Non-Ideal MHD effects on the MRI, with an example: Electron Heating in Hot Accretion Flows

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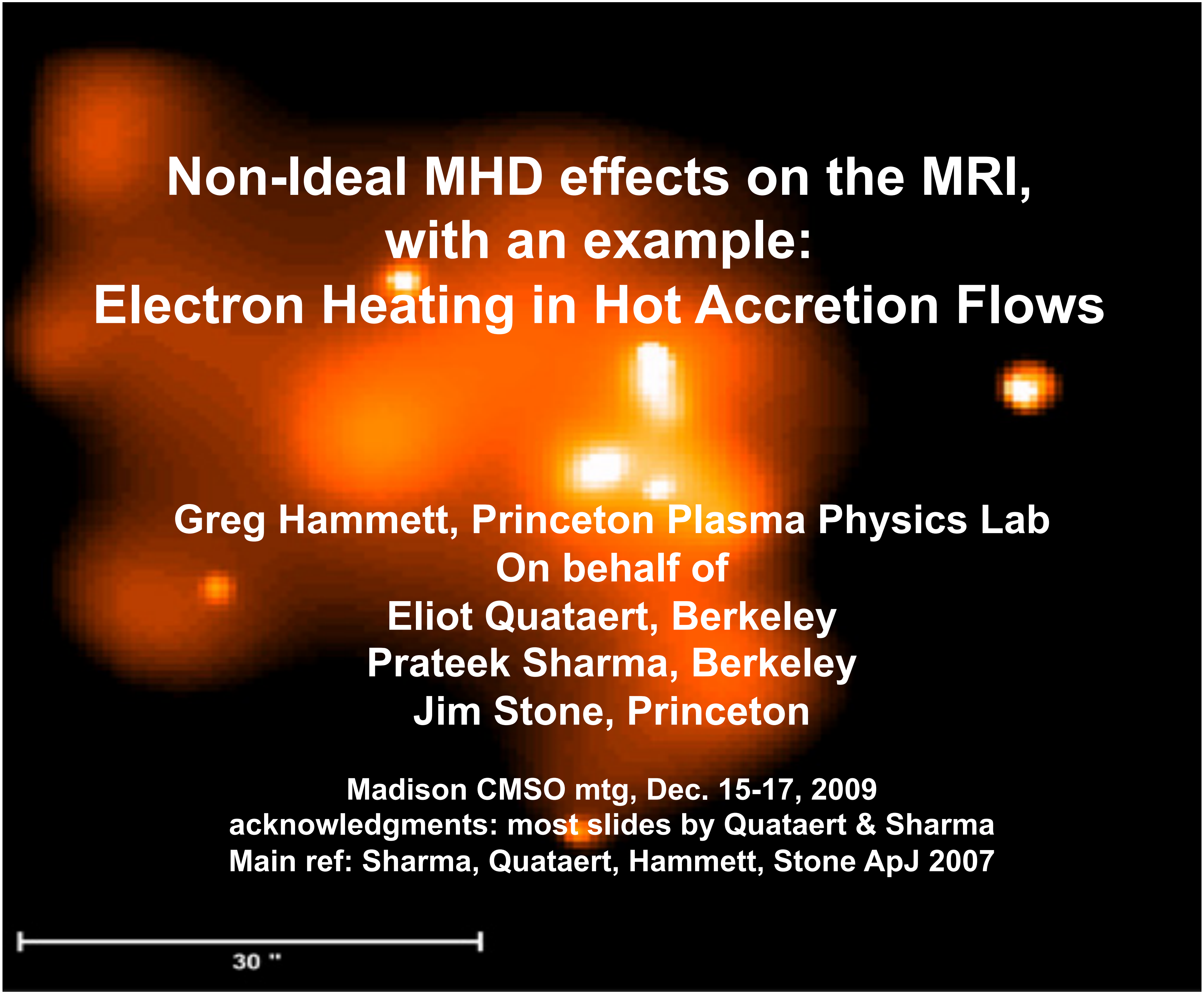
acknowledgments: most slides by Quataert & Sharma

Main ref: Sharma, Quataert, Hammett, Stone ApJ 2007



Non-Ideal MHD Effects

Non-ideal MHD effects w/o FLR:

1. Magnetic Prandtl # dependence (low in metals, high in many plasmas)
2. Magnetized plasmas very anisotropic:
   Braginskii anisotropic viscosity $\mu_{||} >> \mu_{\perp}$
3. Low collisionality: long mean-free path, need drift-kinetic-MHD instead of regular fluid MHD or even Braginskii-MHD.
4. But: get velocity-space anisotropy that drive mirror/cyclotron/firehose microinstabilities at very fine scales and high frequencies

Non-ideal MHD effects involving FLR ($k\,\rho_i$)

5. Hall term corrections to ideal Ohm's law, neutrals and ambipolar diffusion, dust grain charge carriers.
6. Ultra-low B, ions unmagnetized: $\omega > \Omega_{ci}$, $\rho_i > L$.

Magnetic Prandtl # dependence of MRI

$$\mathrm{Pm} = \frac{\text{momentum diffusivity}}{\text{magnetic diffusivity}} \propto \frac{\text{viscosity}}{\text{resistivity}} \approx \frac{\nu_{ii}\lambda_{mfp}^2}{\nu_{ei}c^2/\omega_{pe}^2}$$

$$\approx \sqrt{\frac{m_i}{m_e}}\frac{\beta_e}{2}\frac{\lambda_{mfp}^2}{\rho_i^2} \propto \frac{T^4}{n}$$

- Pm = D_u / D_B ∝ viscosity/resistivity << 1 in liquid metals, some plasmas (stellar interior, cold accretion disks, low-ionization?
- Pm >> 1 in many hot, lower density plasmas (hot accretion flows, ISM, galactic clusters, Pm <~ 10^{29})
- IAS MRI 08 meeting: MRI dynamo w/o net B flux depends on Pm?, turbulence dies away at low Pm? (or if Rm < F(Re) ?)
- MRI more robust with net B flux. Source of large scale B? Beta dependence?

Most plasmas highly anisotropic

Most plasmas, even with fairly weak B, have parallel transport >> perpendicular transport.

Plasma viscosity is isotropic only if $\lambda_{mfp} \ll \rho_i$. In anisotropic case, $Pm_{||}$ is given by previous Pm, and Braginskii's $Pm_{\perp}$ is:

$$\mathrm{Pm}_{\perp} = \frac{\text{perp. momentum diffusivity}}{\text{magnetic diffusivity}} \approx \frac{\nu_{ii}\rho_i^2}{\nu_{ei}c^2/\omega_{pe}^2} \approx \sqrt{\frac{m_i}{m_e}}\frac{\beta_e}{2}$$

In longer mean-free-path regime, Braginskii's fluid closures break down, and one should use Kulsrud/CGL drift-kinetic-MHD, as we will discuss.

Both Braginskii and drift-kinetic-MHD are incomplete by themselves, esp. @ high beta: velocity-space anisotropies drive firehose/mirror/cyclotron instabilities → enhances effective scattering, maybe closer to MHD in a sense, but get strong heating (hard to keep electrons cold),

Non-Ideal MHD Effects (w/ FLR)

- Beyond ideal/resistive Ohm's law: Hall/FLR terms: grad(p_e) + j x B, sometimes referred to as "FLR stabilization"
- Investigated by at least several, incl.:
  - Sano & Stone '02
  - Ferraro '07
  - Mikhailovskii et al. '09

- Neutrals and ambipolar-diffusion effects

- Ultra-low B regime, effects of dust, etc.: Krolik and Zweibel

Accretion

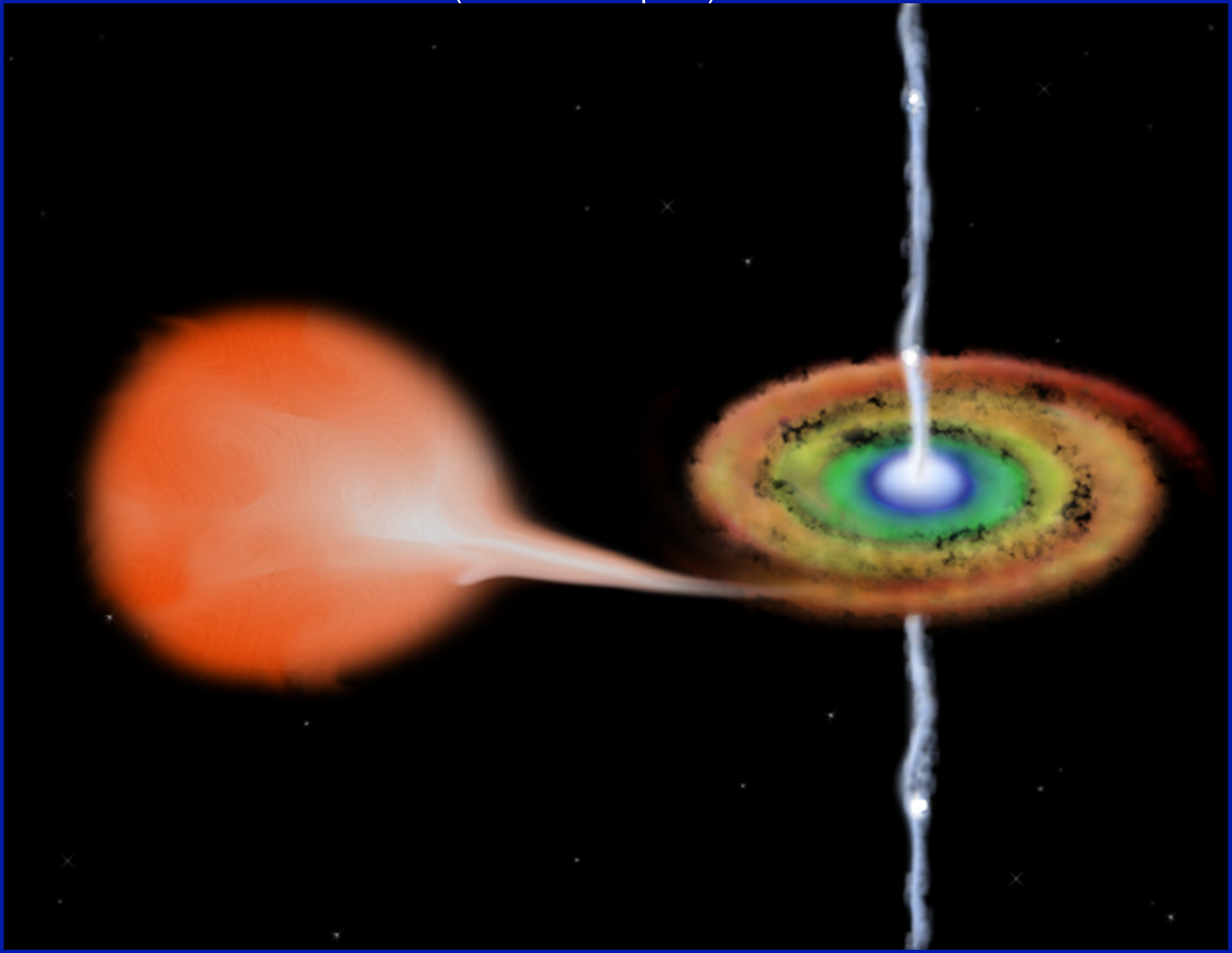
- Inflow of matter onto a central object (generally w/ angular momentum)

- Central to
  - Star & Planet Formation
  - Galaxy Formation
  - Compact Objects: Black Holes, Neutron Stars, & White Dwarfs

- Energy Released: $$\dot{E} = \frac{GM\dot{M}}{2R} \equiv \epsilon \dot{M} c^2$$
  - sun: $\varepsilon \sim 10^{-6}$
  - BH ($R \sim 2GM/c^2$): $\varepsilon \sim 0.25$ (can be << 1; more later)
  - Fusion in Stars: $\varepsilon \sim 0.007$
  - Accretion onto Black Holes & Neutron Stars is Responsible for the Most Energetic Sources of Radiation in the Universe

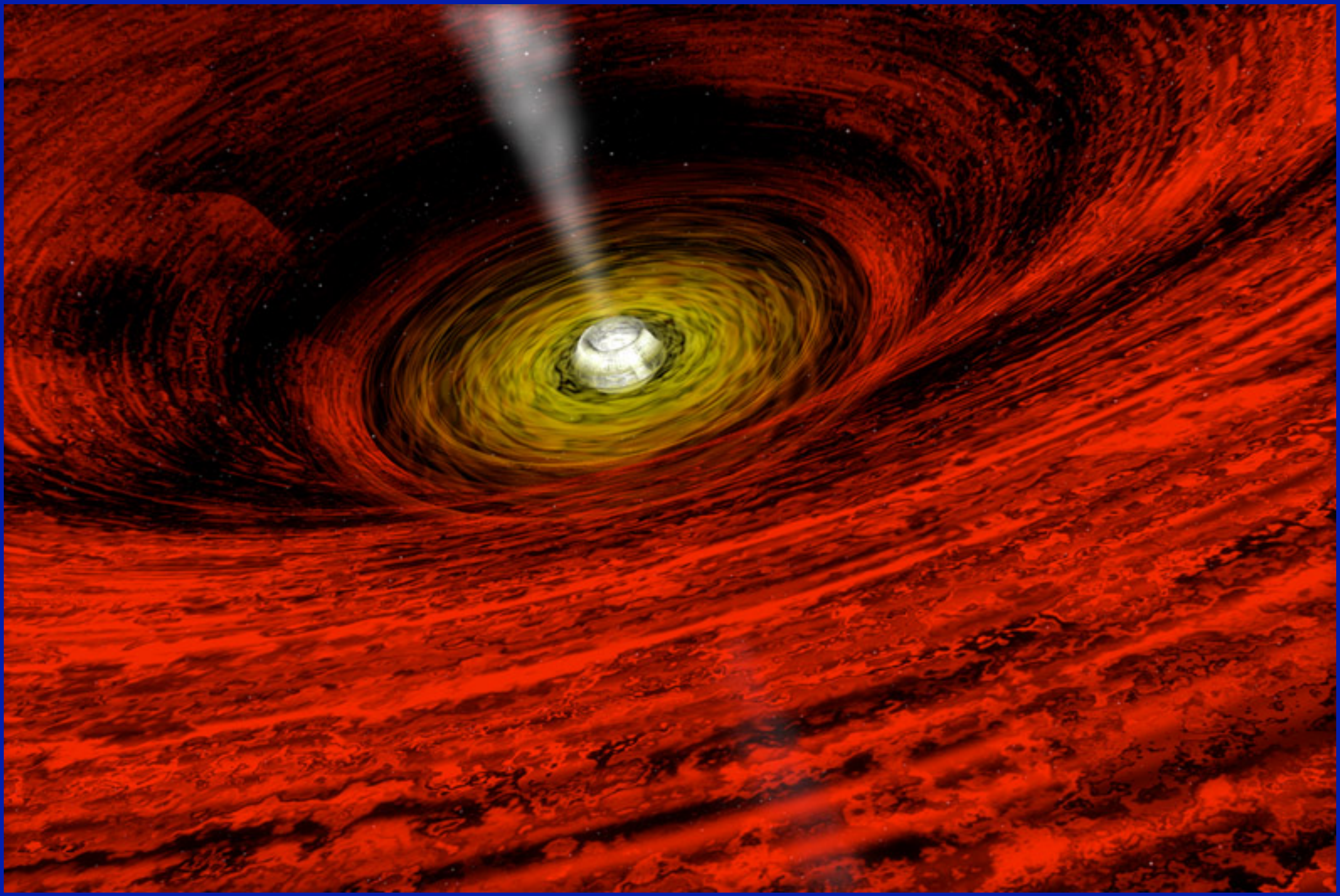
Star orbiting black hole & feeding accretion disk

(artist's conception)



NASA/CXC/SAO A.Hobart http://chandra.harvard.edu/resources/illustrations.html

Black Hole Neighborhood. (artist's conception)



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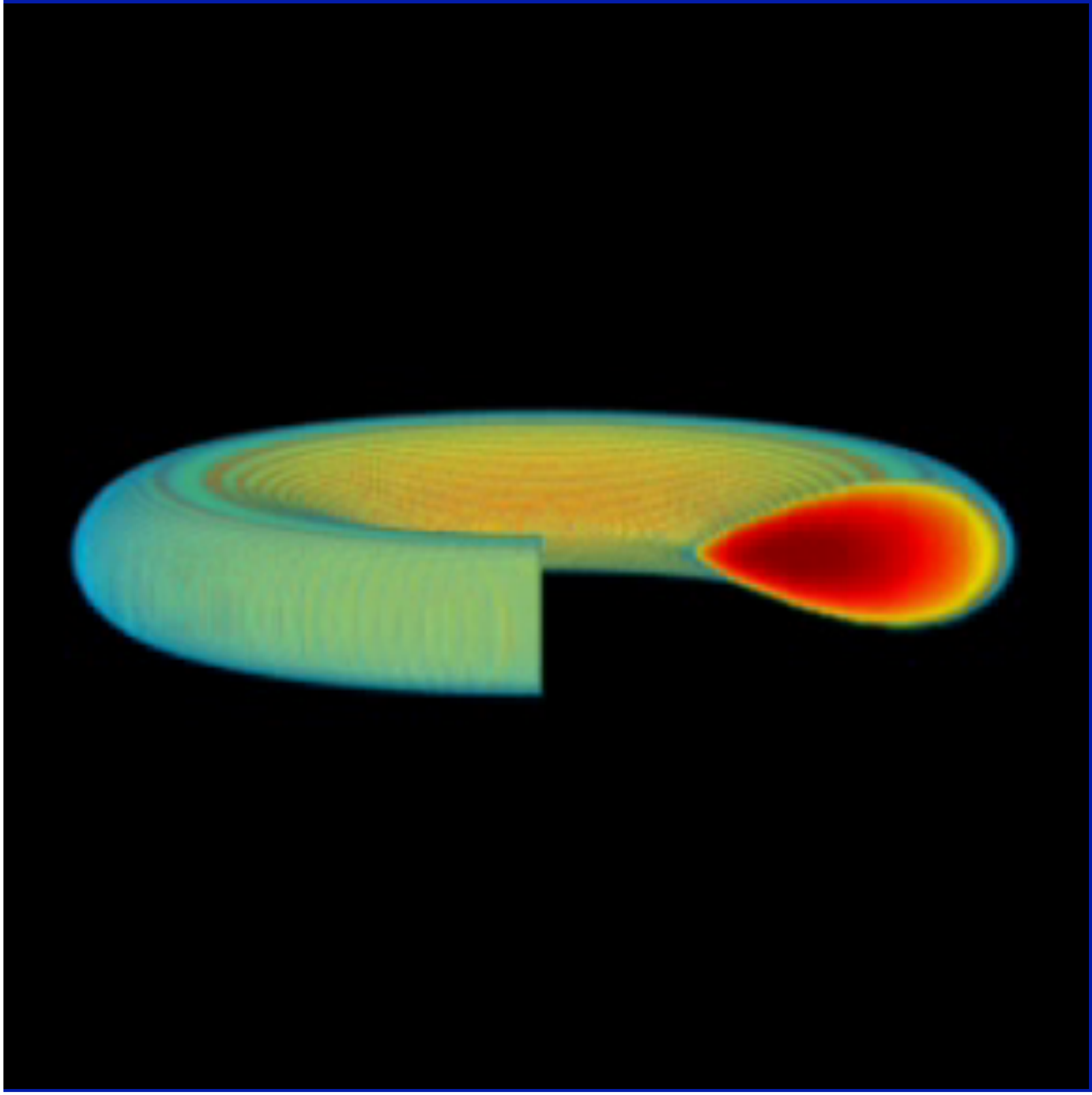
A 3-D Global MHD Simulation



Simulation by Hawley et al.
http://astsun.astro.virginia.edu/~jh8h/

MHD simulations of MRI turbulence very successful. Need to study it in collisionless regime applicable to Sgr A*

Outline

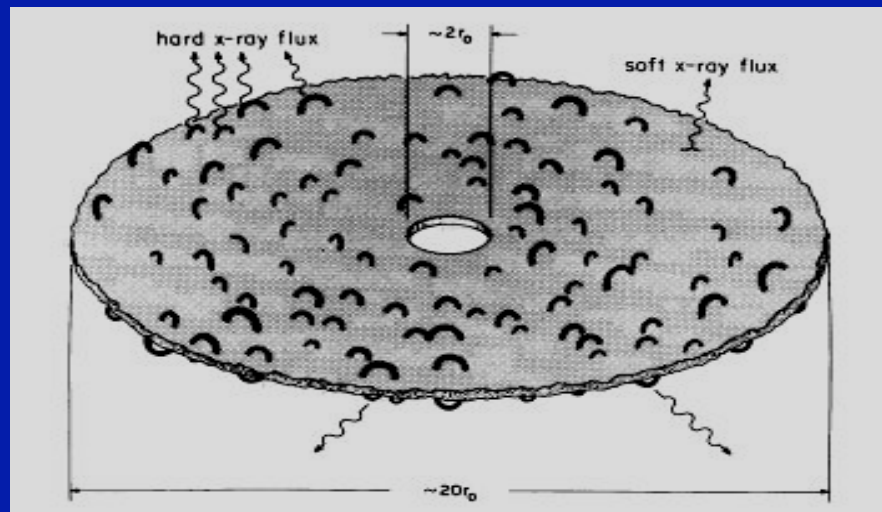
- Accretion Disks: Basic Physical Picture
- MHD of Disks: Angular Momentum Transport
- Collisionless Accretion Flows (BHs & NSs)
  - Astrophysical Motivation
  - Disk Dynamics in Kinetic Theory
    - A mechanism for strong electron heating
      (Sharma et al. astro-ph 07)

Accretion: Physical Picture

- Simple Consequences of Mass, Momentum, & Energy Conservation
- Matter Inspirals on Approximately Circular Orbits
  - $V_r << V_{orb}$ $t_{inflow} >> t_{orb}$
  - t_{inflow} ~ time to lose angular momentum ~ viscous diffusion time
  - $t_{orb} = 2\pi/\Omega$; $\Omega = (GM/r^3)^{1/2}$ (Keplerian orbits; like planets in solar system)
- Disk Structure Depends on Fate of Released Gravitational Energy
  - t_{cool} ~ time to radiate away thermal energy of plasma
  - Thin Disks: $t_{cool} << t_{inflow}$ (plasma collapses to the midplane)
  - Thick Disks: $t_{cool} >> t_{inflow}$ (plasma remains a puffed up torus)

Geometric Configurations

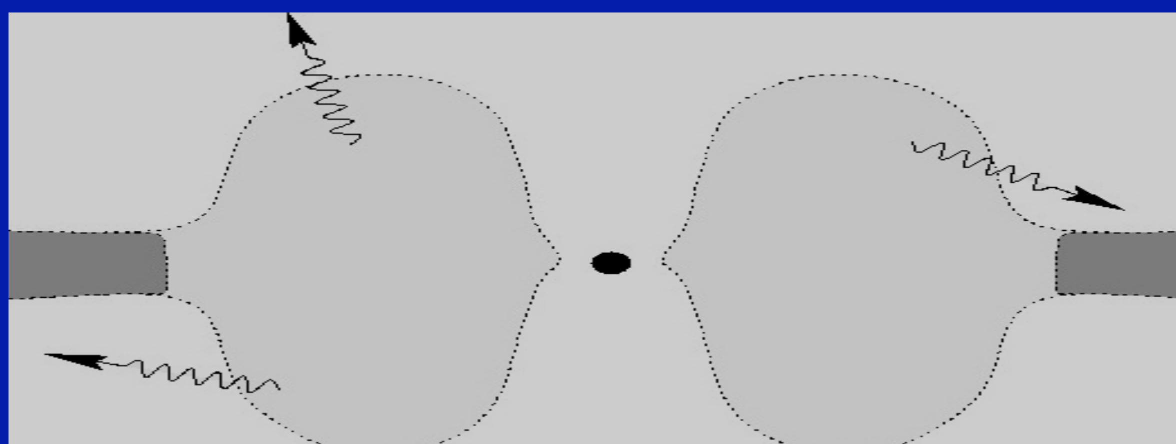
Galeev et al. 1979



e.g., solar system
Milky Way disk

thin disk: energy radiated away

(relevant to star & planet formation, galaxies, and luminous BHs/NSs)

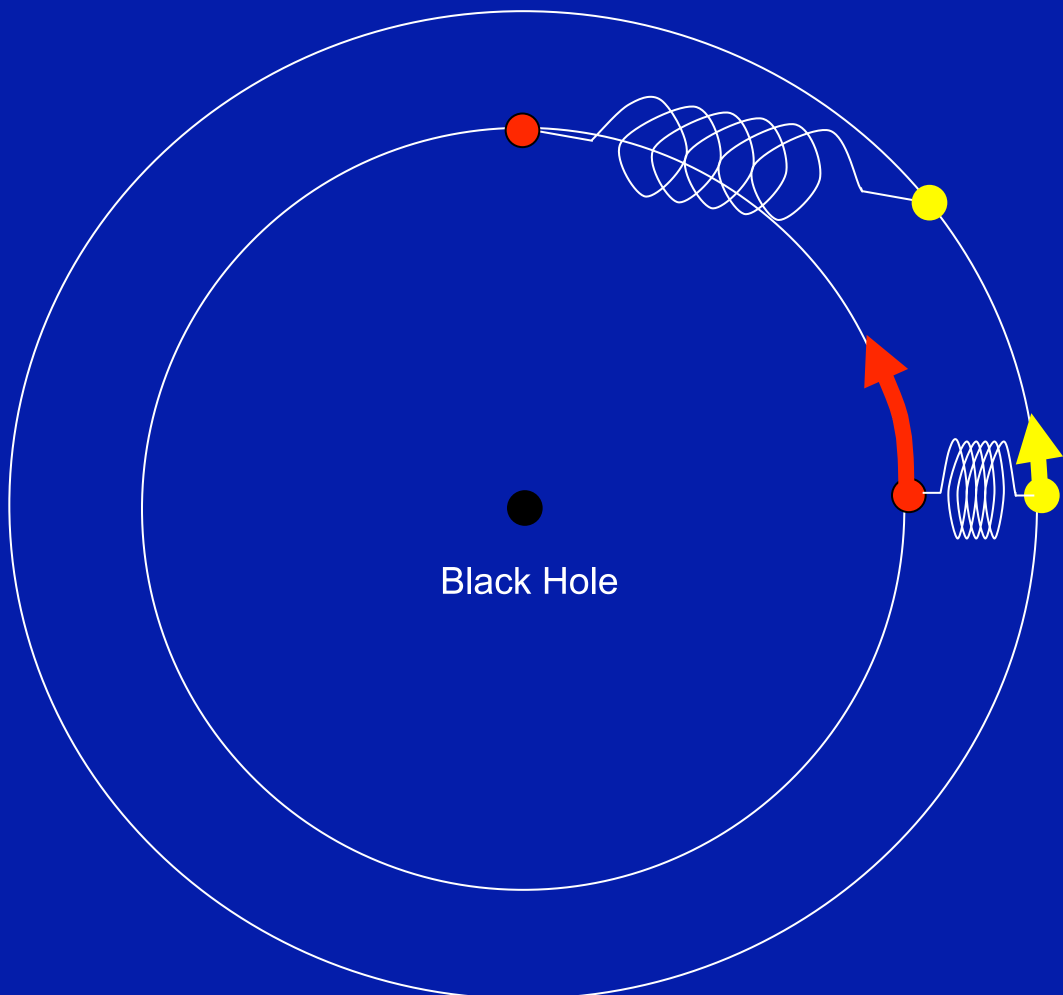


e.g., our Galactic Center (more on this soon)

thick disk (torus; ~ spherical): energy stored as heat

(relevant to lower luminosity BHs/NSs)

Magneto-Rotational Instability explains how accretion disks accrete (Balbus & Hawley, 1991)

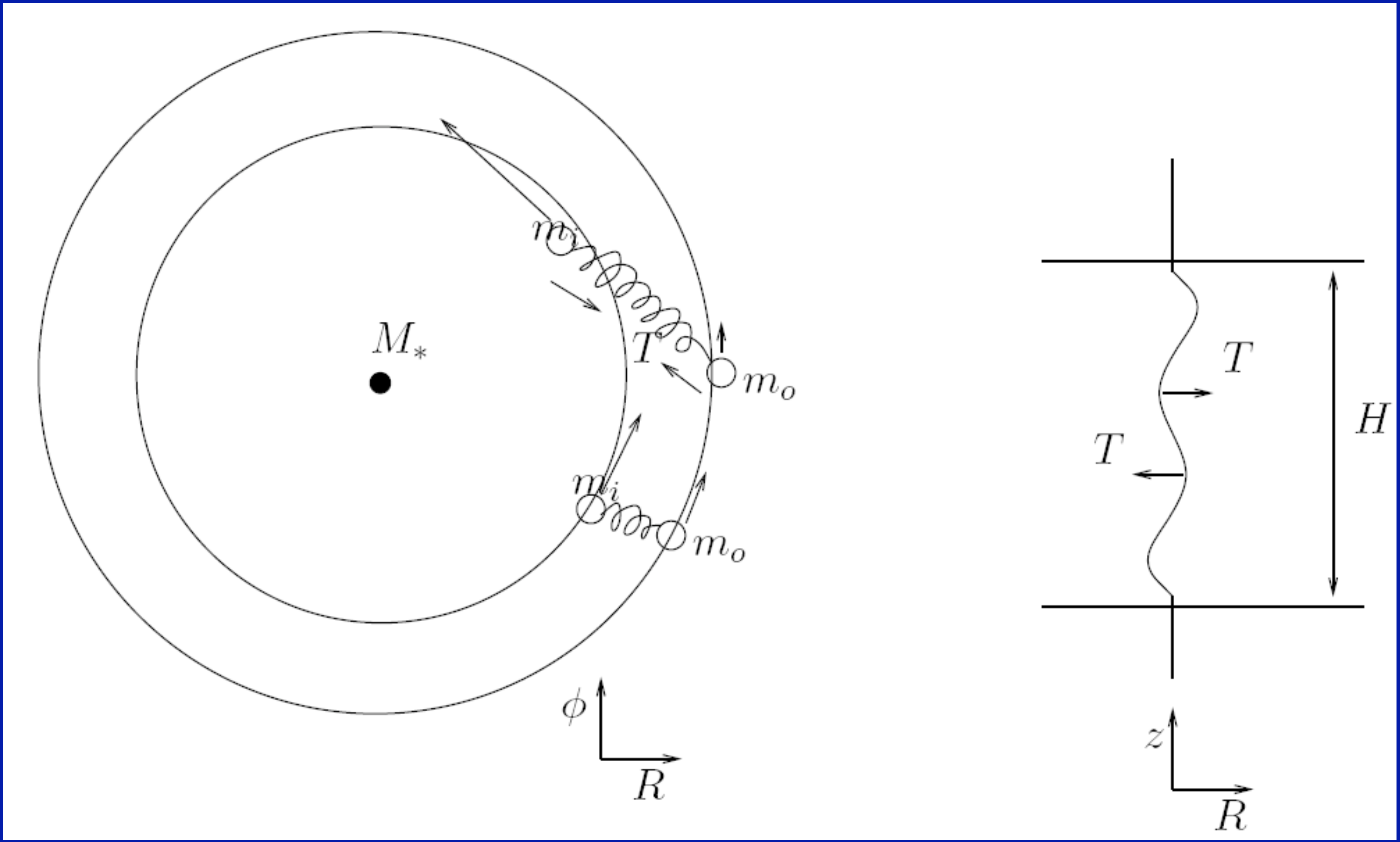


Inner particle orbits faster,
Spring stretches out
Spring force slows inner particle
and accelerates outer particle
Causing inner particle to fall in
and outer particle to go out
Exponentially amplified.

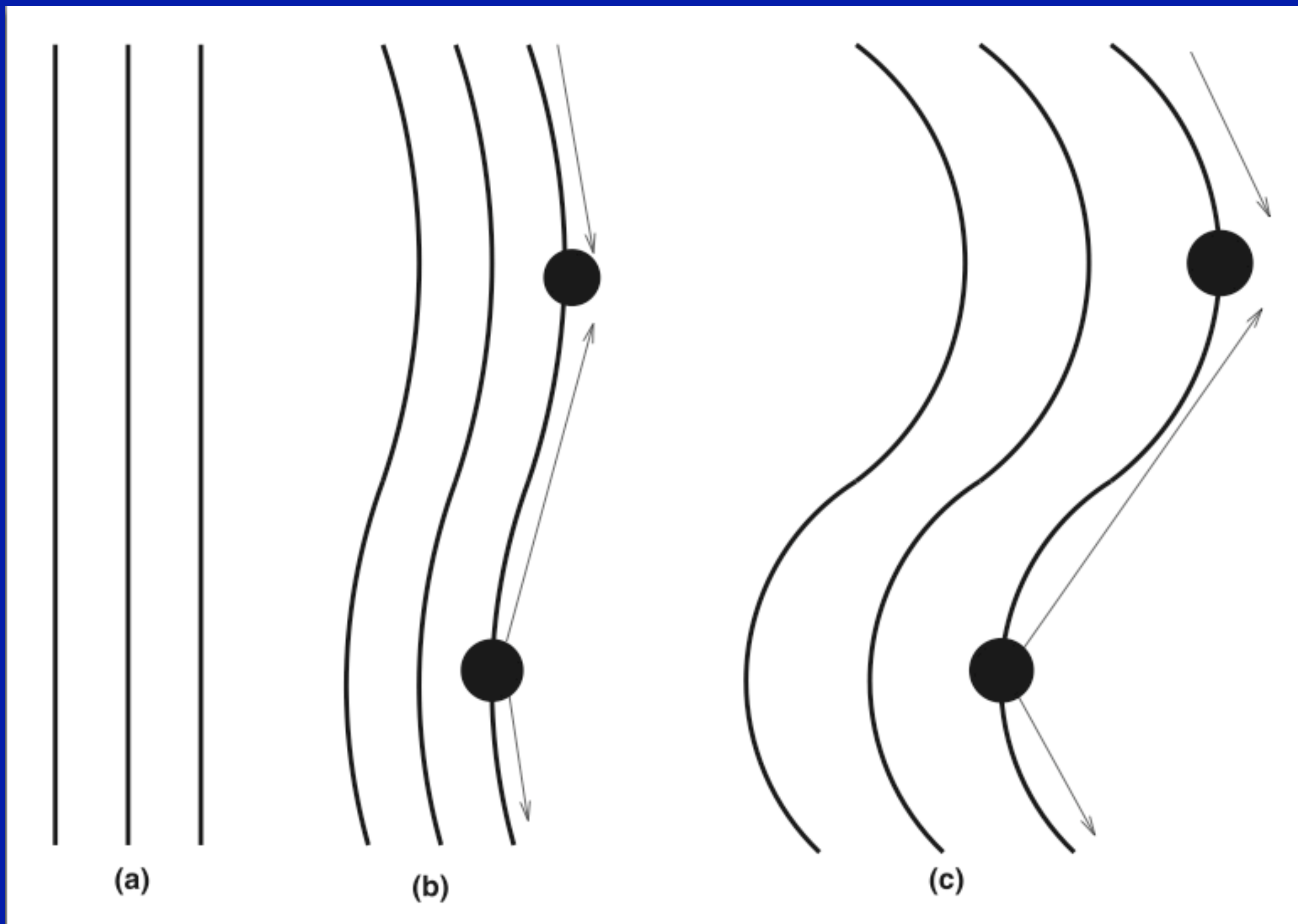
Magnetic fields
Are like springs

spring analogy by Toomre

Side view: magnetic field stretching acts like springs
& transfer angular momentum



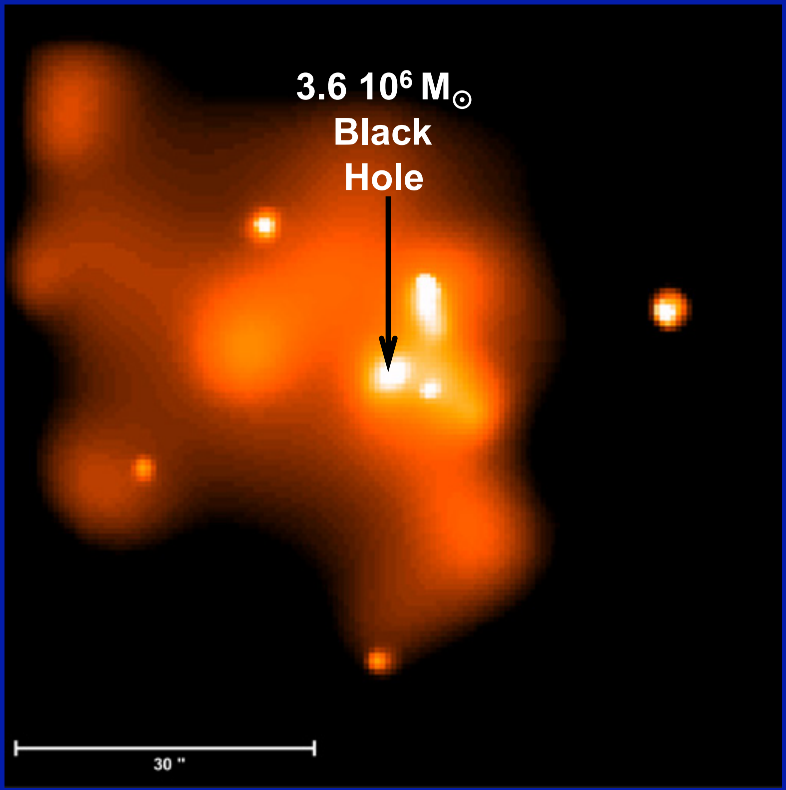
Balbus picture of kinetic-MRI / “Magneto-Viscous Instability” (MVI)



Balbus ApJ ‘04

An Astrophysical Context: Our Galactic Center

Galactic Center (*Chandra*)



Ambient Gas: $n \sim 10\text{-}100\ \mathrm{cm}^{-3}$
$T \sim 1\text{-}2$ keV

- Ambient gas should be grav. captured by the BH
- Estimates (Bondi) give

$$\dot{M}_{\rm captured} \approx 10^{-5}\, \mathrm{M}_\odot\, \mathrm{yr}^{-1}$$

(rate at which gas is captured at large radii)

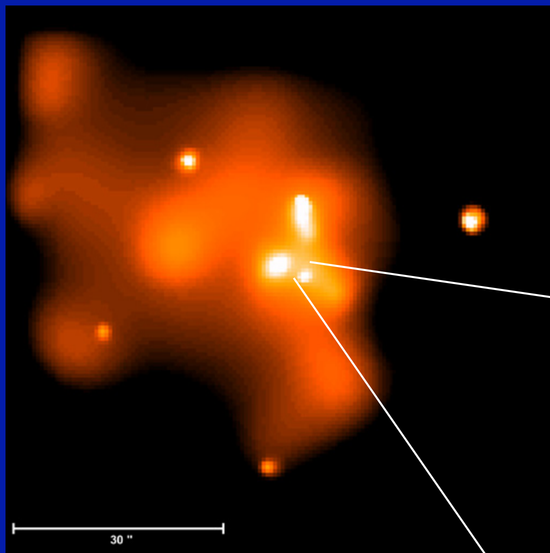
- But then

$$L_{\rm observed} \approx 10^{-5} \times \left(0.1 \dot{M} c^2\right)$$

Either radiation efficiency is x10^{-5} smaller than in quasars (hot ion ADAF regime, Ichimaru, Rees, Narayan), or net accretion $\dot{M}$ much smaller than Bondi estimate.

http://chandra.harvard.edu/

Galactic Center BH



Chandra

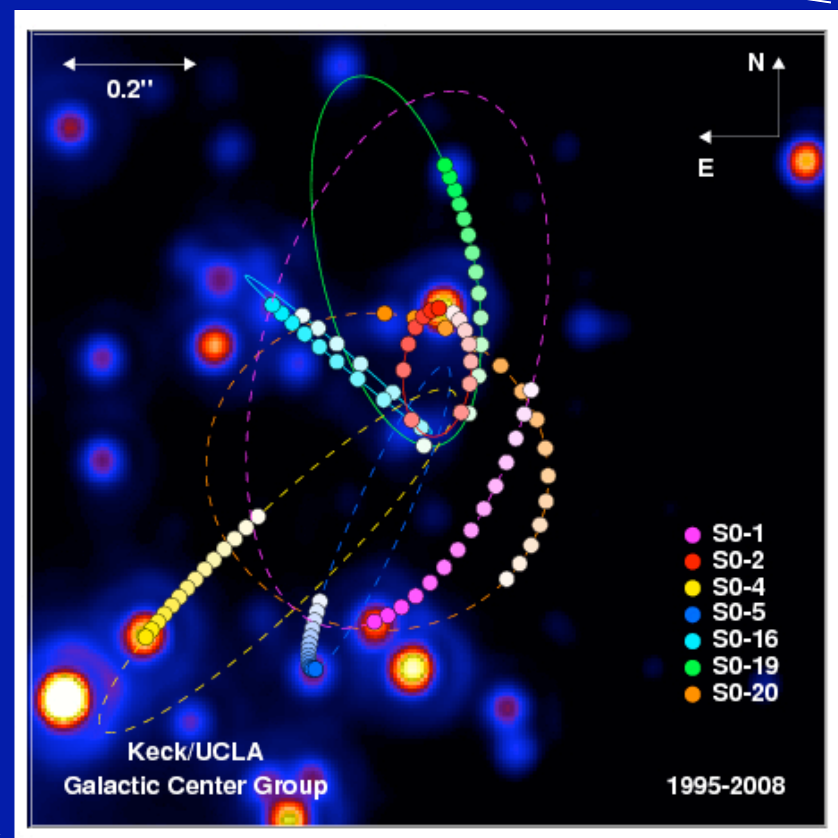
$3.6 \times 10^6\ M_\odot$ black hole

Bondi radius ~ 0.07 pc (2''),
n~100/cc, T~1-2 keV

$\dot{M} \sim 10^{-5}\ M_\odot$ /yr by stellar outflows

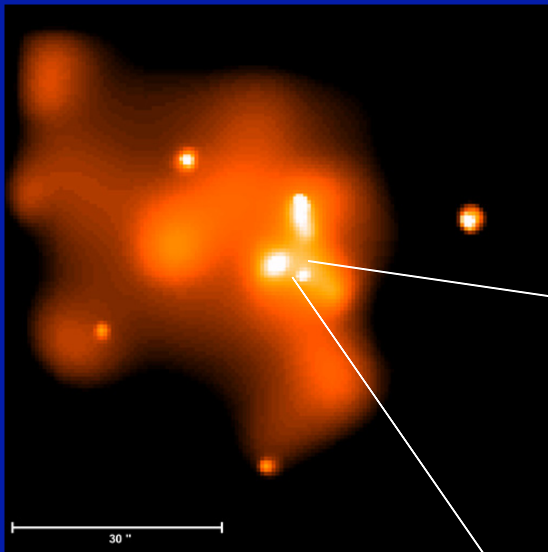
$L_{obs} \sim 10^{-5} \times (0.1\ \dot{M} c^2)$
Why low luminosity? low $\dot{M}$ or low radiative efficiency

Collisionless, magnetized plasma at
R ~ Bondi radius; $r_i \ll H$, $\ell_{mfp} \gg H$

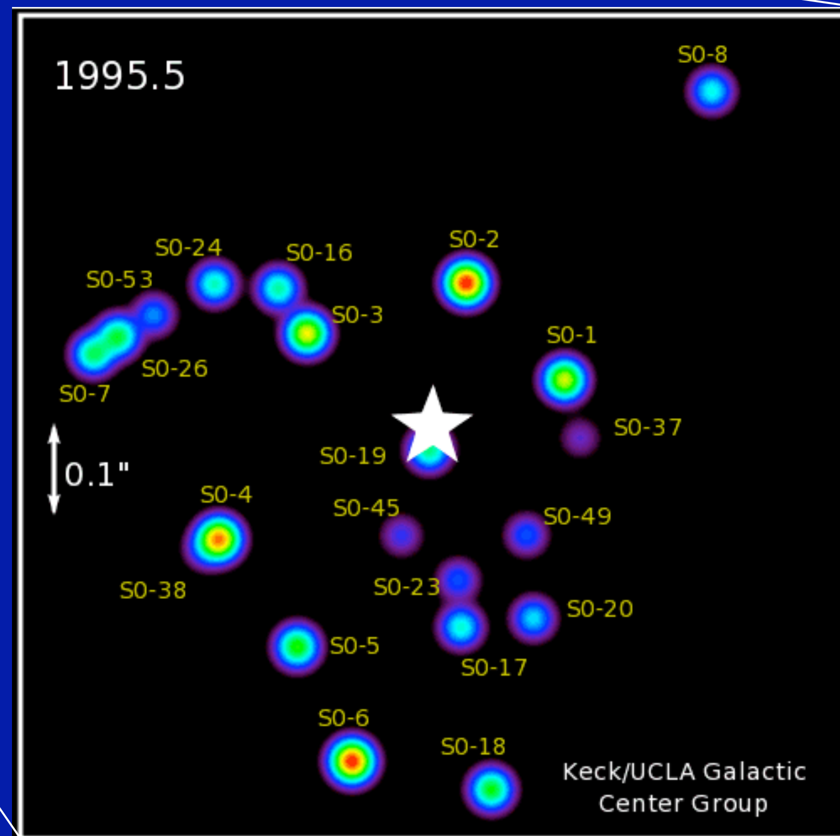


Schödel et al., 2002, A.M. Ghez et al. 2003 http://www.astro.ucla.edu/~ghezgroup/gc

Galactic Center BH



Chandra



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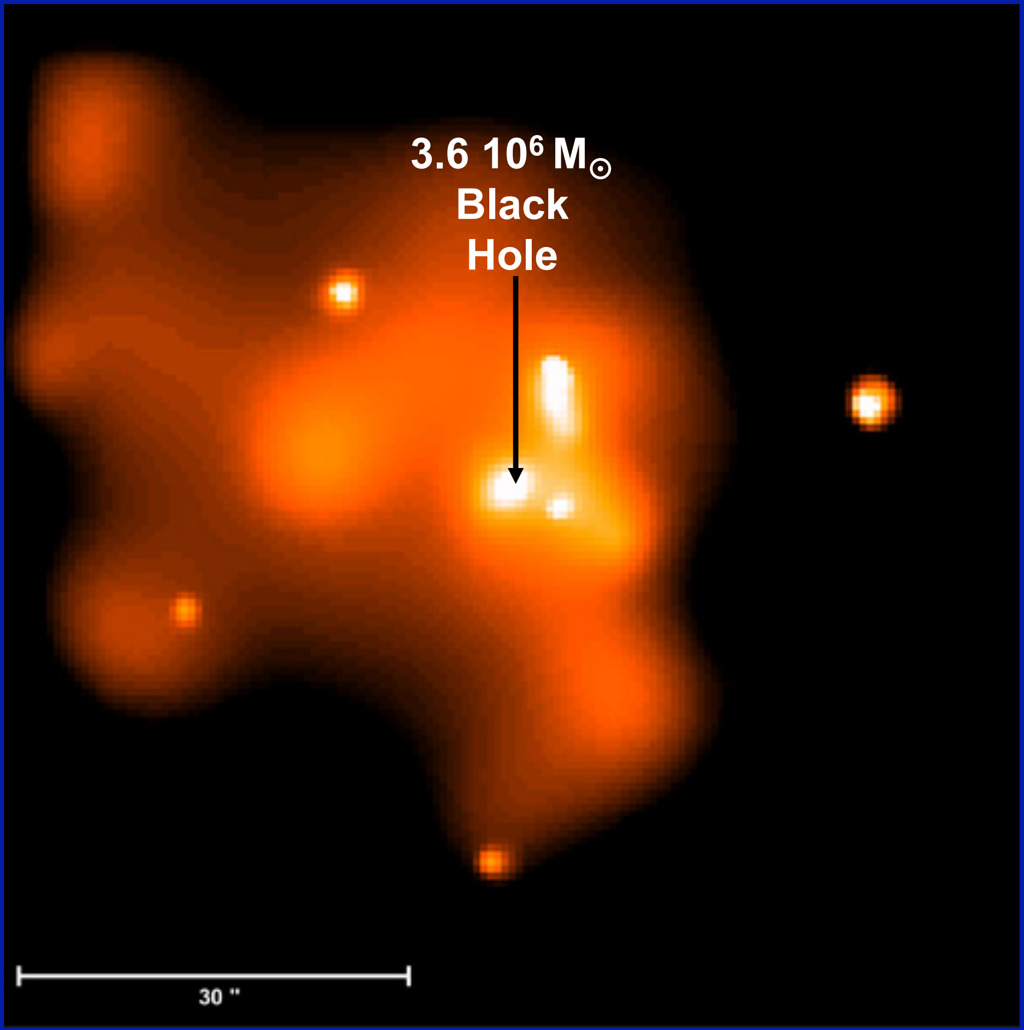
The (In)Applicability of MHD?



Hot Plasma Gravitationally Captured
By BH → Accretion Disk

Observed Plasma

(R ~ 10^{17} cm ~ 10^5 $R_{horizon}$)

T ~ few keV   n ~ 100 cm^{-3}

mfp ~ 10^{16} cm ~ 0.1 R

e-p thermalization time ~ **1000 yrs**

>>

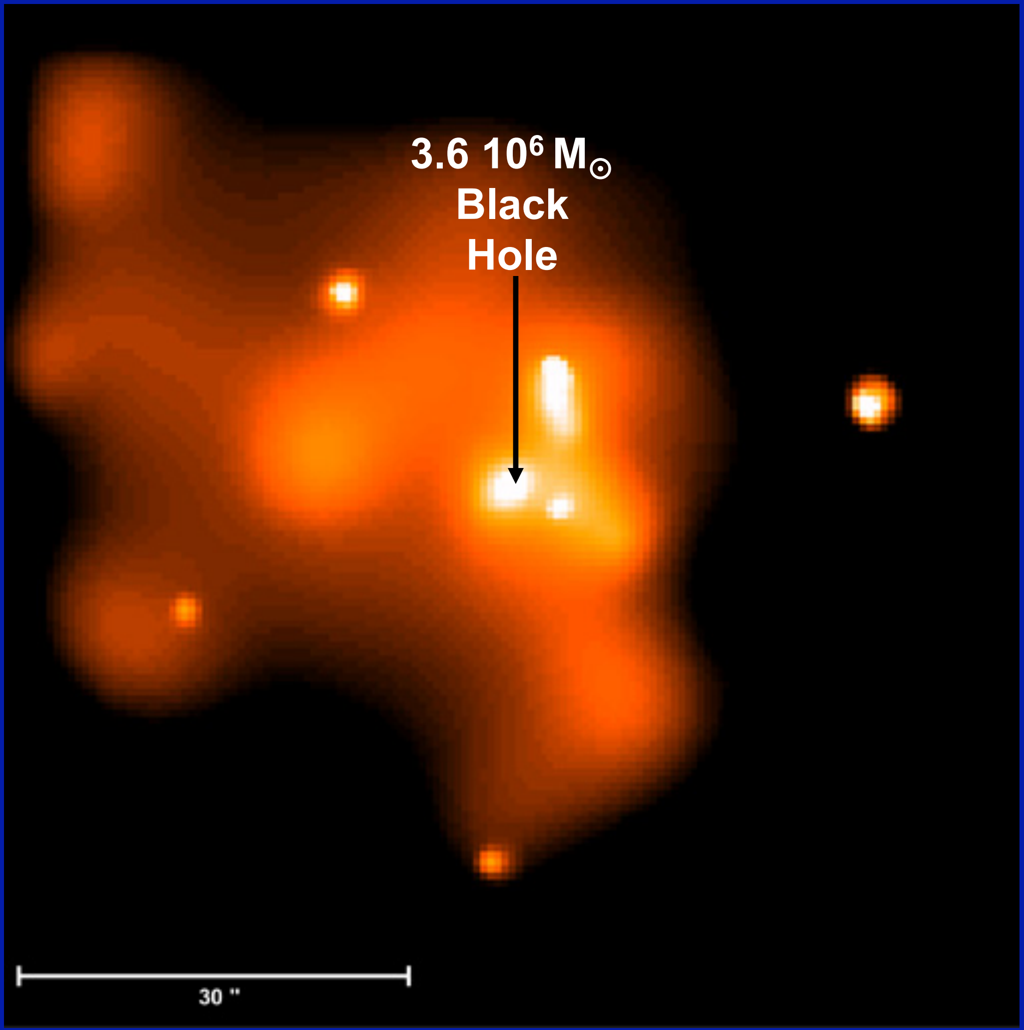
inflow time ~ R/c_s ~ **100 yrs**

electron conduction time ~ **10 yrs**

<<

inflow time ~ R/c_s ~ **100 yrs**

The (In)Applicability of MHD?



Hot Plasma Gravitationally Captured
By BH → Accretion Disk

Estimated Conditions
Near the BH

$T_p \sim 10^{12}$ K
$T_e \sim 10^{11}$ K
$n \sim 10^6$ cm^{-3}
$B \sim 30$ G

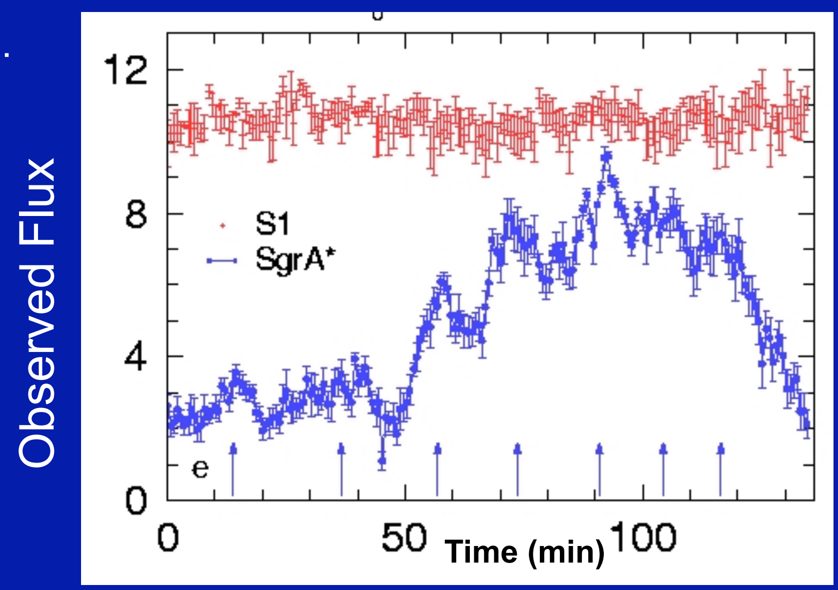
proton mfp ~ 10^{22} cm
>>> $R_{horizon}$ ~ 10^{12} cm

→

need to understand
accretion of a magnetized
collisionless plasma

Major Science Questions

- Macrophysics: Global Disk Dynamics in Kinetic Theory
  - e.g., how adequate is MHD, influence of heat conduction, …
- Microphysics: Physics of Plasma Heating
  - MHD turbulence, reconnection, weak shocks, …
  - **electrons produce the radiation we observe**
- Analogy: Solar Wind
  - macroscopically collisionless
  - thermally driven outflow w/ T_p & T_e determined by kinetic microphysics



MHD Drift Kinetic Eq. for $f_{0s}(\vec{x}, v_{\|}, \mu, t)$

plasma is collisionless, hot w. H~r

Larmor radius << disk height

drift kinetic equation: approx. for Vlasov eq. if $k\rho_i \ll 1$, $\omega \ll \Omega_i$

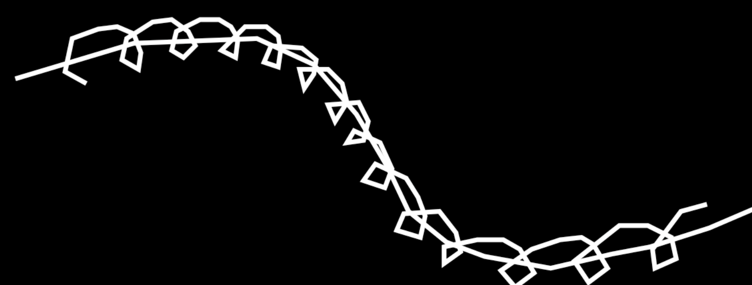
Table 1.2: Plasma parameters for Sgr A*

Parameter	$r = r_{acc}$ 2.2×10^{17} cm	$r = \sqrt{r_{acc} R_S}$ 4.2×10^{14} cm	$r = R_S$ 7.8×10^{11} cm
$\nu_{i,\mathrm{ADAF}}/\Omega_K \sim r^{3/2}$	11.4	9.4×10^{-4}	7.6×10^{-8}
$\nu_{i,\mathrm{CDAF}}/\Omega_K \sim r^{3/2+p}$	11.4	1.81×10^{-6}	2.62×10^{-13}
$\rho_{i,\mathrm{ADAF}}/H \sim r^{-1/4}$	2×10^{-11}	9.94×10^{-11}	4.59×10^{-10}
$\rho_{i,\mathrm{CDAF}}/H \sim r^{-1/4-p/2}$	2×10^{-11}	2.23×10^{-9}	2.48×10^{-7}

$$\frac{\partial f_{0s}}{\partial t} + (\mathbf{V}_E + v_{\|}\hat{\mathbf{b}}) \cdot \nabla f_{0s} + \left(-\hat{\mathbf{b}} \cdot \frac{D\mathbf{V}_E}{Dt} - \mu \hat{\mathbf{b}} \cdot \nabla B + \frac{1}{m_s}(q_s E_{\|} + F_{g\|}) \right) \frac{\partial f_{0s}}{\partial v_{\|}} = 0$$

$\mu = v_\perp^2/B \propto T_\perp/B$ is conserved; $V_E = c(E \times B)/B^2$

mfp >> disk height scales >> Larmor radius



Kulsrud's '61 version of unpublished Chew-Goldberger-Low MHD-drift-kinetic equations

Nonlinear Evolution Simulated Using Kinetic-MHD

- Large-scale Dynamics of collisionless plasmas: expand Vlasov equation retaining "slow timescale" (compared to cyclotron period) & "large lengthscale" (compared to gyroradius) assumptions of MHD (e.g., Kulsrud 1983)

- Particles efficiently transport heat and momentum along field-lines

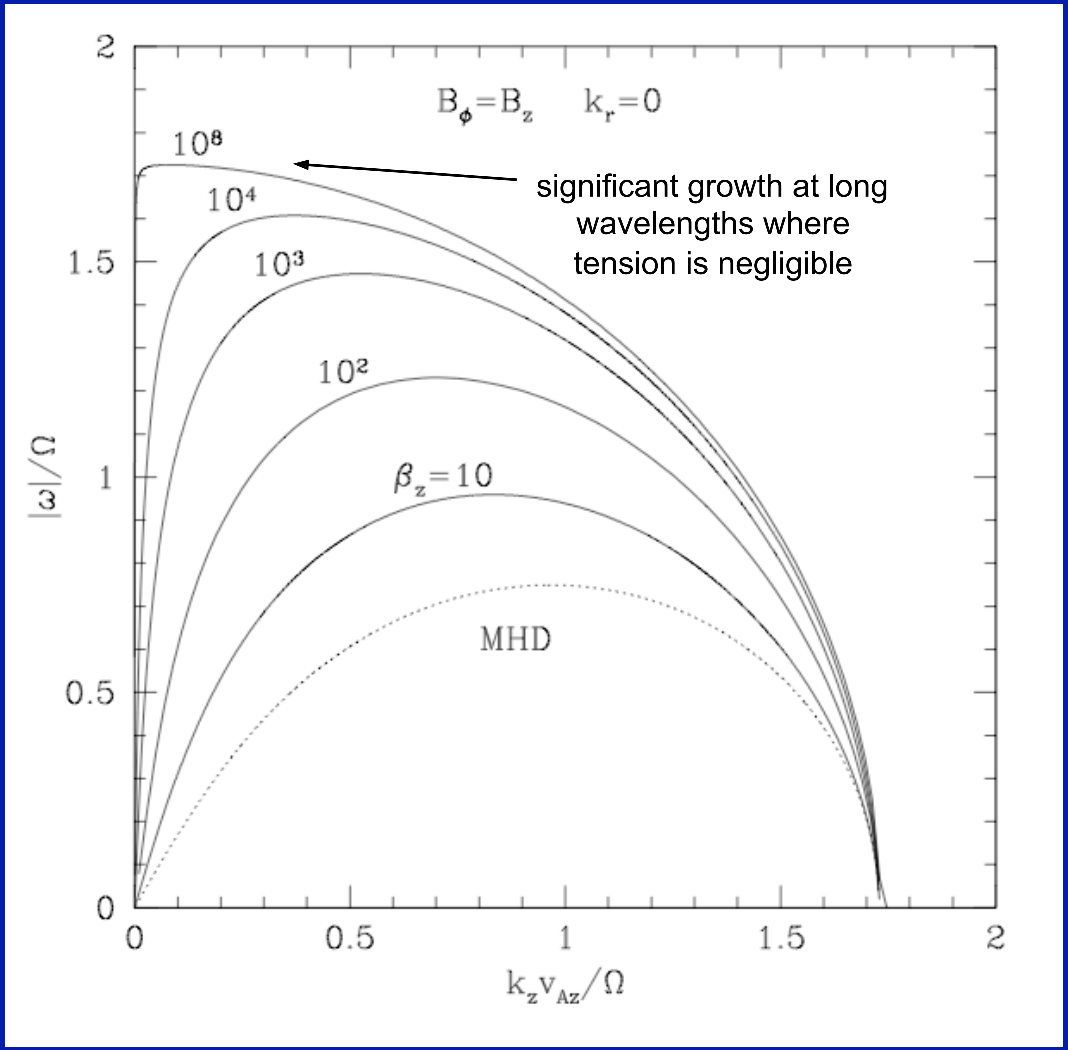
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0,$$

$$\rho \frac{\partial \mathbf{V}}{\partial t} + \rho \left(\mathbf{V} \cdot \nabla\right) \mathbf{V} = \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi} - \nabla \cdot \mathbf{P} + \mathbf{F_g},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{V} \times \mathbf{B}\right),$$

$$\mathbf{P} = p_\perp \mathbf{I} + \left(p_{||} - p_\perp\right) \hat{\mathbf{b}}\hat{\mathbf{b}},$$

The MRI in a Collisionless Plasma

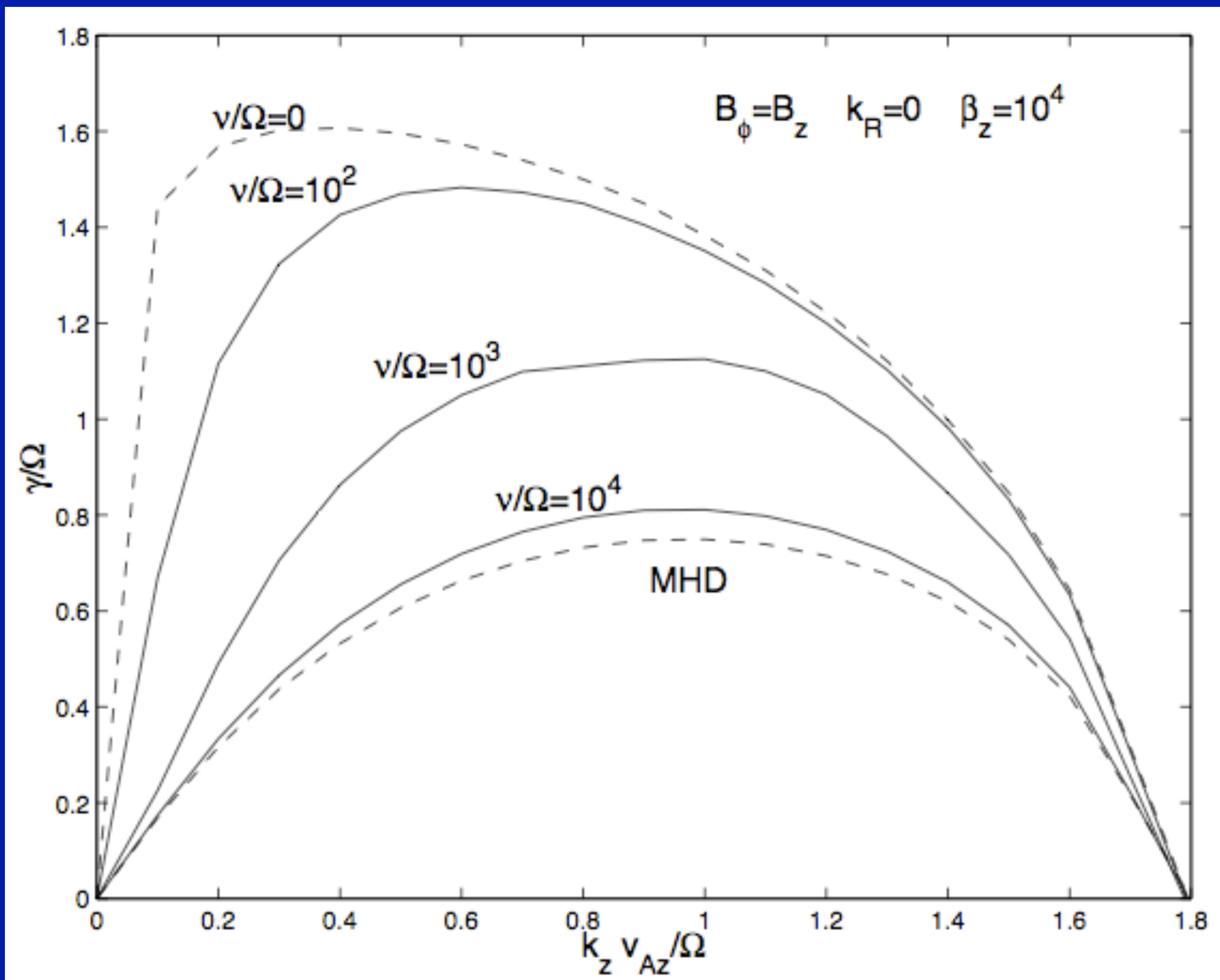


Quataert, Dorland, Hammett 2002; also Sharma et al. 2003; Balbus 2004

angular momentum transport via anisotropic pressure (viscosity!) in addition to magnetic stresses

$$F_\varphi \propto \left(\frac{B_z B_\varphi}{B^2}\right)\left(\delta p_{\parallel} - \delta p_{\perp}\right)$$

Transition from kinetic-MHD to Braginskii-MHD to isotropic MHD as collisionality increases

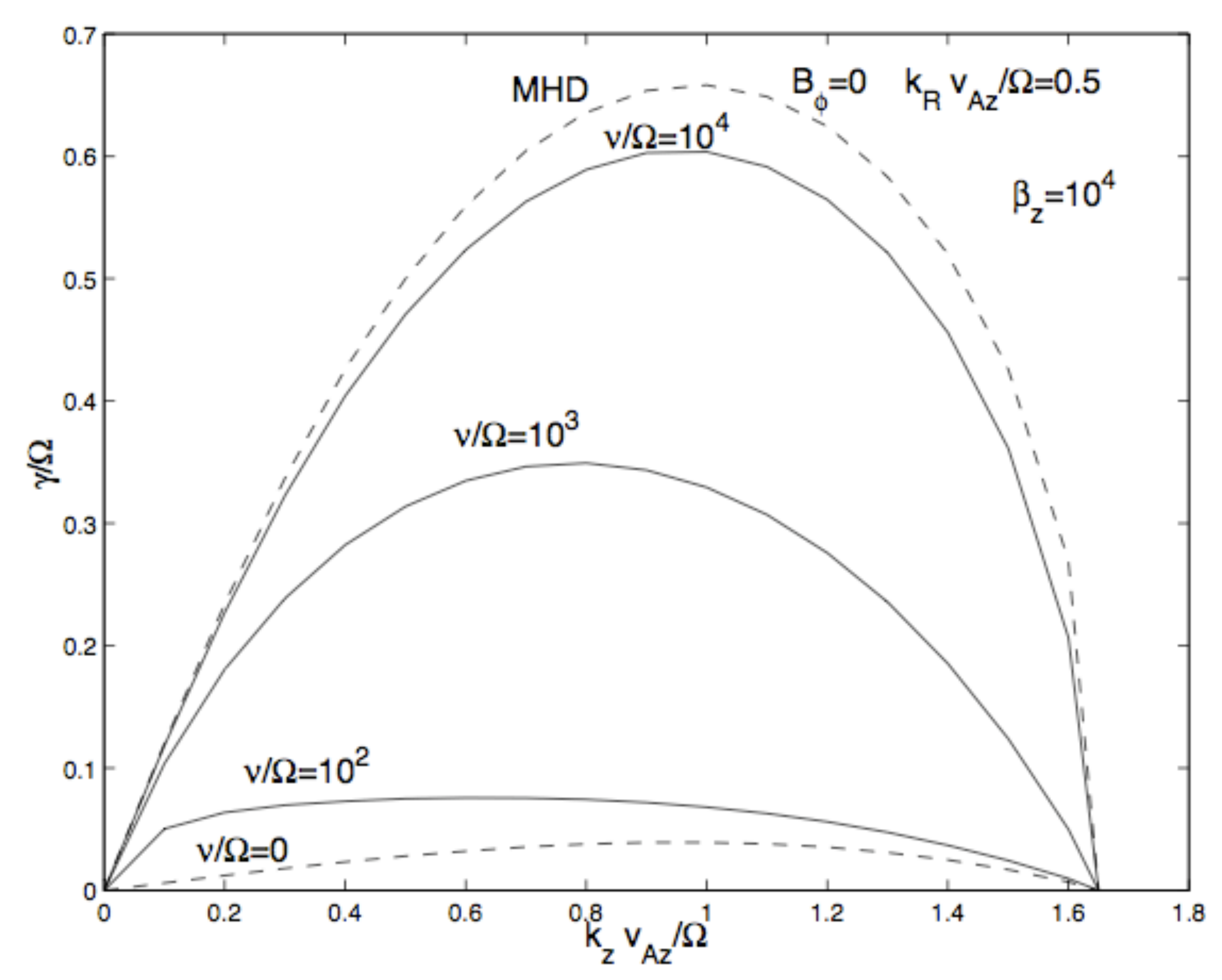


ν = collision frequency
Ω = rotation frequency

Braginskii valid if
$k\,L_{mfp} \sim k v_{ti}/\nu << 1$ &
$\omega / \nu << 1$

Sharma, Hammett, Quataert ApJ 03

Kinetic effects stabilizing if initial $B_\varphi=0$



Different than last 2 slides, where kinetic effects enhance growth rate if initial $B_\varphi= B_z$

Sharma, Hammett, Quataert ApJ 03

Evolution of the Pressure Tensor

$$\rho B \frac{d}{dt}\left(\frac{p_\perp}{\rho B}\right) = -\nabla\cdot(\hat{\mathbf{b}} q_\perp) - q_\perp \nabla\cdot\hat{\mathbf{b}}$$

adiabatic invariance of $\mu \sim mv^2_\perp/B \sim T_\perp/B$

$$\frac{\rho^3}{B^2}\frac{d}{dt}\left(\frac{p_\parallel B^2}{\rho^3}\right) = -\nabla\cdot(\hat{\mathbf{b}} q_\parallel) + 2q_\perp \nabla\cdot\hat{\mathbf{b}},$$

$q_\perp = q_\parallel = 0$ CGL or Double Adiabatic Theory

$$q_{\perp,\parallel} \approx \frac{n \mathrm{v}_{th}}{|k_\parallel|} \nabla_\parallel T_{\perp,\parallel}$$

Closure Models for Heat Flux (temp gradients wiped out on ~ a crossing time)

Pressure Anisotropy

$$\mu \propto T_\perp / B = \text{constant} \;\Rightarrow\; T_\perp > T_\parallel \text{ as B} \uparrow$$

- $T_\perp \neq T_\parallel$ unstable to small-scale (~ gyroradius) modes that *might* act to isotropize the pressure tensor (velocity space anisotropy)
  - e.g., mirror, firehose, ion cyclotron, electron whistler instabilities
  - Some uncertainties, particularly near marginal stability: might saturate w/o breaking μ

- waves w/ Doppler-shifted frequencies ~ Ω_{cyc} violate μ invariance & cause pitch-angle scatter
  - **Increases effective collisions & reduces mean free path of particles in the disk**
  - **Breaking μ invariance critical to making magnetic pumping irreversible and getting net particle heating**
  - impt in other macroscopically collisionless astro plasmas (solar wind, clusters, …)

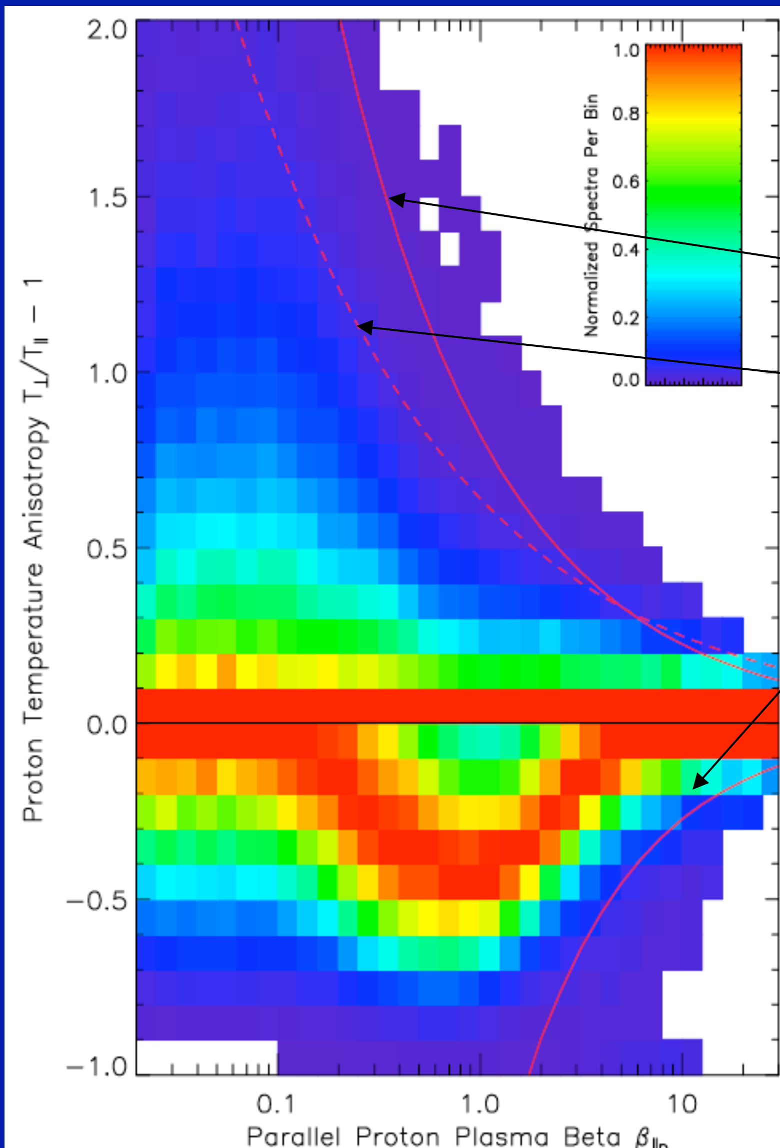
- Assume "subgrid" scattering model in disk simulations

$$\frac{\partial p_\perp}{\partial t} = \ldots - \nu(p_\perp, p_\parallel, \beta)\left[p_\perp - p_\parallel\right]$$

$$\frac{\partial p_\parallel}{\partial t} = \ldots - \nu(p_\perp, p_\parallel, \beta)\left[p_\parallel - p_\perp\right]$$

Mirror/cyclotron/firehose instabilities will also limit Braginskii anisotropic transport coefficients.

Limits on Pressure Anisotropy



$$\left|\frac{p_\perp}{p_\parallel} - 1\right| \leq \frac{S}{\beta^\alpha}$$

mirror: $S=7, \alpha=1$ (to break adiabatic invariance)

ion-cyclotron: $S=0.35$, $\alpha=0.45$ for $\gamma/\Omega_i=10^{-4}$

mirror dominates IC for $\beta \sim 10\text{-}100$

firehose: $S>2$, $\alpha=1$

Pressure anisotropy reduced by pitch-angle scattering if anisotropy exceeds threshold.

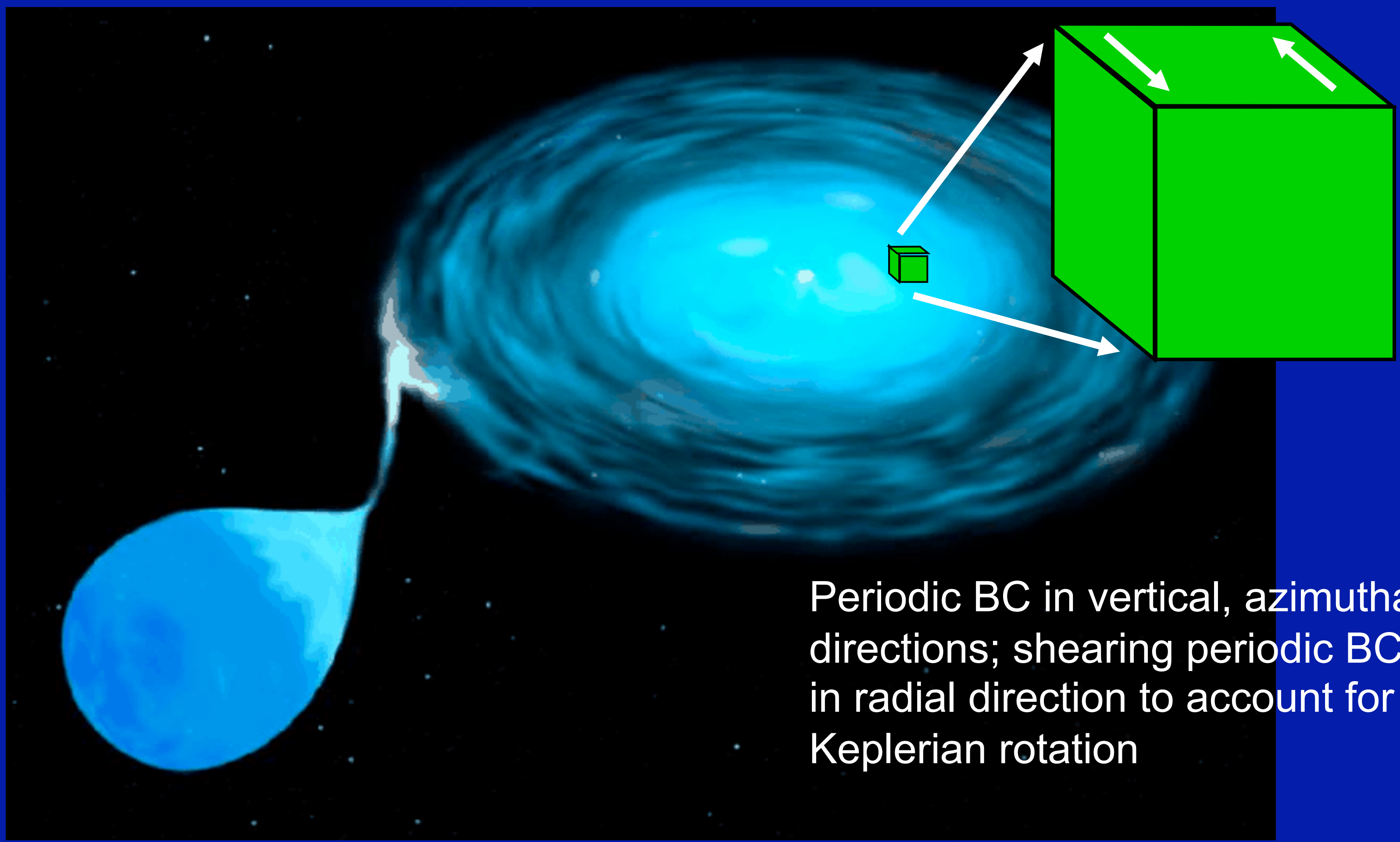
For electrons with $p_\perp > p_\parallel$ electron whistler instability will isotropize: $S=0.13$, $\alpha=0.55$ (γ/Ω = 5x10^{-8}) *[using WHAMP code]*

[Kasper et al. 2003, Gary & coworkers]

Examples from Space Physics

- Solar wind at 1 AU statistically at firehose instability threshold *[Kasper et al., Wind]*
- Magnetic Holes in SW & magnetopause, a signature of mirror modes *[Winterhalter et al.,Ulysses]*
- Mirror mode signatures at Heliopause, *[Liu et al.,Voyager1]*
- Above can be interpreted from μ conservation in expanding/ compressing plasmas
- Small-scale instabilities driven by pressure anisotropy mediate shock transition in collisionless plasmas
- SW an excellent laboratory for collisionless plasma physics
- Since much of astrophysical plasma (except in stars) is collisionless, a lot of applications in astrophysics; e.g., X-ray clusters, accretion disks, collisionless shocks.

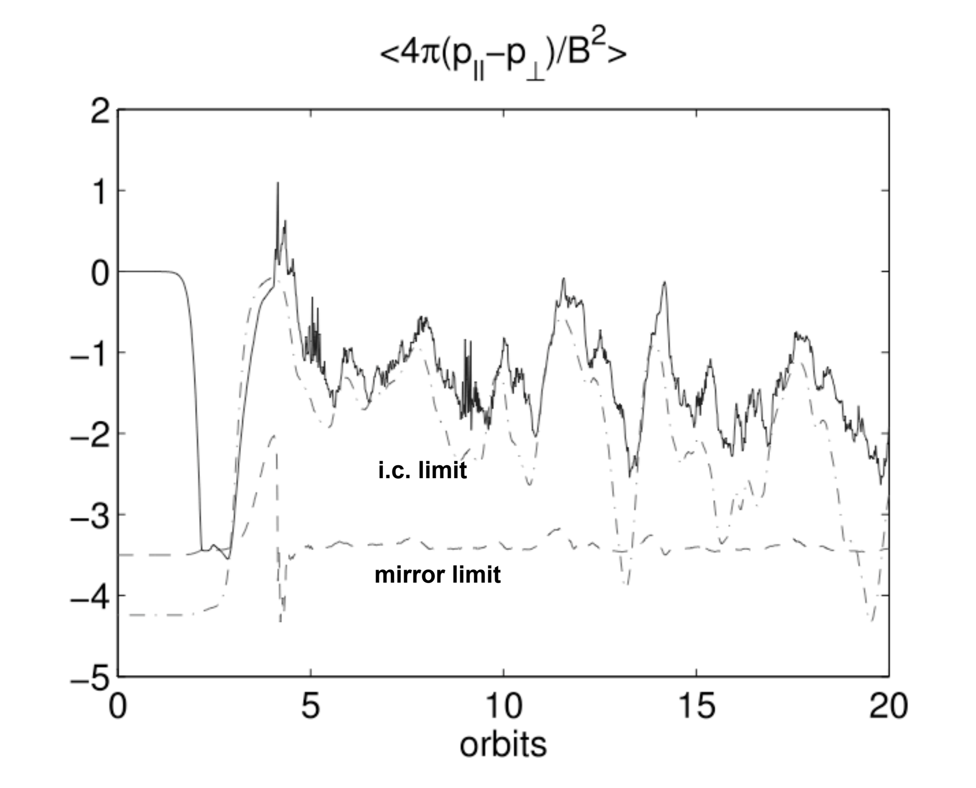
Shearing Box Simulations



Periodic BC in vertical, azimuthal directions; shearing periodic BC in radial direction to account for Keplerian rotation

Local Simulations of the MRI in a Collisionless Plasma

volume-averaged pressure anisotropy



Sharma et al. 2006

Rate of Angular Momentum Transport Enhanced Relative to MHD (by factor ~ unity)

Net Anisotropic Stress (i.e, viscosity) ~ Maxwell Stress

anisotropic stress is a significant source of plasma heating

Heating by Anisotropic Stress

$$\frac{3}{2}\frac{dp}{dt} = -\mathbf{P} : \nabla \vec{\mathrm{v}} + \ldots$$

Pressure tensor heating

$$= -\frac{p_{\parallel} - p_{\perp}}{B^2} \vec{B}\vec{B} : \nabla \vec{\mathrm{v}}$$

$$= -\frac{p_{\parallel} - p_{\perp}}{2B^2} \frac{dB^2}{dt}$$

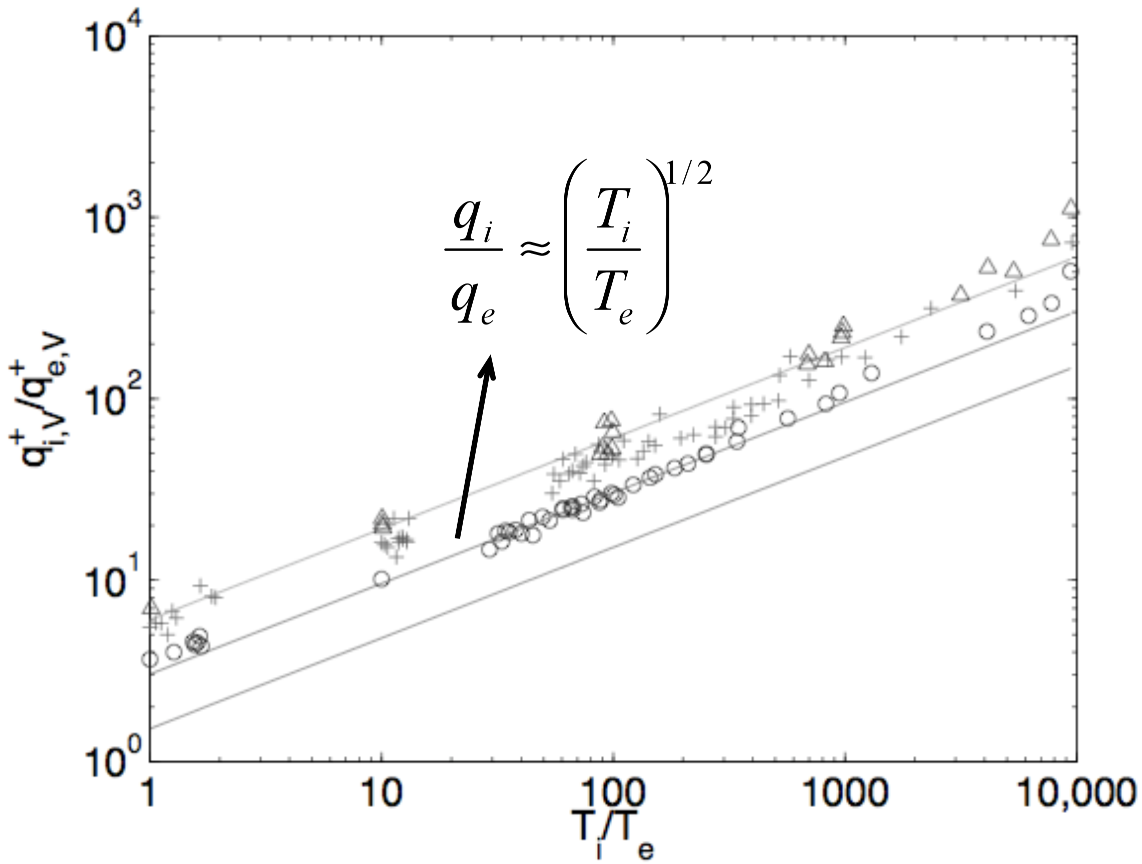
$$\propto \frac{\sqrt{p}}{B} \vec{B}\vec{B} : \nabla \vec{\mathrm{v}}$$

Anisotropy limit set by
Velocity-space instabilities

$$\frac{1}{T_e}\frac{dT_e}{dt} \propto \frac{1}{\sqrt{T_e}}$$

Even if electrons start cold, they will be rapidly heated to a temperature independent of initial conditions, becoming comparable to ion temperature

Heating by Anisotropic Stress



Sharma et al. 2007

$$q_s \propto P_{r\phi} \frac{d\Omega}{d \ln r} \propto \Delta p_s$$

ion cycl. & e- whistler instability thresholds

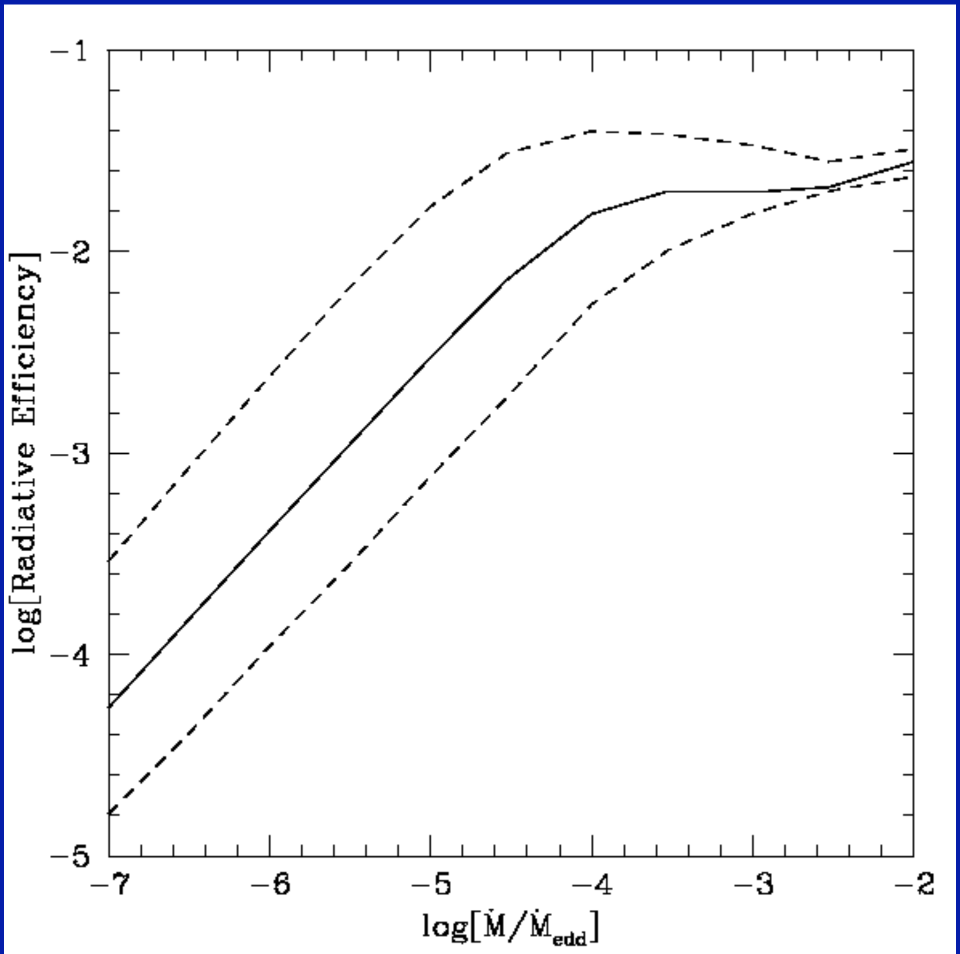
$$\frac{\Delta p_s}{p_s} \sim \beta_s^{-1/2}$$

$$\rightarrow \Delta p_s \propto T_s^{1/2}$$

$$\frac{1}{T_e}\frac{dT_e}{dt} \propto \sqrt{\frac{T_i}{T_e}}\frac{1}{T_i}\frac{dT_i}{dt}$$

Electron heating rate faster than ions in cold electron limit

Final result: predicted radiative efficiency vs. accretion rate



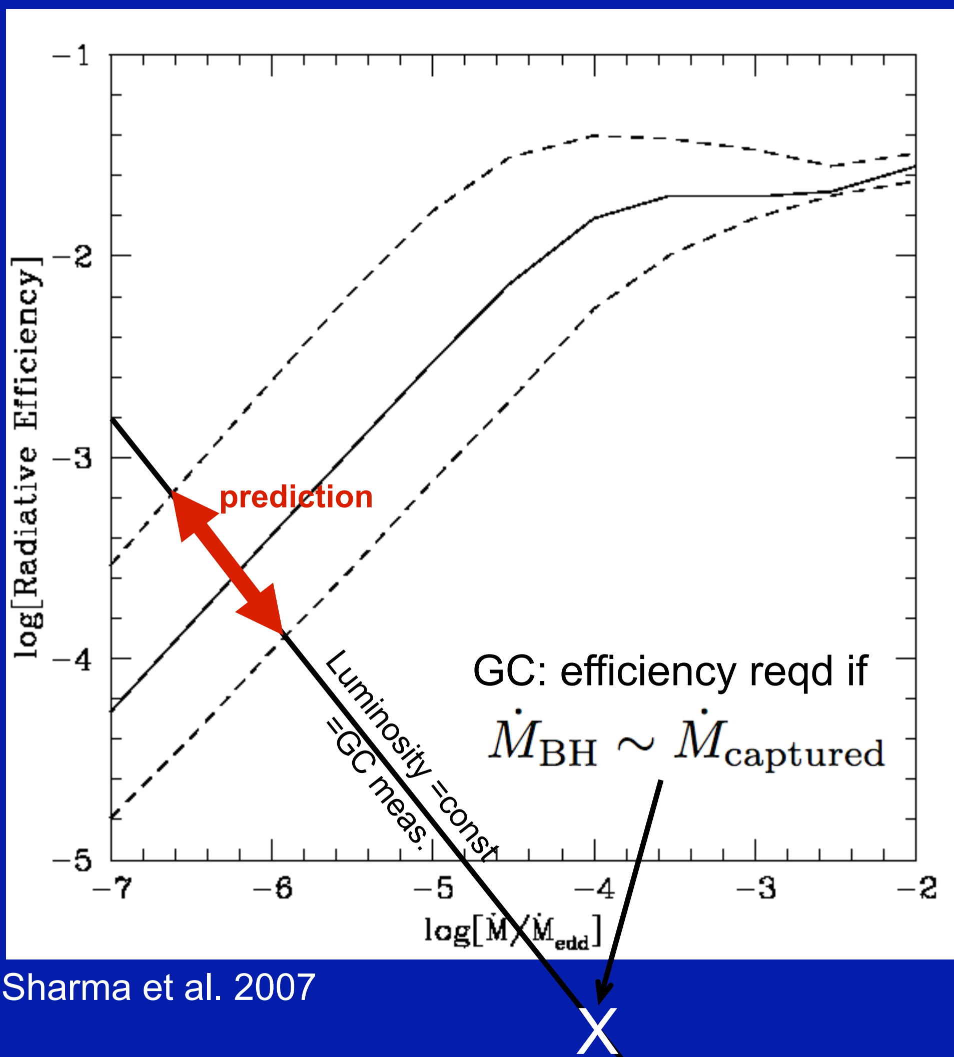
Sharma et al. 2007

'viscous' heating mediated by high freq. instabilities crucial source of electron heating in hot accretion flows

x2 uncertainties from previous page.

(this is a lower bound on electron heating & thus radiative efficiency, might also be resistive heating, and heating from kinetic Alfven tail of cascade)

Astrophysical Implications



Sharma et al. 2007

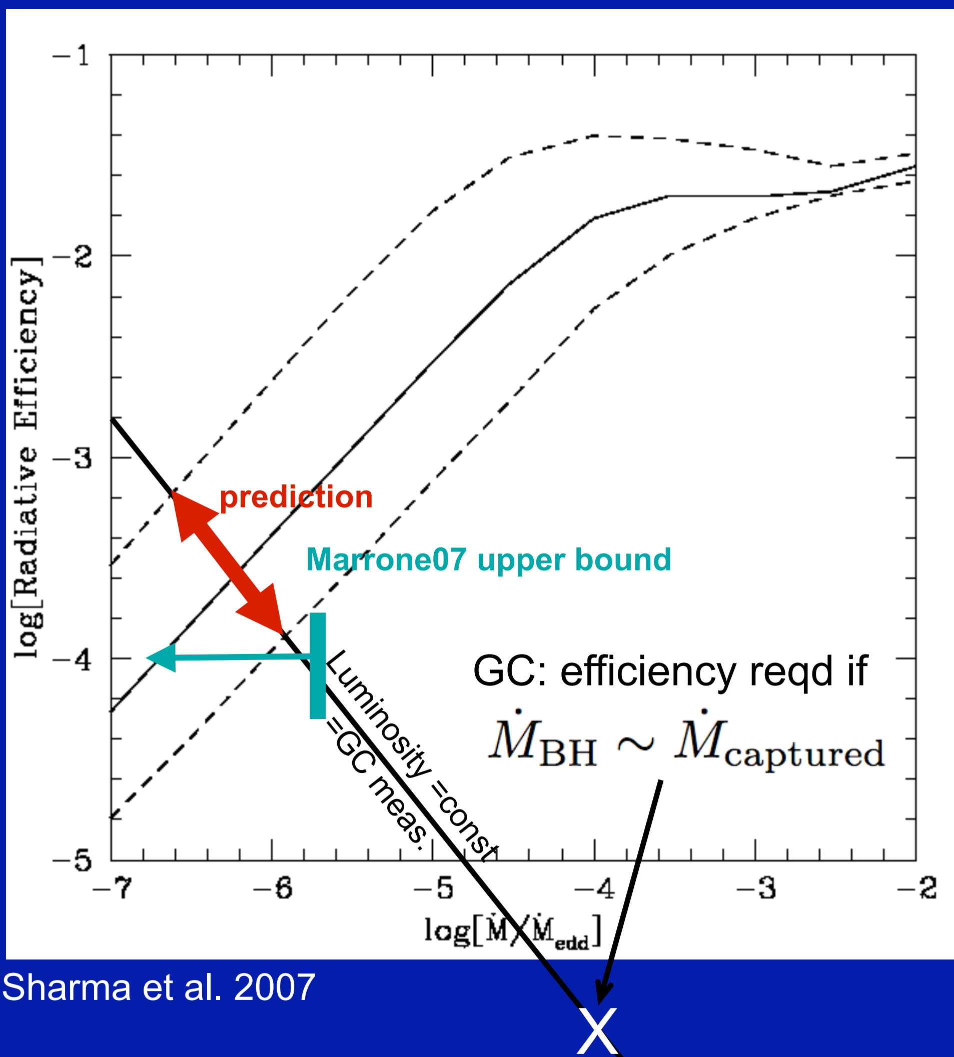
'viscous' heating mediated by high freq. instabilities crucial source of electron heating in hot accretion flows

→ **low accretion rate** required to explain the low luminosity of most accreting BHs

consistent w/ inferences from global MHD sims

$$L_{obs} = \varepsilon \dot{M} c^2$$

Predicted low accretion rate within bounds set by observations



Sharma et al. 2007

'viscous' heating mediated by high freq. instabilities crucial source of electron heating in hot accretion flows

→ **low accretion rate** required to explain the low luminosity of most accreting BHs

consistent w/ inferences from global MHD sims
and with upper bound estimate from Faraday rotation measurements.

Marrone et al. 07 ApJ 654, L57
Faraday rotation measurements.

Summary

- Prandtl # dependence of MRI, Dynamo, and other plasma processes is an interesting & subtle problem.
- In many plasma cases, the problem is even more subtle because of the highly anisotropic viscosity with a magnetic field.

- Long mean-free path regimes require going beyond standard fluid-MHD, to Braginskii-MHD ($L \gg L_{mfp} \gg \rho_i$) or full drift-kinetic MHD ($L \sim L_{mfp} \gg \rho_i$), or fluid-approximations to drift-kinetic MHD.

- Velocity-space microinstabilities (firehose, mirror, cyclotron, and electron whistler versions) will probably limit the amount of allowable pressure anisotropy ($|p_{||} - p_{\perp}| \sim B^2$) . This is crucial for sustaining MRI turbulence, enhances the effective collision frequency (pitch-angle scattering rate), reduces parallel transport coefficients, and provides a mechanism for strong electron heating.

- This strong electron heating makes a cold-ion ADAF scenario unlikely for explaining the low luminosity of some accretion flows, such as on the massive black hole in the galactic center.