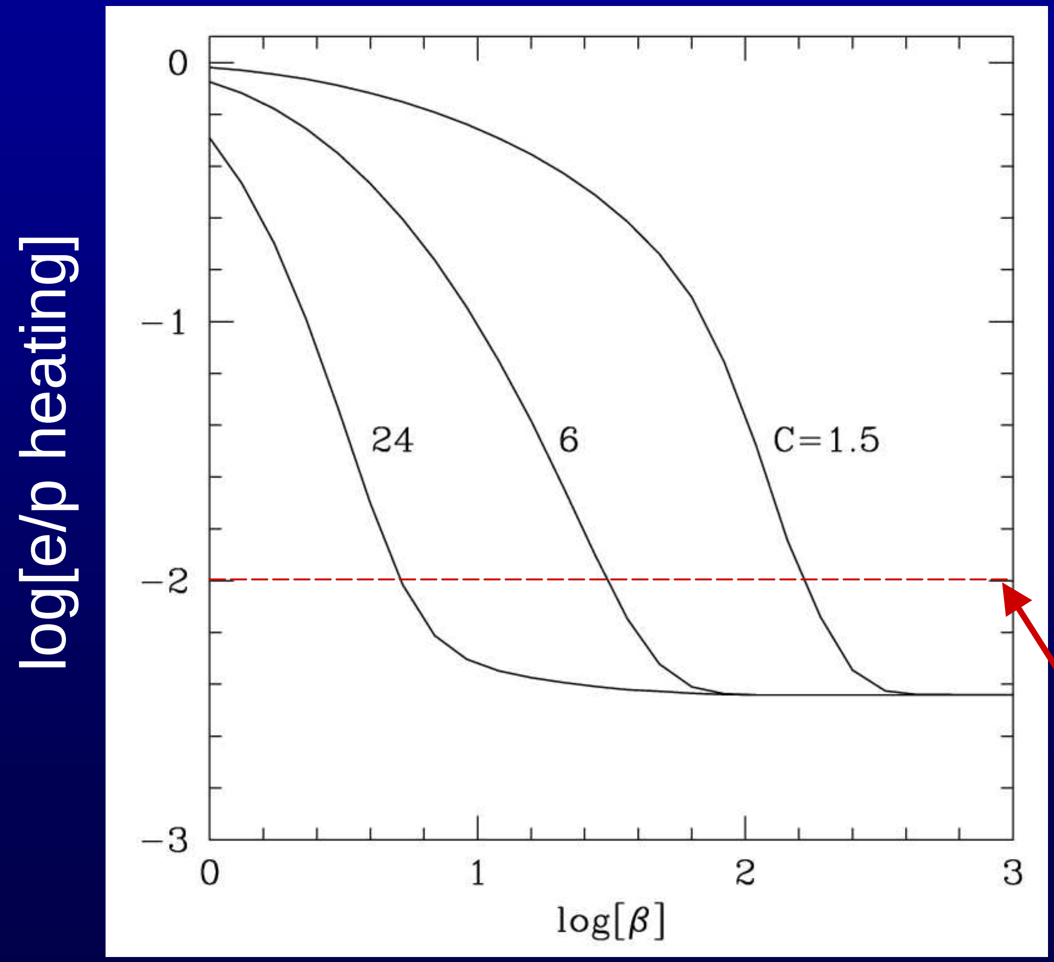
Quataert 1998

Linear collisionless damping of Alfven waves with $k_\perp >> k_{||}$

Strong damping requires
γ ~ nonlinear freq. ~ ω

Damping sets inner scale & $\Rightarrow$ Heating of Plasma

Analytic Estimates of Electron Heating Are Indeterminate



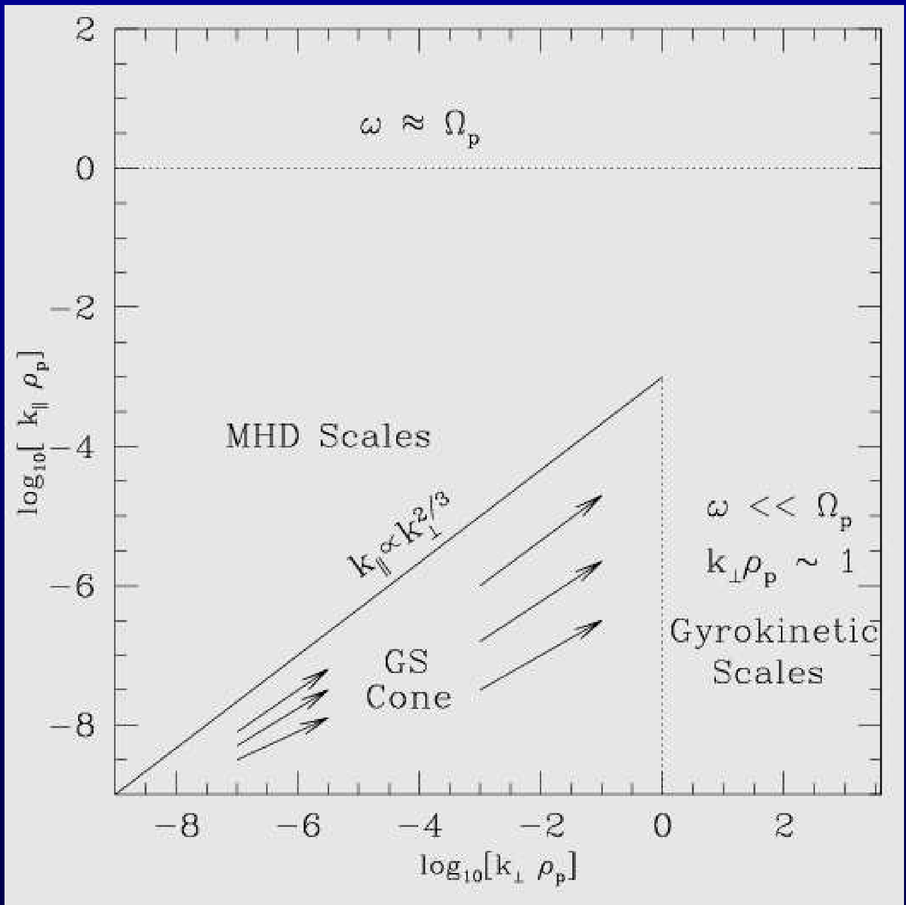
$$C \approx \frac{T_{nonlinear}}{T_{linear}}$$

uncertain because damping occurs at $k_\perp \rho_p > 1$ outside MHD regime $\Rightarrow$ need Gyrokinetic simulations

Low electron heating reqd for ADAF models to explain low luminosity of some black holes

Quataert & Gruzinov 1999

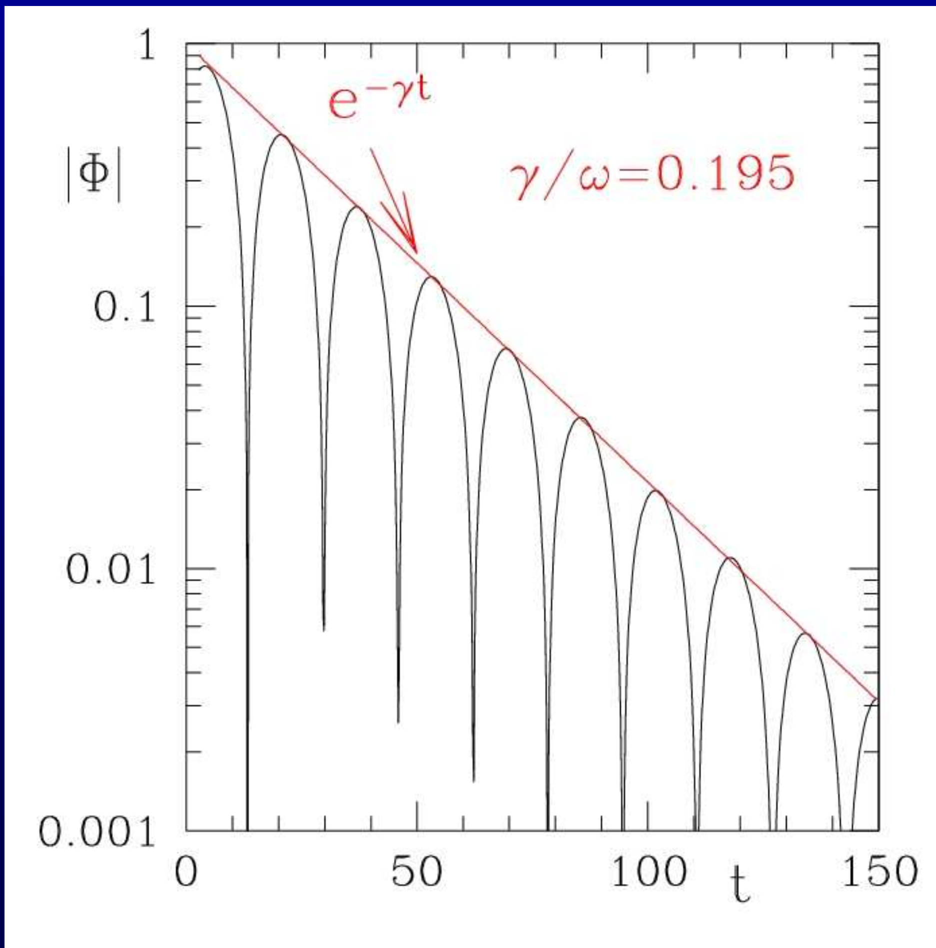
Program: Simulate Gyrokinetic Turbulence in Astrophysics Context



- Very Difft. From Fusion Applications
- Turbulence driven by cascade from large scales not by ~ ρ_i instabilities
  $\Rightarrow$ "stir" box at outer scale
- Simple geometry (triply periodic slab)
  no background plasma gradients
- $\beta << 1$ & $>> 1$, $T_p/T_e \sim 1$ & $>> 1$
- Diagnostics: electron vs. proton heating
  density/vel/B-field power spectra

Using **GS2** code developed by Dorland, Kotschenreuther, & Liu, & utilized extensively in the fusion program

Linear Tests



β = 100; $k_\perp\rho_p$= 0.4

Compare GS2 damping of linear Alfven wave with linear kinetic code

Excellent agreement over entire parameter space

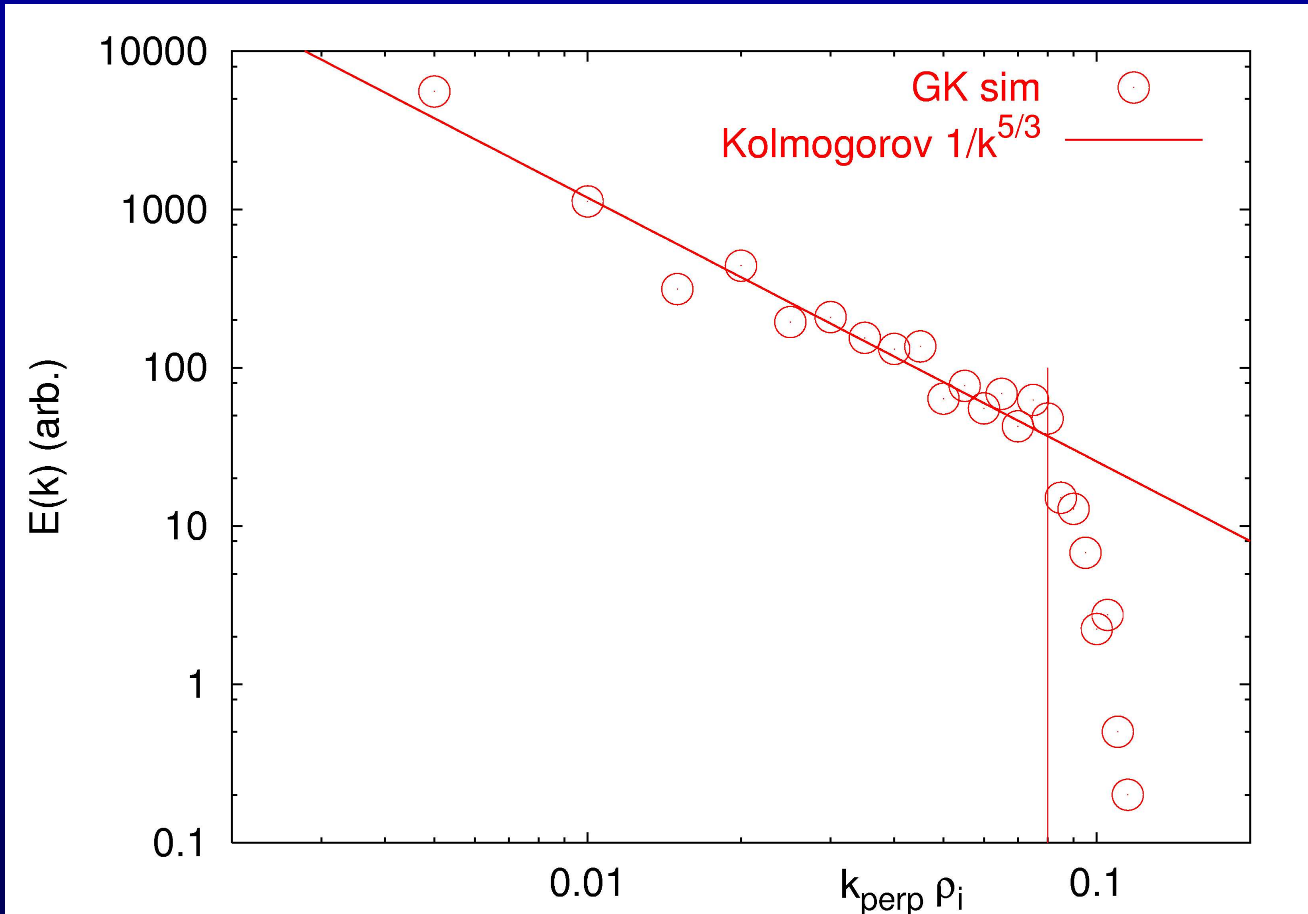
$\beta << 1$ & $\beta >> 1$

$T_p/T_e \sim 1$ & $T_p/T_e >> 1$

$k_\perp\rho_p << 1$ & $k_\perp\rho_p >> 1$

Important test of $\delta B_{||}$ physics (transit-time damping)

First Nonlinear Alfven Tests



- Simulate turbulence in a box >> ρ_i, negligible gyro effects.
- **Reproduces MHD results (Goldreich-Sridhar)**
- Kolmogorov power spectrum and anisotropy

β = 8 (800%). x*y*z*Energy*(Vpar/V)= 50*50*100*12*20

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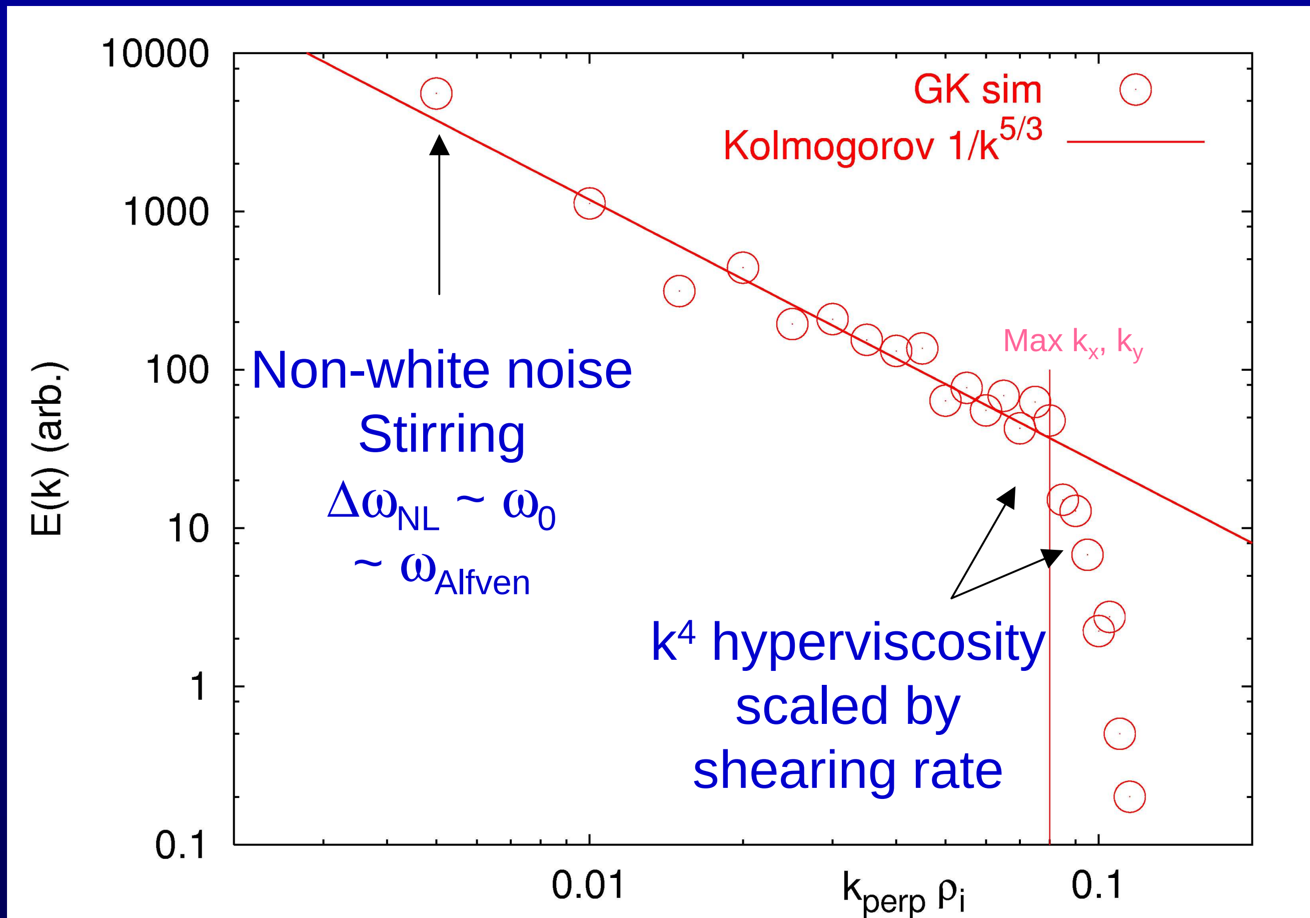

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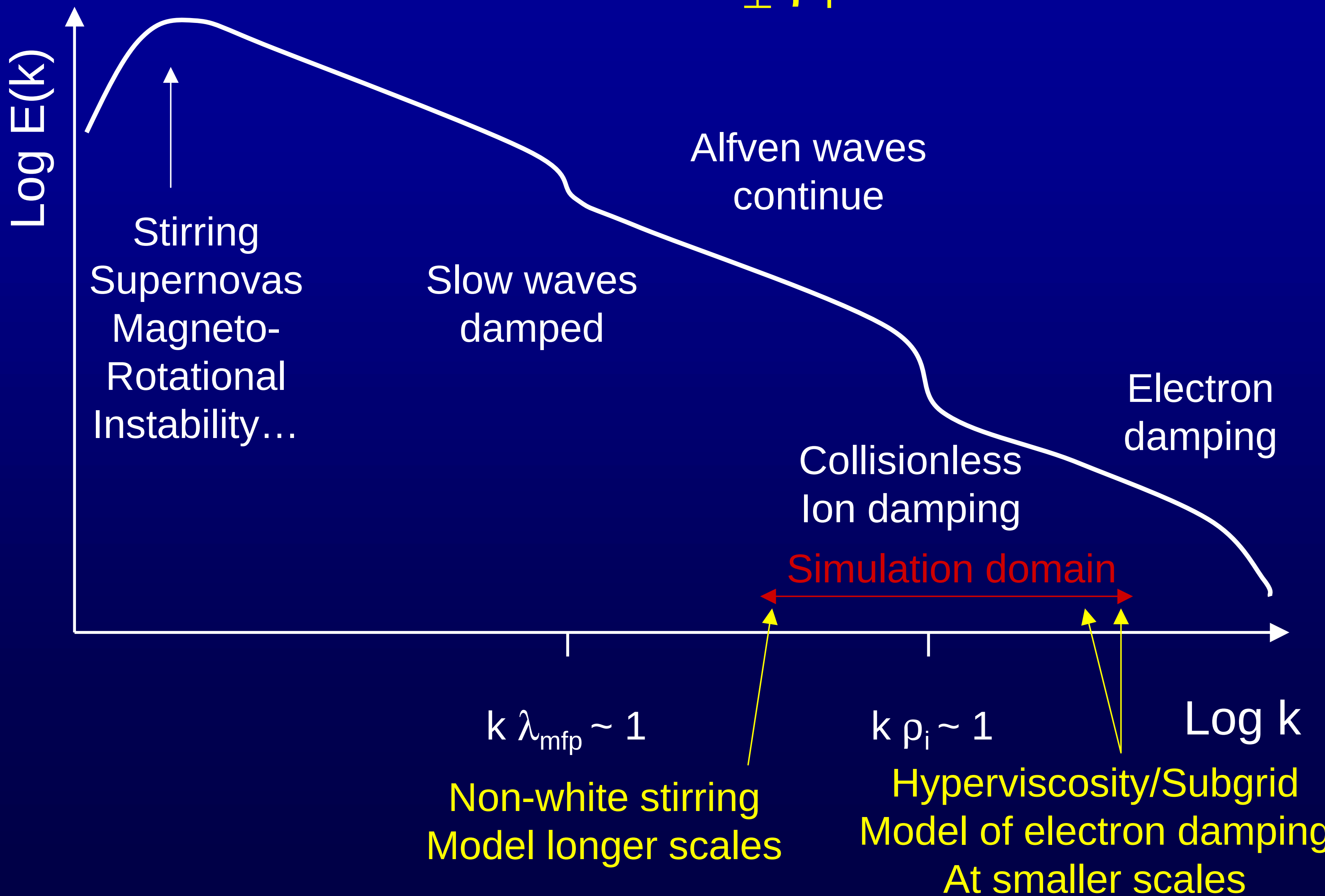
To study amount of ion damping, focus on $0.1 < k_\perp \rho_i < 10.0$



Kinetic effects on Magneto-Rotational Instability

Get feet wet by looking at linear kinetic effects on MRI:

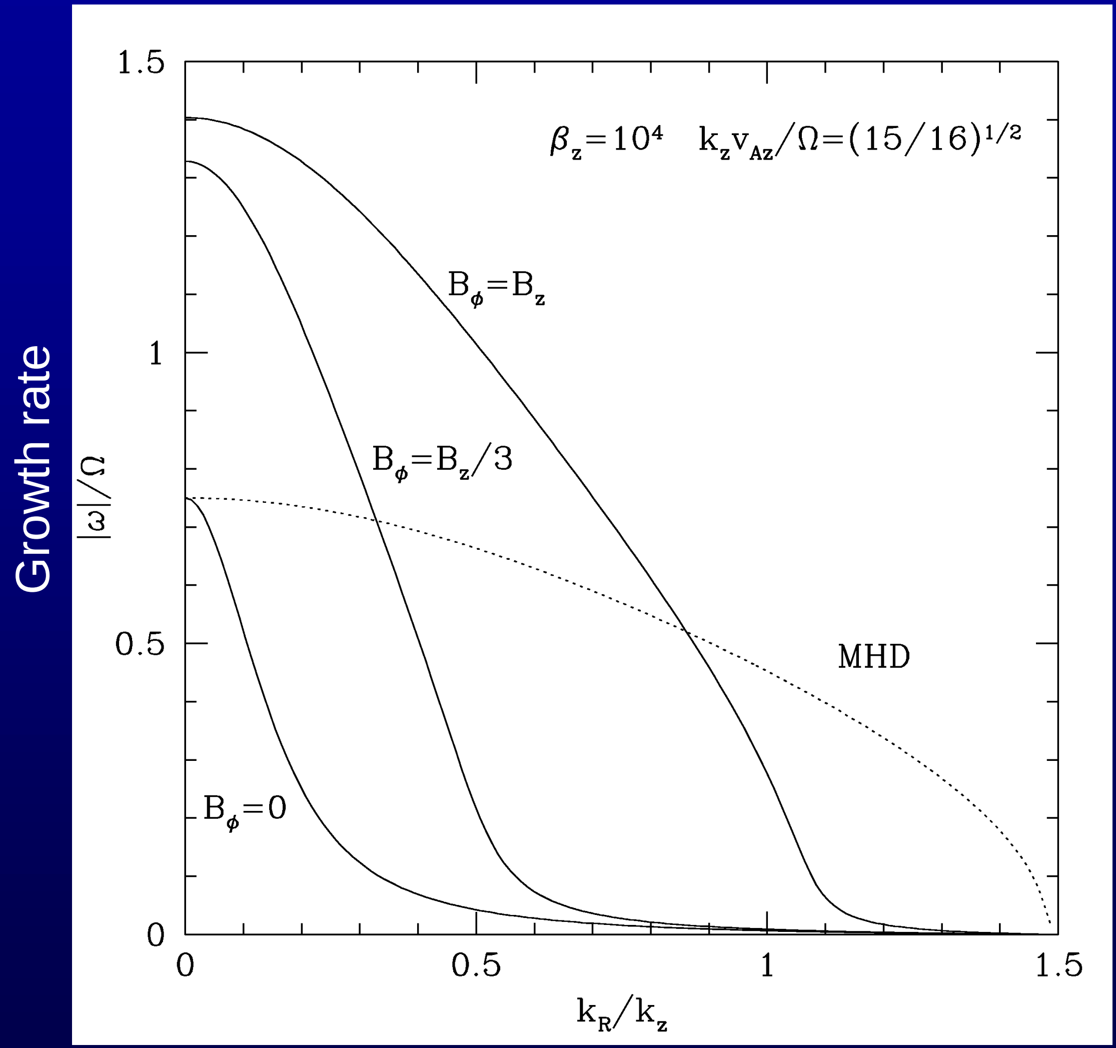
Looking at classic limit done by Balbus & Hawley:

- Axisymmetric (k_ϕ=0)
- B_r=0

?? Put in figure showing geometry, defining R,z,ϕ coordinates

More info: Quataert, Dorland, Hammett, Astro-ph/0205492 (Ap.J. 2002)

Kinetic effects on Magneto-Rotational Instability



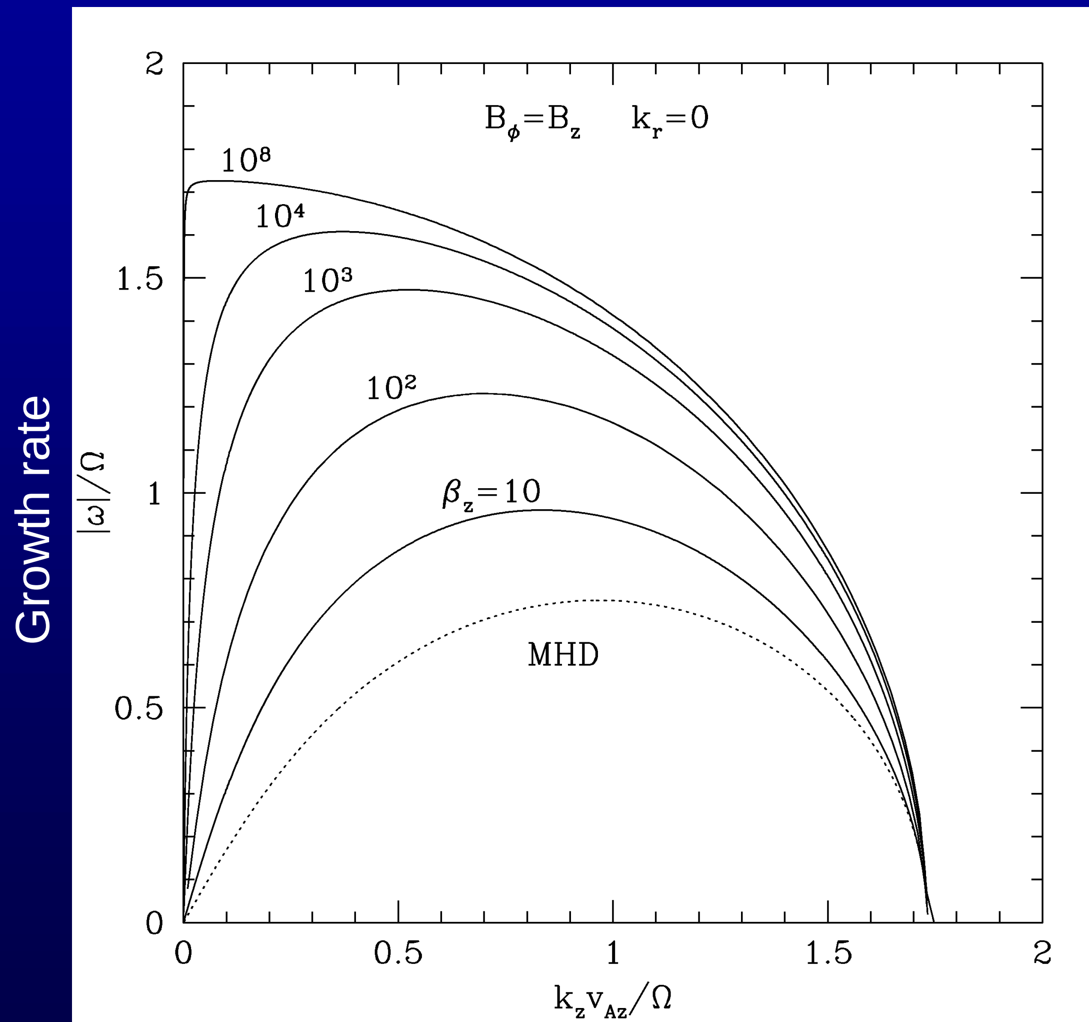
Using Kulrud's version Kinetic MHD

Qualitatively similar Trends to Balbus-Hawley Results.

Kinetic effects can be destabilizing or stabilizing.

More info: Quataert, Dorland, Hammett, Astro-ph/0205492 (Ap.J. 2002)

Kinetic MHD -> regular MHD at lower β (for linearly unstable modes)



Quataert, Dorland, Hammett, Astro-ph/0205492 (Ap.J. 2002)

At high β, fastest growing mode shifts to lower k_z

What happens nonlinearly?

Regular MHD: Viscous damping only at high |k|

Kinetic MHD: collisionless damping of sound & slow magnetosonic waves occurs at any scale (depends on direction of k)

Alters nonlinear state? $\Uparrow$ ion & $\Downarrow$ e heating?

Summary

- MHD turb. $\Rightarrow$ Gyrokinetic turb. on small scales
- Astrophysical applications abound, including

1. Predicting density/velocity/B-field power spectra
   - Compare with observations of ISM & solar wind turb.
2. Predicting plasma heating by gyrokinetic turb.
   - Applications to black hole physics, solar physics, …
3. Possible modifications of Magnetorotational Instability