Kinetic Effects on the MRI, And Rotation in Tokamaks

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With thanks to
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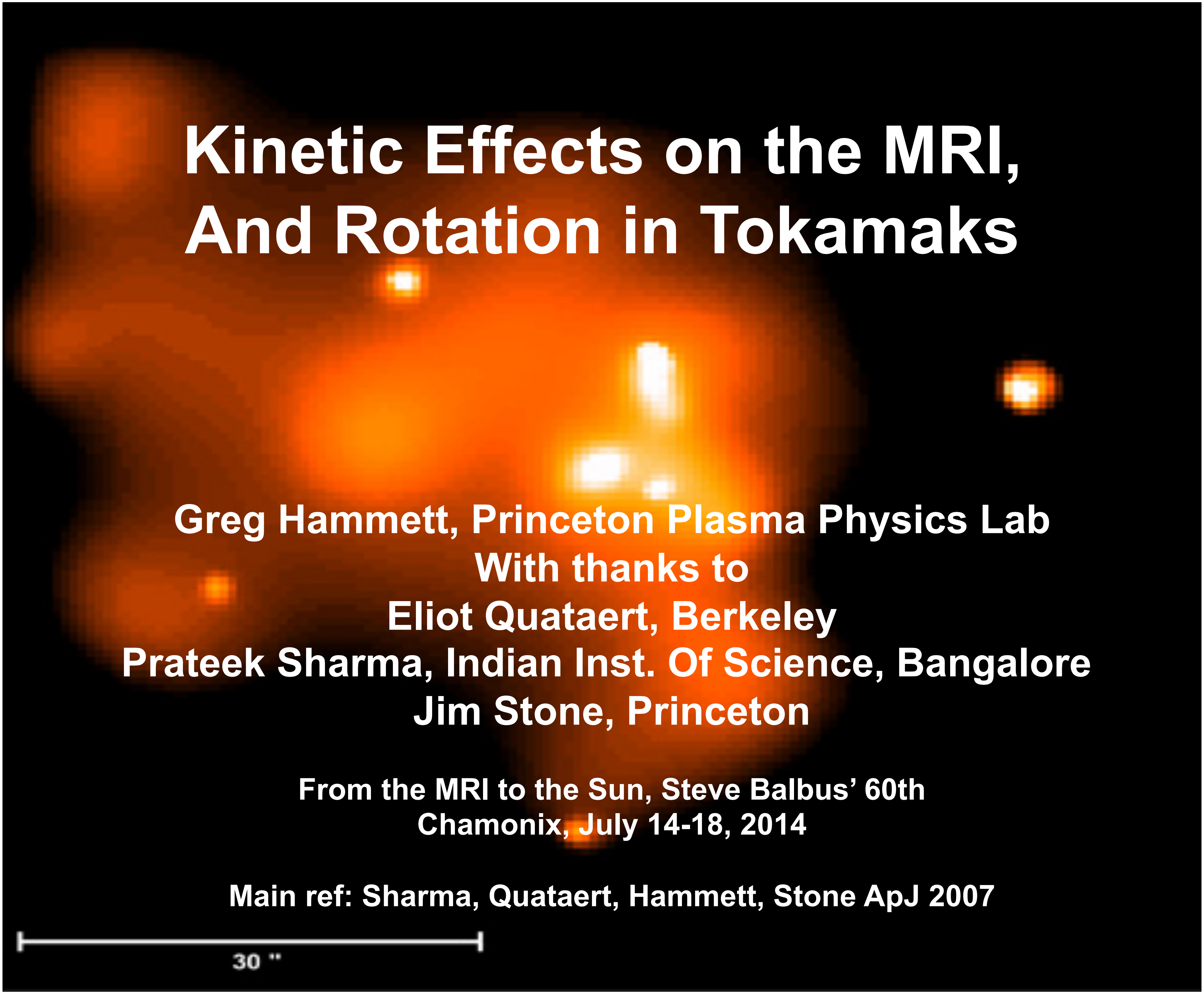
Prateek Sharma, Indian Inst. Of Science, Bangalore
Jim Stone, Princeton

From the MRI to the Sun, Steve Balbus' 60th
Chamonix, July 14-18, 2014

Main ref: Sharma, Quataert, Hammett, Stone ApJ 2007



Kinetic Effects on MHD

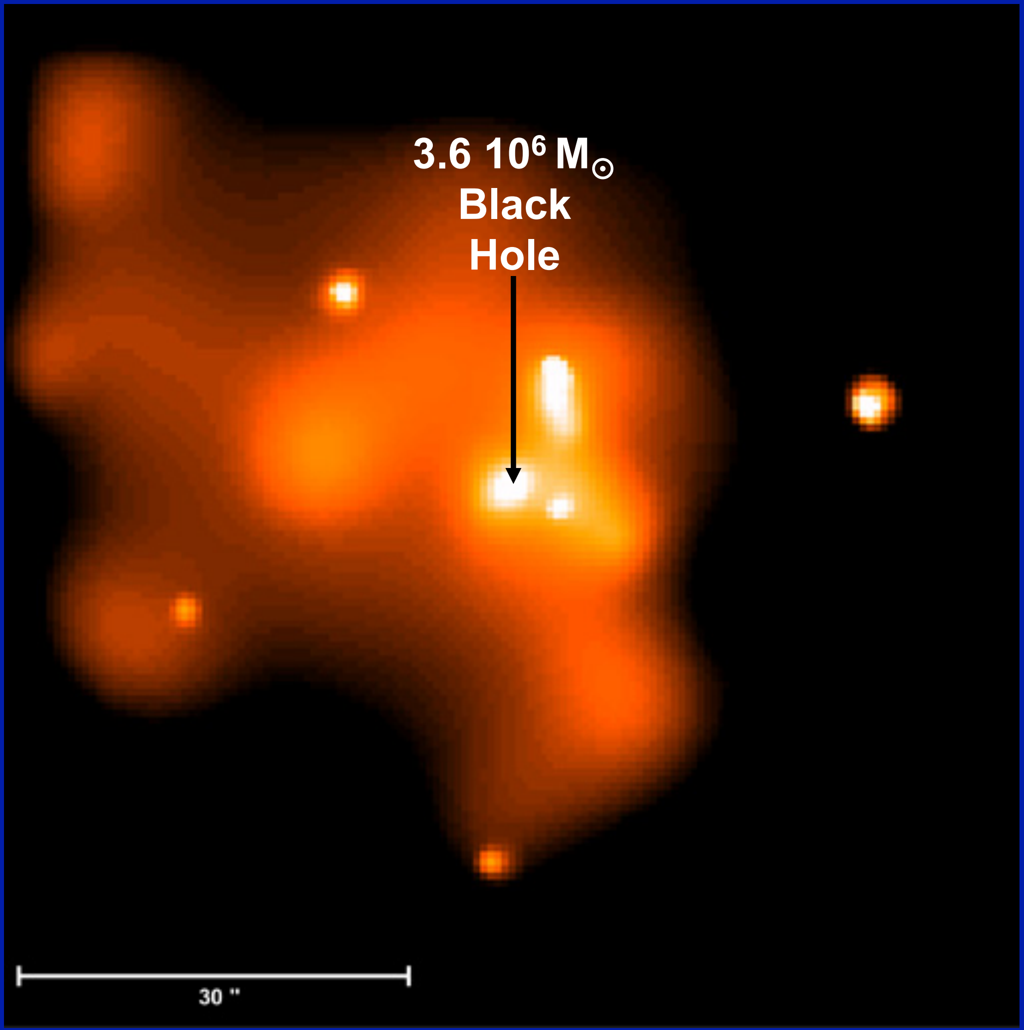
1. MHD works great in high-collisionality, short-mean-free path regimes.

2. Intermediate collisionality ("dillute plasma"): $L \sim 1/k >> \lambda_{mfp} >> \rho_i$, should use MHD w/ Braginskii's anisotropic transport: $\mu_{||} >> \mu_\perp$
$$\mu_{||} \sim \nu \ \lambda^2_{mfp} \ \sim (\Delta x)^2 / \Delta t$$
$$\mu_\perp \sim \nu \ \rho^2_i$$

3. Low collisionality: $\lambda_{mfp} \sim> L \sim 1/k$, then should use Kulsrud's formulation of kinetic-MHD (or Landau-fluid approximations to it).

4. But: in either regime 2 or 3, can get velocity-space anisotropies that (particularly at high beta) can drive mirror/cyclotron/firehose microinstabilities at very fine scales (~gyroradius) and high frequencies (~cyclotron frequency). These microinstabilities can sometimes be treated as giving an enhanced collision frequency. (caveats: Kunz et al., PRL 2014.) They appear to give strong electron heating, so it's hard to keep electrons cold, as assumed in the original ADAF scenario. (Sharma, Quataert, Hammett, Stone ApJ 2007.)

Accretion Region of Milky Way's Black Hole: Low Collisionality



Hot Plasma Gravitationally Captured
By BH → Accretion Disk

Observed Plasma,
near outer bdy
($R \sim R_{Bondi} \sim 10^{17}$ cm $\sim 10^5\ R_{horizon}$)

$T \sim$ few keV $n \sim 100\ \mathrm{cm}^{-3}$

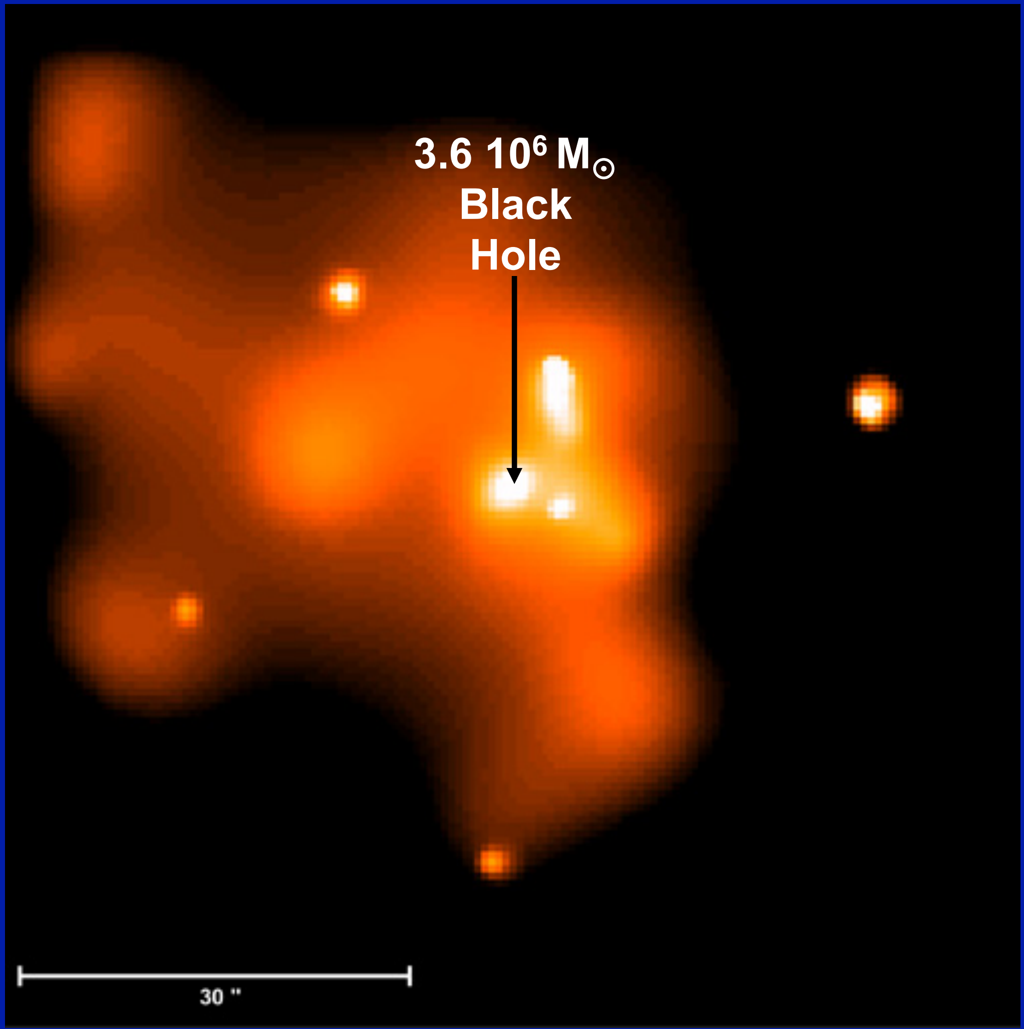
mfp $\sim 10^{16}$ cm $\sim 10^{10}\ \rho_i \sim 0.1\ R$

e-p thermalization time ~ **1000 yrs**
>>
inflow time ~ R/c_s ~ **100 yrs**

electron conduction time ~ **10 yrs**
<<
inflow time ~ R/c_s ~ **100 yrs**

http://chandra.harvard.edu/

Accretion Region of Milky Way's Black Hole: Low Collisionality



Hot Plasma Gravitationally Captured
By BH → Accretion Disk

Estimated Plasma in main accretion region

$R \sim (R_{Bondi}\ R_{horizon})^{1/2}$
$\sim 10^{2.5}\ R_{horizon}$

mfp $\sim 10^3 - 10^6$ R

Very collisionless
(even more so near
the event horizon).

http://chandra.harvard.edu/

Accretion

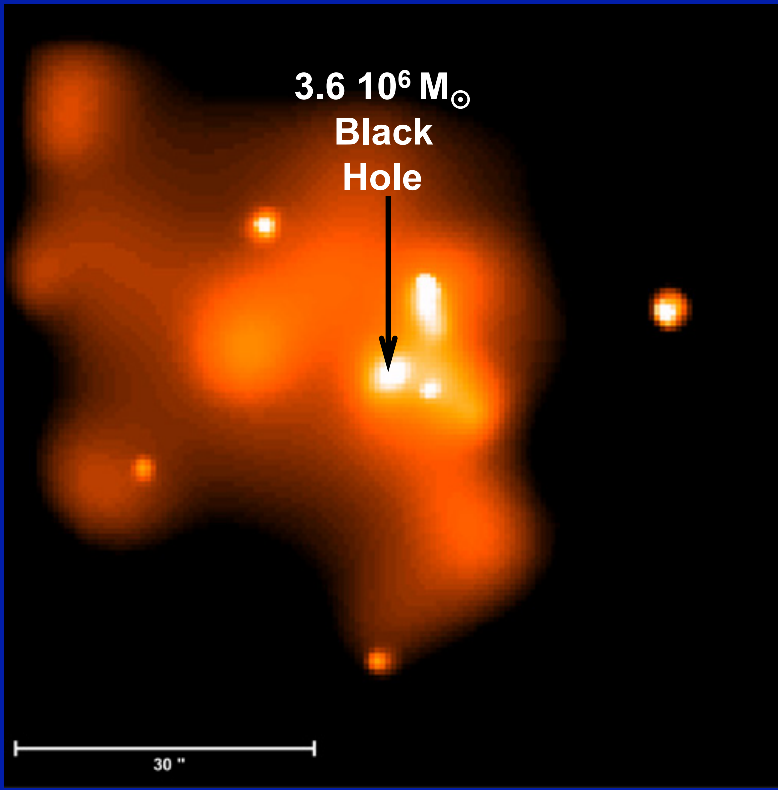
- Inflow of matter onto a central object (generally w/ angular momentum)
- Central to
  - Star & Planet Formation
  - Galaxy Formation
  - Compact Objects: Black Holes, Neutron Stars, & White Dwarfs
- Energy Released:

$$\dot{E} = \frac{GM\dot{M}}{2R} \equiv \epsilon \dot{M} c^2$$

  - sun: $\varepsilon \sim 10^{-6}$
  - BH ($R \sim 2GM/c^2$): $\varepsilon \sim 0.25$ (can be << 1; more later)
  - Fusion in Stars: $\varepsilon \sim 0.007$
  - Accretion onto black holes & neutron stars is responsible for the most energetic sources of radiation in the universe

An Astrophysical Context: Our Galactic Center

Galactic Center (*Chandra*)



Ambient Gas: $n \sim$ 10-100 cm^{-3}
T ~ 1-2 keV

- Ambient gas should be grav. captured by the BH

- Estimates (Bondi) give

$$\dot{M}_{\rm captured} \approx 10^{-5}\, \mathrm{M}_\odot\, \mathrm{yr}^{-1}$$

(rate at which gas is captured at large radii)

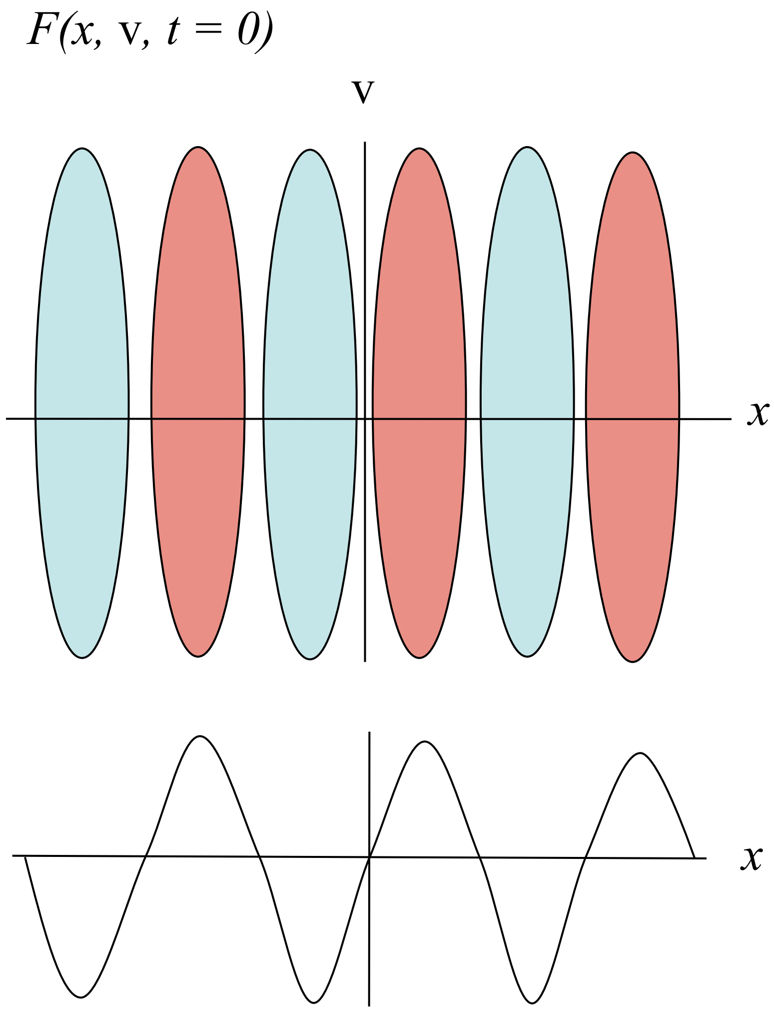
- But then

$$L_{\rm observed} \approx 10^{-5} \times \left(0.1 \dot{M} c^2\right)$$

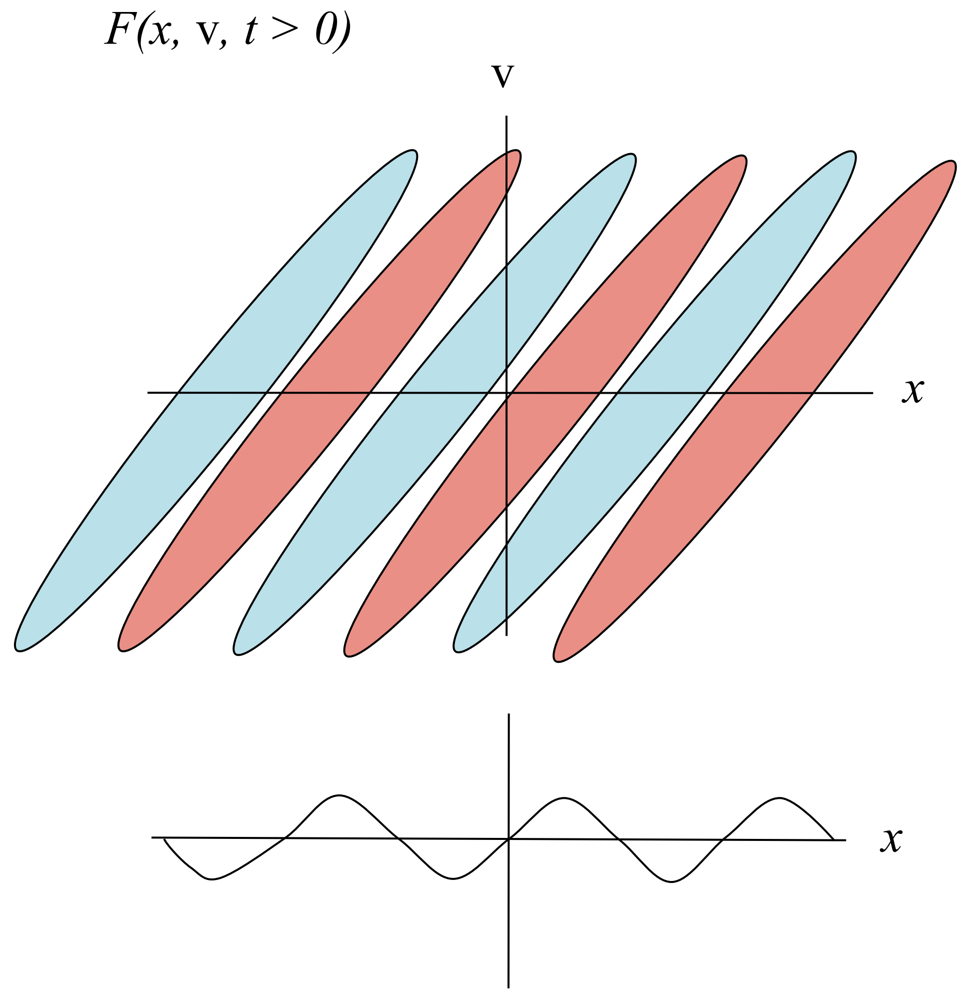
Either radiation efficiency is x10^{-5} smaller than in quasars (hot ion ADAF regime, Ichimaru, Rees, Narayan), or net accretion $\dot{M}$ much smaller than Bondi estimate.

http://chandra.harvard.edu/

Phase-mixing: perturbations decay without collisions



$$n(x, t=0) = \int dv F(x, v, t=0)$$



$$n(x, t) = \int dv F(x, v, t)$$

Landau-Fluid Closures Enable Fluids Eqs. to Approximate Kinetic Effects Like Landau Damping

Landau fluid closure of the general form:

Hammett et al., 92
http://w3.pppl.gov/~hammett/papers/

$$
\begin{aligned}
q &= -n\chi \partial T/\partial z \\
&= -n \frac{v_t^2}{\nu + |k_{||}| v_t} \partial T/\partial z
\end{aligned}
$$

Recover Braginskii
if ν large

Phase-mixing (when $\nu \to 0$)
modelled by effective $\nu \sim |k_{||}|\, v_t$

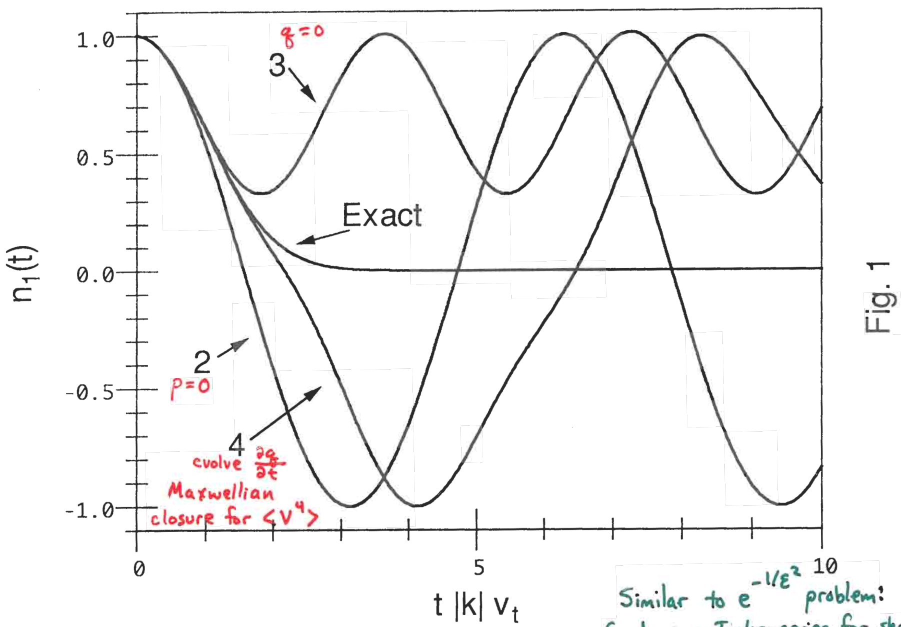
Fast evaluation with FFTs or fast multipole methods (after mapping to field-line following grid). Heat flux is nonlocal integral operator in real space (a Hilbert transform):

$$
q(z) \approx -n_0 v_t \int_0^{\lambda_{\rm mfp}} dz' \frac{T(z+z') - T(z-z')}{z'}
$$

For Prateek Sharma's nonlinear work on the MRI, we just used a constant $k_{||} = k_L$ (in combination with anisotropic pressures) and varied k_L to study sensitivity.

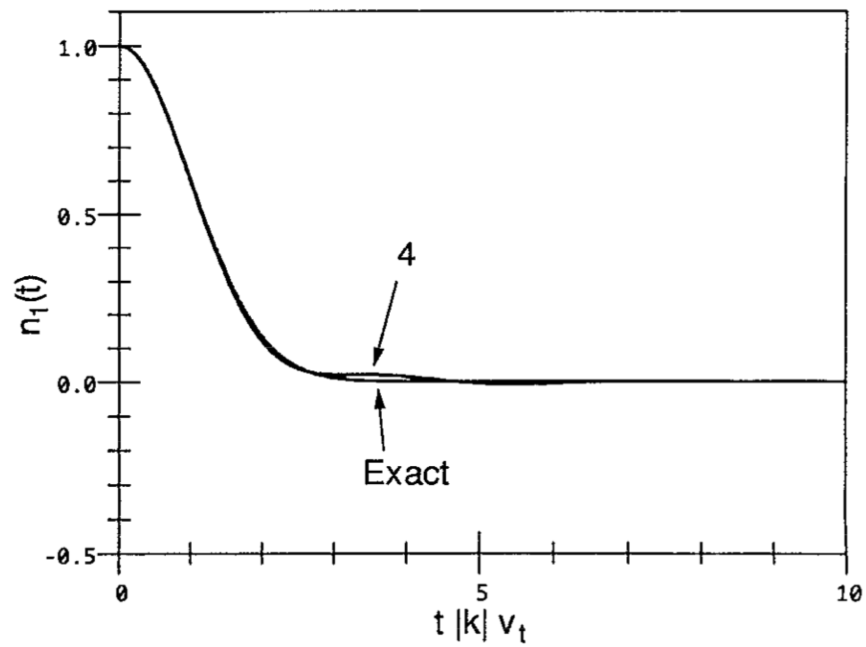
Phase-mixing test of Landau closures

Traditional (quasi-normal) fluid closures fail to reproduce phase-mixing:



Similar to e^{-1/ε^2} problem!
Good as a Taylor-series for short time,
breaks down for long time...

Landau-fluid closures provide an n-pole approximation to the Z-function in the plasma kinetic response. (Here n=4)



Landau-fluid closures are often a good approximation, but are not perfect and may require a large number of moments for some phenomena. Certain subtleties, including if there are general (non-slab) magnetic fields. See Hammett et al., 92, Beer and Hammett, 1998., http://w3.pppl.gov/~hammett/papers/

CGL Kinetic-MHD history

Chew-Goldberger-Low (1956, unpublished Los Alamos report): MHD fluid equations systematically closed by pressure tensor from MHD-ordered drift-kinetic equation ($v_{ExB} \sim v_t$) . (CGL published only the simplified "CGL" $p_{\|}$, $p_{\perp}$ fluid closure approximation.)

Basic ordering: large charge limit "*e* >> *1*", or:

$$\epsilon \sim \frac{\text{frequency}}{\text{gyrofrequency}} \sim \frac{\omega}{\Omega_c} \sim \frac{\text{gyroradius}}{\text{gradientLength}} \sim \frac{\rho}{L} \ll 1$$

CGL Kinetic-MHD published with clear derivation (and some details clarified) in Kulsrud (1962, 1983), based on earlier work also by Kruskal & Oberman, & by Rosenbluth & Rostoker (Steve Cowley led a Princeton grad student journal club in early 80's that covered Kulsrud 62.)

R. M. Kulsrud, in *Proc. of the Int. School of Physics Enrico Fermi, Course XXV, Advanced Plasma Theory*, edited by M. N. Rosenbluth (North Holland, Varenna, Italy, 1962).R. M. Kulsrud, in *Handbook of Plasma Physics*, edited by M. N. Rosenbluth and R. Z. Sagdeev (North Holland, New York, 1983).
see also summary in P. B. Snyder, G. W. Hammett, W. Dorland, Phys. Plasmas **4** (1997), 3974.

Kulsrud's (1983) final equations are summarized on p. 129. His drift kinetic equation, Eq.(37) on p.129 can be simplified a lot by going to ($v_{\|}$, μ) coordinates instead of ($v_{\|}, v_{\perp}$) coordinates, resulting in his Eq. 51. To get this, one has to use Eq. 47 to replace things like the Div($U_{\perp}$) terms in Eq. 37 with dB/dt terms, to get a final version of Eq. 37 that makes use of $d\mu/dt=0$. One other subtlety is that $E_{\|}$ is small but non zero, and appears in his drift-kinetic equation. $E_{\|} / E_{\perp} \sim O(\varepsilon)$ and so it still satisfies the MHD ordering, but it needs to have a non-zero value to insure quasineutrality. His quasineutrality constraint, Eq. 38b on p. 129, leads to an equation that determines $E_{\|}$, as given by his Eq. 49.

CGL Kinetic-MHD

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \mathbf{U}) = 0,$$

$$\rho \left(\frac{\partial \mathbf{U}}{\partial t} + \mathbf{U} \cdot \boldsymbol{\nabla} \mathbf{U} \right) = \frac{(\boldsymbol{\nabla} \times \mathbf{B}) \times \mathbf{B}}{4\pi} - \boldsymbol{\nabla} \cdot \mathbf{P},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \boldsymbol{\nabla} \times (\mathbf{U} \times \mathbf{B}),$$

$$\mathbf{P} = p_{||} \hat{\mathbf{b}}\hat{\mathbf{b}} + p_{\perp} (\mathbf{1} - \hat{\mathbf{b}}\hat{\mathbf{b}}) = p\,\mathbf{1} + (p_{||} - p_{\perp})(\hat{\mathbf{b}}\hat{\mathbf{b}} - (1/3)\mathbf{1})$$

$$p_{\perp} = \sum_s \frac{m_s}{2} \int f_{0_s} v_{\perp}^2 d^3 v,$$

$$p_{\|} = \sum_s m_s \int f_{0_s} (v_{\|} - \mathbf{U} \cdot \hat{\mathbf{b}})^2 d^3 v,$$

quasineutrality constraint leads to:

$$E_{\|} = \sum_s (4\pi e_s / m_s) \hat{\mathbf{b}} \cdot \nabla \cdot \mathbf{P}_s \Big/ \sum_s \omega_{ps}^2$$

$$\frac{\partial f_{0_s}}{\partial t} + (v_{\|} \hat{\mathbf{b}} + \mathbf{v}_E) \cdot \boldsymbol{\nabla} f_{0_s} + \left(-\hat{\mathbf{b}} \cdot \frac{D\mathbf{v}_E}{Dt} - \mu \hat{\mathbf{b}} \cdot \boldsymbol{\nabla} B + \frac{e_s}{m_s} E_{\|} \right) \frac{\partial f_{0_s}}{\partial v_{\|}} = 0$$

Evolution of the Pressure Tensor

$$\rho B \frac{d}{dt}\left(\frac{p_\perp}{\rho B}\right) = -\nabla\cdot(\hat{\mathbf{b}} q_\perp) - q_\perp \nabla\cdot\hat{\mathbf{b}}$$

adiabatic invariance of $\mu \sim mv^2_\perp/B \sim T_\perp/B$

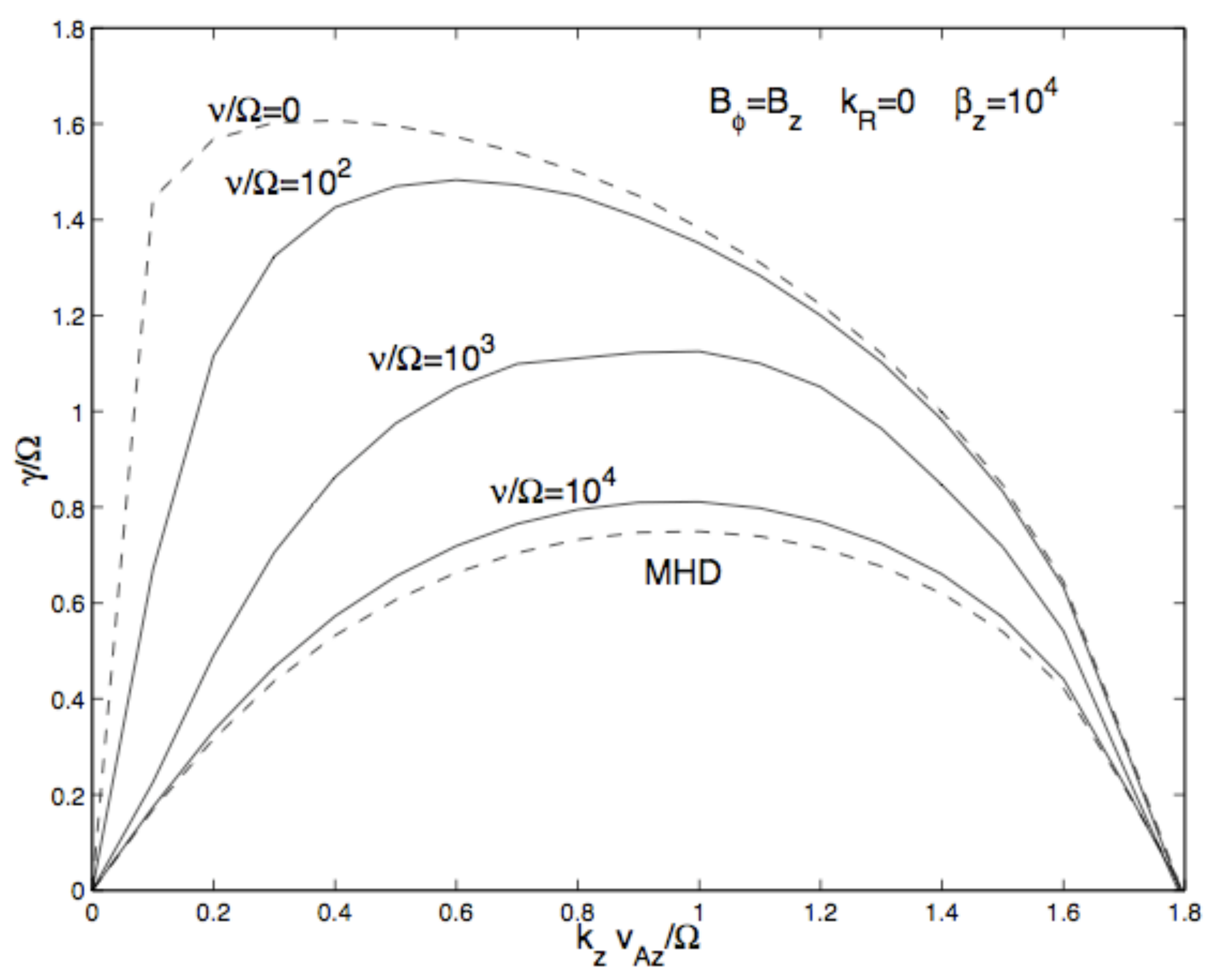
$$\frac{\rho^3}{B^2}\frac{d}{dt}\left(\frac{p_{||}B^2}{\rho^3}\right) = -\nabla\cdot(\hat{\mathbf{b}} q_{||}) + 2q_\perp \nabla\cdot\hat{\mathbf{b}},$$

$q_\perp = q_{||} = 0$ CGL or Double Adiabatic Theory

$$q_{\perp,||} \approx \frac{n\mathrm{v}_{th}}{|k_{||}|}\nabla_{||}T_{\perp,||}$$

Closure Models for
Heat Flux (temp gradients
wiped out on ~ a crossing time)

Transition from kinetic-MHD to Braginskii-MHD to isotropic MHD as collisionality increases



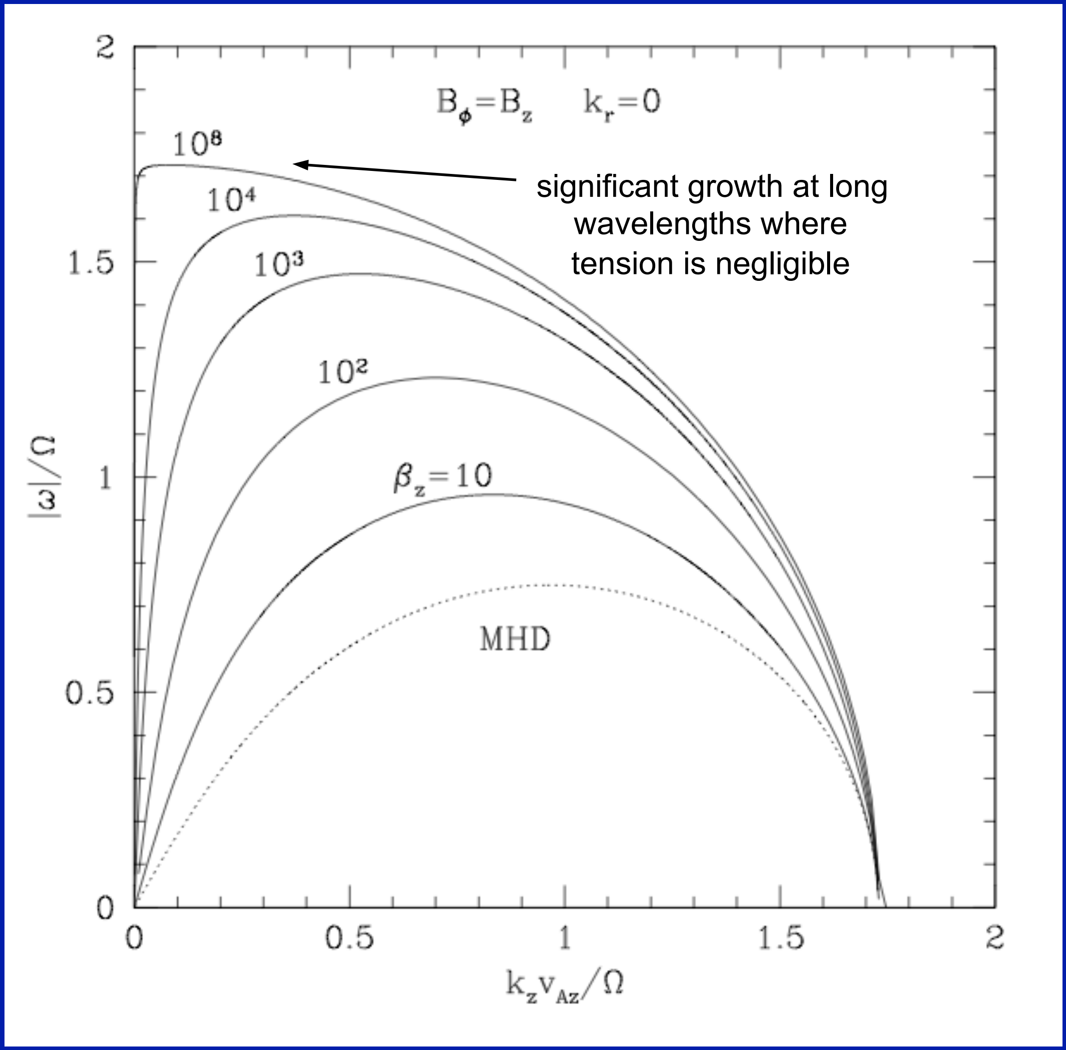
ν = collision frequency
Ω = rotation frequency

Braginskii valid if
$k\,L_{mfp} \sim kv_{ti}/\nu << 1$ &
$\omega / \nu << 1$

With his characteristically elegant insights, Steve Balbus showed how one can reproduce this with simpler Braginskii-MHD equations (and can even throw away jxB force). Magnetoviscous instability, Balbus 2004, Islam & Balbus 2005).

Sharma, Hammett, Quataert ApJ 03

The MRI in a Collisionless Plasma

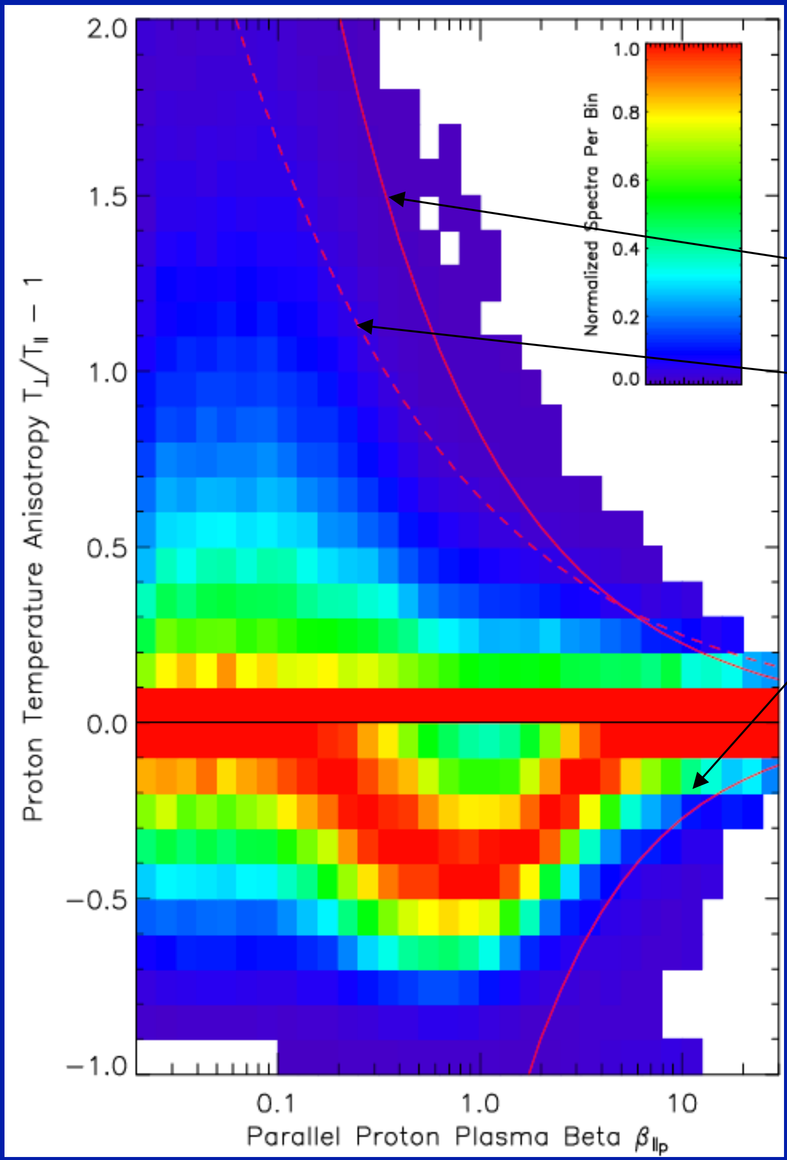


Quataert, Dorland, Hammett 2002; also Sharma et al. 2003; Balbus 2004

angular momentum transport
via anisotropic pressure (viscosity!)
in addition to magnetic stresses

$$F_\varphi \propto \left(\frac{B_z B_\varphi}{B^2}\right)\left(\delta p_{\parallel} - \delta p_{\perp}\right)$$

Limits on Pressure Anisotropy



$$\left| \frac{p_\perp}{p_\parallel} - 1 \right| \le \frac{S}{\beta^\alpha}$$

mirror: S=7, α=1 (to break adiabatic invariance)

ion-cyclotron: S=0.35, α=0.45 for $\gamma/\Omega_i = 10^{-4}$

mirror dominates IC for β˜10-100

firehose:S>2, α=1

Pressure anisotropy reduced by pitch-angle scattering if anisotropy exceeds threshold.

For electrons with $p_\perp > p_\parallel$ electron whistler instability will isotropize: S=0.13, α=0.55 (γ/Ω = 5x10^{-8}) *[using WHAMP code]*

[Kasper et al. 2003, Gary & coworkers]

Examples from Space Physics

- Solar wind at 1 AU statistically at firehose instability threshold *[Kasper et al., Wind]*
- Magnetic Holes in SW & magnetopause, a signature of mirror modes *[Winterhalter et al.,Ulysses]*
- Mirror mode signatures at Heliopause, *[Liu et al.,Voyager1]*
- Above can be interpreted from μ conservation in expanding/ compressing plasmas
- Small-scale instabilities driven by pressure anisotropy mediate shock transition in collisionless plasmas
- SW an excellent laboratory for collisionless plasma physics
- Since much of astrophysical plasma (except in stars) is collisionless, a lot of applications in astrophysics; e.g., X-ray clusters, accretion disks, collisionless shocks.

Pressure Anisotropy

$$\mu \propto T_\perp / B = \text{constant} \;\Rightarrow\; T_\perp > T_\parallel \text{ as B } \uparrow$$

- $T_\perp \neq T_\parallel$ unstable to small-scale (~ gyroradius) modes that *might* act to isotropize the pressure tensor (velocity space anisotropy)
  - e.g., mirror, firehose, ion cyclotron, electron whistler instabilities
  - Some uncertainties, particularly near marginal stability: might saturate w/o breaking μ
- waves w/ Doppler-shifted frequencies ~ Ω_{cyc} violate μ invariance & cause pitch-angle scatter
  - **Increases effective collisions & reduces mean free path of particles in the disk**
  - **Breaking μ invariance critical to making magnetic pumping irreversible and getting net particle heating**
  - impt in other macroscopically collisionless astro plasmas (solar wind, clusters, …)
- Assume "subgrid" scattering model in disk simulations

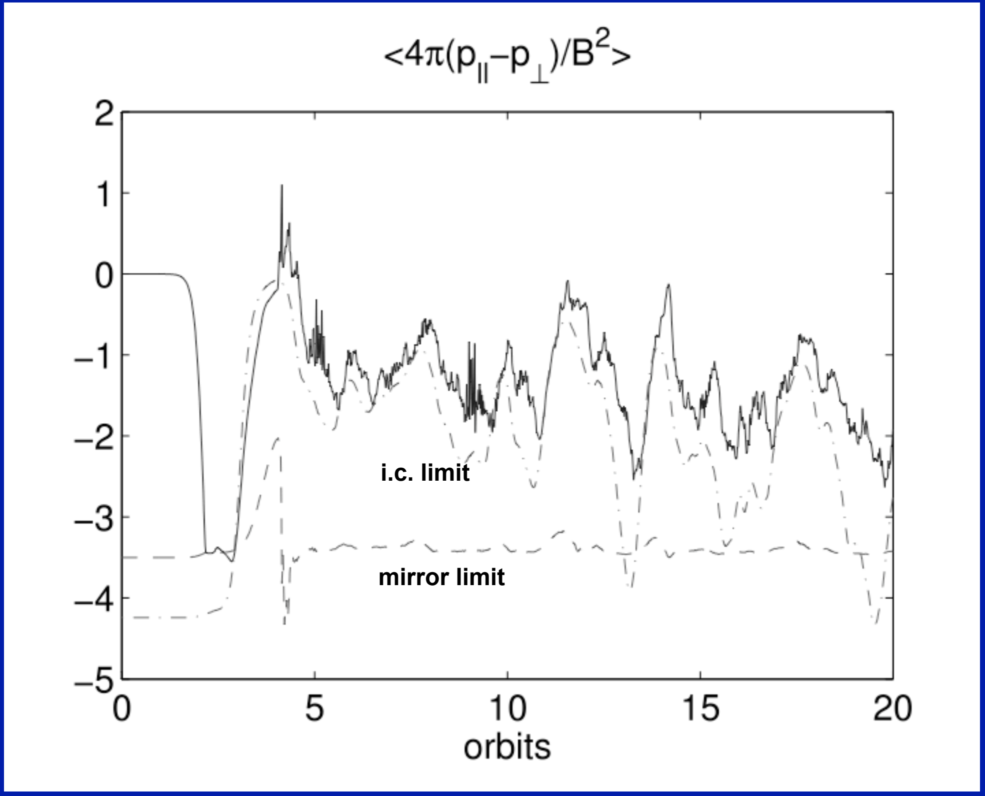
$$\frac{\partial p_\perp}{\partial t} = \ldots - \nu(p_\perp, p_\parallel, \beta)\left[p_\perp - p_\parallel\right]$$

$$\frac{\partial p_\parallel}{\partial t} = \ldots - \nu(p_\perp, p_\parallel, \beta)\left[p_\parallel - p_\perp\right]$$

Mirror/cyclotron/firehose instabilities will also limit Braginskii anisotropic transport coefficients.

Local Simulations of the MRI in a Collisionless Plasma

volume-averaged pressure anisotropy



Sharma et al. 2006

Rate of Angular Momentum Transport Enhanced Relative to MHD (by factor ~ unity)

Net Anisotropic Stress (i.e, viscosity) ~ Maxwell Stress

anisotropic stress is a significant source of plasma heating

Heating by Anisotropic Stress

$$\frac{3}{2}\frac{dp}{dt} = -\mathbf{P} : \nabla\vec{\mathrm{v}} + \ldots$$

Pressure tensor heating

$$= -\frac{p_{\parallel} - p_{\perp}}{B^2}\vec{B}\vec{B} : \nabla\vec{\mathrm{v}}$$

$$= -\frac{p_{\parallel} - p_{\perp}}{2B^2}\frac{dB^2}{dt}$$

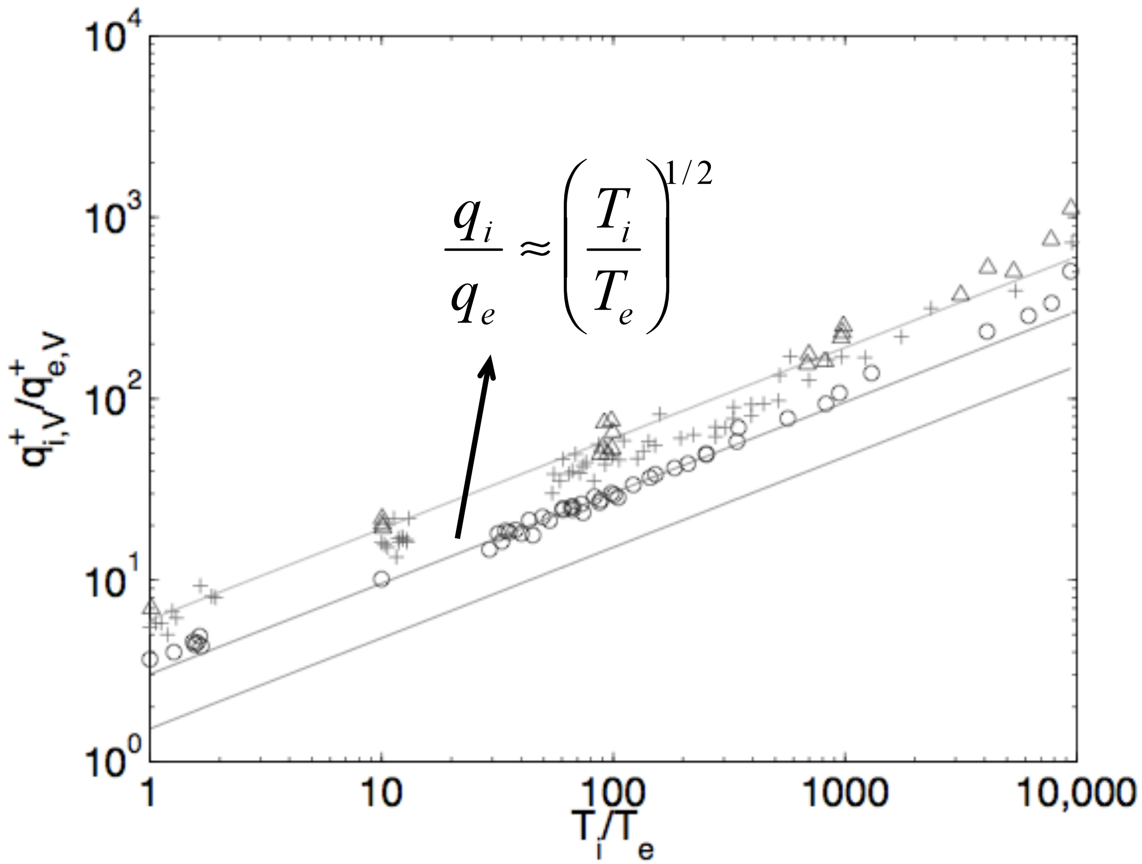
$$\propto \frac{\sqrt{p}}{B}\vec{B}\vec{B} : \nabla\vec{\mathrm{v}}$$

Anisotropy limit set by
Velocity-space instabilities

$$\frac{1}{T_e}\frac{dT_e}{dt} \propto \frac{1}{\sqrt{T_e}}$$

Even if electrons start cold, they will be rapidly heated to a temperature somewhat independent of i.c.s, comparable to ion temperature

Heating by Anisotropic Stress



Sharma et al. 2007

$$q_s \propto P_{r\phi} \frac{d\Omega}{d\ln r} \propto \Delta p_s$$

ion cycl. & e- whistler instability thresholds

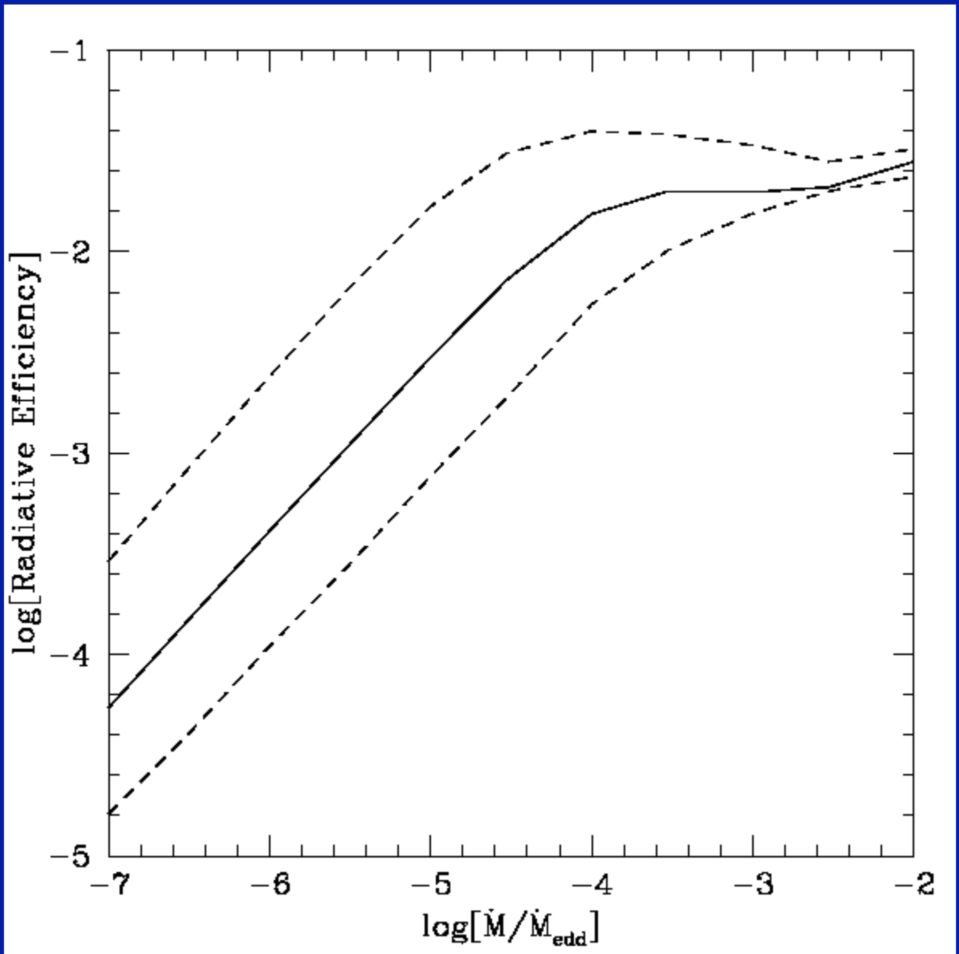
$$\frac{\Delta p_s}{p_s} \sim \beta_s^{-1/2}$$

$$\rightarrow \Delta p_s \propto T_s^{1/2}$$

$$\frac{1}{T_e}\frac{dT_e}{dt} \propto \sqrt{\frac{T_i}{T_e}} \frac{1}{T_i}\frac{dT_i}{dt}$$

Electron heating rate faster than ions in cold electron limit

Final result: predicted radiative efficiency vs. accretion rate



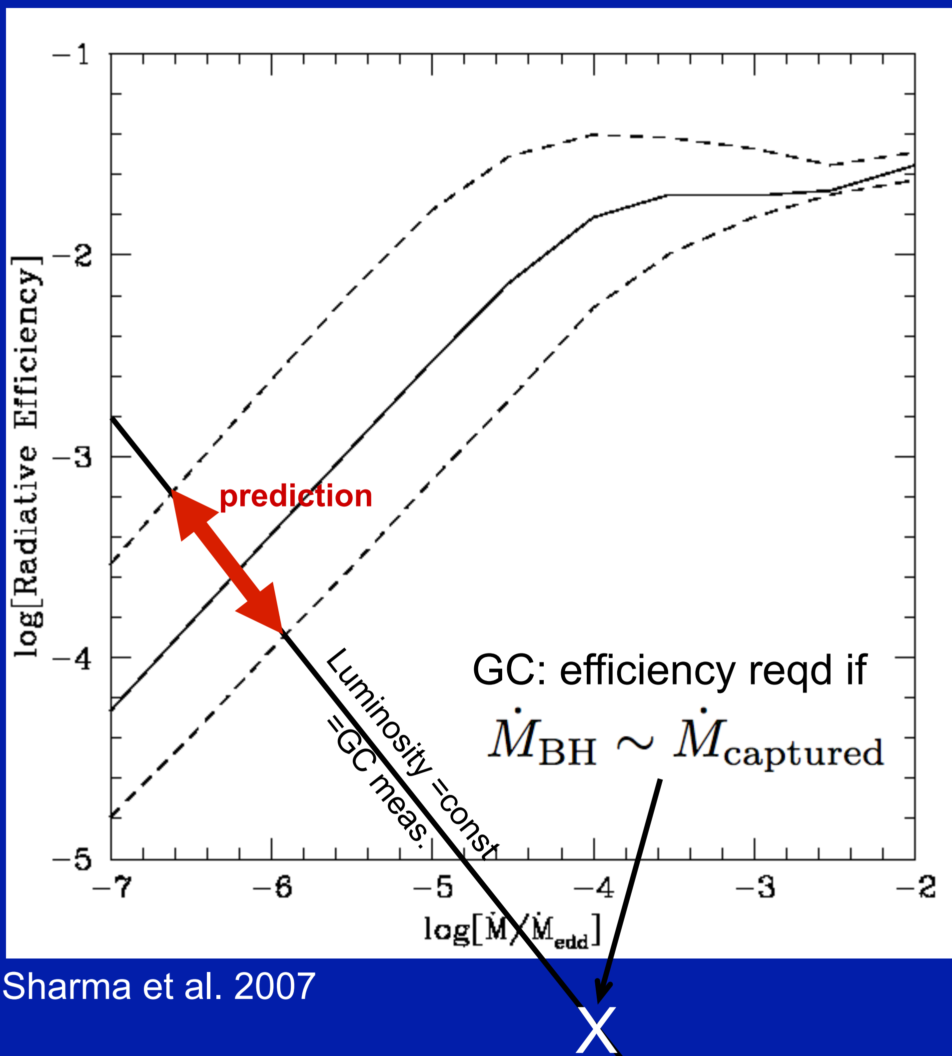
Sharma et al. 2007

'viscous' heating mediated by high freq. instabilities crucial source of electron heating in hot accretion flows

x2 uncertainties from previous page.

(this is a lower bound on electron heating & thus radiative efficiency, might also be resistive heating, and heating from kinetic Alfven tail of cascade)

Astrophysical Implications



Sharma et al. 2007

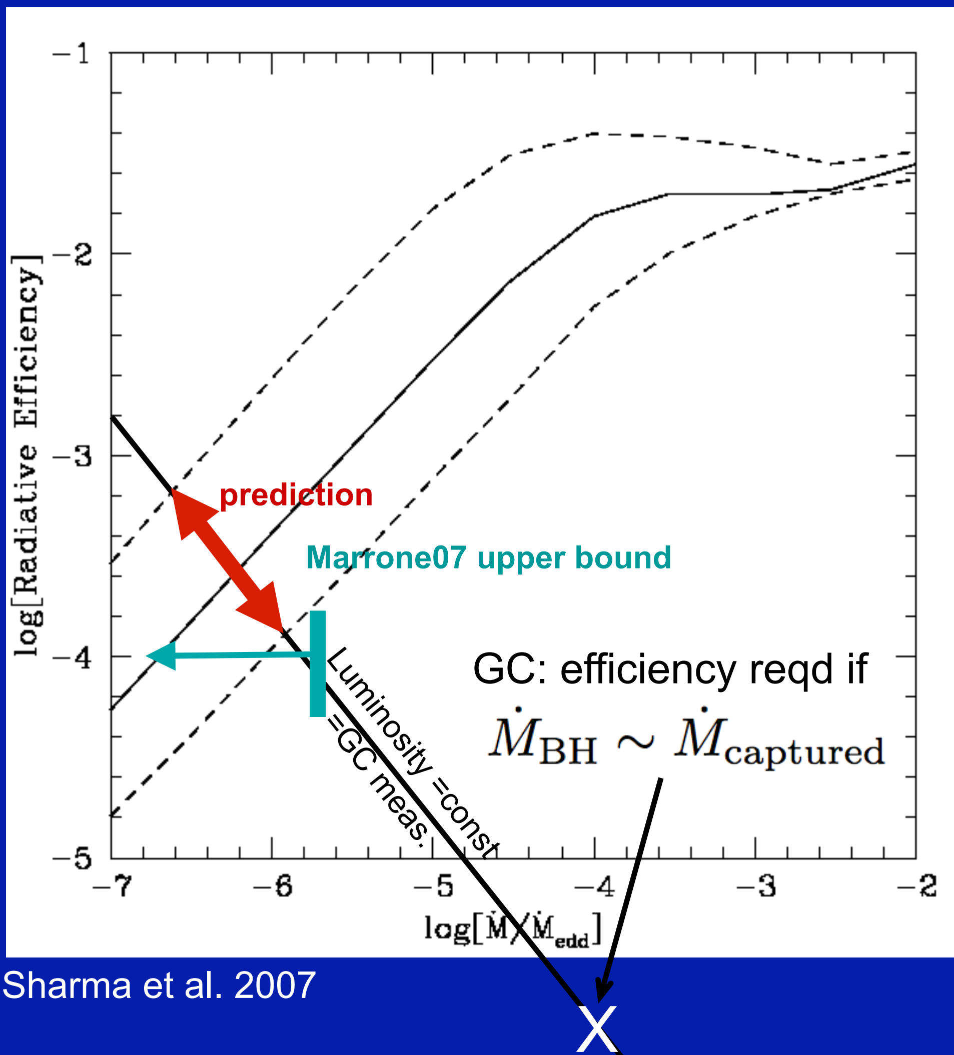
'viscous' heating mediated by high freq. instabilities crucial source of electron heating in hot accretion flows

→ **low accretion rate** required to explain the low luminosity of most accreting BHs

consistent w/ inferences from global MHD sims

$$L_{obs} = \varepsilon \dot{M} c^2$$

Predicted low accretion rate within bounds set by observations



Sharma et al. 2007

'viscous' heating mediated by high freq. instabilities crucial source of electron heating in hot accretion flows

→ **low accretion rate** required to explain the low luminosity of most accreting BHs

consistent w/ inferences from global MHD sims
and with upper bound estimate from Faraday rotation measurements.

Marrone et al. 07 ApJ 654, L57
Faraday rotation measurements.

Microinstabilities Driven by Anisotropies Give Alternative Heating Mechanism

MHD instabilities at long wavelengths

Nonlinear cascade to smaller scales (Kolmogorov / Goldreich-Sridhar)

Collisional Viscous Dissipation (& Landau Damping)

μ conservation leads to anisotropies

Firehose/Mirror/… instabilites driven at gyro-scales, scatters particles

Summary

- The MRI is a rich problem to study in its many forms. The MRI is still robustly unstable in the kinetic regime (long mean-free-path). Long-wavelength modes grow very fast.

- Velocity-space microinstabilities (firehose, mirror, cyclotron, and electron whistler versions) limit the amount of pressure anisotropy ($|p_{||} - p_{\perp}| \sim B^2$) . This is crucial for sustaining MRI turbulence, enhances the effective collision frequency (pitch-angle scattering rate), reduces parallel transport coefficients, and provides a mechanism for strong electron heating.

- This strong electron heating makes a cold-ion ADAF scenario unlikely for explaining the low luminosity of some accretion flows, such as on the massive black hole in the galactic center.

Improving Confinement Can Significantly ↓ Size & Construction Cost of Fusion Reactor

Well known that improving confinement & β can lower Cost of Electricity / kWh, at fixed power output.

Even stronger effect if consider smaller power: better confinement allows significantly smaller size/cost at same fusion gain Q ($nT\tau_E$).

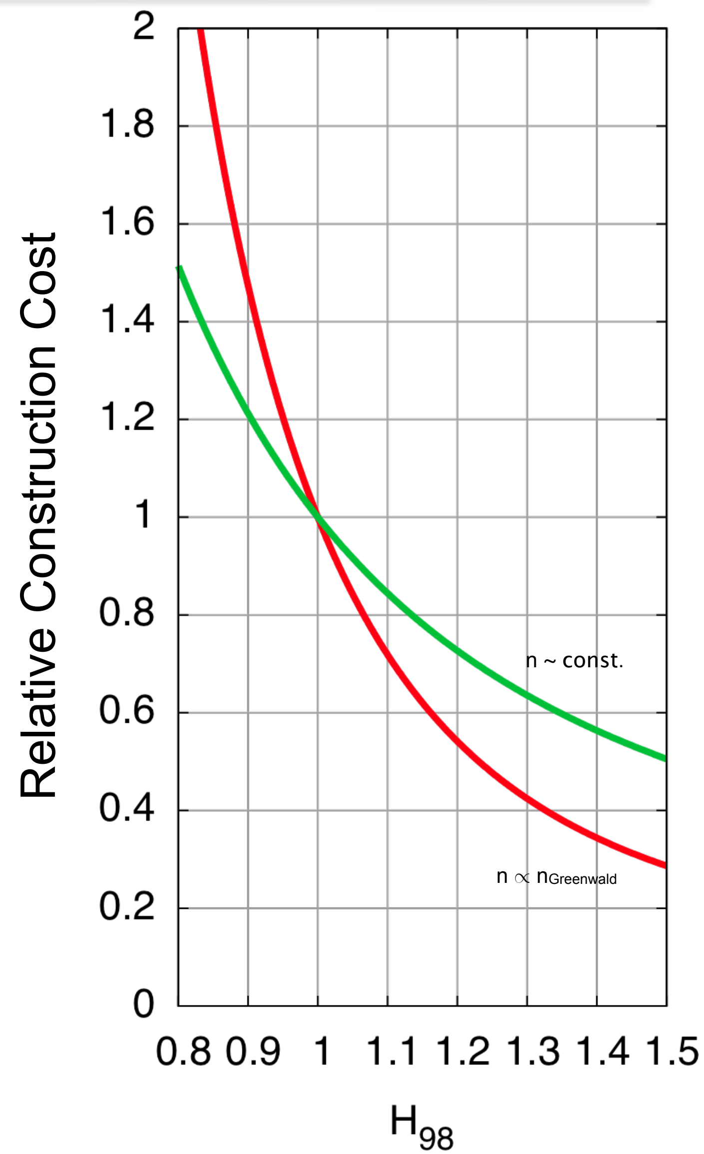
Standard H-mode empirical scaling:

$$\tau_E \sim H\, I_p^{0.93}\, P^{-0.69}\, B^{0.15}\, R^{1.97} \ldots$$

(and assuming fixed $nT\tau_{E,}$ q_{95}, β_N, $n/n_{Greenwald}$):

$$R \sim 1 / (H^{2.4}\, B^{1.7})$$

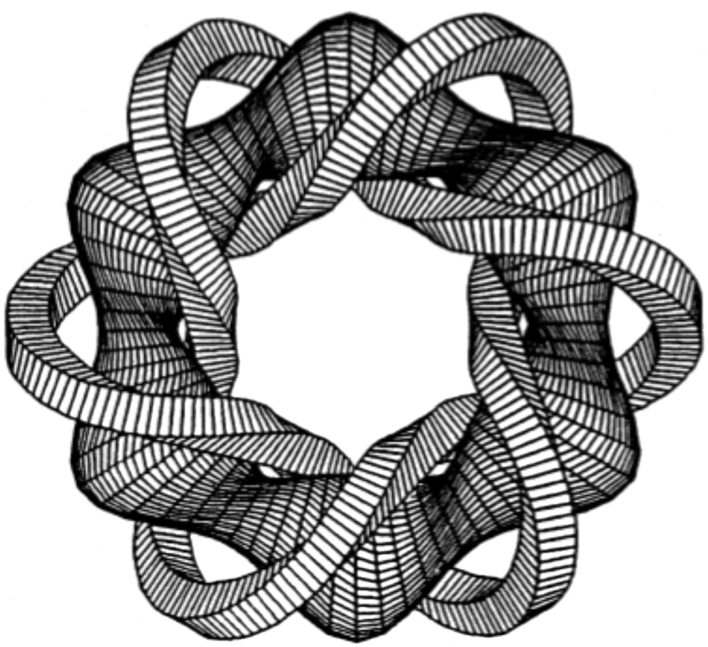
ITER std $H=1$, steady-state $H{\sim}1.5$
ARIES-AT $H{\sim}1.5$
MIT ARC $H_{89}/2 \sim 1.4$



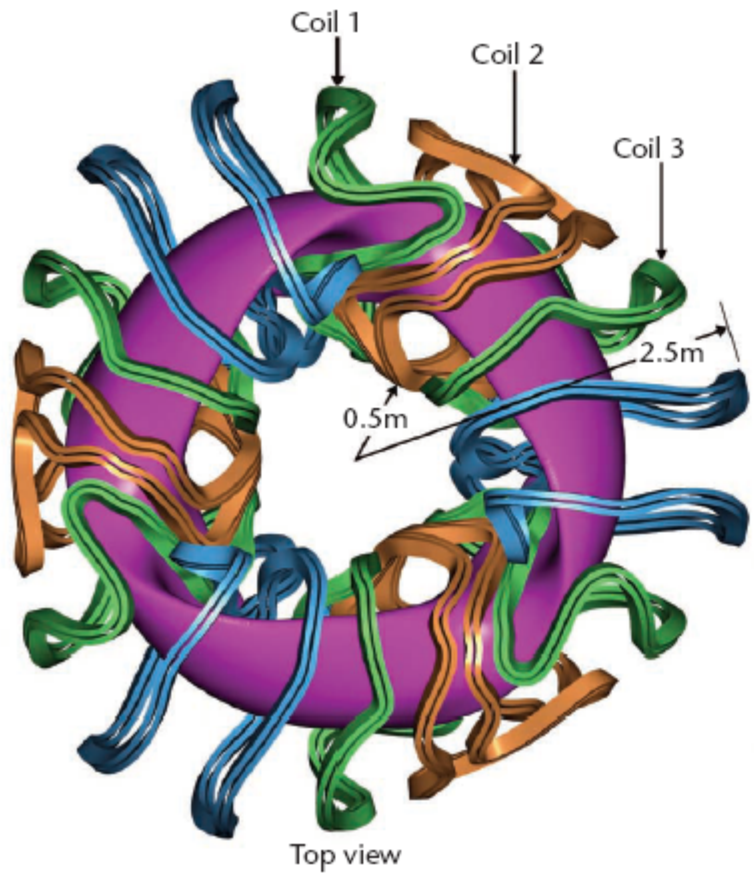
(Plots assumes $a/R=0.25$, cost $\propto R^2$ roughly. Plot accounts for constraint on B @ magnet with 1.16 m blanket/shield. Several caveats: empirical scaling is uncertain. Need more detailed engineering design code to account for various constrains, including current drive requirements.)

Improved Stellarators Being Studied

- Originally invented by Spitzer ('51). Mostly abandoned for tokamaks in '69. But computer optimized designs now much better than slide rules.
- Quasi-symmetry discovered in late 90's: don't need vector ***B*** exactly symmetric toroidally, |***B***| symmetric in field-aligned coordinates sufficient to be as good as tokamak.
- Magnetic field twist & shear provided by external coils, not plasma currents, inherently steady-state. Stellarator expts. don't have hard beta limit & don't disrupt.
- Robotic advances could bring down manufacturing cost.

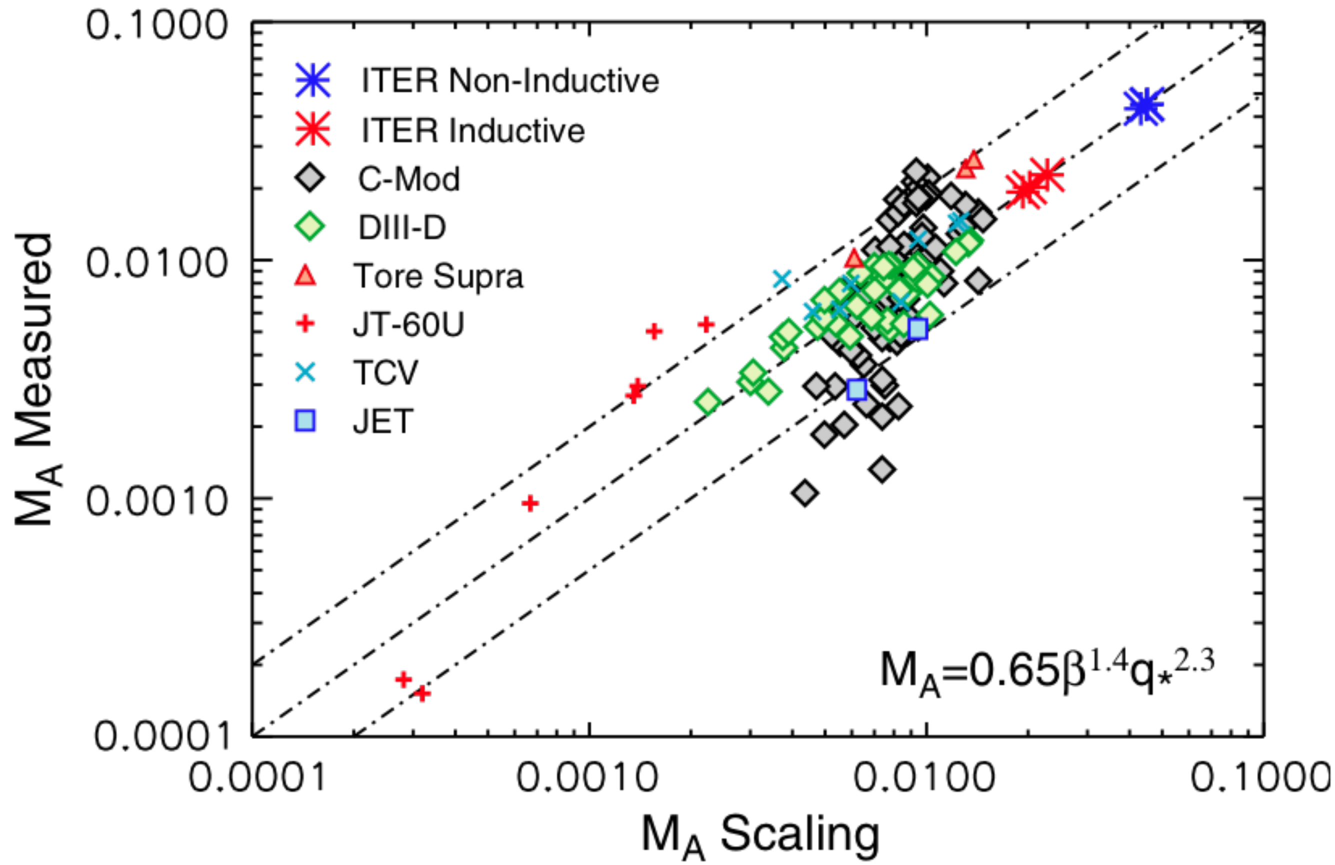


JF Lyon et al., 1997 http://aries.ucsd.edu/LIB/REPORT/SPPS/FINAL/chap2.pdf



Princeton Quasar (Quasi-axisymmetric Stellarator)

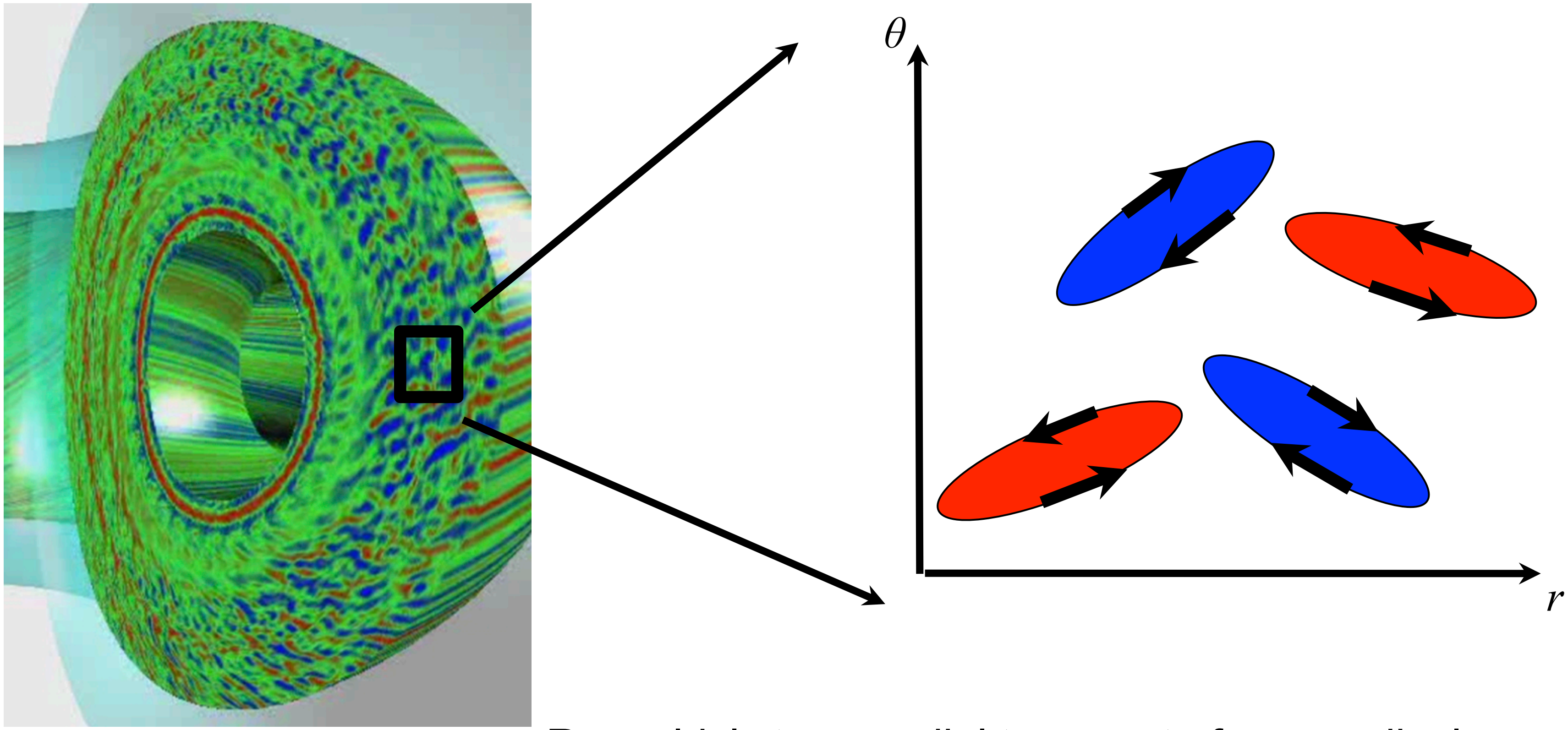
Tokamaks observed to spontaneously spin without (direct) external torque



This spontaneous rotation helps improve long-wavelength (MHD) stability, and can reduce the small-scale (gyrokinetic) turbulence.

(Debate about proper scaling without up-down asymmertry and edge effects. Probably weaker, as in Parra 2012. Good overall review: Peeters et al. Nucl. Fusion, 51, 094027 (2011))

Intuitive picture of Reynolds' stress: asymmetry needed to drive net rotation



Reynolds' stress radial transport of perpendicular momentum

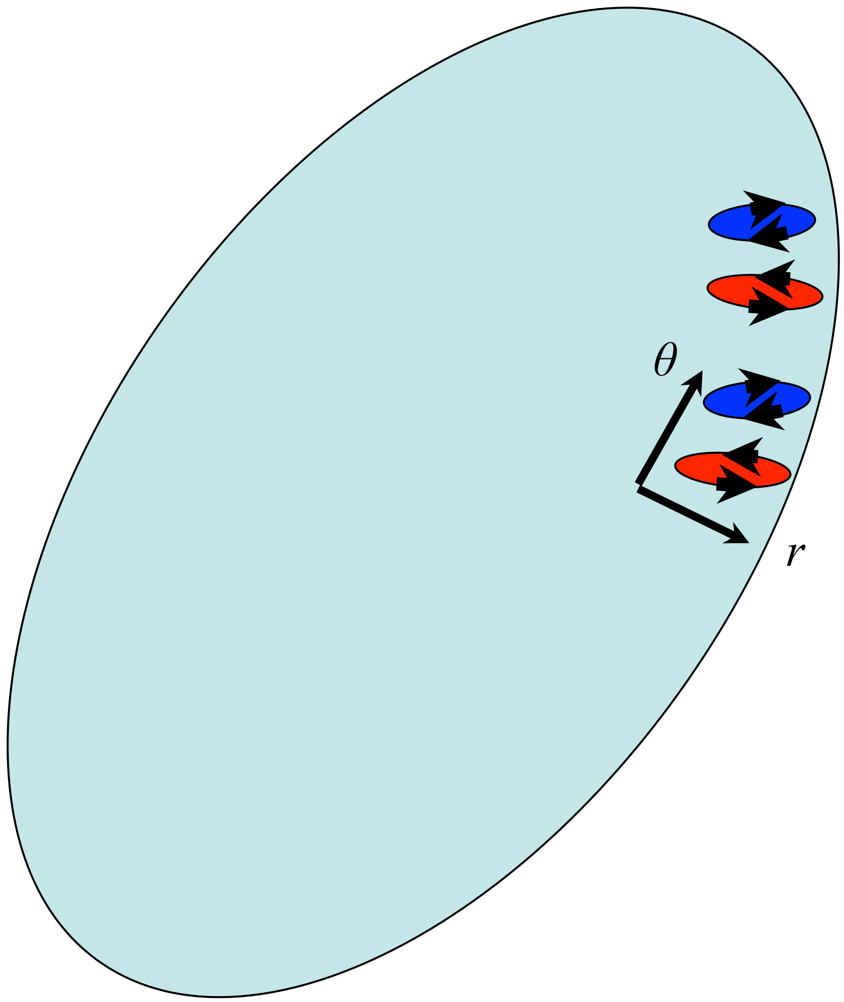
= $\langle v_r v_\theta \rangle$ > 0 for eddy tilted up

$\langle v_r v_\theta \rangle$ < 0 for eddy tilted down

$\langle v_r v_\theta \rangle$ averages to zero with up-down symmetry

Rigorous, complete proof by Parra et al. 2011

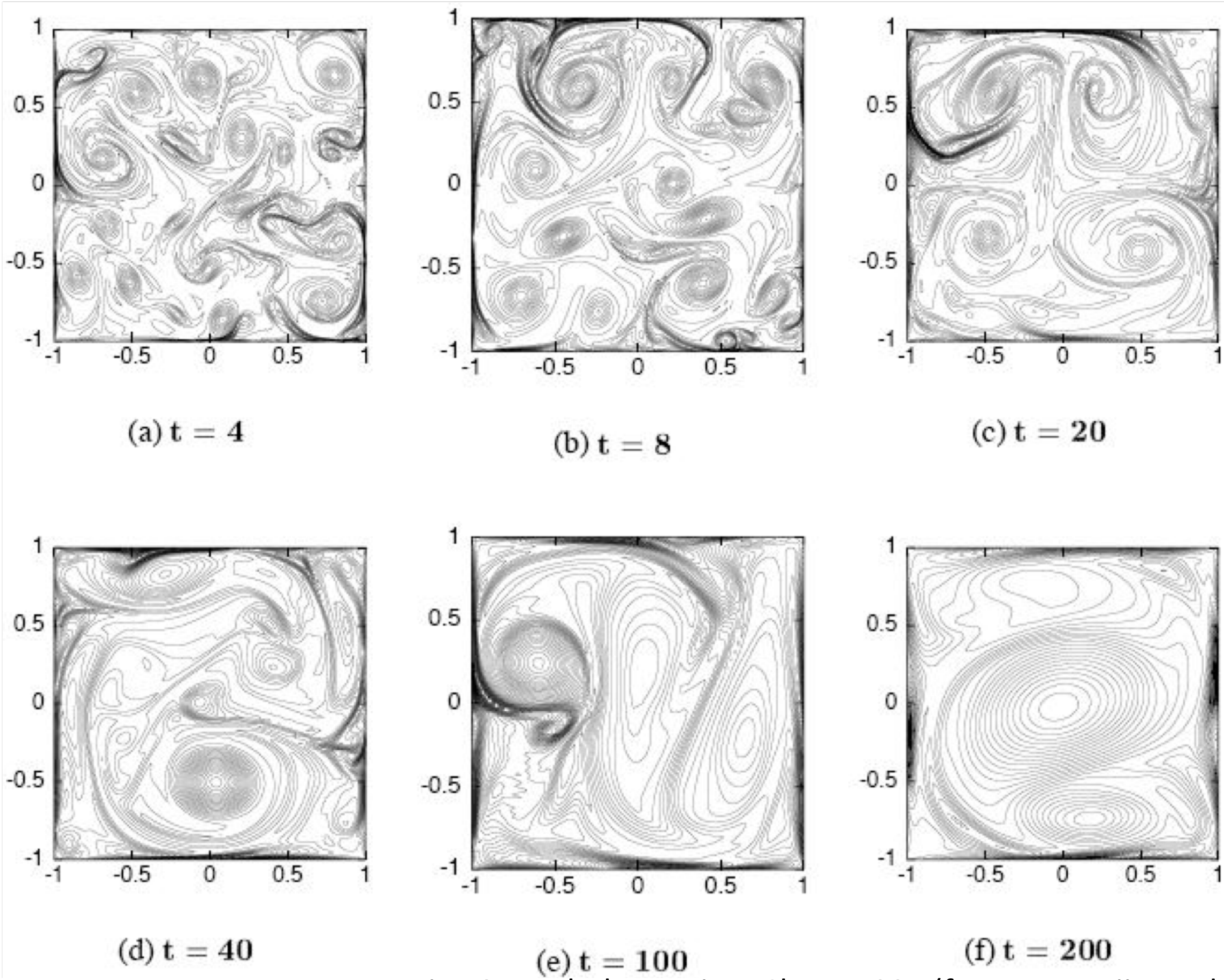
How much spin driven by up-down asymmetry?



Eddies tilted relative to local radial (r), poloidal (θ) coordinates. → net momentum flux. Observed in TCV tokamak (Lausanne, Switzerland, Camenen PRL 2010). How strong can it be?

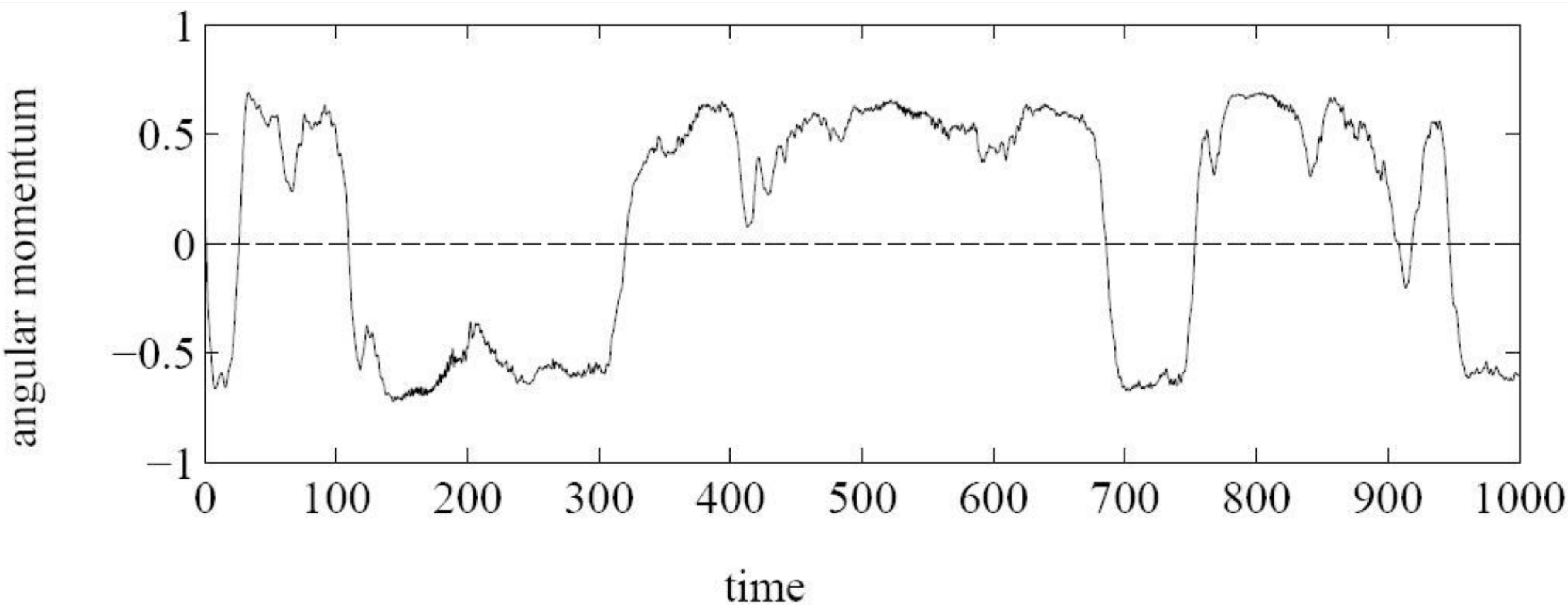
Initial gyrokinetic results (Ball, Parra, et al. 2014) find v_φ/v_{ti} ~ 5%, perhaps too weak. Stronger in stellarators? Other optimizations? Possible edge torque mechanism: T. Stoltzfus-Dueck, PRL 2012.

Spontaneous spin-up in 2-D bounded hydro is large: ~25% of kinetic energy in net solid body rotation



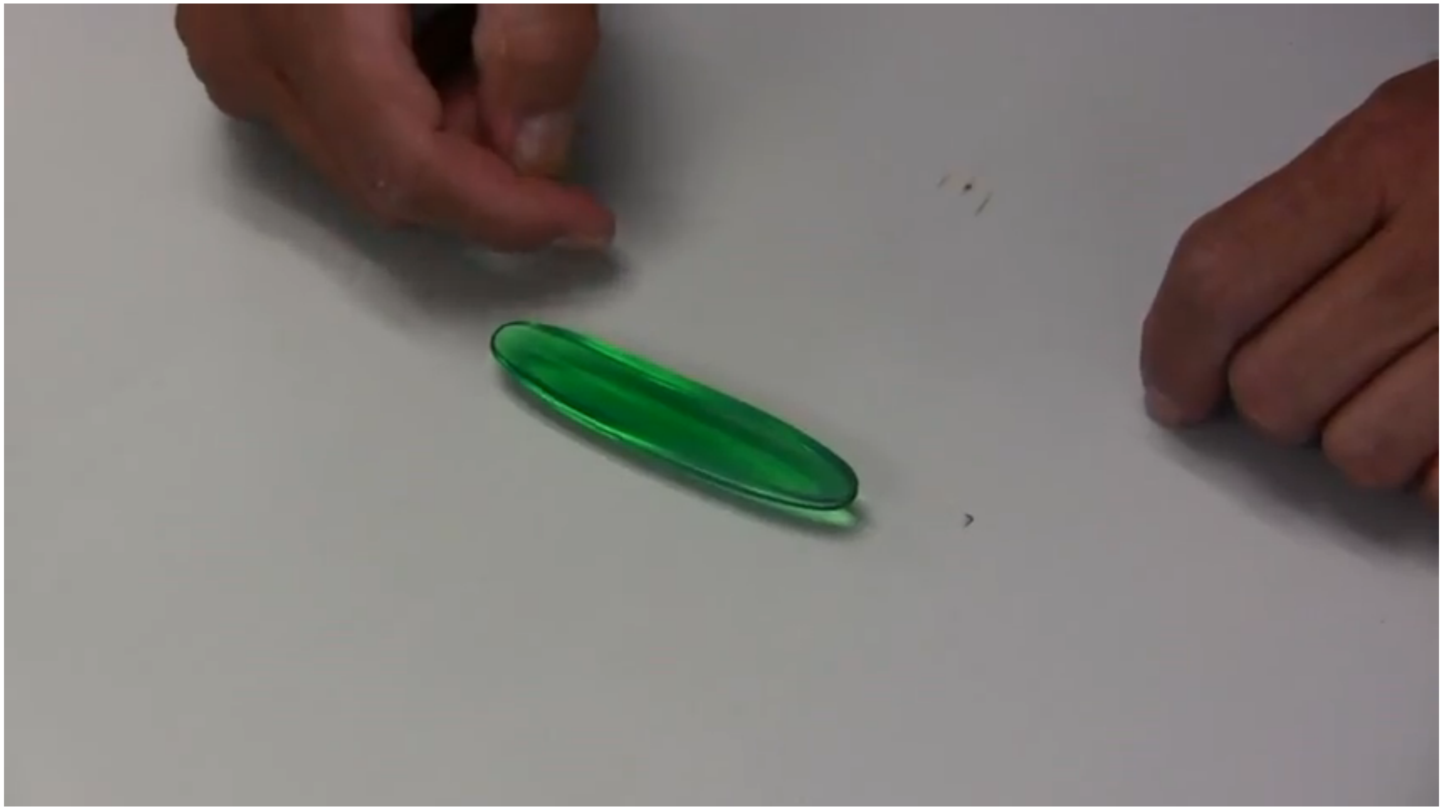
Decaying 2D turbulence sim., Clercx 1997 (from van Heijst and Clercx 2009)

Spontaneous spin-up in 2-D bounded hydro is large: ~25% of kinetic energy in net solid body rotation



Driven 2D turbulence sim., Molenaar et al. 2004(from van Heijst and Clercx 2009)

Rattleback spinning toy



http://www.youtube.com/watch?v=o2nURFQ-m5g

"Rattleback" toy: spin it one way, and it eventually reverses:

- San Jose Scientific rattleback (concise): http://www.youtube.com/watch?v=o2nURFQ-m5g
- longer, entertaining demo by Dr. Tadashi Tokieda (rattleback example starts at t=1:20. He mentions the general property of chirality and the example of the earth's geodynamo):
  - http://www.youtube.com/watch?v=AcQMoZr_x7Q

LAST

SLIDE

Magnetic Prandtl # dependence of MRI

$$\mathrm{Pm} = \frac{\text{momentum diffusivity}}{\text{magnetic diffusivity}} \propto \frac{\text{viscosity}}{\text{resistivity}} \approx \frac{\nu_{ii}\lambda_{mfp}^2}{\nu_{ei}c^2/\omega_{pe}^2}$$

$$\approx \sqrt{\frac{m_i}{m_e}}\frac{\beta_e}{2}\frac{\lambda_{mfp}^2}{\rho_i^2} \approx \left(\frac{T}{1eV}\right)^4 \left(\frac{6.5\times 10^{10}\,cm^{-3}}{n}\right)$$

- Pm = D_u / D_B ∝ viscosity/resistivity << 1 in liquid metals, some plasmas (stellar interior, cold accretion disks, low-ionization?
- Pm >> 1 in many hot, lower density plasmas (hot accretion flows, ISM, galactic clusters, Pm <~ 10^{29})
- IAS MRI 08 meeting: MRI dynamo w/o net B flux depends on Pm?, turbulence dies away at low Pm? (or if Rm < F(Re) ?)
- MRI more robust with net B flux. Source of large scale B? Beta dependence?

Most plasmas highly anisotropic

Most plasmas, even with fairly weak B, have parallel transport >> perpendicular transport.

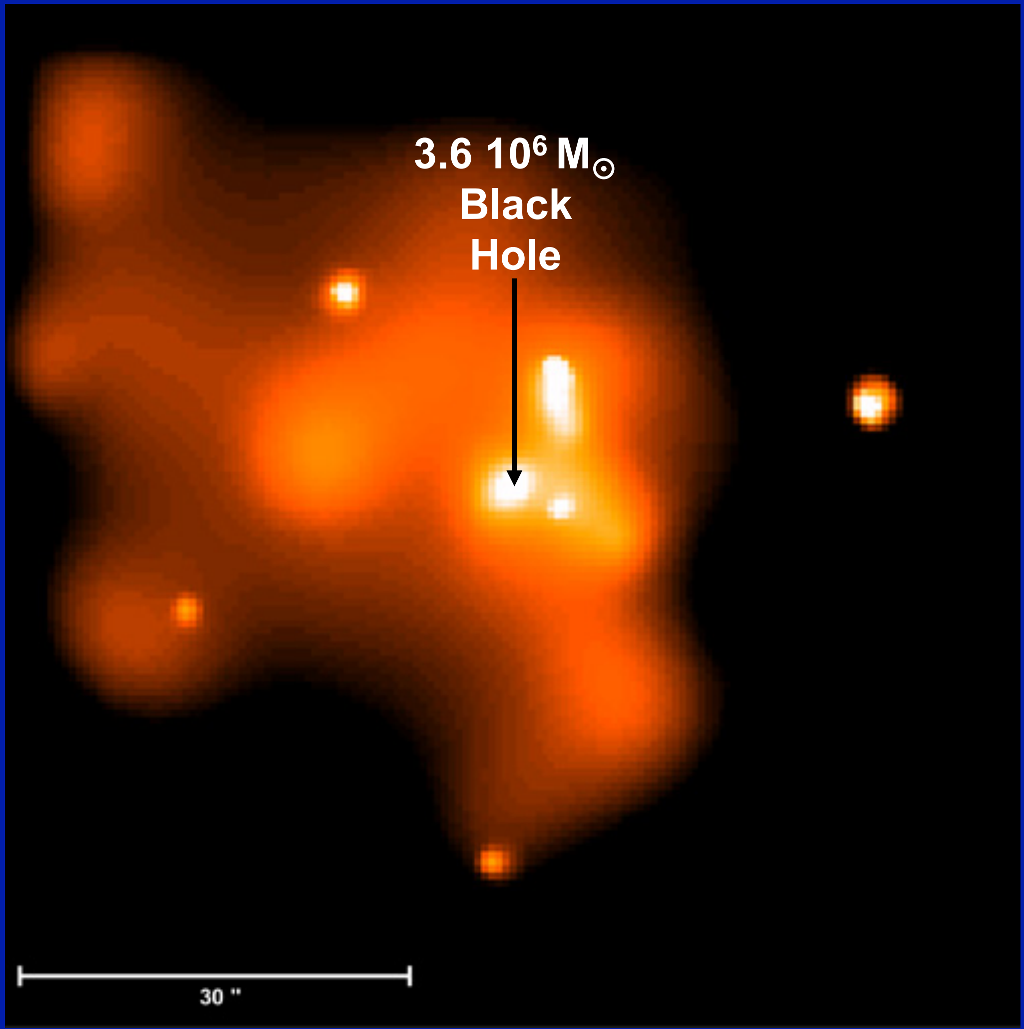
Plasma viscosity is isotropic only if $\lambda_{mfp} << \rho_i$. In anisotropic case, $Pm_{||}$ is given by previous Pm, and Braginskii's $Pm_\perp$ is:

$$\mathrm{Pm}_\perp = \frac{\text{perp. momentum diffusivity}}{\text{magnetic diffusivity}} \approx \frac{\nu_{ii}\rho_i^2}{\nu_{ei}c^2/\omega_{pe}^2} \approx \sqrt{\frac{m_i}{m_e}}\frac{\beta_e}{2}$$

In longer mean-free-path regime, Braginskii's fluid closures break down, and one should use Kulsrud/CGL drift-kinetic-MHD, as we will discuss.

Both Braginskii and drift-kinetic-MHD are incomplete by themselves, esp. @ high beta: velocity-space anisotropies drive firehose/mirror/cyclotron instabilities → enhances effective scattering, maybe closer to MHD in a sense, but get strong heating (hard to keep electrons cold),

Accretion Region of Milky Way's Black Hole: Low Collisionality



Hot Plasma Gravitationally Captured
By BH → Accretion Disk

Estimated Conditions
Near the BH

$T_p \sim 10^{12}$ K
$T_e \sim 10^{11}$ K
$n \sim 10^6\ cm^{-3}$
$B \sim 30$ G

proton mfp ~ 10^{22} cm
>>> $R_{horizon} \sim 10^{12}$ cm

→

need to understand
accretion of a magnetized
collisionless plasma

Outline

- MHD of Disks: Angular Momentum Transport
- Collisionless Accretion Flows (BHs & NSs)
  - – Astrophysical Motivation
  - – Disk Dynamics in Kinetic Theory
    - A mechanism for strong electron heating
      (Sharma et al. astro-ph 07)

Accretion: Physical Picture

- Simple Consequences of Mass, Momentum, & Energy Conservation

- Matter Inspirals on Approximately Circular Orbits

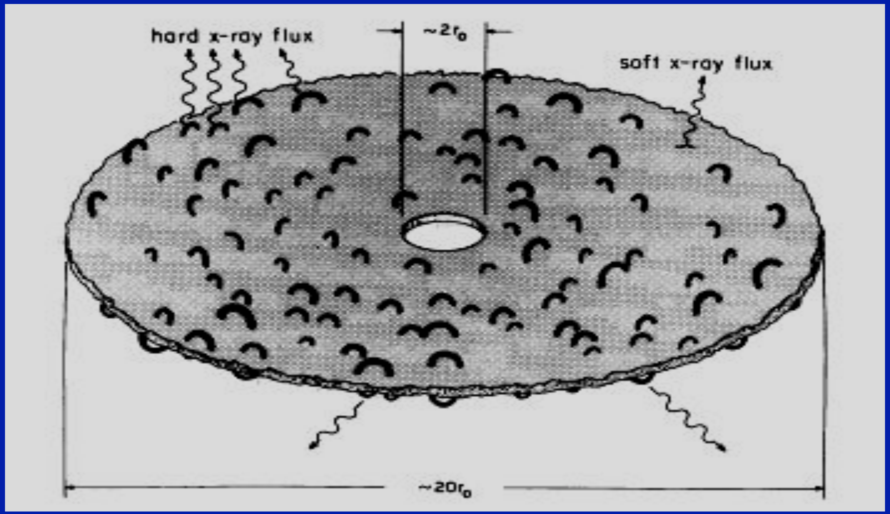
  - $V_r << V_{orb}$ $t_{inflow} >> t_{orb}$
  - t_{inflow} ~ time to lose angular momentum ~ viscous diffusion time
  - $t_{orb} = 2\pi/\Omega$; $\Omega = (GM/r^3)^{1/2}$ (Keplerian orbits; like planets in solar system)

- Disk Structure Depends on Fate of Released Gravitational Energy

  - t_{cool} ~ time to radiate away thermal energy of plasma

  - Thin Disks: $t_{cool} << t_{inflow}$ (plasma collapses to the midplane)
  - Thick Disks: $t_{cool} >> t_{inflow}$ (plasma remains a puffed up torus)

Geometric Configurations

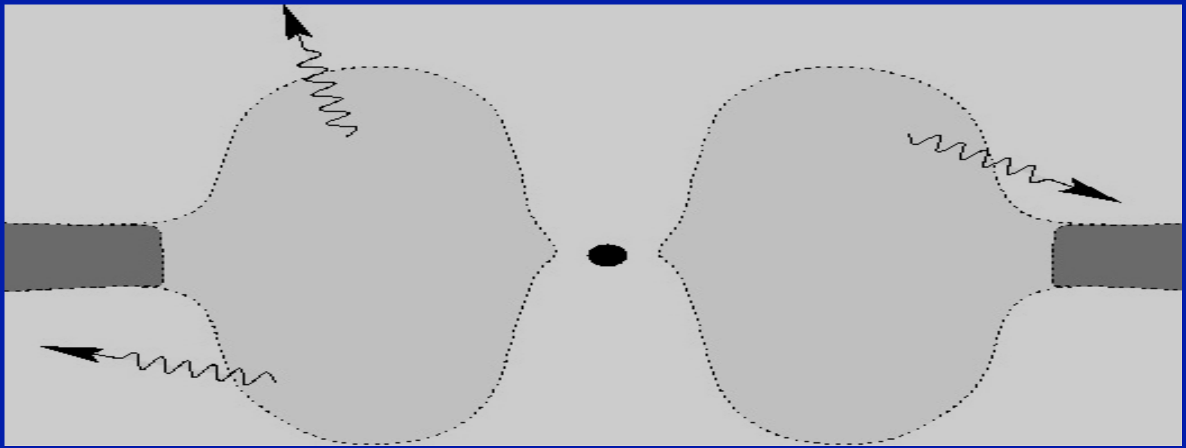


Galeev et al. 1979

e.g., solar system
Milky Way disk

thin disk: energy radiated away

(relevant to star & planet formation, galaxies, and luminous BHs/NSs)



e.g., our Galactic Center (more on this soon)

thick disk (torus; ~ spherical): energy stored as heat

(relevant to lower luminosity BHs/NSs)

MHD Drift Kinetic Eq. for $f_{0s}(\vec{x}, \mathrm{v}_{\|}, \mu, t)$

plasma is collisionless, hot w. H~r

Larmor radius << disk height

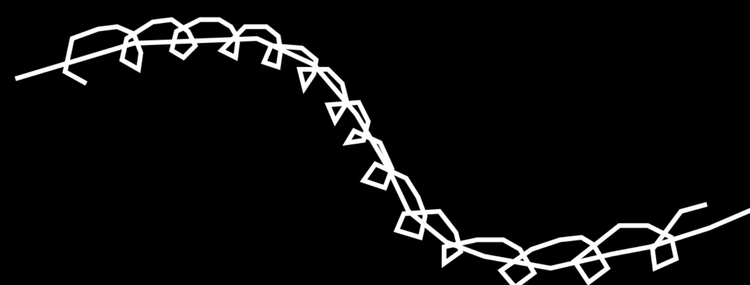
drift kinetic equation: approx. for Vlasov eq. if $k\rho_i \ll 1$, $\omega \ll \Omega_i$

Table 1.2: Plasma parameters for Sgr A*

Parameter	$r = r_{acc}$ 2.2×10^{17} cm	$r = \sqrt{r_{acc} R_S}$ 4.2×10^{14} cm	$r = R_S$ 7.8×10^{11} cm
$\nu_{i,\mathrm{ADAF}}/\Omega_K \sim r^{3/2}$	11.4	9.4×10^{-4}	7.6×10^{-8}
$\nu_{i,\mathrm{CDAF}}/\Omega_K \sim r^{3/2+p}$	11.4	1.81×10^{-6}	2.62×10^{-13}
$\rho_{i,\mathrm{ADAF}}/H \sim r^{-1/4}$	2×10^{-11}	9.94×10^{-11}	4.59×10^{-10}
$\rho_{i,\mathrm{CDAF}}/H \sim r^{-1/4-p/2}$	2×10^{-11}	2.23×10^{-9}	2.48×10^{-7}

$$\frac{\partial f_{0s}}{\partial t} + (\mathbf{V}_E + v_{\|}\hat{\mathbf{b}}) \cdot \nabla f_{0s} + \left(-\hat{\mathbf{b}} \cdot \frac{D\mathbf{V}_E}{Dt} - \mu \hat{\mathbf{b}} \cdot \nabla B + \frac{1}{m_s}(q_s E_{\|} + F_{g\|}) \right) \frac{\partial f_{0s}}{\partial v_{\|}} = 0$$

$\mu = v_{\perp}^2/B \propto T_{\perp}/B$ is conserved; $V_E = c(E \times B)/B^2$

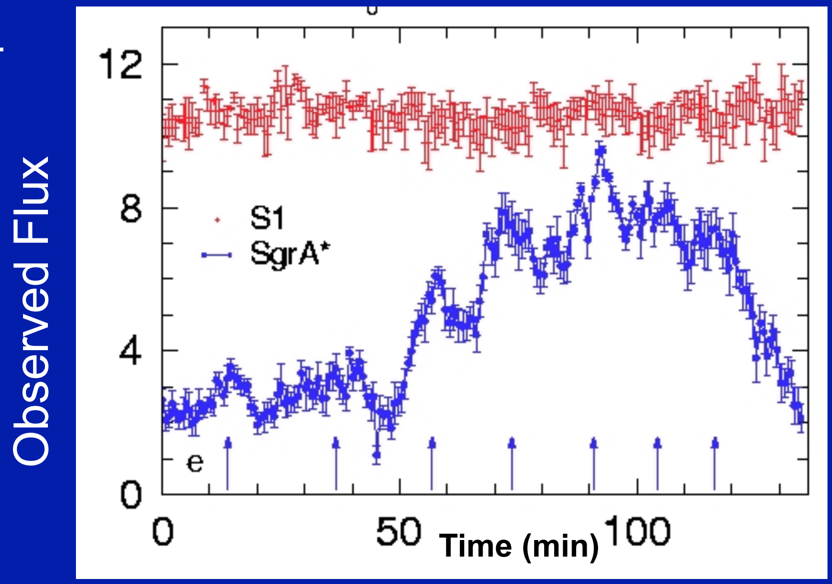


mfp >> disk height scales >> Larmor radius

Kulsrud's '61 version of unpublished Chew-Goldberger-Low MHD-drift-kinetic equations

Major Science Questions

- Macrophysics: Global Disk Dynamics in Kinetic Theory
  - e.g., how adequate is MHD, influence of heat conduction, …
- Microphysics: Physics of Plasma Heating
  - MHD turbulence, reconnection, weak shocks, …
  - **electrons produce the radiation we observe**
- Analogy: Solar Wind
  - macroscopically collisionless
  - thermally driven outflow w/ T_p & T_e determined by kinetic microphysics



Nonlinear Evolution Simulated Using Kinetic-MHD

- Large-scale Dynamics of collisionless plasmas: expand Vlasov equation retaining "slow timescale" (compared to cyclotron period) & "large lengthscale" (compared to gyroradius) assumptions of MHD (e.g., Kulsrud 1983)
- Particles efficiently transport heat and momentum along field-lines

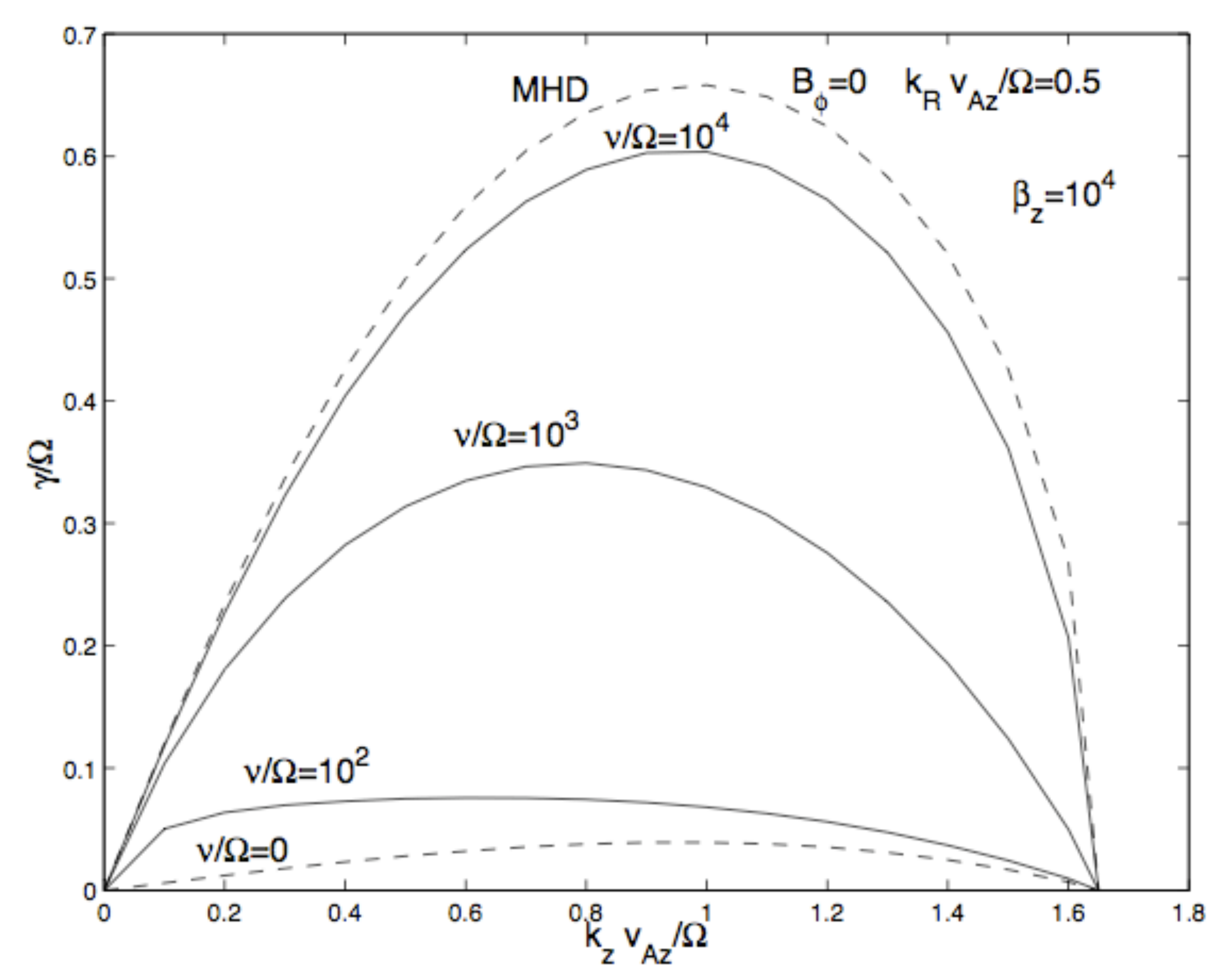
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0,$$

$$\rho \frac{\partial \mathbf{V}}{\partial t} + \rho \left(\mathbf{V} \cdot \nabla \right) \mathbf{V} = \frac{(\nabla \times \mathbf{B}) \times \mathbf{B}}{4\pi} - \nabla \cdot \mathbf{P} + \mathbf{F_g},$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left(\mathbf{V} \times \mathbf{B} \right),$$

$$\mathbf{P} = p_\perp \mathbf{I} + \left(p_\| - p_\perp \right) \hat{\mathbf{b}}\hat{\mathbf{b}},$$

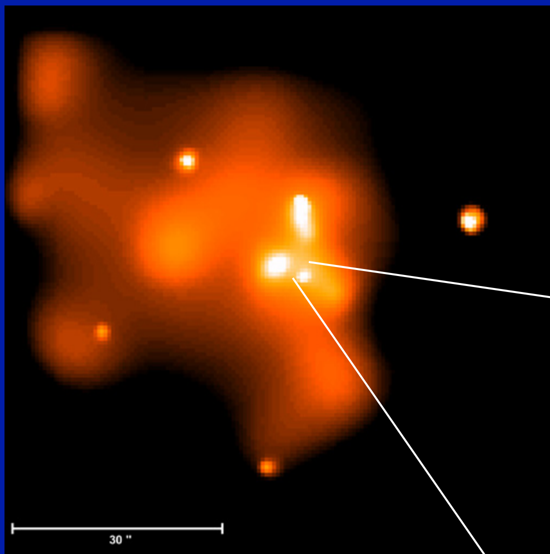
Kinetic effects stabilizing if initial $B_\varphi=0$



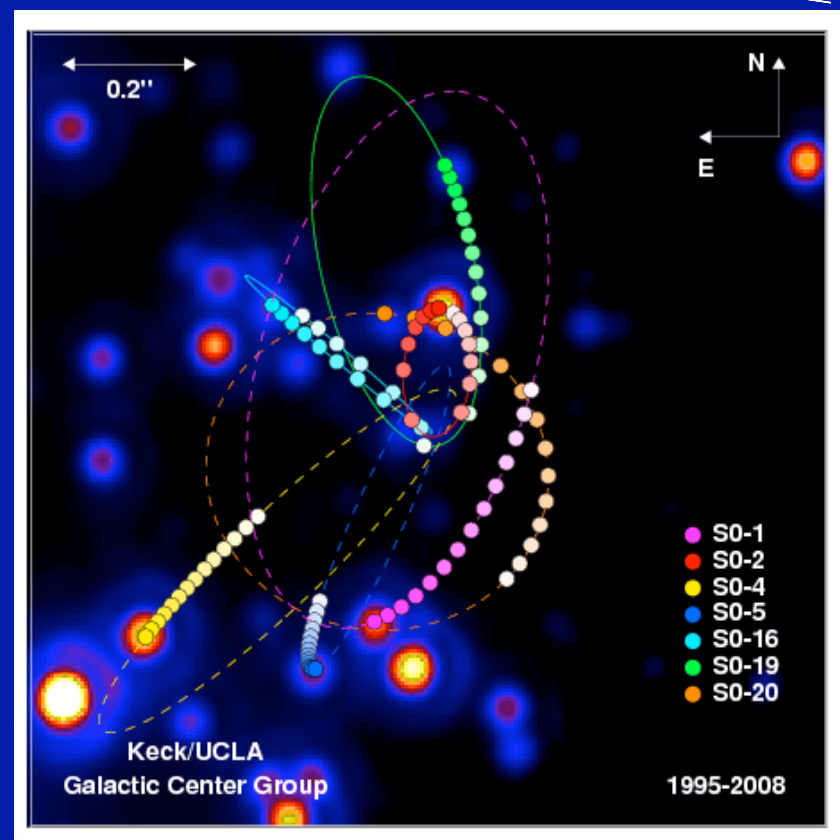
Different than last slide, where kinetic effects enhance growth rate if initial $B_\varphi = B_z$

Sharma, Hammett, Quataert ApJ 03

Galactic Center BH



Chandra



$3.6 \times 10^{6}\ M_{\odot}$ black hole

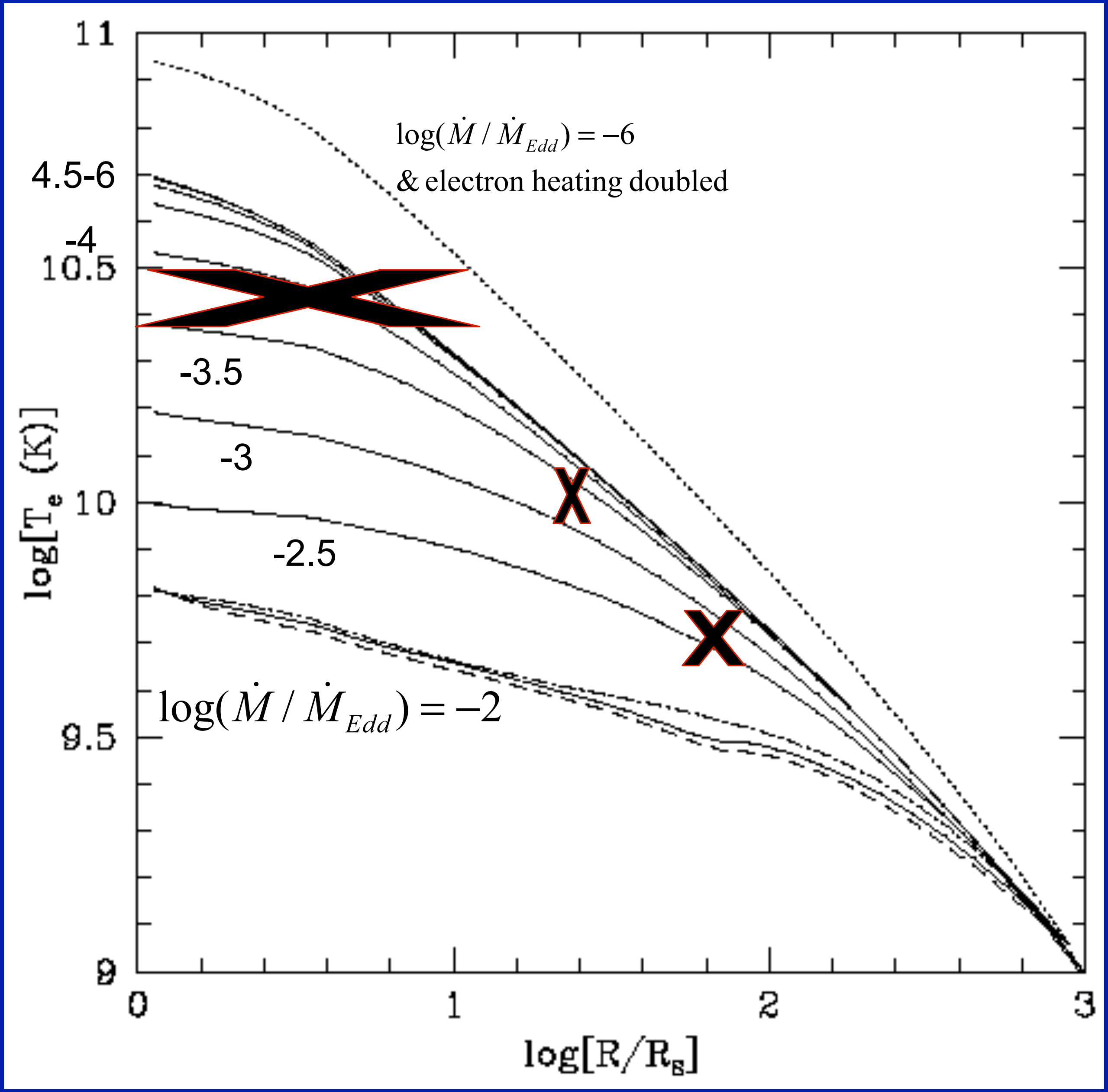
Bondi radius ~ 0.07 pc (2''),
n~100/cc, T~1-2 keV

$\dot{M} \sim 10^{-5}\ M_{\odot}$ /yr by stellar outflows

$L_{obs} \sim 10^{-5} \times (0.1\ \dot{M}c^2)$
Why low luminosity? low $\dot{M}$ or low radiative efficiency

Collisionless, magnetized plasma at R ~ Bondi radius; $r_i \ll H$, $\ell_{mfp} \gg H$

Schödel et al., 2002, A.M. Ghez et al. 2003 http://www.astro.ucla.edu/~ghezgroup/gc



Predicted vs. measured temperature profiles for various accretion rates

Predicted curves from Fig. 8 of Sharma et al. ApJ 2007

with data points added from Bower et al. Science 04 w/ approx. error bars. Caveats: this slide needs more careful assessment of translation from “size” to “radius” in brightness temperature measurements, and of meaning of error bars in both size and brightness temperature?

Electron temperature profile not a strong way to distinguish (in this case) between Bondi accretion M_dot/ M_edd=1.e-4 and our predicted accretion rate M_dot/M_Edd ~ 1.e-7-1.e-6, because already in the radiatively inefficient regime.