Simulating the Effects of Stellarator Geometry on Gyrokinetic Drift-wave Turbulence

Jessica A. Baumgaertel

Princeton University Final Public Oral Examination

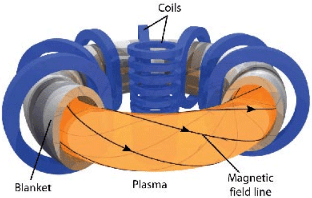
May 23, 2012

Advisors: G. W. Hammett, D. R. Mikkelsen

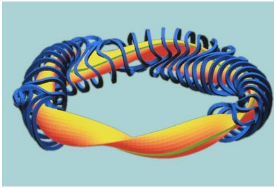
Outline

Outline

Stellarators are an attractive fusion energy design

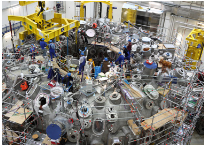


- Tokamaks: field created by external coils and plasma current
- Large inductive internal current leads to poor profile control and disruptions
- External current drive needed for steady state

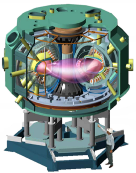


- Stellarators: field created almost entirely by external coils
- Mostly disruption-free, reduced MHD instabilities
- Good profile control
- Inherently steady-state

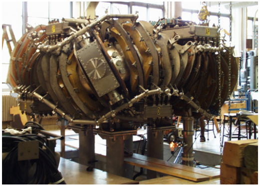
Modern stellarators exploring ideas to reduce neoclassical transport



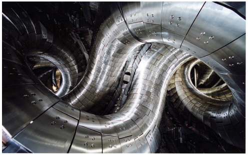
W7-X: quasi-omnigenous



NCSX: quasi-axisymmetric



HSX: quasi-helical



LHD

Ways to improve MFE: control turbulent transport

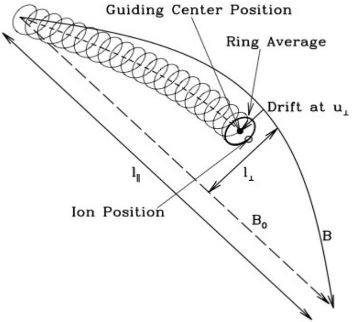
- Plasma turbulence is believed to be due to drift-wave instabilities
- Drift waves: fluctuations in density or temperature, driven by
  - density or temperature gradients
  - magnetic curvature or ∇B
- Extensively studied in tokamaks:
  - Theoretically, computationally, experimentally

Turbulence should be studied in stellarators

- Modern stellarators are optimized (subject to certain constraints) for neoclassical transport
- Neoclassical transport levels have been exceeded
- Turbulence could be the cause of higher transport levels
- Stellarators have large parameter space of configurations
- Opportunity for optimizing for turbulence

Gyrokinetics: mathematical description of strongly-magnetized turbulence

- Gyrokinetic equation: gyro-angle averaged Fokker-Planck equation
- $\frac{\partial h}{\partial t} + v_{||}\mathbf{z}\cdot\frac{\partial h}{\partial \mathbf{R}} + \frac{c}{B_0}[\langle\chi\rangle_{\mathbf{R}}, h] - (\frac{\partial h}{\partial t})_{coll} = \frac{q}{T_0}\frac{\partial\langle\chi\rangle_{\mathbf{R}}}{\partial t}F_0$
- Describes evolution of perturbed distribution function, $\delta f = h - F_0\frac{q<\phi>_R}{T}$+smaller order terms, for
    - small-fluctuations $(\frac{\delta f}{F_0} \sim \frac{\delta B}{B_0} \sim \frac{\rho_i}{l_{||}} \sim \frac{\rho_i}{L} \ll 1)$
    - low-frequencies $(\omega \ll \Omega_i)$
- Reduces $(x, y, z, v_x, v_y, v_z) \rightarrow (x, y, z, v_{||}, v_\perp)$



Gyrokinetics in stellarators is an active, but small, area of research

- FULL (Rewoldt): Linear eigenvalue code
  - Used to compare nine configurations for linear ITG/TEM stability
  - NCSX case with higher β had growth rates lower than standard NCSX case
- GENE (Xanthopoulos, Jenko): Nonlinear initial-value or eigenvalue code
  - Coupled to STELLOPT to investigate optimization of stellarators for turbulent transport
- GKV-X (Sugama, Watanabe, Nunami): nonlinear code, adiabatic electrons
  - Mainly used to simulate LHD plasmas
  - Started comparisons with experiment: ITG unstable in regions of high transport

GS2 was briefly used for stellarators a few years ago

- GS2 (Dorland, Kotschenreuther) is nonlinear initial-value gyrokinetic turbulence code that
  - uses flux-tube geometry
  - uses Eulerian finite difference and spectral methods in position space and spectral methods in velocity space
  - returns linear growth rates, real frequencies, eigenfunctions and nonlinear heat and particle fluxes
  - has been benchmarked with FULL, GENE, and other gyrokinetic codes
  - was used for validation studies with tokamak and ST experimental data
- Original studies by Belli/Dorland: FULL/GS2 NCSX benchmark
  - unresolved questions of geometry normalizations
  - my initial thesis research improved upon this study
- Guttenfelder: HSX linear studies

Scope of My Thesis Research

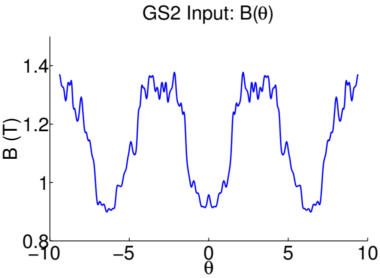
- Because...
  - the knowledge base of turbulence in non-axisymmetric geometries is tiny compared to axisymmetric geometries,
  - the complexity of the problem highlights the need for multiple codes,
  - turbulence codes must be benchmarked and tested,
- I have...
  - developed GS2 and related geometry packages for stellarator turbulence,
    - upgraded magnetically trapped particle treatment
    - written a new computational grid generator
  - linearly benchmarked GS2 with FULL, GENE, GKV-X for stellarator geometry,
  - used GS2 to study microstability dependence on geometry and plasma parameters in NCSX and W7-AS.

Outline

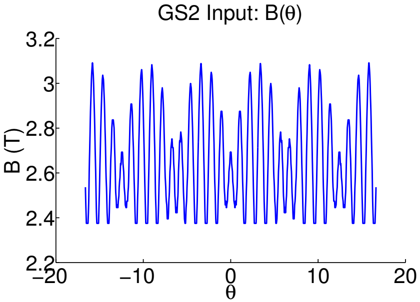
Outline

Initial W7-X studies revealed numerical instability/bug related to complicated |B| structure

- NCSX

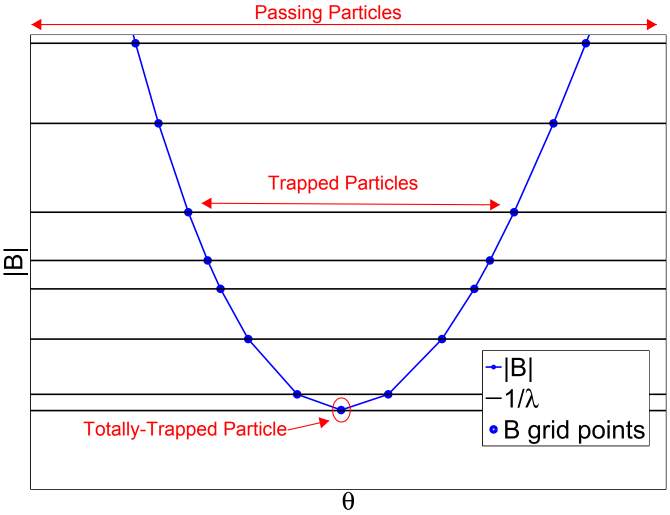


- W7-X

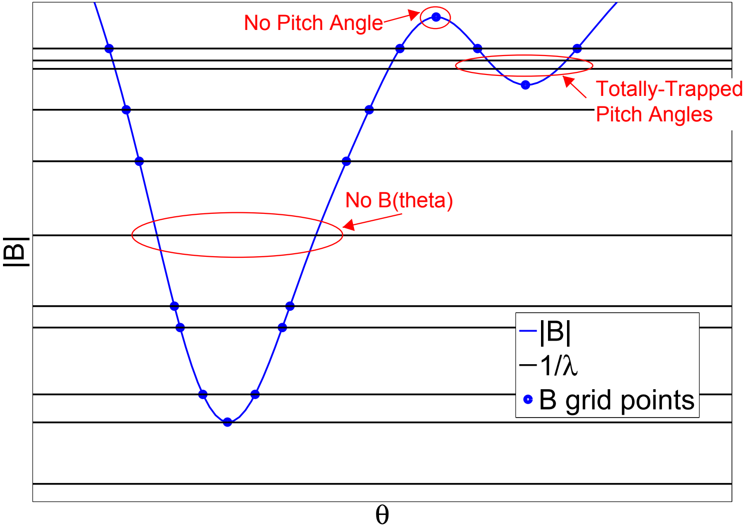


GS2's grid generator should have coupled the θ and λ grids

- $\lambda = \mu/E = 1/B_{tp}$; $v_{||}/v = \sqrt{1 - \lambda B(\theta)}$



However, GS2's grid generator improperly handled complicated geometries



Trapped particle treatment now allows for such flexible grids

- Allows for multiple "totally-trapped pitch angles" in a well
- Treats barely-passing or barely-trapped particles consistently
- Fixed handling of the boundary conditions for trapped particles at turning points
- **Now the pitch angle grid is allowed to be independent of the spatial grid**

- All of these changes are buried in GS2's implicit solver

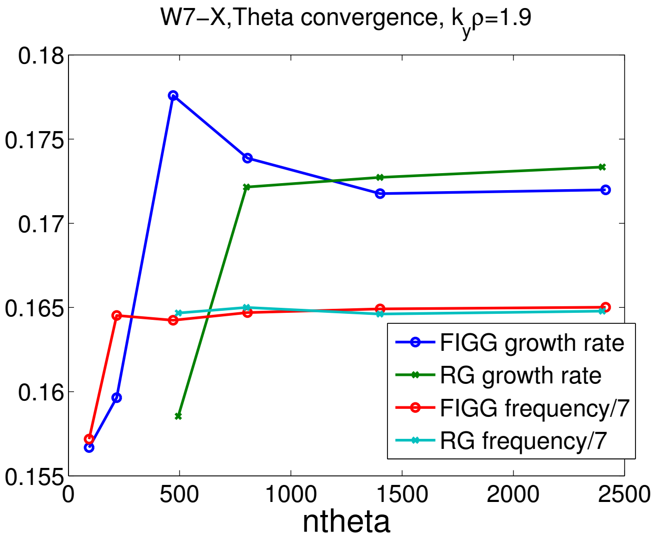
- There were continued issues with Rungridgen (the grid generator):
  - trying to satisfy original conditions, occasionally it would fail to create any grid at all
  - sometimes create grids with unrelated points in the domain

Outline

New Grid Generator for GS2

- **FIGG: Flexible Improved Grid Generator**
- Input:
  - 3D VMEC MHD equilibrium
  - GIST (Terpsichore/VVBAL)
    - high-res single flux-tube ballooning coefficients
    - $|B|$, ∇B, curvature drift components, etc
- Output: lower-res coefficients and calculated pitch-angle grid
  - Initial FIGG θ grid is tied to λ grid: satisfying original condition
  - Depending on user input, θ points are added or subtracted for final FIGG θ grid
- Written in MATLAB
- Reliable, tested in several geometries

GS2 convergence studies using FIGG geometry



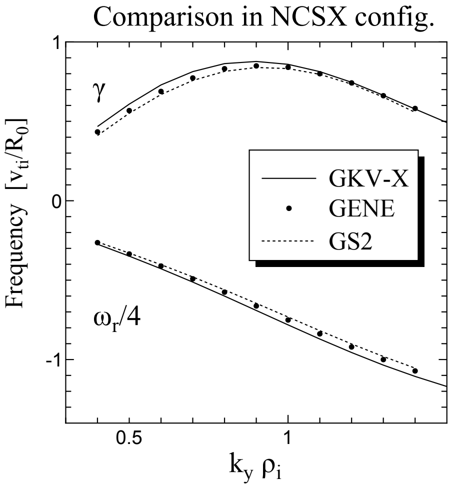
Outline

GS2 benchmark against FULL improved

- In 2000, E. Belli and W. Dorland conducted the first linear GS2 studies with non-axisymmetric geometries (NCSX QAS3-C82)
- My initial thesis research was improving the study
  - troubleshooting geometry chain, reproducing geometry input
    - bug fixes (Guttenfelder, etc)
    - clarifying definitions of parameters
  - re-benchmarking with the modern GS2
    - newer energy grid
    - my trapped particle modifications
- Results published: Baumgaertel, et al, Phys. Plasmas 18, 122301 (2011)

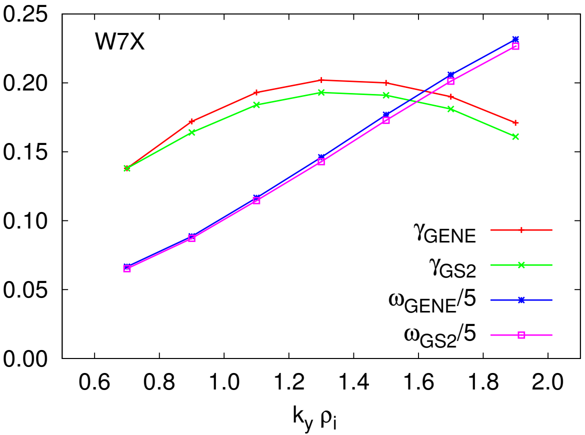
GS2, GENE, and GKV-X agree well in NCSX benchmark

- ITG, adiabatic electrons
- Radial variable: $\sqrt{s}$
- $s = \Phi_T/\Phi_0 = 0.515$
- $q = 2.162$
- $a = 0.345$ m
- $T_i/T_e = 1.0$
- $a/L_T = 3.0$
- $a/L_n = 0.0$



GS2 and GENE agree well for W7-X $k_y\rho_i$ spectrum

- ITG, adiabatic electrons
- Radial variable: $\sqrt{s}$
- $s = \Phi_T/\Phi = 0.2$
- averaged minor radius $a = 0.5$ m
- $T_i/T_e = 1.0$
- $a/L_T = 3.0$
- $a/L_n = 0.0$

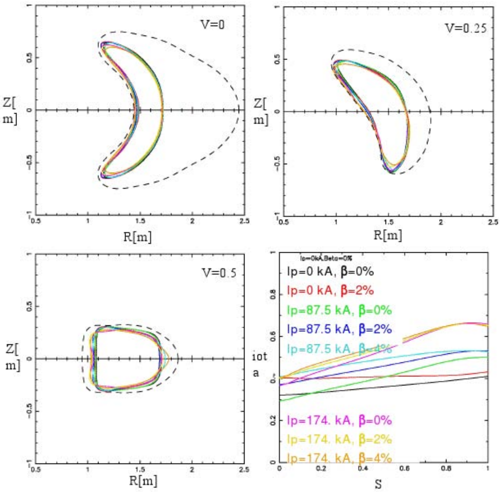


Outline

Outline

NCSX Flexibility Studies allow the study of single effect

- Given a set of coils, currents were varied to discover good configurations
- Sets of configurations in which only one quantity was varied significantly
  - study isolated effects on drift-wave stability
  - discuss optimal running of NCSX

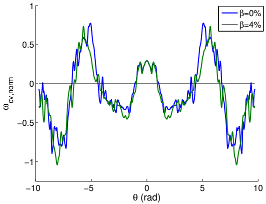


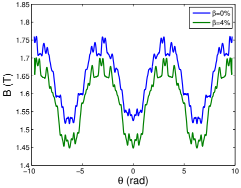
Will β be as stabilizing in stellarators?

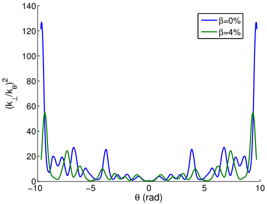
- $\beta =$(plasma pressure)/(magnetic pressure)
  - MFE needs high $\beta \approx nT/(B^2/2\mu_0)$
- In tokamaks, higher β is stabilizing to drift waves to a certain extent
  - it changes the Shafranov shift
- In stellarators, will β be stabilizing? Will it have any effect?
  - the Shafranov shift shouldn't change as much
  - equilibrium set by external coils

NCSX Geometry

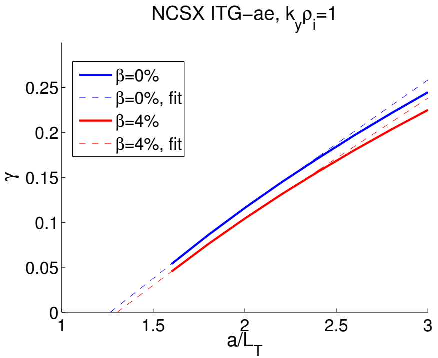
	$\beta = 0\%$	$\beta = 4\%$
$s \approx (r/a)^2$	0.25	0.25
a_N	0.322m	0.322m
α, θ_0	0, 0	0, 0
$\hat{s}$	-0.36	-0.28
$\iota = 1/q$	0.46	0.50





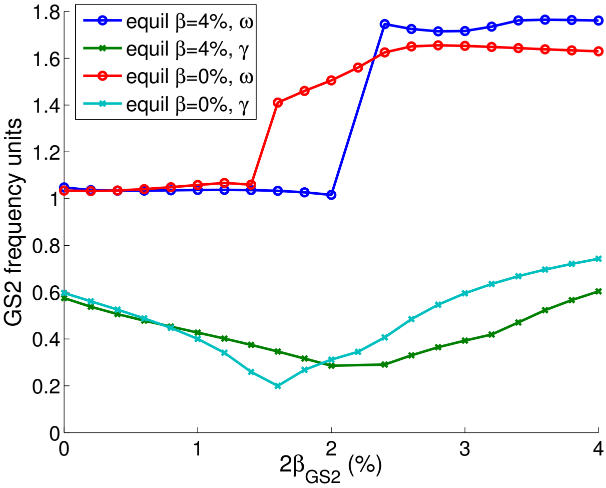


ITG-ae slightly more stable in higher β equilibrium



- Electrostatic fluctuations; $j_{||} = 0$
  - $j_{||} \propto \int d^3 v \delta f v_{||} \propto \beta_{GS2}$
- β parameters: β_{equil} vs. β_{GS2} vs. β_{local}
  - $\beta_{GS2} = 2\mu_0 n_{ref} T_{ref} / B_{ref}^2$ scales $\delta B_{||}, \delta A_{||}$
  - For consistent calculations, GS2's internal β_{GS2} must be set to $\beta_{local}/2$

Electromagnetic results



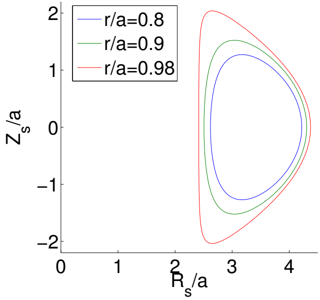
- $\delta B_{||}$ found to be more important at higher β than in typical tokamak simulations
- Additional parameter scans in Dissertation

Outline

Narrow NCSX cross-section might signal greater instability

- Tokamak: hybrid ARIES-AT/JET
  - took a well-studied JET case
  - extrapolated to ARIES-AT's higher κ

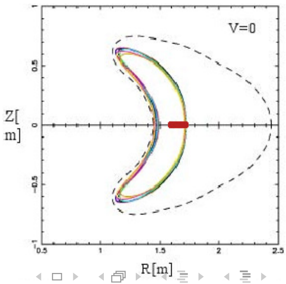
r/a	R/a	$q_{0.8}$	$\hat{s}$	κ	δ
0.8	3.42	2.03	1.62	1.59	0.31



- Compare edge temperature T_{ped} to T_0: more spread out in tokamak

$$(\nabla T)_{local} = \frac{\partial T}{\partial \rho}(\nabla \rho)_{local}$$
$$= -T\frac{a}{L_T}(\nabla \rho)_{local}$$

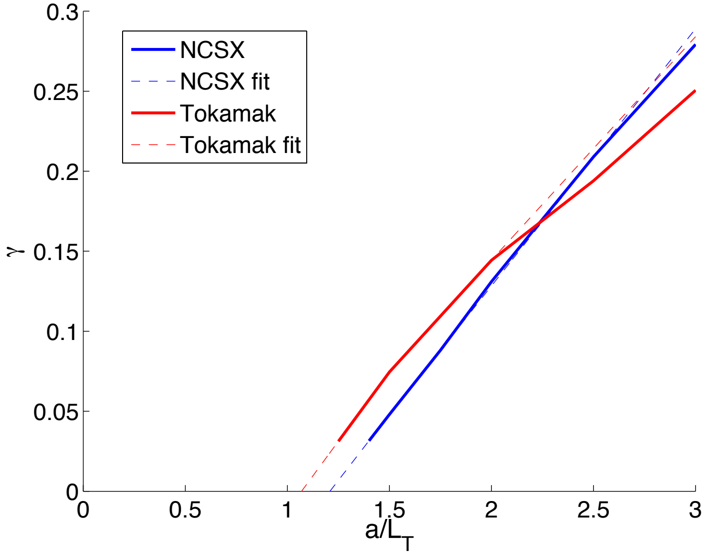
- ∇T is locally much larger in some places in NCSX $\Longrightarrow$ stronger instability drive



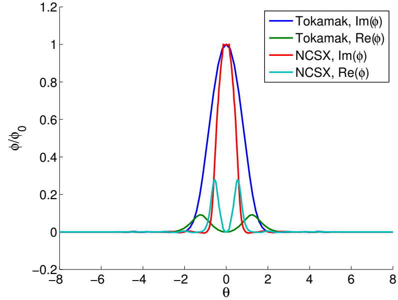
Simple comparison metric for device performance: $a/L_{T,crit}$

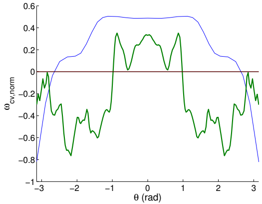
- MFE devices need T_0/T_{ped} to be very high
- Near marginal stability: $T(r) = T_0 e^{-r/L_{T,crit}}$
- $T_{min} = T(a) = T_{ped} = T_0 e^{-a/L_{T,crit}}$
- So $T_0/T_{ped} = e^{a/L_{T,crit}}$
  - In this simple situation: high $a/L_{T,crit} \implies$ high T_0/T_{ped}

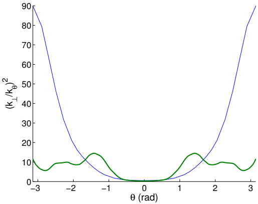
ITG-ke mode more stable in NCSX than in tokamak



Greater stability due to more localized NCSX eigenfunctions





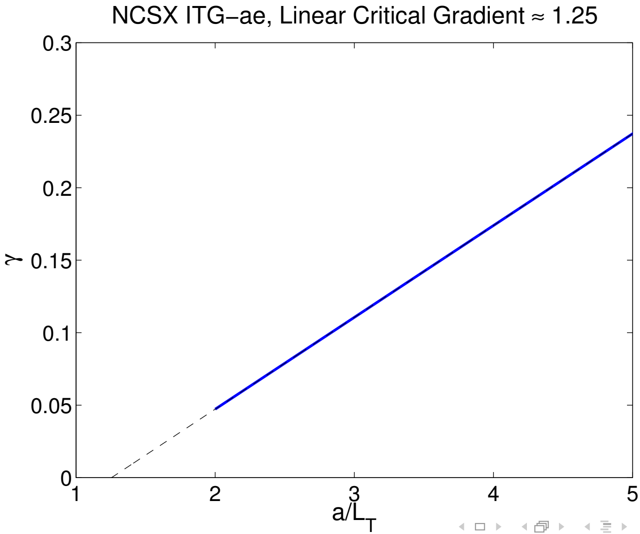


(Blue=tokamak, Green=NCSX)

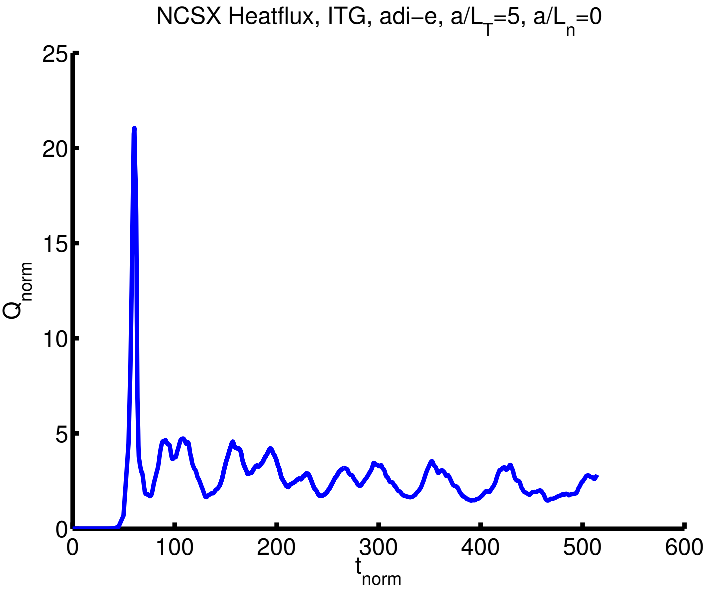
Outline

First GS2 nonlinear non-axisymmetric studies: NCSX

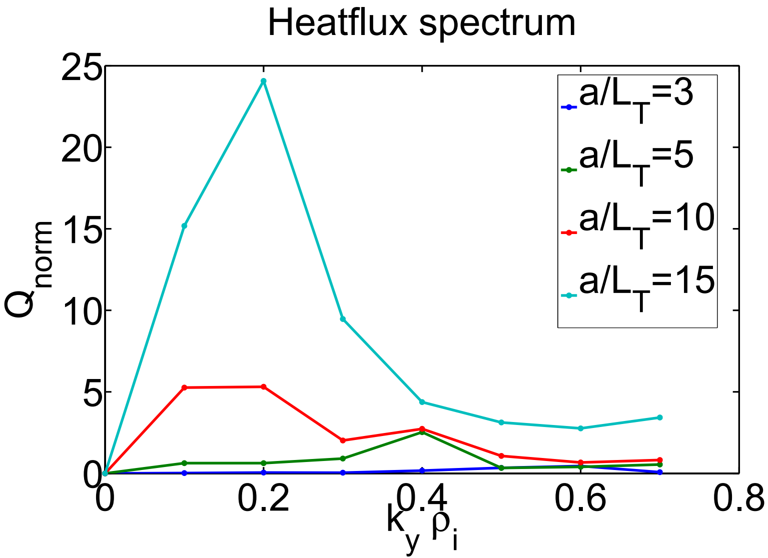
- Low resolution: $ntheta = 292 \in (-5\pi, 5\pi)$, $nlambda = 15$, $negrid = 12$, $n_x = 32$, $n_y = 24$ $(nk_x \approx 21, nk_y \approx 8)$, $L_x \approx 3$, $L_y = 10$



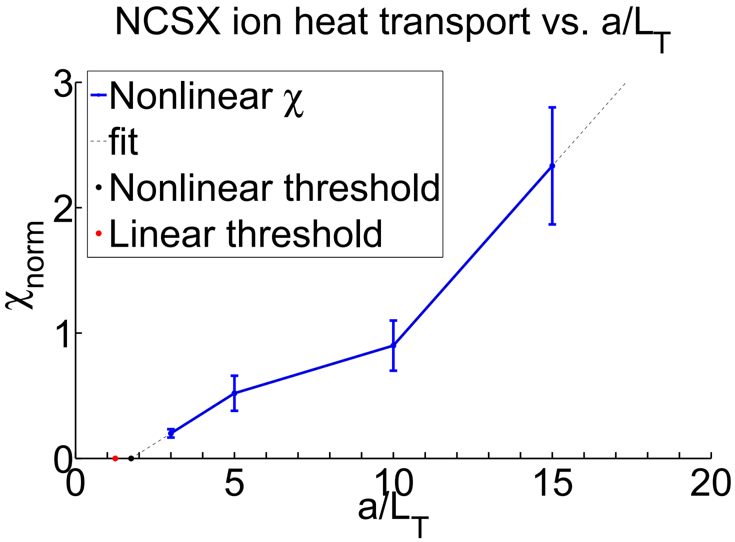
Nonlinear heat flux behaves as expected



Peak heat flux moves to lower $k_y \rho_i$ at higher a/L_T

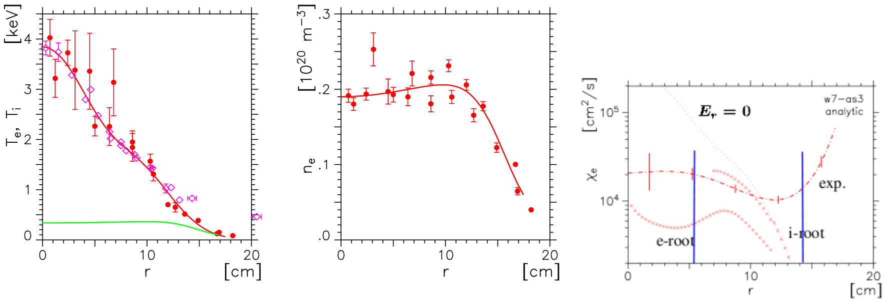


Possible evidence of a Dimits shift



Outline

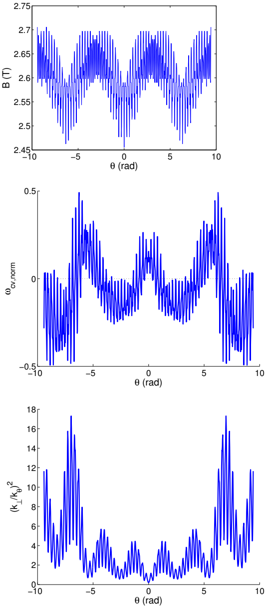
W7-AS: transport neoclassical in core; turbulent near edge?



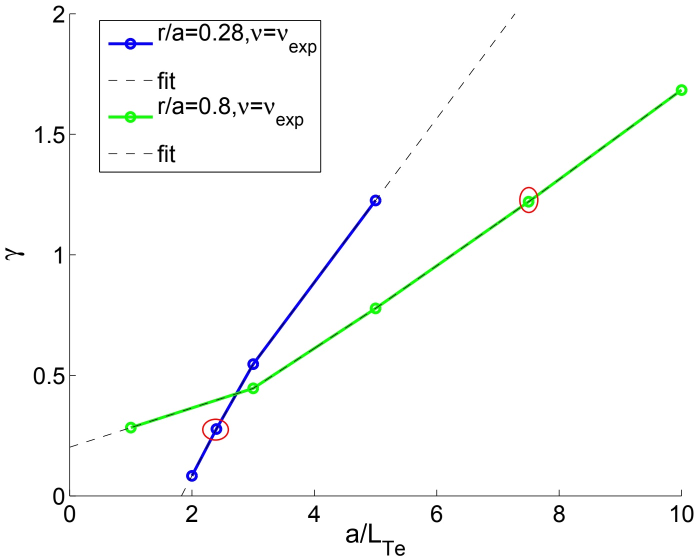
- Neoclassical levels can often account for transport in the core, but not out at the edge
  - The core is hotter: expect more neoclassical transport
  - The edge is cooler with strong gradients: potential for turbulence

W7-AS geometry and experimental parameters

	$r/a = 0.28$	$r/a = 0.8$
T_i	0.35keV	0.24keV
T_e	2.6keV	0.4keV
Z_{eff}	3	3
n_e	$2 \times 10^{19} m^{-3}$	$1.6 \times 10^{19} m^{-3}$
a_N/L_{Ti}	0	3.75
a_N/L_{Te}	2.4	7.5
a_N/L_n	0	2
a_N	0.175m	0.175m
α, θ_0	$\pi/5, 0$	$\pi/5, 0$
$\langle\beta\rangle$	0.14	0.14
$\hat{s}$	0.24	0.12
$\iota = 1/q$	0.23	0.34



Growth rates for $r/a = 0.28$ less than $r/a = 0.8$ at $a/L_{Te,exp}$



- Other scans (a/L_{Ti}, a/L_n, ν) available in the Dissertation.

Mixing-length approximation for heat diffusivity

- Nonlinear simulations are required to accurately calculate heat diffusivities
- Mixing-length theories can approximate diffusivities

$$\chi_{mix} = \frac{\gamma}{k_{\theta,loc}^2}$$

- $k_{\theta,loc} = k_\perp(\theta = 0) = k_y\sqrt{g_1(0)}$
  - k_y is an average poloidal wavenumber
  - $g_1(\theta)$ is one of the metric coefficients
- The experimentally-measured values of total χ_e are about
  - $\chi_{exp,inner} \approx 2 \times 10^4 \ \mathrm{cm}^2/\mathrm{s}$
  - $\chi_{exp,outer} \approx 10^4 \ \mathrm{cm}^2/\mathrm{s}$

Diffusivity using γ_{peak} within a factor of $0.8\chi_{exp}$ at $r/a = 0.8$

- With the experimental values for $T_i, T_e, a/L_{Ti,e,n},$ *and* ν,

- $\gamma_{inner} \approx 2.9 \times 10^5 \ \mathrm{sec}^{-1}$ at $k_y \rho_i = 1.4$
  - ($k_{\theta,loc}\rho_i \approx 0.6$, with $g_1(0) \approx 0.21$)
- $\gamma_{outer} \approx 7.5 \times 10^5 \ \mathrm{sec}^{-1}$ at $k_y \rho_i = 2.2$
  - ($k_{\theta,loc}\rho_i \approx 0.8$, with $g_1(0) \approx 0.14$)

- $\chi_{mix,inner} \approx 0.7 \times 10^4 \ \mathrm{cm}^2/\mathrm{s} \approx 0.35\chi_{exp,inner}$
- $\chi_{mix,outer} \approx 0.8 \times 10^4 \ \mathrm{cm}^2/\mathrm{s} \approx 0.8\chi_{exp,outer}$

Diffusivity using γ at $\frac{1}{2}k_{y,peak}$ within a factor of $2.0\chi_{exp}$

- Turbulence typically peaks at $k_y\rho_i = \frac{1}{2}(k_y\rho_i)_{linear\ \ peak}$

- $\gamma_{inner} \approx 0.9 \times 10^5 \ \mathrm{sec}^{-1}$ at $k_y\rho_i = 0.7$
- $\gamma_{outer} \approx 4.1 \times 10^5 \ \mathrm{sec}^{-1}$ at $k_y\rho_i \approx 1.1$

- $\chi_{mix,inner-half} \approx 0.9 \times 10^4 \ \mathrm{cm}^2/\mathrm{s} \approx 0.4\chi_{exp}$
- $\chi_{mix,outer-half} \approx 2.0 \times 10^4 \ \mathrm{cm}^2/\mathrm{s} \approx 2.0\chi_{exp}$

Turbulence could contribute to heat flux seen in W7-AS

- Turbulence is comparable to neoclassical estimate in the core
- Turbulence may dominate heat transport in the outer regions
  - $\chi_{mix,outer} \approx 2\chi_{exp,outer}$
- Caveats:
  - Rigorous nonlinear gyrokinetic studies needed to quantify further
  - Only one α (flux tube) was checked; it should be the fastest growing
  - The value of ι and global shear in the geometry for the inner radius is uncertain; new, possibly more-accurate equilibrium is now available.

Outline

Summary

- Upgraded GS2 trapped particle treatment to allow for more flexible grids
- Tested new geometry framework for 3D GS2 simulations
- Wrote new grid generator for GS2
- Improved linear benchmark with FULL in NCSX geometry
- Linearly benchmarked GS2, GENE, GKV-X for NCSX, W7-X
- Showed marginal improvement in stability in higher β NCSX cases
- Demonstrated the need to include $\delta B_{||}$ in high β studies
- Compared NCSX and a highly-shaped tokamak for linear stability: NCSX comparable or slightly more stable
- Ran first nonlinear GS2 simulations for non-axisymmetric geometry
- Used experimental parameters and compared stability for two locations in W7-AS
    - the outer location had much higher growth rates and χ_{mix} than the inner location
    - mixing-length estimates for nonlinear heat flux are within a factor of 2-3 of experiment

Future Work: Development

- Improvements to GS2
  - add $v_{||}/v = \sqrt{1 - \lambda B(\theta)} = 0$ interpolation for grid points without $B(\theta) = 1/\lambda$
    - ★ the integral of the distribution function $\int f \frac{dv_{||}}{v}$ integrates from $v_{||,1}$ to $v_{||,end}$
    - ★ if $v_{||,1} \neq 0$, the integral misses the piece of the function from $[0, v_{||,1}]$
    - ★ this difference should be small, but interpolating the result at $v_{||} = 0$ would improve accuracy
- Modifications to FIGG
  - add bounce/orbit-averaging of ω_{drift} terms over cell width to decrease resolution needed

Future Work: Physics

- Benchmarks with GENE, GKV-X in NCSX geometries
  - Zonal Flows (attempts discussed in Dissertation)
  - Nonlinear heat fluxes
- Further NCSX studies
  - nonlinear comparisons of β equilibria
  - studies with additional scans of equilibria from flexibility studies: $\hat{s}$, ι
  - nonlinear heat fluxes and growth rates at other αs should be compared for NCSX and tokamak cases
  - possible Dimits shift investigated
- Expand W7-AS studies
  - Only one α (flux tube) was checked; it should be the fastest growing: could check more.
  - The value of ι and global shear in the geometry for the inner radius is uncertain; new, possibly more-accurate equilibrium is now available.
  - nonlinear heat fluxes should be compared at both radii in W7-AS to experimental values
  - computationally challenging due to higher parallel resolution needed

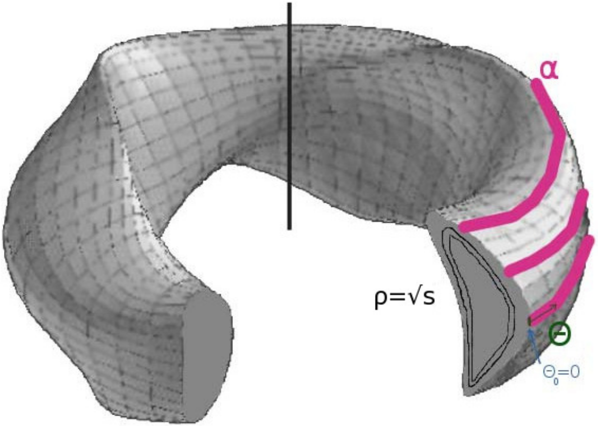
Acknowledgements

- G.W. Hammett, D.R. Mikkelsen
- P. Xanthopoulos, M. Nunami, E.A. Belli, W. Dorland, W. Guttenfelder, M.A. Barnes
- D. Gates, S. Jardin, C. Phillips, J. Krommes
- Barbara Sarfaty, Jennifer Jones
- Martha Redi, James Morgan, Andrew Zwicker, John DeLooper, Patti Wieser
- Luc Peterson, Craig Jacobson, Erik Granstedt, Jongsoo Yoo, Eisung Yoon
- Josh Kallman, Laura Berzak, Steffi Diem, Nate Ferraro, Tim Gray, Dave Smith
- Jong-Kyu Park, Nik Logan
- Kelsey Tresemer, Meghan Peterson, Audrey Sederberg, Rachel Loer, Erica Caden, Sarah Angelini
- Dad, Mom, David, Amy, and Jenny

PPPL
PRINCETON
PLASMA PHYSICS
LABORATORY

Additional Slides

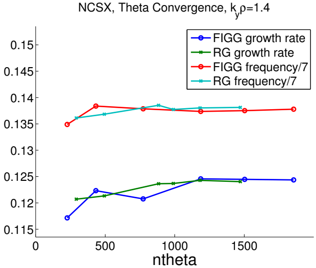
Geometry

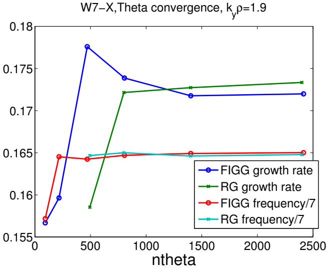


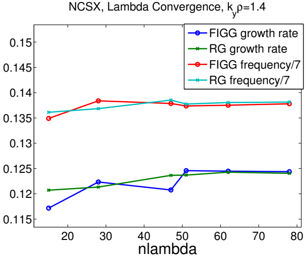
3D geometry builder chain for GS2

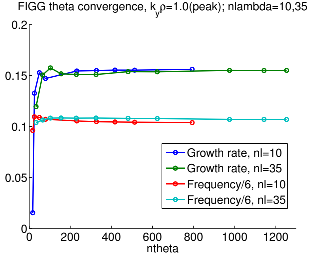
- Starting from a VMEC 3D MHD equilibrium:
- Historically:
  - Terpsichore
    - ★ Boozer coordinate transformation of global equilibrium
    - ★ Radial coordinate: normalized toroidal flux
  - VVBAL
    - ★ Chooses surface and single flux tube
    - ★ Calculates ballooning coefficients
    - ★ Radial coordinate: normalized poloidal flux
  - Rungridgen: GS2's grid generator
- New:
  - GIST (Xanthopoulos)
    - ★ Packages Terpsichore and VVBAL
    - ★ Radial coordinate: either poloidal or toroidal flux
  - FIGG: GS2's new grid generator

GS2 convergence studies using FIGG geometry

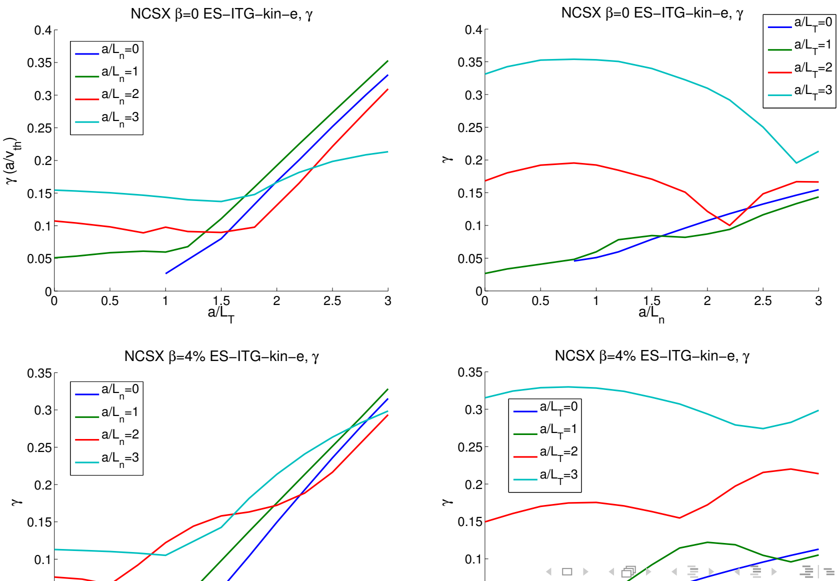






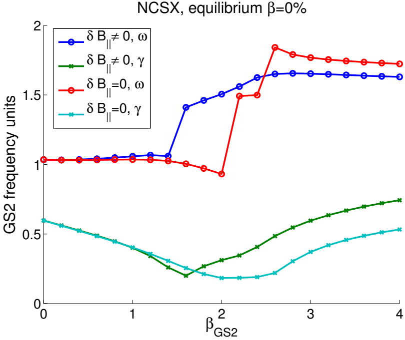


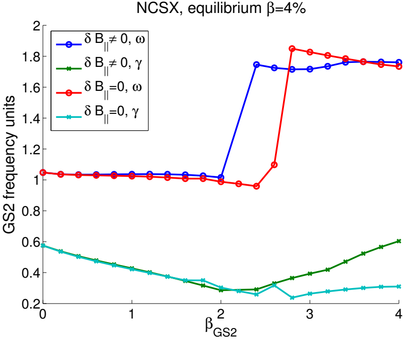
NCSX: Electrostatic ITG-TEM growth rates lower for higher β



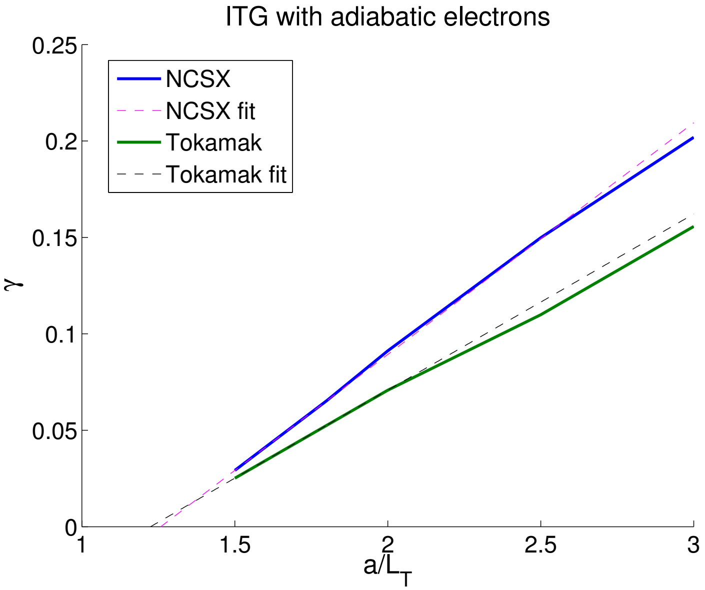
Electromagnetic results: including $\delta B_{||}$

- Including $\delta B_{||}$ is important for high β

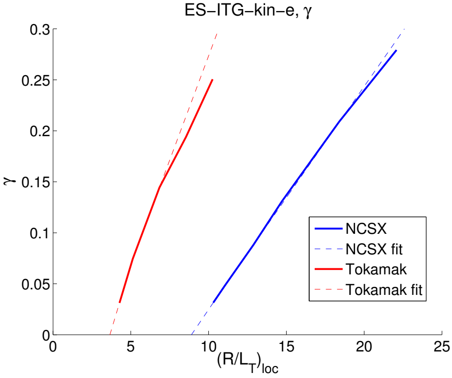




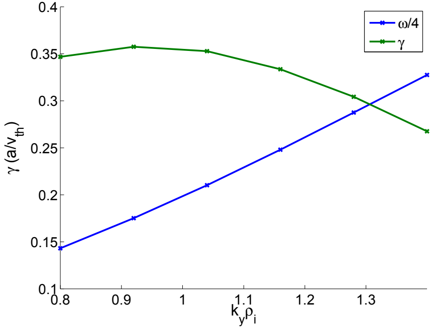
ITG-ae stability similar for NCSX and Tokamak

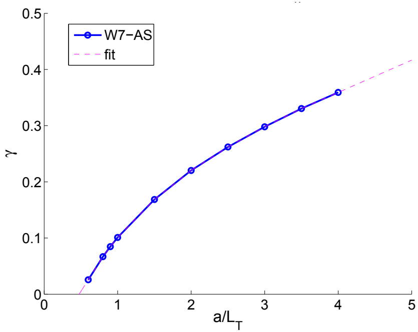


ITG-ke mode more stable in NCSX than in tokamak

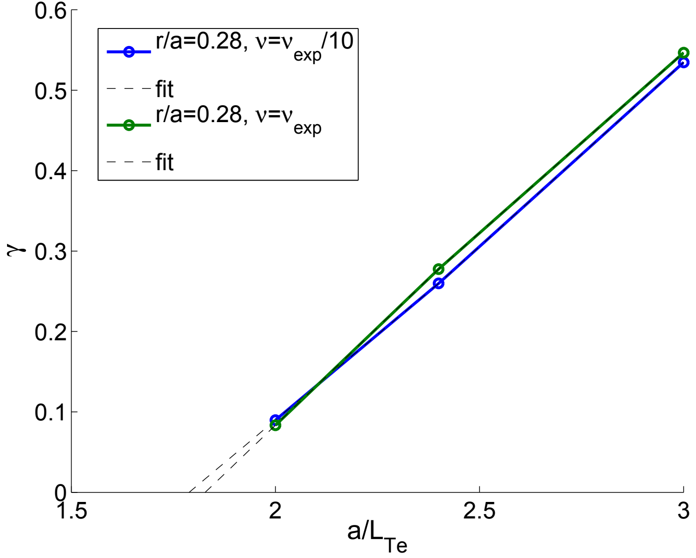


ITG-ae mode threshold for $r/a = 0.8$ is $a/L_{Ti} \approx 0.5$.

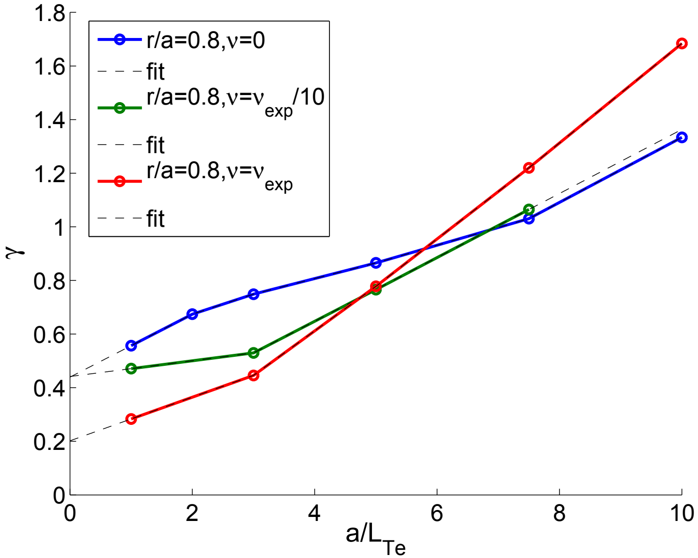




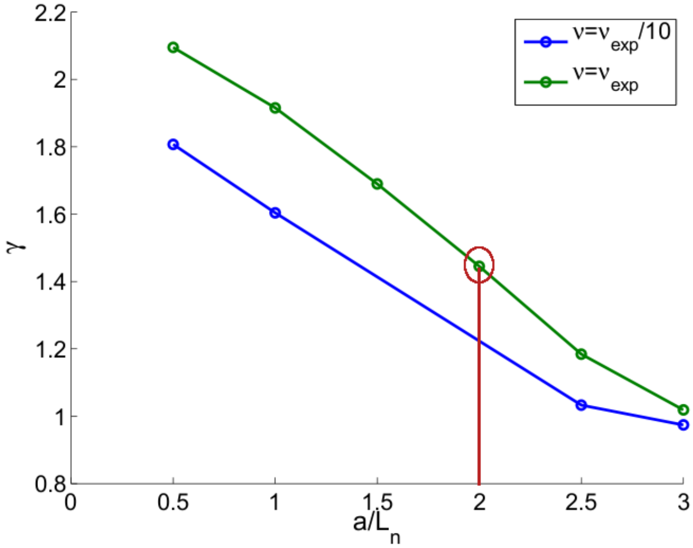
TEM-ETG threshold changes little with collisionality for $r/a = 0.28$



Growth rates increase at $a/L_{Te,exp}$ with collisionality for $r/a = 0.8$



TEM-ETG growth rate dependence on a/L_n for $r/a = 0.8$



TEM-ETG growth rate dependence on ν for $r/a = 0.28, 0.8$