Single Particle Motion

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(thanks to Karney & many others...)

Assume a given $\underset{\sim}{E}$ & $\underset{\sim}{B}$, as generated by external coils & perhaps including currents & charges of all other particles in the plasma.

Study single particle motion in this specified $\underset{\sim}{E}$ & $\underset{\sim}{B}$ field.

Motivation:

* Fusion: use magnetic field to confine a plasma.

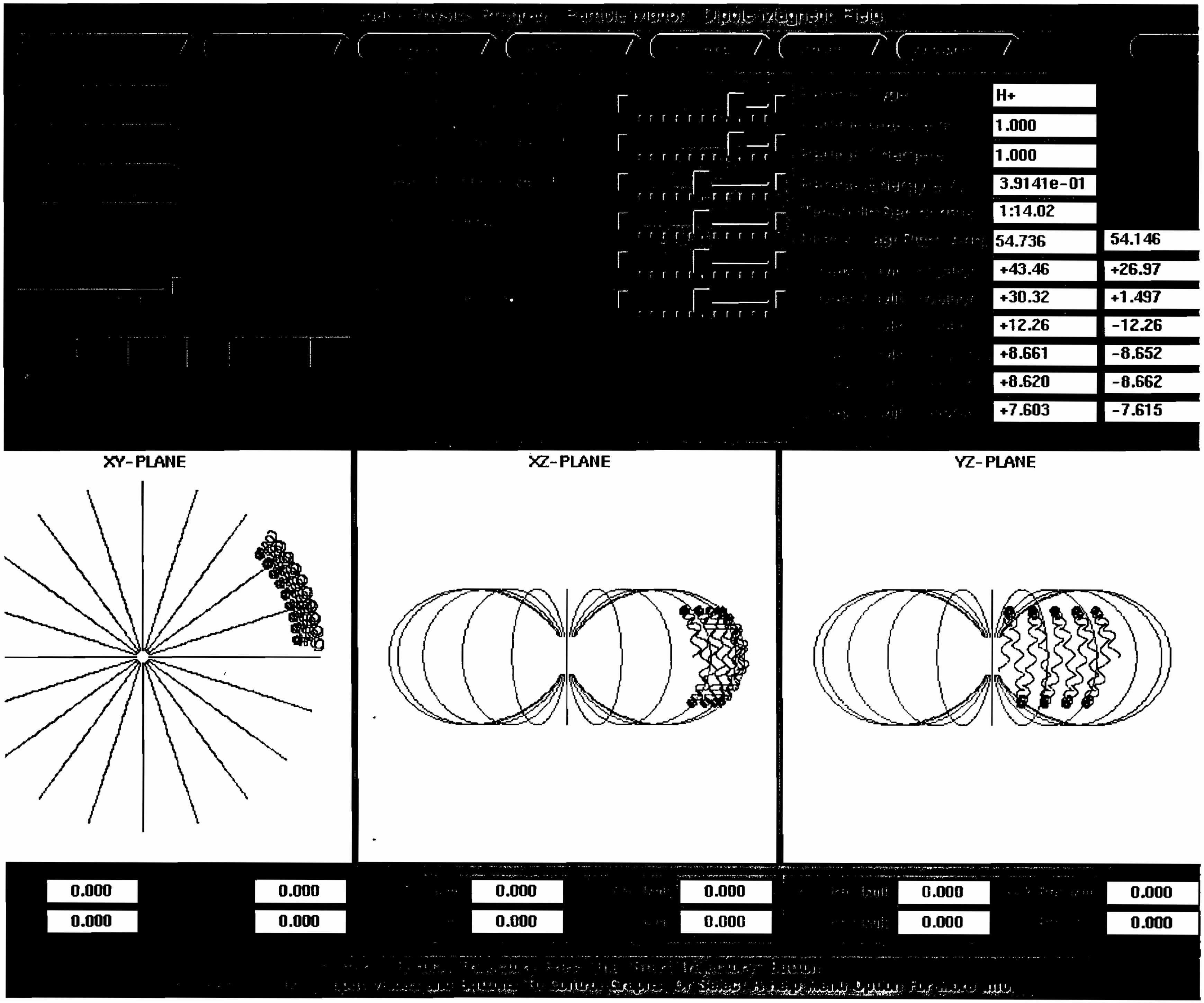
Lawson's criterion $n\tau > 10^{20}\ m^{-3}\ s$

typical $n \sim 10^{20}/m^3 \Rightarrow \tau > 1$ sec.

Single particle confinement is a necessary (but not sufficient) condition for plasma confinement.

* Other applications:

Microwaves, radar, free-electron lasers
Particle accelerators
Van Allen Belts around earth.
Solar Wind, Solar flares, sunspots.
Cosmic rays, Pulsars, galactic magnetic field...
Plasma processing: semiconductor etching, material coatings
Chemistry, medicine: ultra high resolution mass spectrometers.


Dipole Magnetic Field
H+
1.000
1.000
3.9141e-01
1:14.02
54.736
54.146
+43.46
+26.97
+30.32
+1.497
+12.26
-12.26
+8.661
-8.652
+8.620
-8.662
+7.603
-7.615
XY-PLANE
XZ-PLANE
YZ-PLANE
0.000
0.000
0.000
0.000
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How to analyze complex problems:

- search for simple cases where exact answers can be found
- small perturbations around simple limits to find approximate, useful solutions.
- search for approximations to simplify equations.
- combine rigorous derivation, back-of-the-envelope estimates, & physical pictures to build intuition & insight into complex physical systems...

Lorentz Force Law

(in cgs-gaussian units, see NRL p. 10-19 for conversions.)

$$\boxed{\begin{aligned} \frac{d\underset{\sim}{v}}{dt} &= \frac{q}{m}\left[\underset{\sim}{E}(\underset{\sim}{r},t) + \frac{\underset{\sim}{v} \times \underset{\sim}{B}(\underset{\sim}{r},t)}{c}\right] \\ \frac{d\underset{\sim}{r}}{dt} &= \underset{\sim}{v} \end{aligned}}$$

(My vector notation $\underset{\sim}{v} = \vec{v}$)

Generally, this is a nonlinear differential equation ($\underset{\sim}{E}$ & $\underset{\sim}{B}$ are nonlinear functions of $\underset{\sim}{r}$)

Exact result (related to Hamiltonian structure of mechanics):

$$\underset{\sim}{v} \cdot \frac{d\underset{\sim}{v}}{dt} = \frac{q}{m}\left[\underset{\sim}{v}\cdot\underset{\sim}{E} + \underbrace{\underset{\sim}{v}\cdot\frac{(\underset{\sim}{v}\times\underset{\sim}{B})}{c}}_{\equiv 0}\right]$$

$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = q\,\underset{\sim}{v}\cdot\underset{\sim}{E}$$

Kinetic Energy is conserved if $\underset{\sim}{E} = 0$.
$\underset{\sim}{B}$ causes $\underset{\sim}{v}$ to change directions, but keeps $|\underset{\sim}{v}|$ = constant.

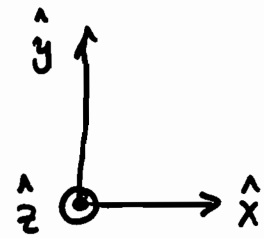
$$\left[\ \underset{\sim}{v}\cdot\frac{d\underset{\sim}{v}}{dt} = v_x\frac{dv_x}{dt} + v_y\frac{dv_y}{dt} + v_z\frac{dv_z}{dt} = \frac{d}{dt}\left(\frac{1}{2}\underset{\sim}{v}\cdot\underset{\sim}{v}\right)\ \right]$$

Simplest Case: $\underset{\sim}{E} = 0$, $\underset{\sim}{B} = \text{const} = B_0 \hat{z}$

$$\frac{d\underset{\sim}{v}}{dt} = \frac{qB_0}{mc} \underset{\sim}{v} \times \hat{z}$$

$$\frac{d\underset{\sim}{r}}{dt} = \underset{\sim}{v}$$

$\hat{z}$ = unit vector (dimensionless, $|\hat{z}| = 1$) pointing in the z-direction

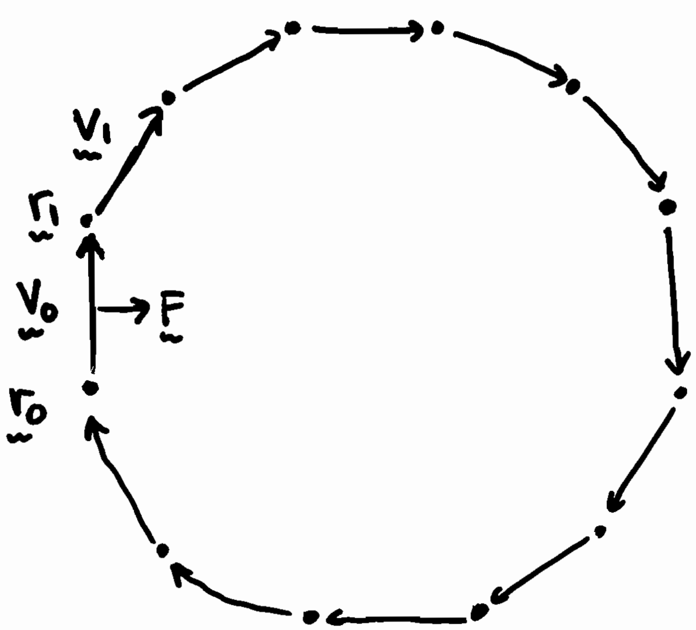


$\underset{\sim}{B} \parallel \hat{z}$ ⊙ Out-of-page

Numerical Integration by Hand:

$$\underset{\sim}{r}_1 = \underset{\sim}{r}(t_0 + \Delta t) = \underset{\sim}{r}_0 + \Delta t\, \underset{\sim}{v}_0$$

$$\underset{\sim}{v}_1 = \underset{\sim}{v}(t_0 + \Delta t) = \underset{\sim}{v}_0 + \Delta t \frac{qB}{mc} \underset{\sim}{v} \times \hat{z}$$

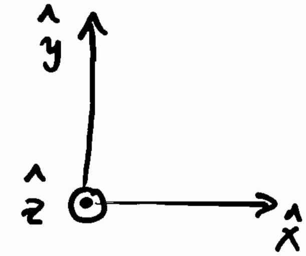


Becomes a circular orbit as $\Delta t \to 0$.

Like circular orbits of planets, or swinging a weight on a string, the force is always towards the center of the circle...

Rigorous Solution of

Simplest Case: $\underset{\sim}{E} = 0$, $\underset{\sim}{B} = \text{const.} = B_0 \hat{z}$



$$\frac{d\underset{\sim}{v}}{dt} = \underbrace{\frac{qB}{mc}}_{\equiv \Omega} \underset{\sim}{v} \times \hat{z}$$

$\equiv \Omega$ $\left\{ \begin{array}{l} \text{cyclotron frequency } \Omega_{ci} = \dfrac{q_i B}{m_i c} \\ \text{gyro-frequency} \\ \text{Larmor frequency} \end{array} \right.$

frequency has units of $\dfrac{1}{\text{time}}$

Component form:

$$\frac{dv_x}{dt} = \Omega v_y \qquad\qquad \frac{dv_z}{dt} = 0$$

$$\frac{dv_y}{dt} = -\Omega v_x$$

Solution:

$$v_x = v_\perp \sin(\Omega t + \alpha)$$
$$v_y = v_\perp \cos(\Omega t + \alpha)$$
$$v_z = v_\parallel$$

Initial Conditions (v_{x0}, v_{y0}, v_{z0})
determine $(v_\perp, \alpha, v_\parallel)$

Integrate $\frac{d\underset{\sim}{r}}{dt} = \underset{\sim}{v} \quad \Rightarrow \quad \underset{\sim}{r} = \int dt\, \underset{\sim}{v}$

$$x = x_0 - \frac{v_\perp}{\Omega}\cos(\Omega t + \alpha)$$

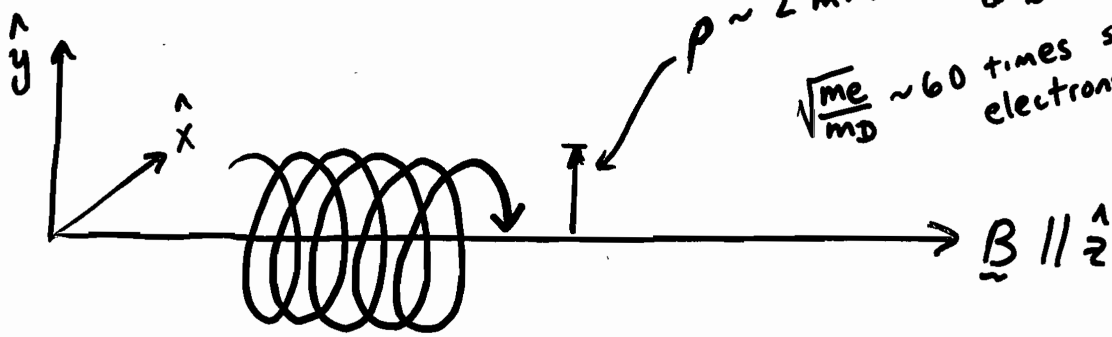
$$y = y_0 + \frac{v_\perp}{\Omega}\sin(\Omega t + \alpha)$$

$$\frac{v_\perp}{\Omega} \equiv \rho = \begin{cases} \text{gyro-radius} \\ \text{Larmor-radius} \end{cases}$$

$$z = z_0 + v_\parallel t$$

$\rho \sim 2$ mm for 10 keV deuterons & $B = 5$ Tesla.

$\sqrt{\frac{m_e}{m_D}} \sim 60$ times smaller for 10 keV electrons



<u>Helical Orbit</u>:

Straight $\underset{\sim}{B}$ confines $\perp$ to $\underset{\sim}{B}$.
Particles still lost $\parallel$ to $\underset{\sim}{B}$.
"end-loss problem"

Particle gyrates around a "guiding center" (or "gyro-center") which follows $\underset{\sim}{B}$ (exactly for straight $\underset{\sim}{B}$, approximately for general $\underset{\sim}{B}$)

$$\underset{\sim}{r} = \underset{\sim}{R}_{gc} + \underset{\sim}{\rho}$$

Particle Position — Guiding Center Moves Smoothly $(x_0, y_0, z_0 + v_\parallel t)$ — Rapidly changing gyro-position vector

Handy formulas in NRL Plasma Formulary (p. 28-29) for

$$\Omega = \frac{qB}{mc} \qquad \rho = \frac{v_\perp}{\Omega}$$

$\frac{\Omega_{ci}}{2\pi} \sim 80$ MHz for protons & B = 5 Tesla
(RF range, close to FM band)

$\frac{\Omega_{ce}}{2\pi} \sim 140$ GHz for electrons " " "
(microwave range, $\lambda \sim 2$ mm)

Heating methods: Launch waves which resonate with particle gyrations.

10 MW Microwave Heating System
$\approx$ 20,000 home microwave ovens (?)
Cooking time for a cow $\approx$ 10 seconds

Diagnostic methods: gyrating particles $\Rightarrow$ little radiating antennas

reabsorption is often strong. black-body radiation $\Rightarrow$ measure temperature

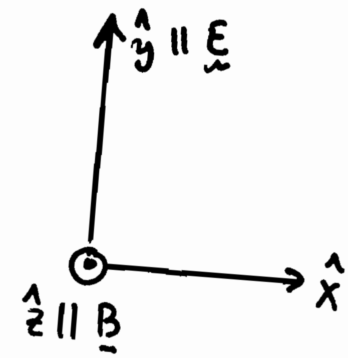
Non-fusion applications:

inject particle beams into RF cavities and/or magnetic fields of various configurations: high power RF tubes, microwave generation, free-electron lasers...

$E \times B$ Drift

$\underset{\sim}{E} = \text{const} = E_0 \hat{y}$ $\qquad$ $\underset{\sim}{B} = \text{const} = B_0 \hat{z}$

$$\frac{d\underset{\sim}{v}}{dt} = \frac{q}{m}\left[\underset{\sim}{E} + \frac{\underset{\sim}{v} \times \underset{\sim}{B}}{c}\right]$$



Substitute:

$$\underset{\sim}{v} = \underbrace{\frac{c}{B^2}\underset{\sim}{E} \times \underset{\sim}{B}} + \underset{\sim}{v}'$$

$$\times \frac{\underset{\sim}{B}}{c} = \frac{1}{B^2}\left[\left(\underset{\sim}{E} \times \underset{\sim}{B}\right) \times \underset{\sim}{B}\right] = -\underset{\sim}{E}_\perp$$

(Component of $\underset{\sim}{E} \perp \underset{\sim}{B}$)

Leaves

$$\frac{d\underset{\sim}{v}'}{dt} = \frac{qB}{mc}\underset{\sim}{v}' \times \hat{z}$$

same oscillatory motion as before with $\underset{\sim}{E} = 0$.

Equivalent to a relativistic transformation to a frame of reference where $\underset{\sim}{E}$ vanishes.

Fast gyration: $\qquad$ $E \times B$ drift:

$$v_x = v_\perp \sin(\Omega t + \alpha) + \frac{c}{B}E_y$$

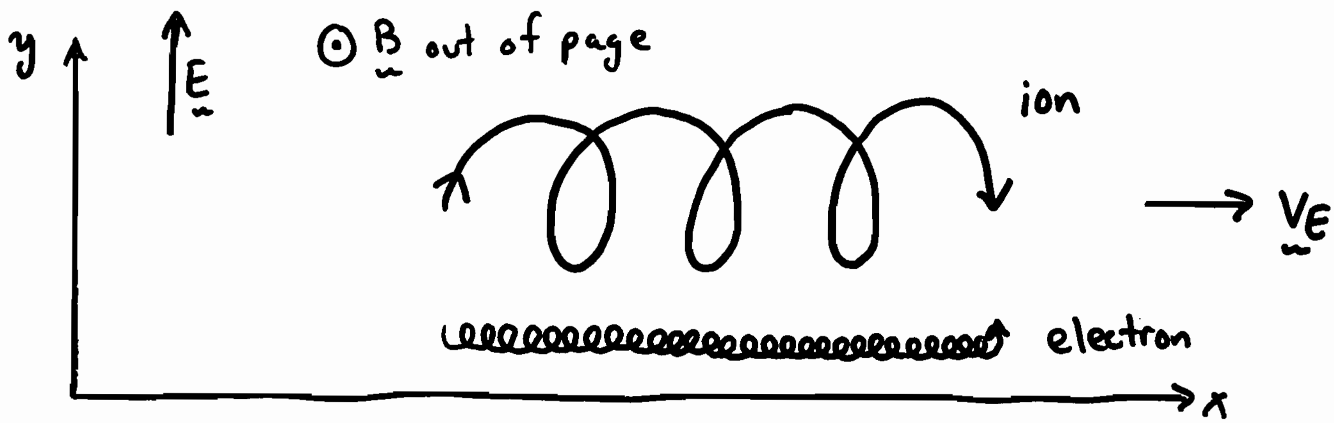
$$v_y = v_\perp \cos(\Omega t + \alpha)$$

$$v_z = v_\parallel$$

More generally, guiding-center moves at the velocity:

$$\frac{d\underset{\sim}{R}_{g.c.}}{dt} = v_\parallel \frac{\underset{\sim}{B}}{B} + \underbrace{\frac{c}{B^2}\underset{\sim}{E} \times \underset{\sim}{B}}_{\underset{\sim}{v}_{E \times B}} + \text{higher-order drifts}$$

Physical Picture of $\underset{\sim}{E}\times\underset{\sim}{B}$ drift:



$\underset{\sim}{E}$ acceleration causes $v_\perp$ to be bigger on top half of orbit for ion
(or for electrons on bottom half of orbit. But electrons gyrate around $\underset{\sim}{B}$ in the opposite direction, $\Omega_{ce} < 0$, $\Omega_{ci} > 0$, since $q_e = -q_i$; so net $E\times B$ drift is in same direction for electrons & ions)

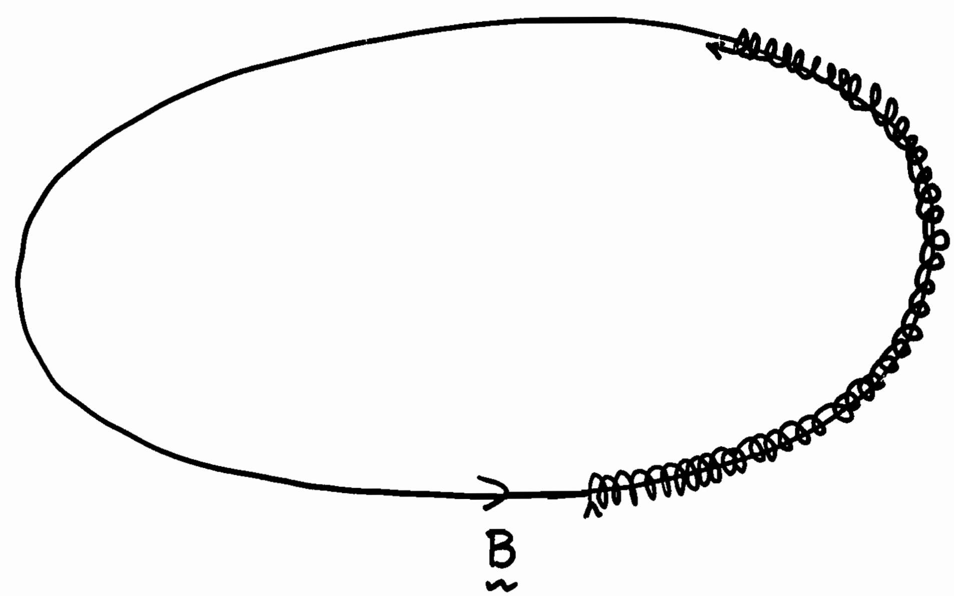
Gyroradius $\rho = \dfrac{v_\perp}{\Omega}$ is $\therefore$ bigger on top half of orbit (for ions)

$\Rightarrow$ Net drift to right

$$V_{\underset{\sim}{E}\times\underset{\sim}{B}} = \frac{c}{B^2}\,\underset{\sim}{E}\times\underset{\sim}{B}$$
is the same for electrons & all types of ions
(V_E independent of q & m of particle species)

Surprise! Net particle motion is not in the direction of $\underset{\sim}{E}$!
(for constant $\underset{\sim}{E}$, $\underset{\sim}{E}\perp\underset{\sim}{B}$)

Can we confine particles (permanently) just by bending the magnetic field into a torus (donut)?



(Not quite...)

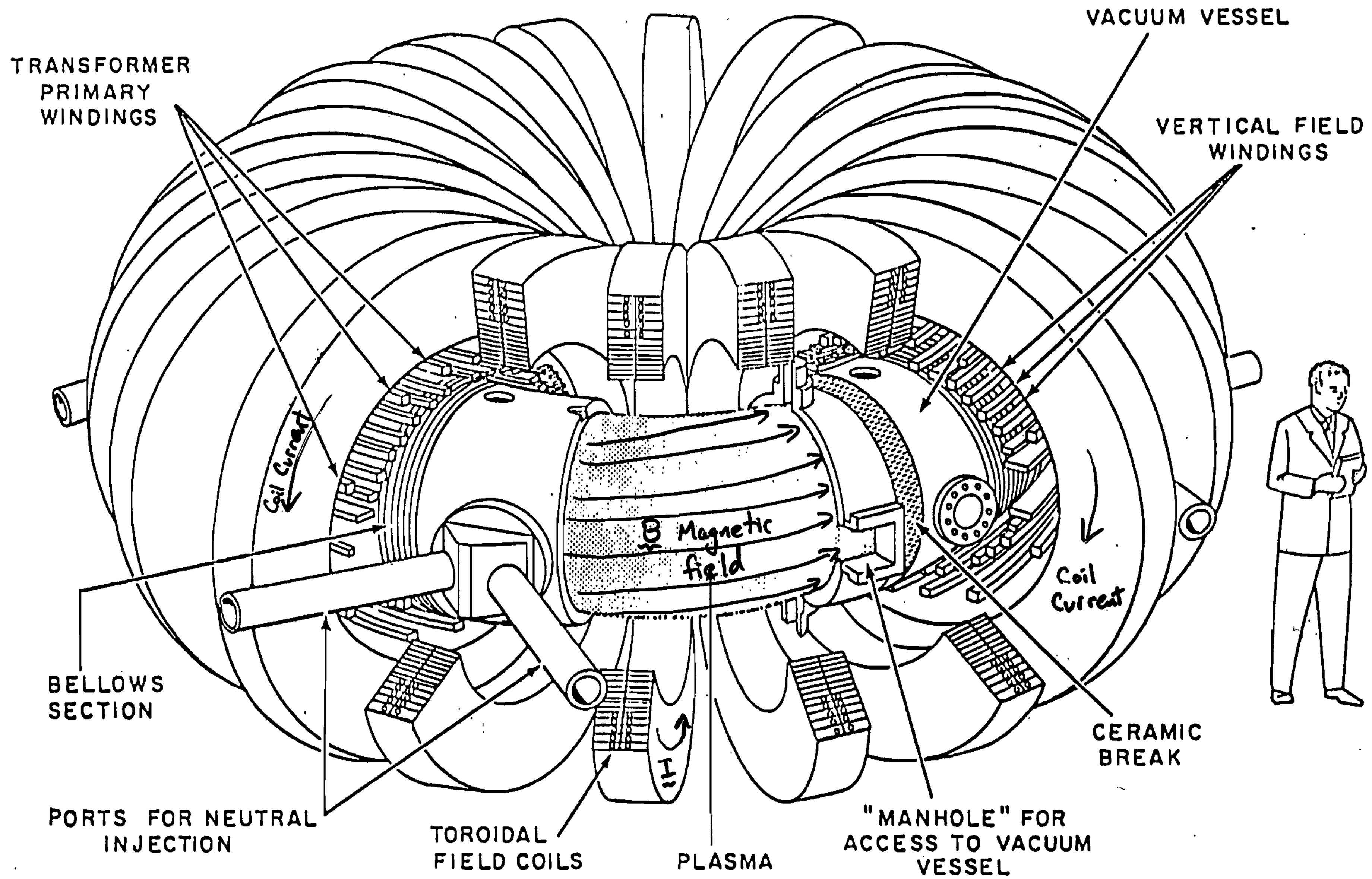
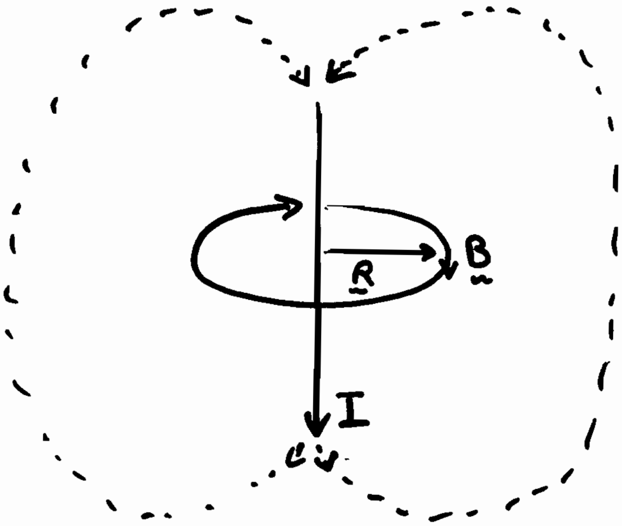


Figure II.8. Schematic of the Princeton ~~Large~~ Little Torus. (PPL 803814)

PLT ~1976-1986

Try solving confinement problem by bending $\underset{\sim}{B}$ into a torus.

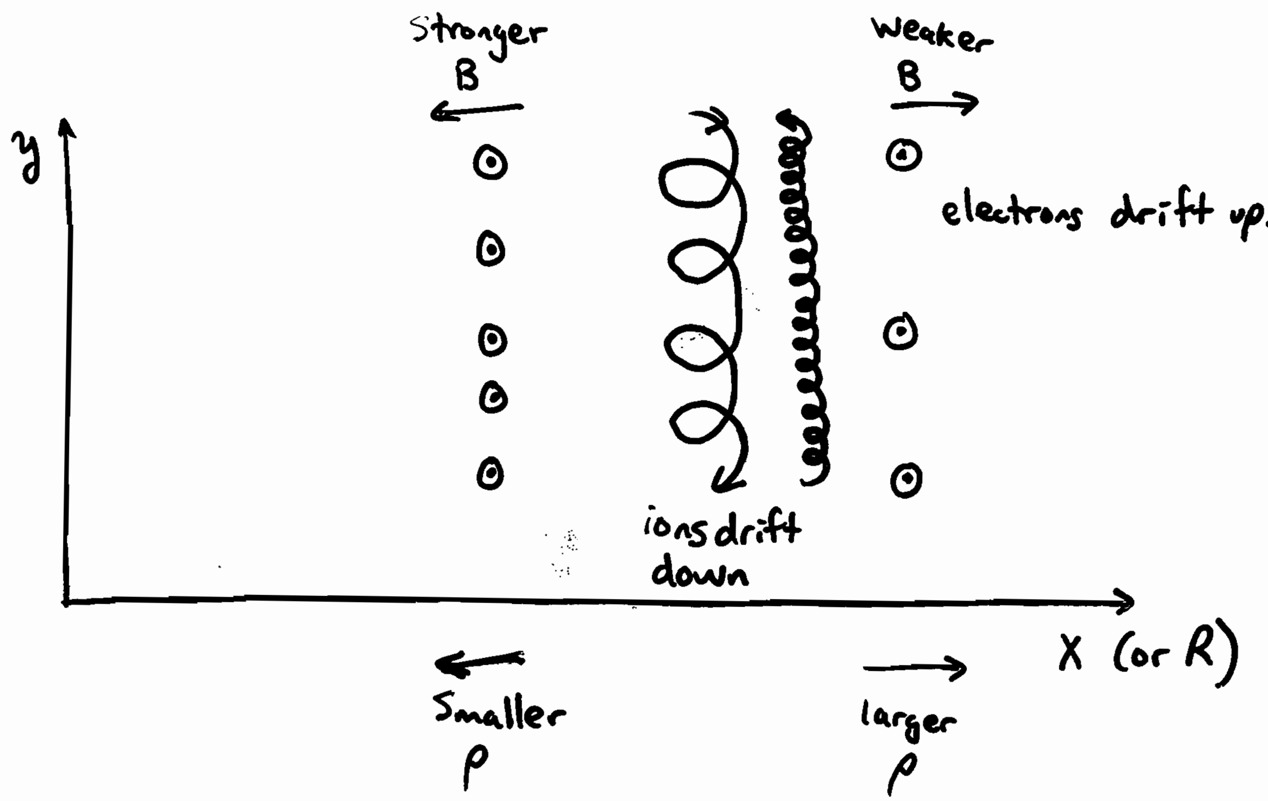


$$\nabla \times \underset{\sim}{B} = \frac{4\pi}{c}\,\underset{\sim}{j}$$

$$\oint d\underset{\sim}{l} \cdot \underset{\sim}{B} = \frac{4\pi}{c}\, I$$

$$2\pi R\, B = \frac{4\pi}{c}\, I$$

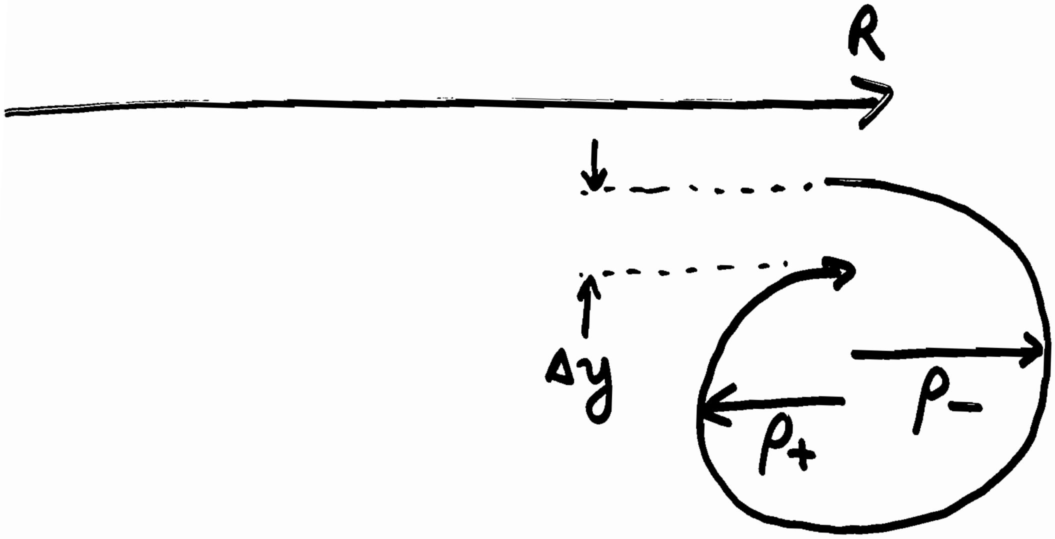
$$B \propto \frac{1}{R} \propto \frac{1}{X}$$



$$\rho = \frac{v_\perp}{\Omega} \propto \frac{1}{B} \propto R \propto X$$

$$\boxed{\text{"}\nabla B \text{ drift"}}$$

Order of Magnitude ∇B drift



$$B \propto \frac{1}{R}$$

In reactor: $\rho \sim 1.4\,\text{mm}$, $R \sim 5\,\text{m}$

Small change in B ($\propto \rho$)

$$\Delta y = 2\rho_- - 2\rho_+$$

$$\rho = \frac{V_\perp}{\Omega} \propto \frac{1}{B} \propto R = \frac{\rho_0}{R_0} R$$

$$\rho_- \approx \frac{\rho_0}{R_0}\left(R_0 + \frac{\rho_0}{2}\right)$$

$$\rho_+ \approx \frac{\rho_0}{R_0}\left(R_0 - \frac{\rho_0}{2}\right)$$

$$\Delta y = 2\frac{\rho_0}{R_0}\rho_0$$

Relative shift per gyration small:

$$\frac{\Delta y}{\rho_0} \sim \frac{\rho_0}{R} \sim 10^{-3}$$

∇B drift velocity:

$$V_{\nabla B} = \frac{\Delta y}{\Delta t} = \frac{\Delta y}{\left(\frac{2\pi}{\Omega}\right)}$$

$$\approx 2\frac{\rho_0}{R_0}\,\rho_0\,\frac{\Omega}{2\pi}$$

$$\approx 2\frac{\rho_0}{R_0}\,\frac{V_\perp}{\cancel{\Omega}}\,\frac{\cancel{\Omega}}{2\pi}$$

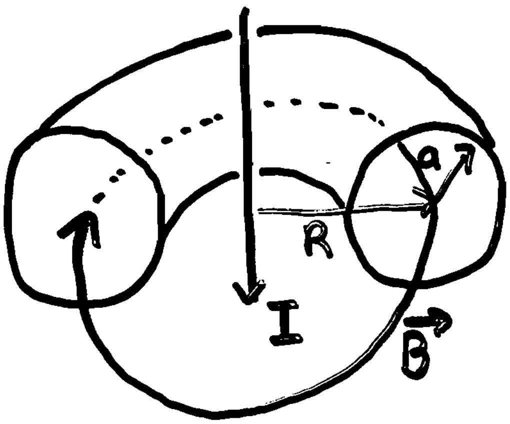
$$\approx \frac{1}{\pi}\frac{\rho_0}{R_0}\,V_\perp$$

$$= \frac{1}{2}\frac{\rho_0}{R_0}\,V_\perp$$ (from more rigorous calc.)

Particle loss rates in a pure toroidal field

Bending $\vec{B}$ into a torus solved the $\parallel \vec{B}$ loss problem.
but now $\perp \vec{B}$ confinement is no longer perfect.
Particles eventually drift up or down out of the machine,
though a thousand times slower than if there had been no $\vec{B}$:

$$V_{\nabla B} = \frac{1}{2} \frac{\rho}{R} V_\perp \sim 3*10^{-4} V_\perp$$



Compare with TFTR achievement:

Major Radius $R \sim 2.5 m$ $\quad B \sim 5$ Tesla
Minor Radius $a \sim 1 m$
Deuterium temperature ~ 10 keV
Deuterium gyroradius $\rho \sim 1.4$ mm
$n \sim 10^{20}/m^3$ $\quad \tau \sim 0.2$ sec

Loss rate with:

No B field $\quad \tau \sim \frac{a}{V_\perp} \sim \frac{1\ m}{7*10^5 m/s} \sim 1.4*10^{-6}$ s

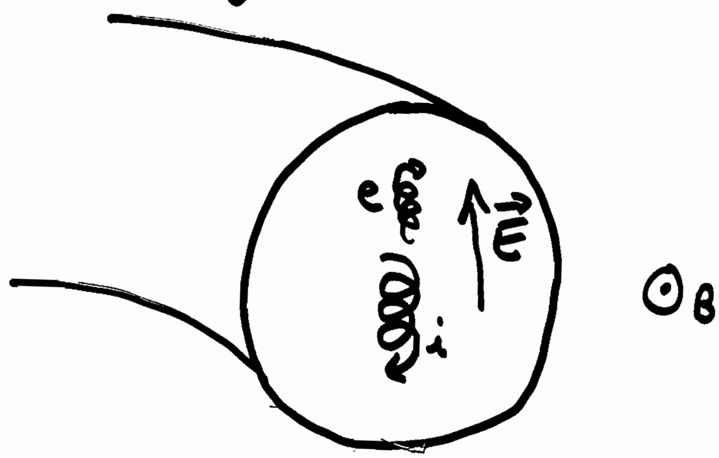
Fusion impossible!
except at astronomically
high n (inertial fusion)

Pure toroidal $\vec{B}$ $\quad \tau_{\nabla B} \sim \frac{a}{V_{\nabla B}} \sim \frac{a}{V_\perp} \frac{2R}{\rho} \sim 5*10^{-3}$ sec

Actual TFTR confinement is ≈ 50 times better!

What is secret to tokamak's confinement?

(Actually, $\tau_{\nabla B} \sim 5 * 10^{-3}$ s overly optimistic)



∇B drift $\rightarrow$ ions drift down & electrons drift up

charge separation $\Rightarrow$ vertical $\vec{E}$

Small charge separation produces large $\vec{E}$

$\Rightarrow$ $\vec{E} \times \vec{B}$ drift expels particles outwards almost as fast as if $\vec{B}$ wasn't there.

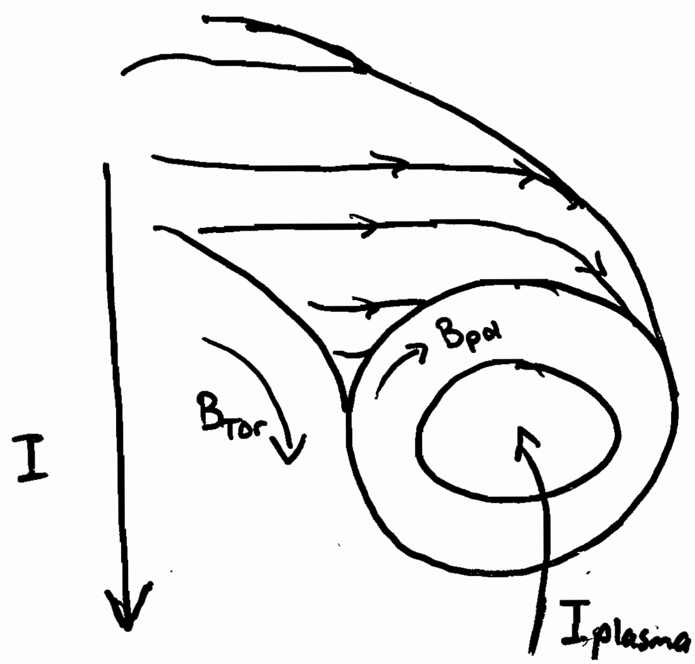
What is secret to tokamak's good confinement?

The Secret to the Tokamak's Success: The Twirling Honey Effect

Similar effect employed in most toroidal devices: stellarators, RFP's, spheromaks...

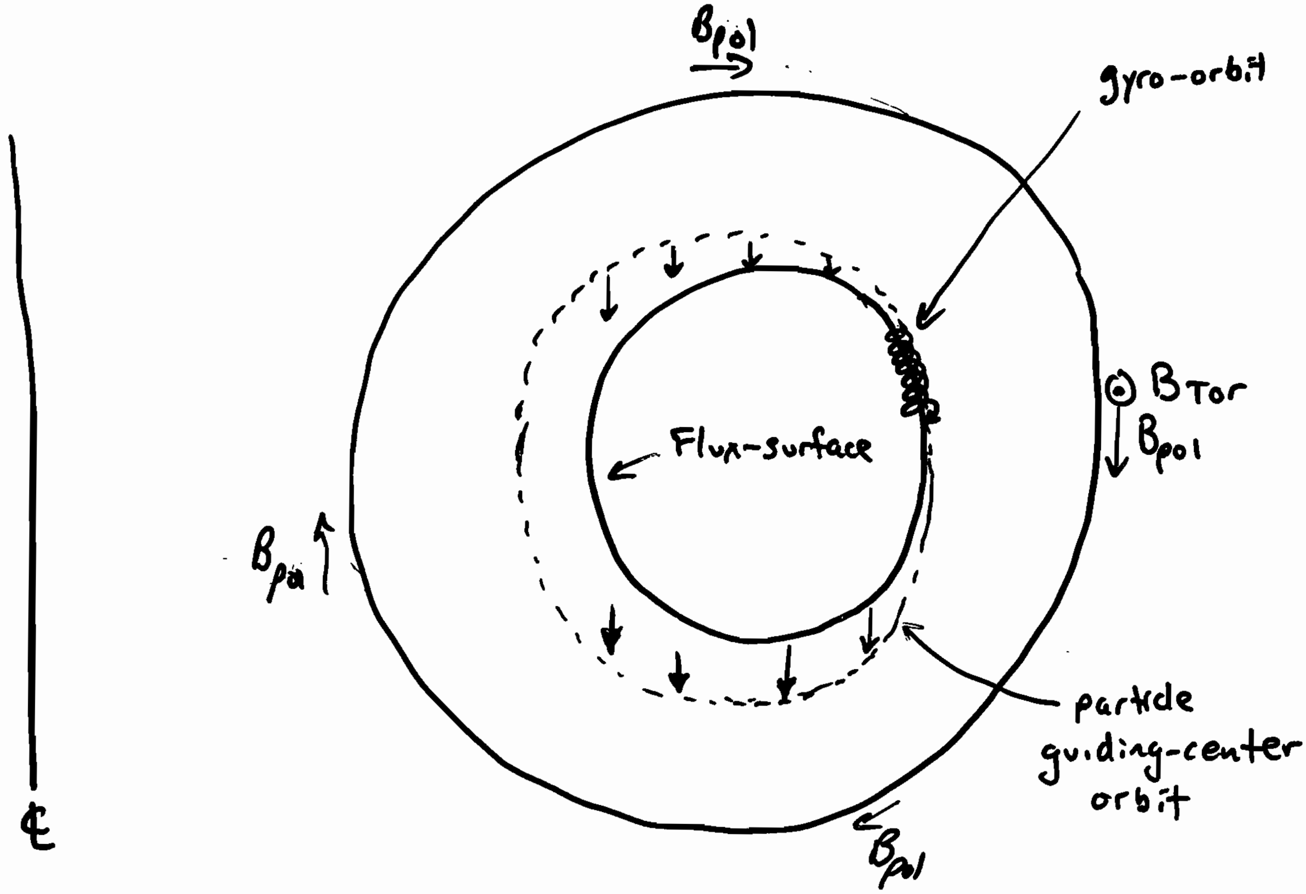
Tokamak has both poloidal (short way) & toroidal (long way) (around tokamak) $\underset{\sim}{B}$ fields:

Superimpose "poloidal" field due to "toroidal" plasma current:



Magnetic field lines lie in nested toroidal "flux surfaces".

Look at ∇B drift in Tokamak!



Bottom half, downward ∇B drift $\Rightarrow$ outwards.

Top half, downward ∇B drifts $\Rightarrow$ inwards.

Net effect is to shift the orbit sideways.

* Theoretical Single particle motion is rigorously confined.

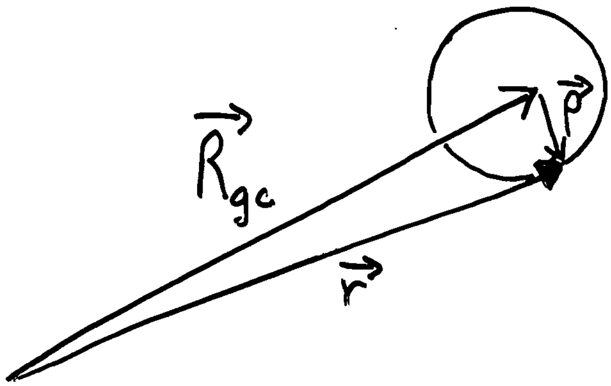
Spitzer in U.S. & Sakharov in U.S.S.R. independently thought of these problems & their solution in early ~~[illegible]~~ 1950's!

* In reality, particles still escape from tokamaks primarily due to small scale, turbulent fluctuations. Finding a way to reduce this turbulence & $\uparrow \tau_E$ by a factor of 2-4 $\Rightarrow$ fusion much more economical!

More Rigorous ∇B drift

$$\frac{d\vec{v}}{dt} = \frac{q}{mc}\,\vec{v}\times\vec{B}(\vec{r}) \approx \frac{q}{mc}\,\vec{v}\times\Big[\underbrace{\vec{B}(\vec{R}_{gc})}_{\vec{B}_0} + \underbrace{\varepsilon\,\vec{\rho}\cdot\nabla\vec{B}}_{+\,\varepsilon\,\vec{B}_1}\Big]$$

(ε to keep track of ordering, $\epsilon \to 1$ in end)



$$\vec{r} = \vec{R}_{gc} + \vec{\rho}$$

Slowly changing guiding-center position

ρ is small, but $\vec{\rho}$ rapidly oscillates in direction, average over rapid gyrations.

$$\vec{v} = \vec{v}_0 + \varepsilon\,\vec{v}_1$$

$$\frac{d\vec{v}_0}{dt} = \frac{q}{mc}\,\vec{v}_0\times\vec{B}_0 \qquad \text{already solved.}$$

$$\frac{d\vec{v}_1}{dt} = \frac{q}{mc}\left[\vec{v}_1\times\vec{B}_0 + \vec{v}_0\times\vec{B}_1\right]$$

(ignoring subtleties for $v_{\parallel 0} \neq 0$ that $\Rightarrow$ curvature drift)

$$\frac{d\vec{V}_1}{dt} = \frac{e}{mc}\left[\vec{V}_1 \times \vec{B}_0 + \underbrace{\vec{V}_0 \times (\vec{\rho}_0 \cdot \nabla \vec{B})}_{\vec{V}_0 \times \left(\rho_{0x} \frac{\partial B}{\partial R}\frac{1}{B}\vec{B}_0\right)}\right]$$

using $\vec{B}_0 = \frac{B_0 R_0}{R}\hat{\phi}$

$\vec{\rho}_0$ + $\vec{V}_0$ oscillate $\propto \cos(\Omega t)$

$$\Rightarrow \vec{V}_1 = \overline{\vec{V}_1} + c_1 \cos(2\Omega t)$$

Oscillatory parts give no net drift of guiding center.
Time-avg. over one gyration!

$$0 = \overline{\vec{V}_1} + \overline{\vec{V}_0\, \rho_{0x}}\, \frac{\partial B}{\partial R}\frac{1}{B}$$

$$\overline{\vec{V}_1} = - v_\perp \overline{(\hat{x}\sin\Omega t + \hat{y}\cos\Omega t)\left(-\frac{v_\perp}{\Omega}\right)\cos(\Omega t)}\, \frac{\partial B}{\partial R}\frac{1}{B}$$

$$= \hat{y}\,\frac{v_\perp^2}{2\Omega B}\frac{\partial B}{\partial R}$$

General ∇B drift! $\vec{V}_{\nabla B} = \frac{v_\perp^2}{2\Omega B^2}\vec{B}\times\nabla B$

note! ions + electrons drift in opposite directions
$\Omega = \frac{qB}{mc}$ opp. sign for e + i.

Adiabatic Invariance of μ

$$\mathbf{B} = B_0(t)\,\hat{z}$$

$\mathbf{B}$ is uniform, but time-varying

(From here on, heavily borrowed from Charles Karney's lecture notes...)

Assume variation of $\mathbf{B}$ is slow:

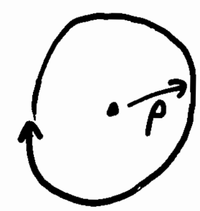
$$\frac{1}{B}\frac{\partial B}{\partial t} \ll \Omega$$

Faraday's law: $\quad \nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}$

$\Rightarrow \mathbf{E} \Rightarrow$ energy of particle is not conserved.

$$\frac{d}{dt}\left(\frac{1}{2}mv_\perp^2\right) = m\mathbf{v}_\perp \cdot \frac{d\mathbf{v}_\perp}{dt} = \mathbf{v}_\perp \cdot q\mathbf{E}$$

Integrate over 1 cyclotron period $\frac{2\pi}{\Omega}$



$$\Delta W_\perp = \Delta \tfrac{1}{2}mv_\perp^2 = \oint \mathbf{v}_\perp \cdot q\,\mathbf{E}\,dt$$

$$= q\oint \mathbf{E}\cdot d\mathbf{s}$$

$$= q\int_A \nabla\times\mathbf{E}\cdot d\mathbf{A} \qquad \text{Stokes theorem}$$

$$= -\frac{q}{c}\int_A \frac{\partial \mathbf{B}}{\partial t}\cdot d\mathbf{A} \qquad \text{Faraday}$$

$$\frac{\Delta B}{\Delta \tau} = \Delta B\,\frac{\Omega}{2\pi}$$

$$A = -\pi\rho^2 = -\pi\left(\frac{v_\perp}{\Omega}\right)^2$$

Sense of area is negative for positive ions.

$$\Delta W_\perp = + \frac{q}{c} \frac{\Delta B}{m} \frac{\cancel{\Omega}}{2\cancel{\pi}} \cancel{\pi} \frac{m v_\perp^2}{\Omega^{\cancel{2}}}$$

$$\Rightarrow \quad \Delta W_\perp = \Delta B \, \frac{W_\perp}{B}$$

$$\frac{\Delta W_\perp}{W_\perp} = \frac{\Delta B}{B}$$

Or $\quad \Delta\left(\frac{W_\perp}{B}\right) = 0$

$$\Delta(\ln W_\perp) = \Delta \ln B$$

$$\Delta(\ln W_\perp - \ln B) = 0$$

$\frac{W_\perp}{B} = \mu \qquad$ - adiabatic invariant. $\qquad \Delta \ln\left(\frac{W_\perp}{B}\right) = 0$

$\mu \equiv$ magnetic moment

Note! $\mu \propto \pi \rho^2 B$ = flux through gyro-orbit.

Gyrating particle ring acts like a superconducting ring, conserving flux through its orbit very accurately, if the rate of change of B is slow compared to Ω

$$\frac{\Delta \mu}{\mu} \sim e^{-\frac{1}{\varepsilon}} \qquad \varepsilon \sim \frac{\omega}{\Omega} \sim 10^{-3}$$

$$e^{-(10^3)}$$

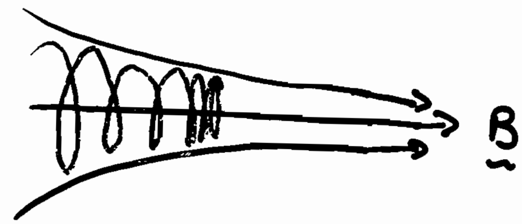
$e^{-\frac{1}{\varepsilon}}$

ε

$$f(\varepsilon) = f(0) + \varepsilon \frac{\partial f}{\partial \varepsilon} + \frac{1}{2}\varepsilon^2 \frac{\partial^2 f}{\partial \varepsilon^2} + \cdots$$

Mirror Effect

$\frac{\partial B}{\partial z} > 0$



$\underset{\sim}{E} = 0$

$W =$ constant

$= \frac{W_\perp}{B} B + W_{\parallel}$

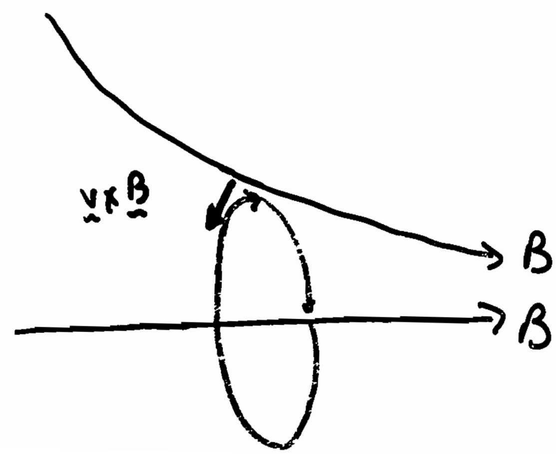
$\Rightarrow$ Kinetic Energy $= W =$ constant

$W = \mu B + \frac{1}{2} m v_{\parallel}^2$

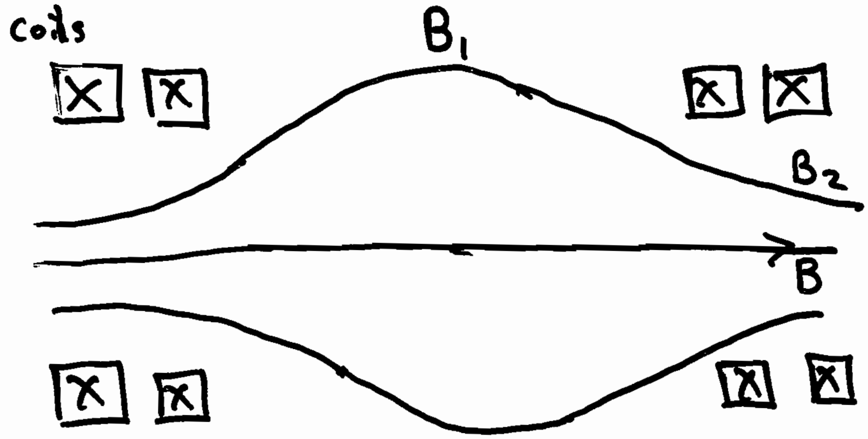
But μ conservation $\Rightarrow$ $W_\perp \propto B$

$\Rightarrow$ $W_{\parallel}$ becomes 0 at some point
- particle is reflected

z component of force comes from radial component of $\underset{\sim}{B}$



Mirror Machine



Mirror force reflects particles near mirror throat.

Conservation of μ & W:

Particle which reflects at B_2 $(v_{\parallel} = 0)$ has energy $W = W_{\perp 2} = \mu B_2$

At B_1 it has the same energy $W = W_{\perp 1} + W_{\parallel 1}$

$$= \mu B_1 + \tfrac{1}{2} m v_{\parallel 1}^2$$

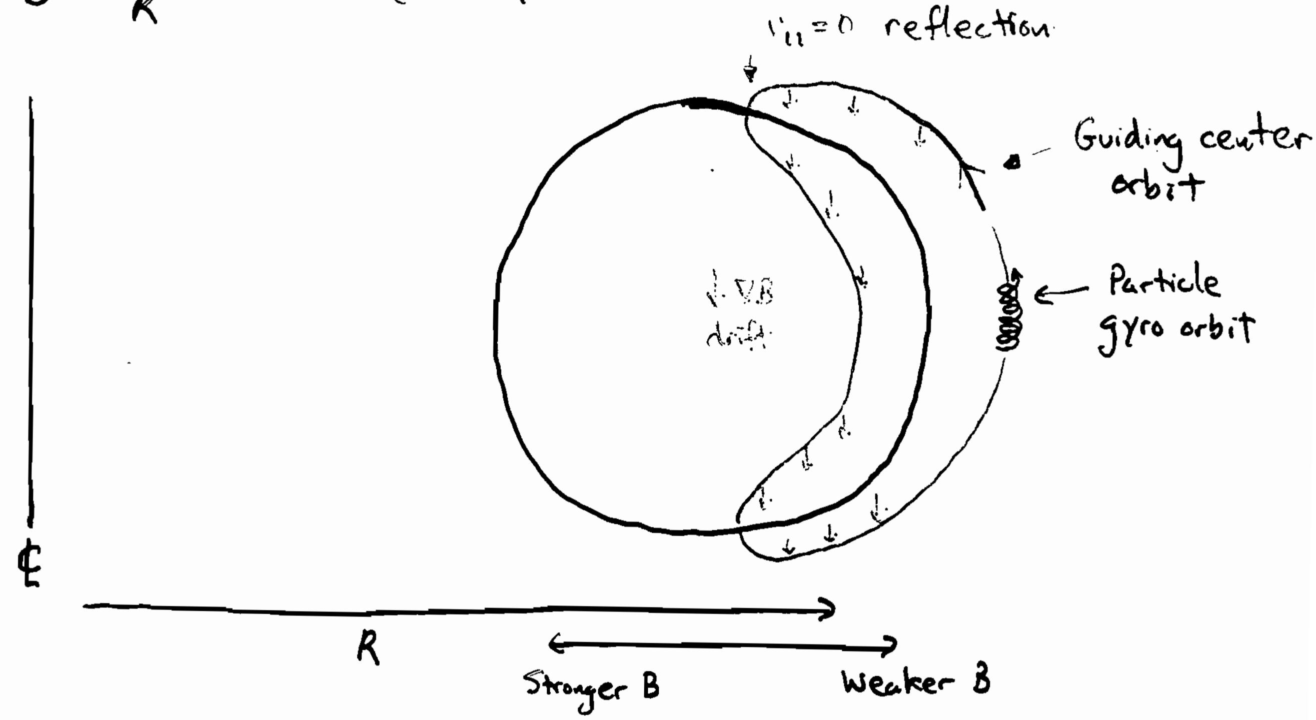
Can show that all particles with

$$\left.\frac{v_\perp}{v}\right|_{\text{at } B_1} > \sqrt{\frac{B_1}{B_2}} \quad \text{are reflected.}$$

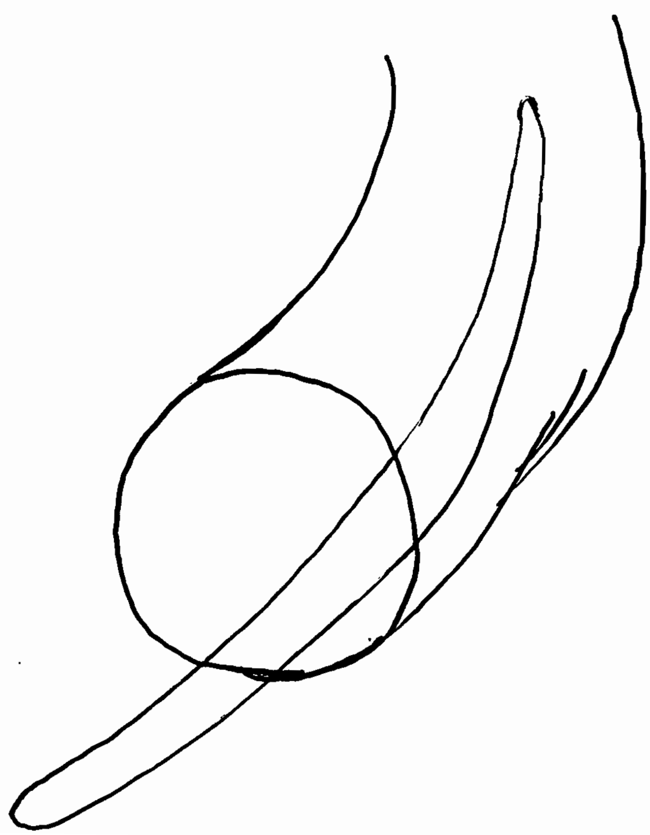
Mirror Effect in Tokamaks: Banana Orbits

$B_{pol} \sim \frac{1}{10} B_{Tor}$

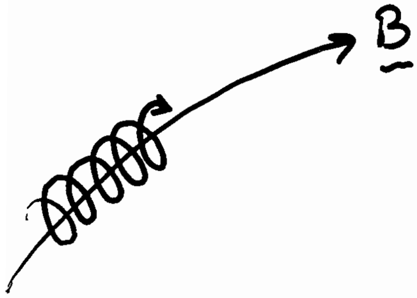
So $B \propto \frac{1}{R}$ still a good approximation (usually, except ST's...)



3D view:



Guiding Center Motion Summary



$$\Omega = \frac{qB}{mc} \quad \text{gyrofrequency}$$

$$\rho = \frac{v_\perp}{\Omega} \quad \text{gyroradius}$$

$$\mathbf{r} = \mathbf{R}_{gc} + \boldsymbol{\rho}$$

Particle Position — Guiding-center position — Rapidly gyrating gyro-radius vector

For static fields (& $\mathbf{E}$ not too strong...), the dominant guiding center drifts are:

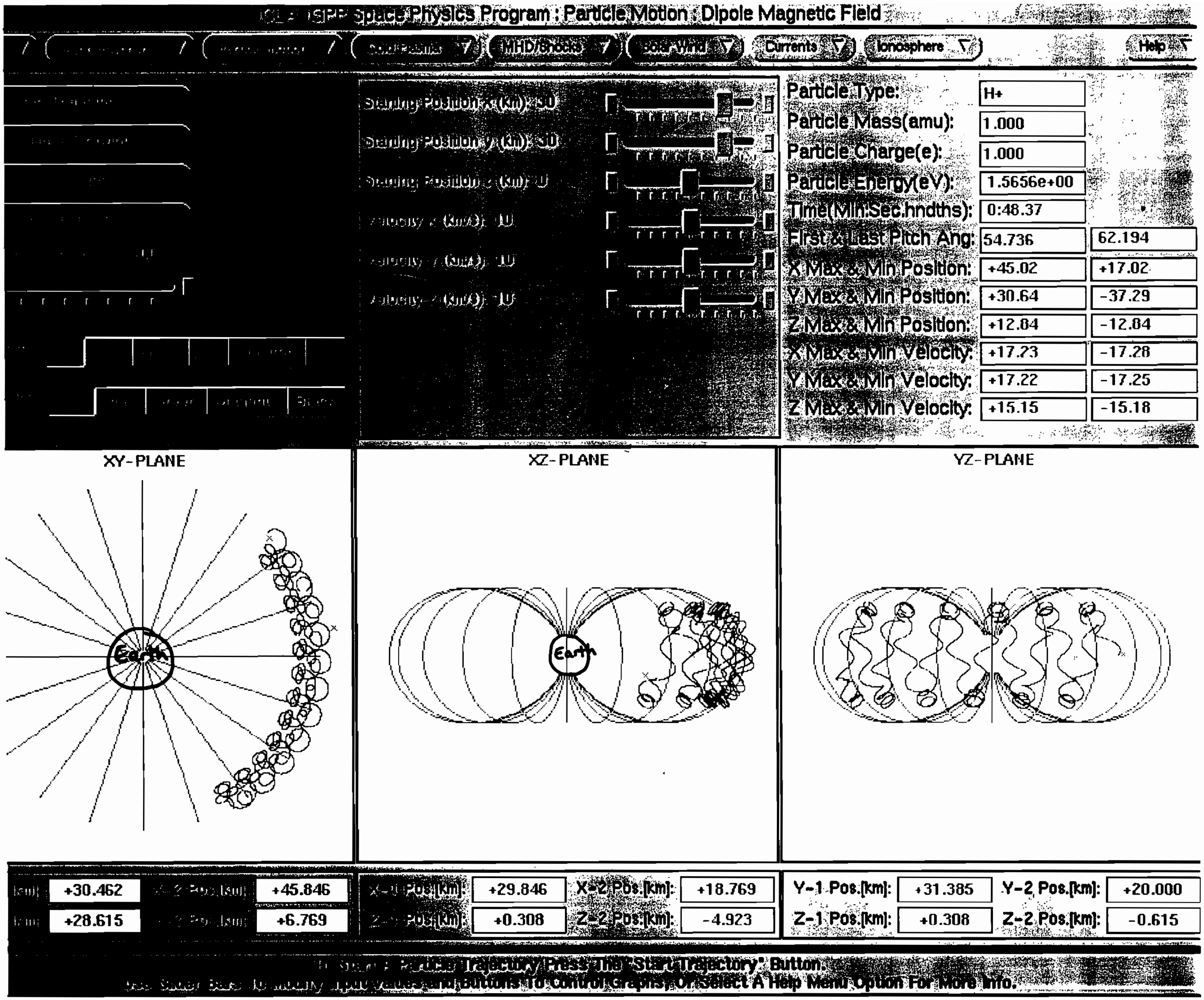
$$\frac{d\mathbf{R}_{gc}}{dt} = v_\parallel \hat{b} + \mathbf{v}_d$$

$$\hat{b} = \frac{\mathbf{B}}{B} \qquad \mu = \frac{W_\perp}{B} = \text{constant}$$

$$W = \mu B + q\Phi + \tfrac{1}{2} m v_\parallel^2 = \text{constant}$$

$$\mathbf{v}_d = \frac{c}{B^2}\,\mathbf{E}\times\mathbf{B} + \frac{\frac{1}{2}v_\perp^2}{\Omega\,B}\,\hat{b}\times\nabla B + \frac{v_\parallel^2}{\Omega}\,\hat{b}\times(\hat{b}\cdot\nabla\hat{b})$$

"E x B" — "∇B" — "Curvature" drift (see textbooks...)



http://www-ssc.igpp.ucla.edu/ssc/education.html

This method can be extended to

Systematic Derivation of Complete Guiding Center Drifts.

For details, see:

George Schmidt, Physics of High Temperature Plasmas (1979)
Kenro Miyamoto, Plasma Physics for Nuclear Fusion (1976)

(Most textbooks derive particle drifts piecemeal, not systematically...)

$\boxed{\text{Modern approach: Hazeltine \& Waelbroeck, The Framework of Plasma Physics (1998)}}$

Basic idea:

$$\underset{\text{Particle Position}}{\underset{\sim}{r}} = \underset{\text{Slowly moving guiding center}}{\underset{\sim}{R}_{gc}} + \underset{\text{fast gyro-motion}}{\underset{\sim}{\rho}}$$

Assume gyroradius ρ is small & expand: $\underset{\sim}{B} \approx \underset{\sim}{B}(\underset{\sim}{R}_{gc}, t) + \underset{\sim}{\rho} \cdot \nabla \underset{\sim}{B}$
etc.

Assume Ω is very large & average equations of motion over fast gyro-motion to obtain an equation for the slower moving guiding-center...

Subtleties: various ways to expand $\underset{\sim}{E}$ → MHD ordering / macro-instabilities
→ gyrokinetic ordering / micro-instab.

Alternative: Modern Hamiltonian methods, non-canonical Lie perturbation theory, useful for going to higher order, investigate accuracy of adiabatic invariants, etc...

* Remember the "twisting honey effect"
  Stabilize a top by spinning it (gyroscope).

  Can turbulence be stabilized by differentially spinning the plasma? (A current hot topic of research.)

* Tokamak confinement is 10^5 times better than having no magnetic field.

  Finding a way to improve confinement another factor of 2-10 would be a great breakthrough + would lead to cheaper fusion reactors.

* Lots of interesting + important problems for clever people to work on...

Skipped topic: I have assumed $\underset{\sim}{B}$ forms simple nested toroidal flux surfaces. Small asymmetries in $\underset{\sim}{B}$ can cause $\underset{\sim}{B}$ and/or the particle orbits to become chaotic, making the loss of particles worse...