Progress in Continuum Gyrokinetic Simulations of Edge Turbulence, in a Helical Model SOL

Greg W. Hammett, July 18, 2017
Eric L. Shi (PPPL graduate student), Ammar Hakim

10th (!) Plasma Kinetics Working Meeting
Wolfgang Pauli Institute, Vienna

First successful continuum gyrokinetic code doing turbulence on open field lines with sheath boundary conditions:
E.L. Shi, G.W. Hammett, T. Stotlzfus-Dueck, A. Hakim, J. Plasma Physics (2017)
http://dx.doi.org/10.1017/S002237781700037X

That was with straight field lines, LAPD-like case. Here we show first extension to the toroidal case, with a helical model of the SOL including bad-curvature drive.

Improving Confinement Can Significantly ↓ Size & Construction Cost of Fusion Reactor

Well known that improving confinement & β can lower Cost of Electricity / kWh, at fixed power output.

Stronger effect if consider smaller power: better confinement allows smaller size & capital cost at same fusion gain Q ($nT\tau_E$).

Standard H-mode empirical scaling:

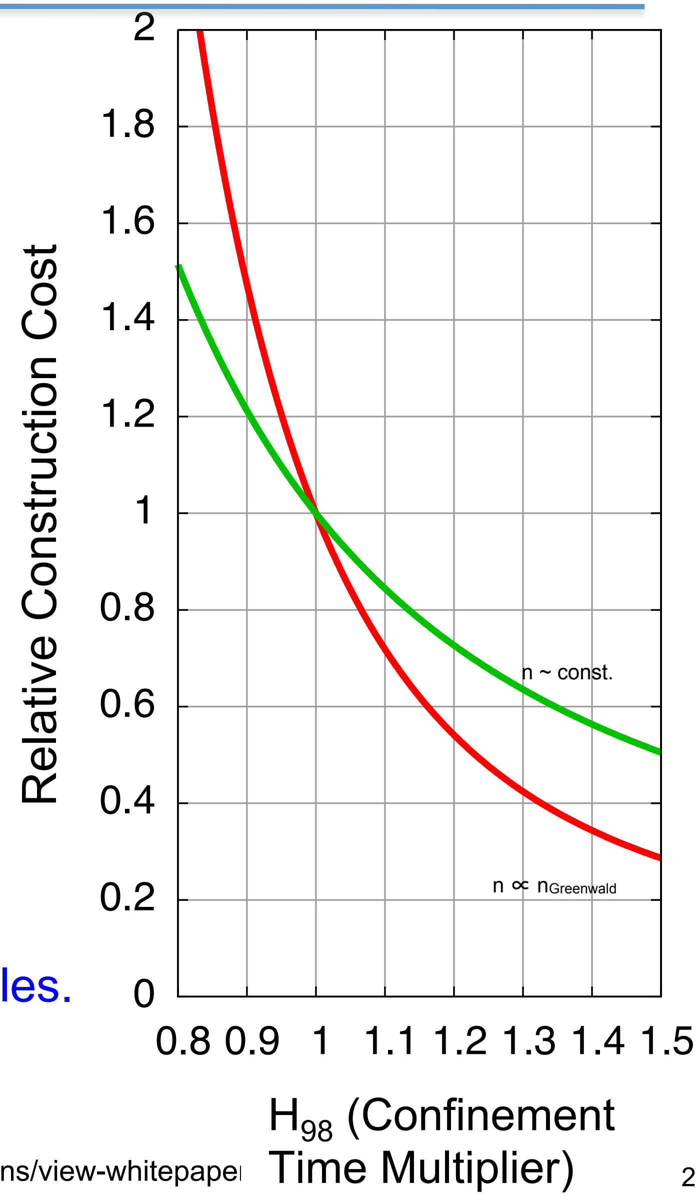
$$\tau_E \sim H\, I_p^{0.93}\, P^{-0.69}\, B^{0.15}\, R^{1.97} \ldots$$

($P = 3VnT/\tau_E$ & assume fixed $nT\tau_E$, q_{95}, β_N, $n/n_{Greenwald}$):

Capital Cost \$ ~ $R^2 \sim 1/(H^{4.8} B^{3.4})$

ITER std $H=1$, steady-state $H\sim1.6$
ARIES-AT $H\sim1.5$
MIT ARC $H_{89}/2 \sim 1.4$

Need comprehensive simulations, validated with experiments, to extrapolate improved H to reactor scales.



(Plots assumes cost ∝ R^2 roughly. Includes constraint on B @ magnet with ARIES-AT 1.16 m blanket/shield, $a/R=0.25$, i.e. $B = B_{mag}(R-a-a_{BS})/R$. Neglects current drive issues.)

Hammett & Dorland, White Paper 2017, https://sites.google.com/site/usmfrstrategicdirections/view-whitepaper

Interesting Ideas To Improve Fusion

* New high-field superconductors (MIT). Dramatic reduction in size & cost (x1/5 ?)

* Liquid metal (lithium, tin) coatings/flows on walls or vapor shielding: (1) protects solid wall (2) absorbs hydrogen ions, reduces recycling of cold neutrals back to plasma, raises edge temperature & improves global performance. TFTR found: ~2 keV edge temperature. NSTX, LTX: more lithium is better, where is limit?

* Spherical Tokamaks (STs) appear to be able to suppress much of the ion turbulence: PPPL & Culham upgrading 1 --> 2 MA to test scaling

* Advanced tokamaks, alternative regimes (reverse magnetic shear / "hybrid"), methods to control ELMs, higher plasma shaping, advanced divertors.

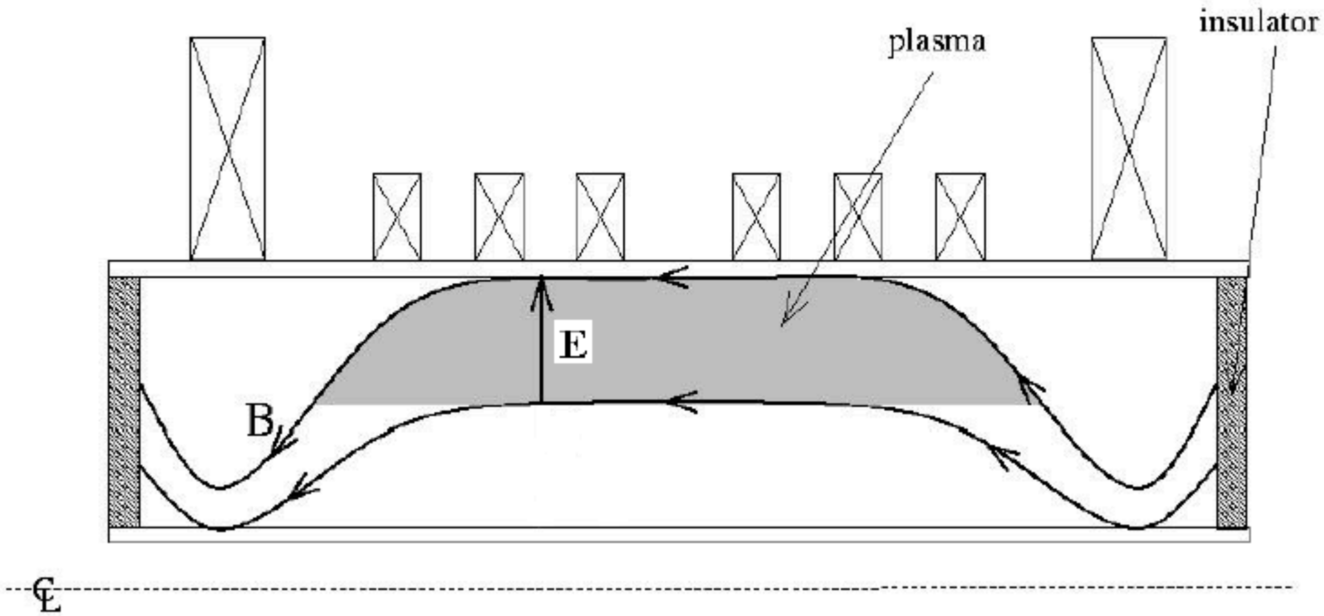
* Tokamaks spontaneously spin: reduce turbulence & improve MHD stability. ITER spins more than previously expected? Up-down-asymmetric tokamaks/stellarators?

* New stellarator designs, room for further optimization: Hidden symmetry discovered after 35+ years of fusion research. Fixes disruptions, steady-state, density limit.

* More speculative concepts: RFPs, FRCs, GDT, rotating mirrors, …

* Robotic manufacturing advances: reduce cost of complex, precision, specialty items

Spinning Mirrors:
MCX Maryland Centrifugal Experiment



- centrifugal forces => axial confinement
- rotation shear => stability to interchanges

Hassam, AB, Comments Plasma Phys Cont. Fus., 18, 263, 1997
Ellis, RF; Hassam, AB; Messer, S; et al. PHYSICS OF PLASMAS 8, 2057, 2001

Supersonic rotation achieved, MHD stable in simulations & expts: *Huang, Y-M, Hassam AB, PoP 2004*
from http://theory.pppl.gov/news/rrseminars/20170630Hassam.pdf (see also refs, 2010 PRL & 2014 PoP)

Gkeyll using novel algorithms, has multiple spinoffs

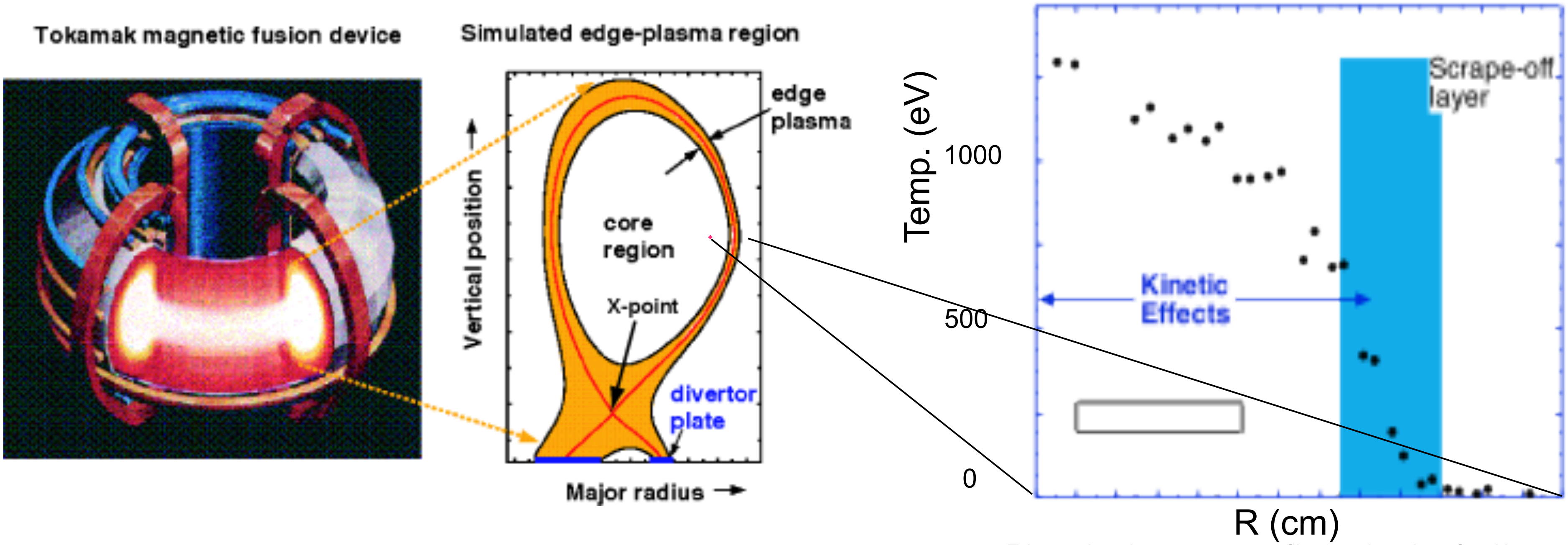
Novel version of Discontinuous Galerkin algorithm, conserves energy for Hamiltonian system even with upwinding. High-order algorithms that reduce communication costs helpful for Exascale computers.

4 Versions / spinoffs:

- Gyrokinetic DG version for edge turbulence in fusion
  LAPD results: E. Shi, Hammett, Stoltzfus-Dueck. Hakim, J. Plasma Physics (2017), Shi et al. PoP 2015
- Vlasov/Poisson DG version for plasma thrusters (AFOSR/Virginia Tech)
  Cagas et al. Phys. Plasmas (2017)
- Vlasov/Maxwell DG version for solar wind turbulence (U. Maryland, NSF)
  J. TenBarge, Sherwood Inv. Talk (2017), J. Juno et al., Arxiv (2007)
- Multi-moment multi-fluid (~extended MHD) finite-volume version, studying reconnection (Princeton Center for Heliophysics). Also coupled with OpenGGCM global magnetosphere code (UNH)
  J. Ng PoP 2015, L. Wang PoP 2015

Also, modeled Lithium Vapor Box ideas by adding evaporation/condensation b.c.s to finite-volume fluid version. Co-authors on Goldston et al. 2017 Nucl. Mat. & Energy

Edge region very difficult



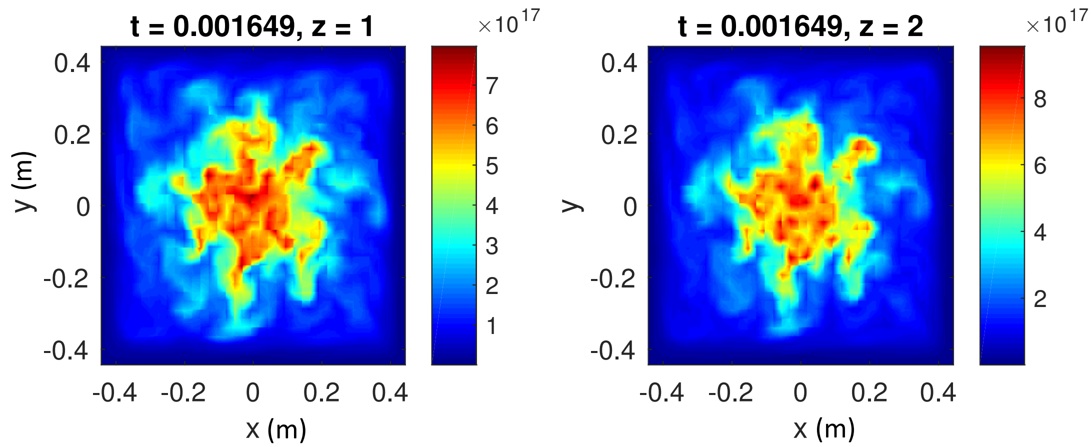
Edge pedestal temperature profile near the edge of an H-mode discharge in the DIII-D tokamak. [Porter*2000*]. Pedestal is shaded region.

Present core gyrokinetic codes are highly optimized for core, need new codes to handle additional complications of edge region of tokamaks (& stellarators):

open & closed field lines, plasma-wall-interactions, large amplitude fluctuations, (positivity constraints, non-Maxwellian full-F), atomic physics, non-axisymmetric RMP / stellarator coils, magnetic fluctuations near beta limit…

Hard problem: but success of core gyrokinetic codes and progress of XGC PIC code makes me believe this is tractable, with a major initiative

First Gkeyll Simulation of 3D+2v Gyrokinetic Turbulence in Scrape Off Layer (SOL).



Density snapshots in LAPD simulation, fluctuation level similar to experiments

- Worried about difficulties in gyrokinetic-sheath interactions and other edge computational challenges (special algorithms helped). Ran into & fixed several problems that drove high frequency, large amplitude ϕ fluctuations. Now appears fairly robust.

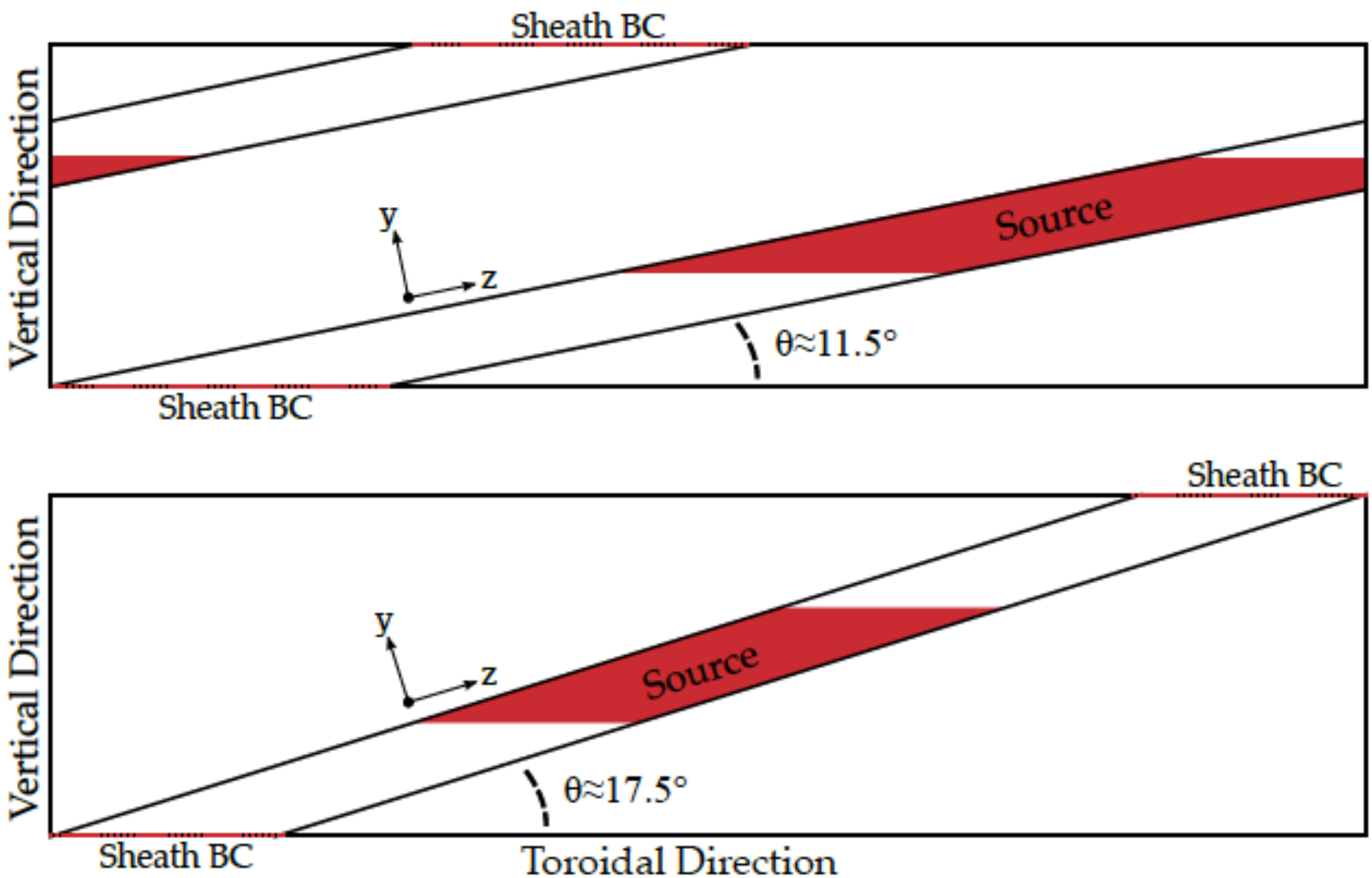
- Present model (kinetic generalization of previous fluid sheath) more general than simple logical sheath, allows currents into and out of walls.

- Gyrokinetic extension of pioneering fluid work (Rogers & Ricci, Umansky, Friedman et al.)

- Simple helical SOL at present (like Torpex, Helimak expts.), no separatrix, but have bad-curvature drive, have done simulations of Torpex. Plan initial NSTX-like simulations soon.

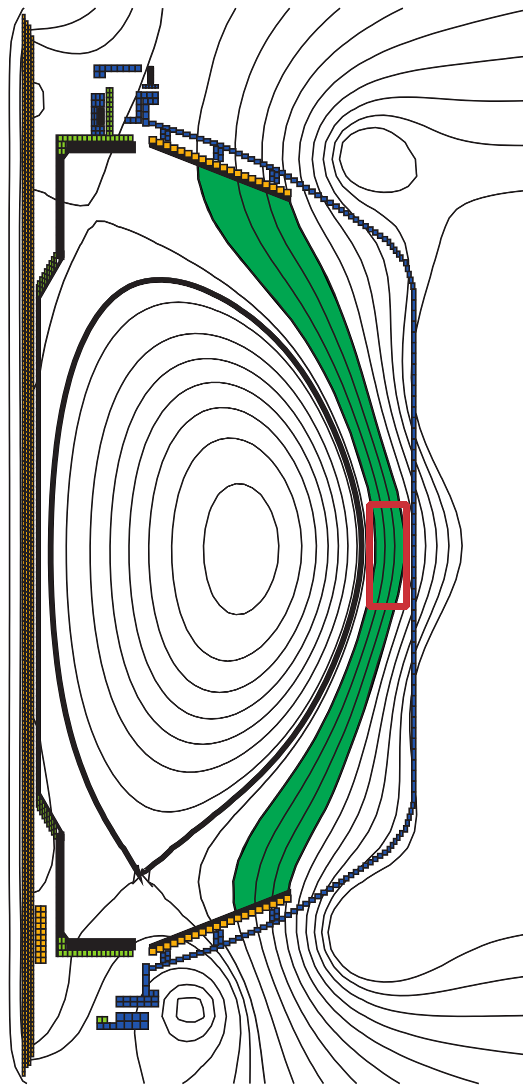
E. L. Shi, G. Hammett, T. Stoltzfus-Dueck, A. Hakim, JPP 2017

Simulating SOL flux-tube / annulus domain

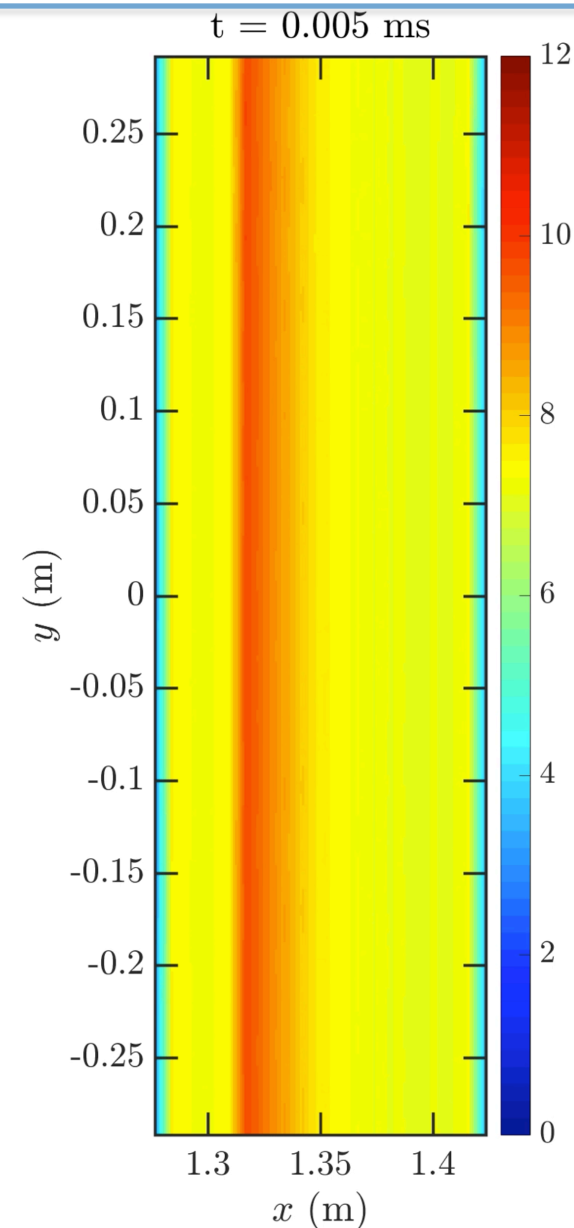


E. Shi (Ph.D. 2017)

Gkeyll: First Continuum 5D Gyrokinetic Simulations of Turbulence in SOL with sheath model boundary conditions



Edge region has been computationally very difficult.

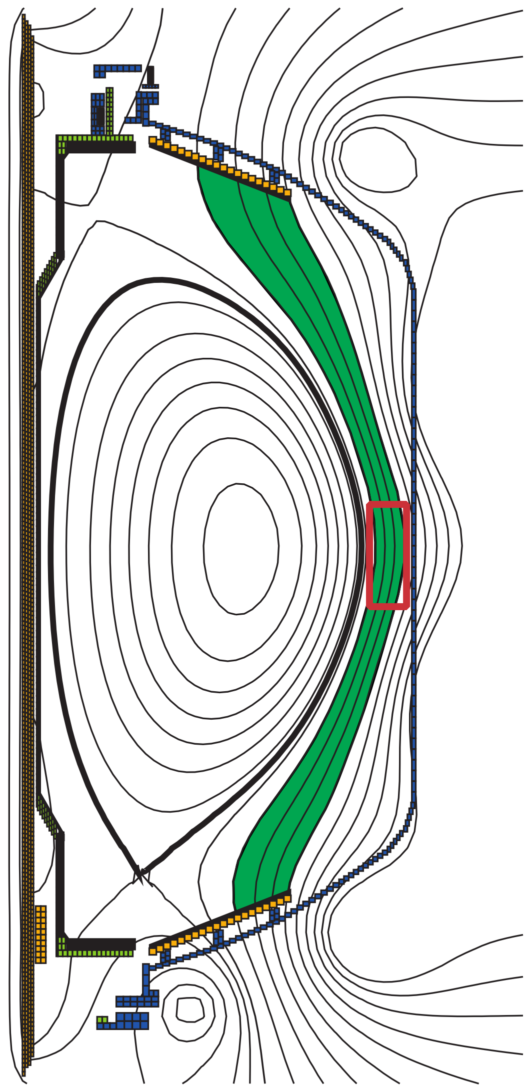


Various simplifications at present, such as helical model of SOL (toroidal + vertical B field).
XGC is only gyrokinetic turbulence code that can handle separatrix at present.

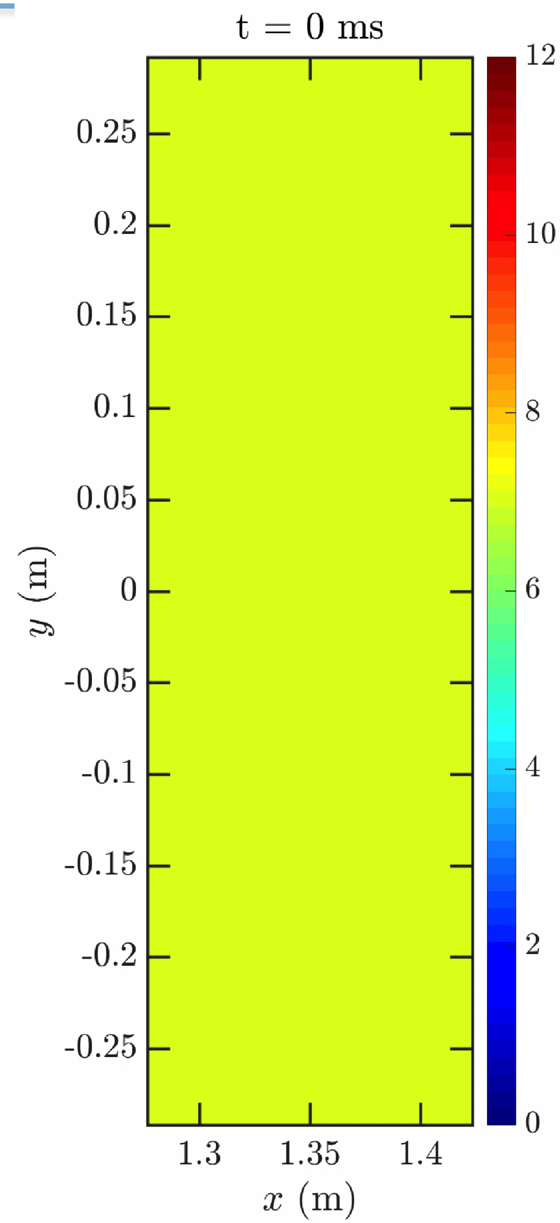
LAPD: E. Shi, A. Hakim, T. Stolzfus-Dueck, J. Plasma Physics (2017, in press; Arxiv)

(red region indicates source location)

Gkeyll: First Continuum 5D Gyrokinetic Simulations of Turbulence in SOL with sheath model boundary conditions



Edge region has been computationally very difficult.

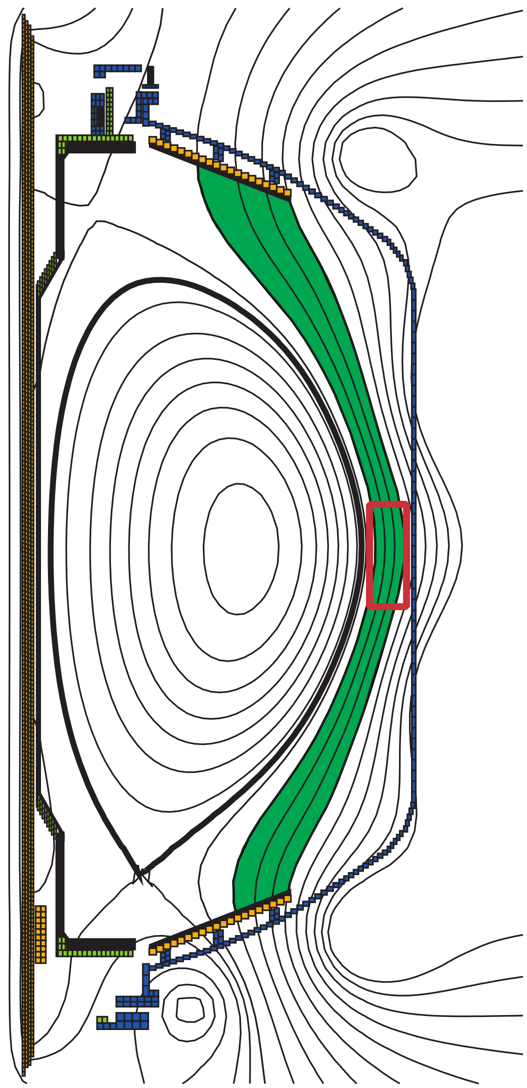


(movie)

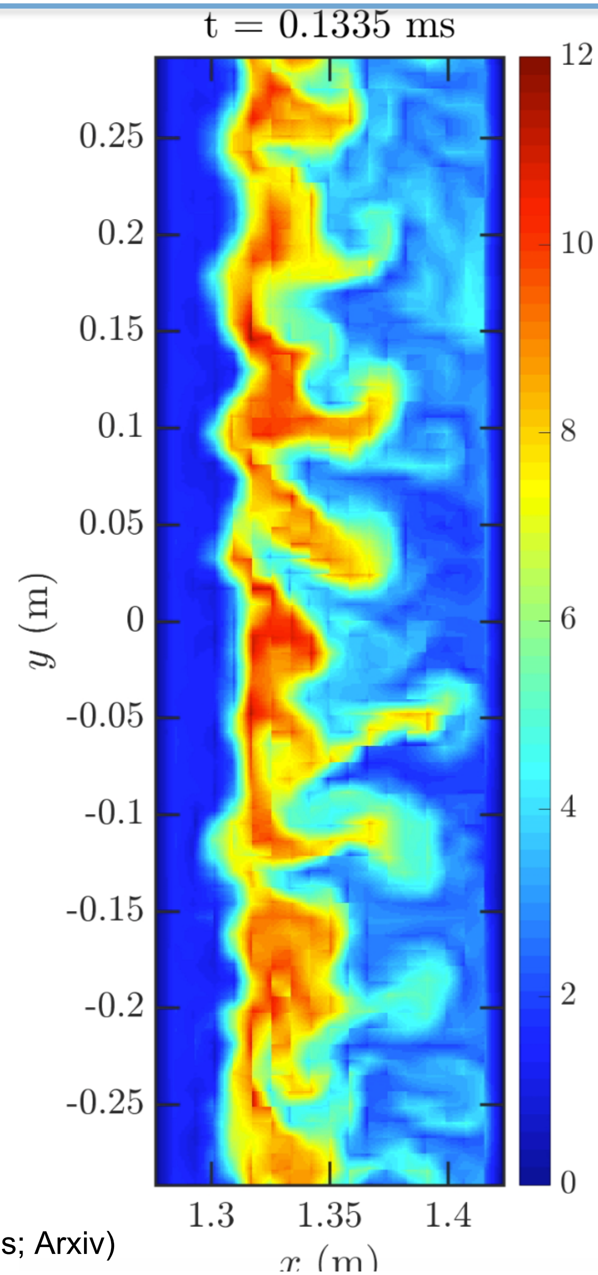
Various simplifications at present, such as helical model of SOL (toroidal + vertical B field).
XGC is only gyrokinetic turbulence code that can handle separatrix at present.

LAPD: E. Shi, A. Hakim, T. Stolzfus-Dueck, J. Plasma Physics (2017, in press;

Gkeyll: First Continuum 5D Gyrokinetic Simulations of Turbulence in SOL with sheath model boundary conditions



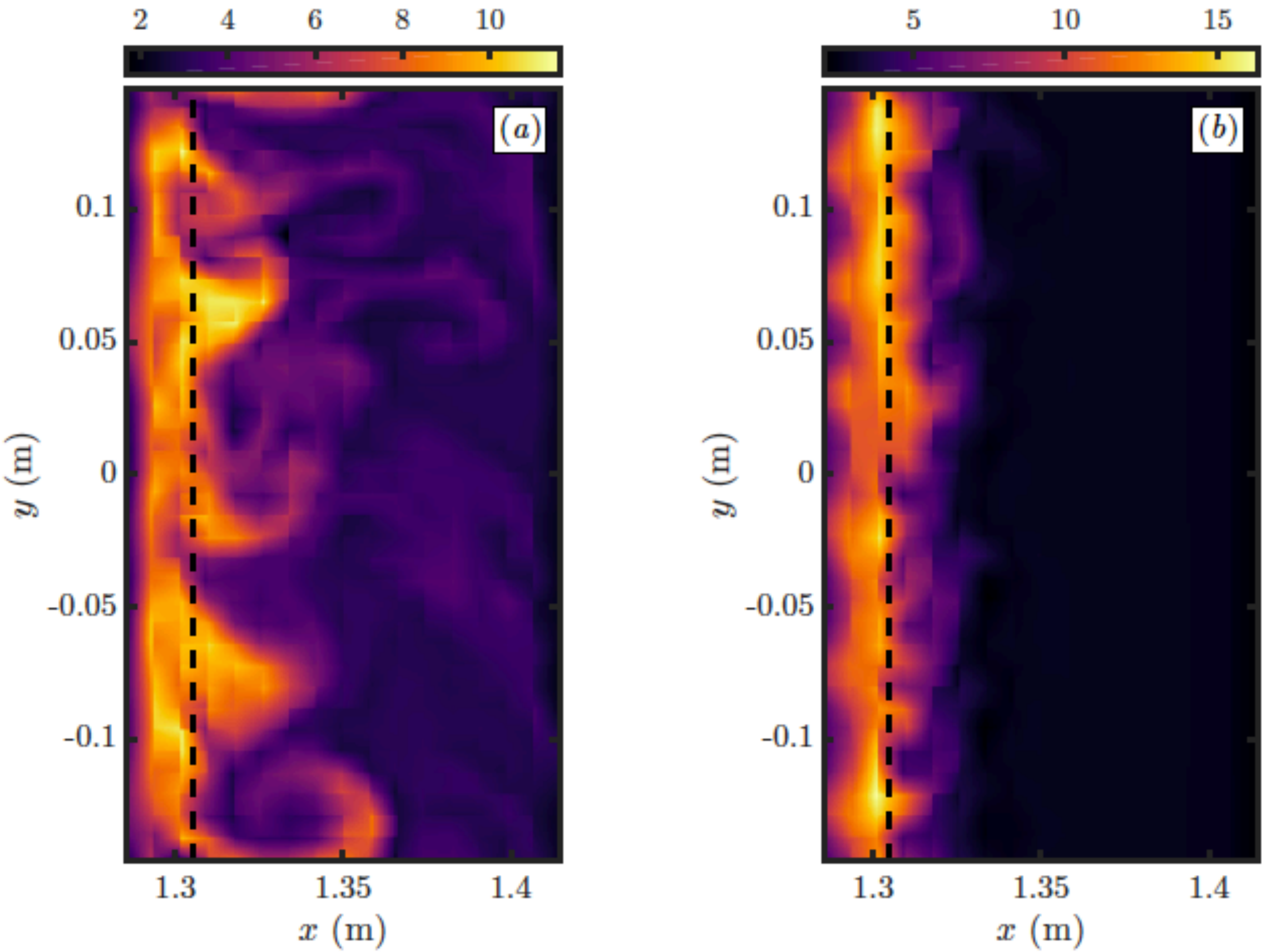
Edge region has been computationally very difficult.



Various simplifications at present, such as helical model of SOL (toroidal + vertical B field).
XGC is only gyrokinetic turbulence code that can handle separatrix at present.

LAPD: E. Shi, A. Hakim, T. Stolzfus-Dueck, J. Plasma Physics (2017, in press; Arxiv)

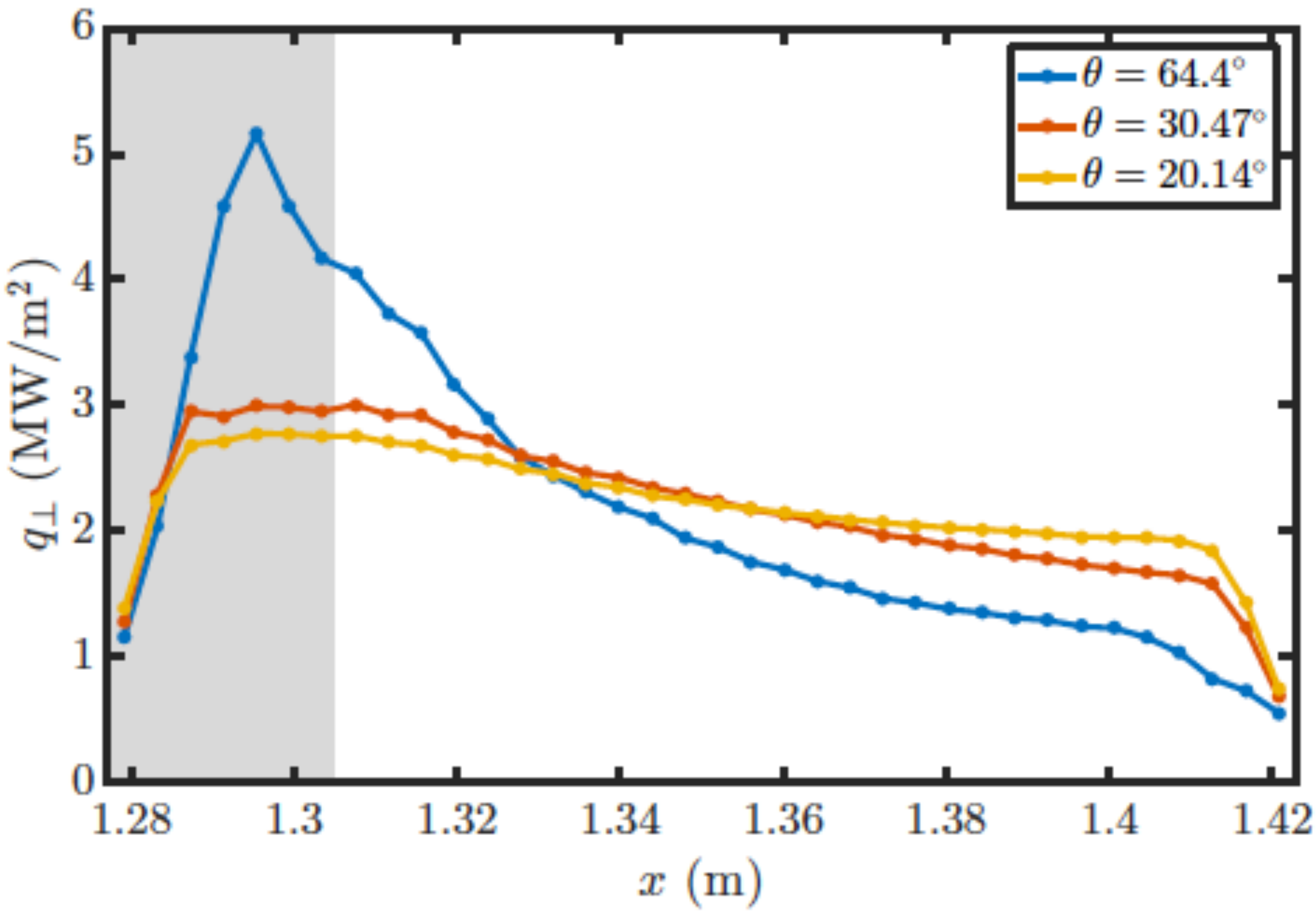
Toroidal case (left) vs. Slab case (right)



Clearly shows bad curvature enhances instability drive

E. Shi (Ph.D. 2017)

Divertor heat flux broadens ~ theta ~ 1/B_pol



E. Shi (Ph.D. 2017)

(Present simulation neglects magnetic shear and related stabilization near x-point, shortened parallel length to divertor plates to approximately compensate.)

Larger amplitude & more intermittent blobs in far SOL

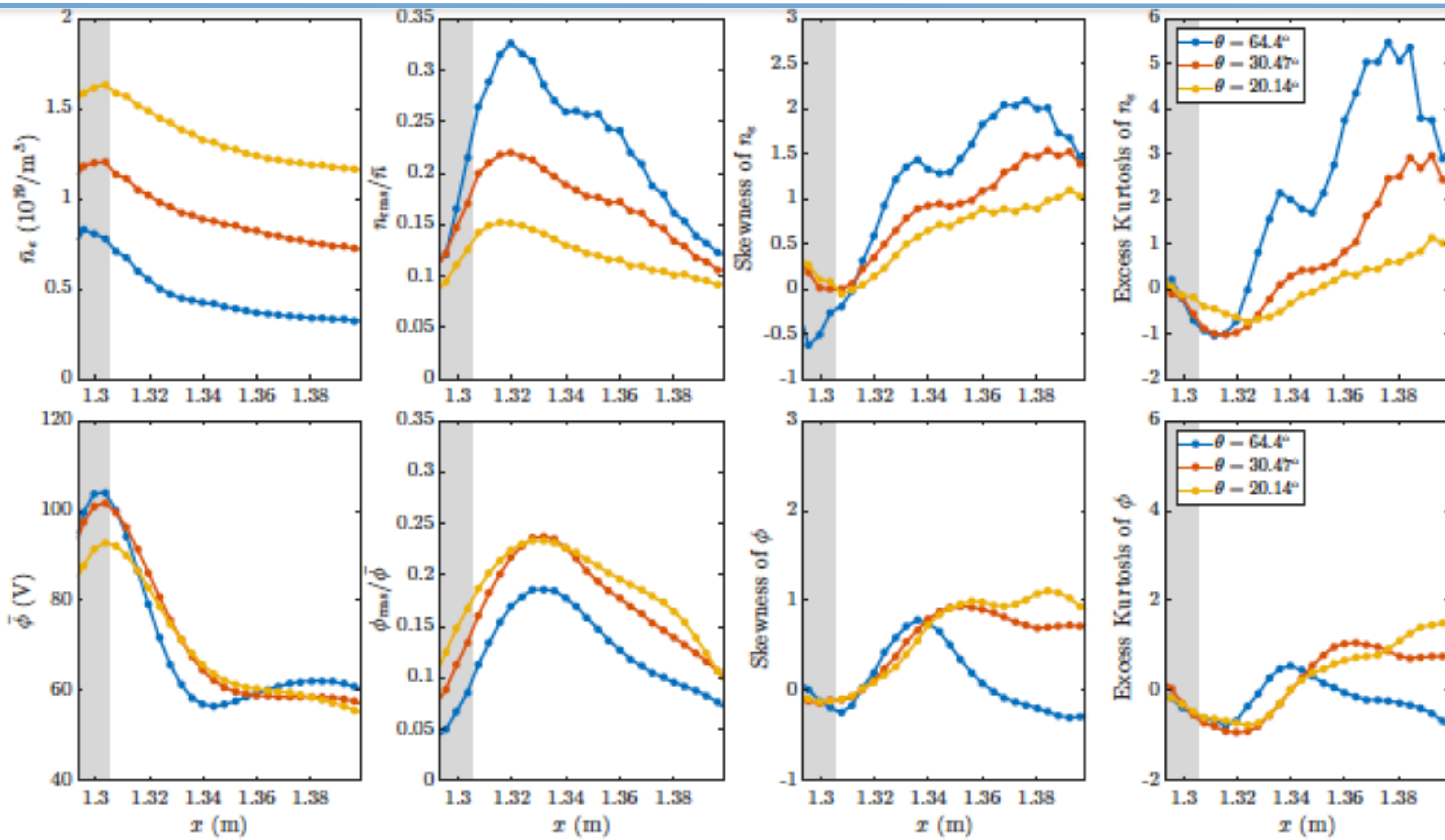


Figure 5.13: Electron density fluctuation statistics (top row) and potential fluctuation statistics (bottom row) computed near the $z = 0$ m plane for three cases with different magnetic field line incidence angle θ. The potential fluctuations are notably less intermittent than the density fluctuations. The shaded area indicates the region in which the source is concentrated.

E. Shi (Ph.D. 2017)

Runge phenomena

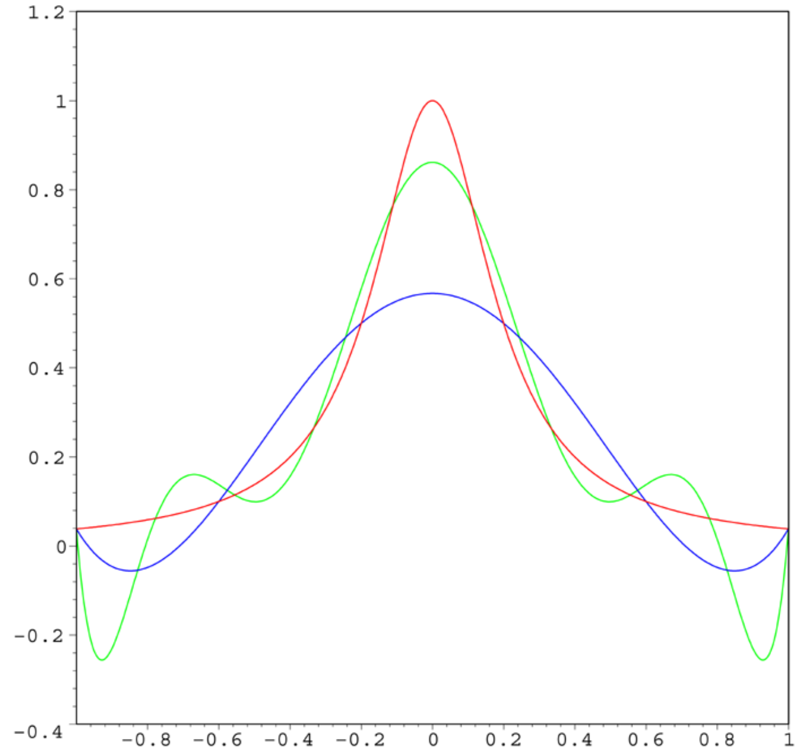
Red: $f(x)=1/(1+25x^2)$

Blue: 5th order polynomial
(6 equally-spaced points)

Green: 9th order polynomial
(10 equally-space points)

Polynomials fit to equally-spaced points actually diverges near bdy as N→infity.

(Taylor-series radius of convergence is distance to closest pole, at x=+/-i/5)



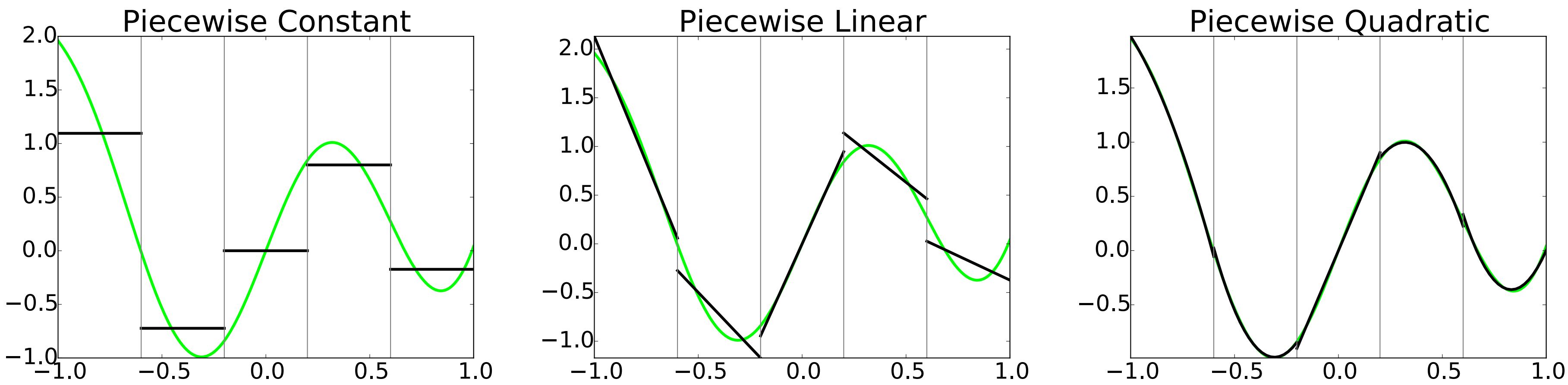
Can be fixed by:

(1) special placement of points: closer to edge. (Chebyshev nodes)
(2) related: do least-squares fit of a nth polynomial over x=[-1,1], not just at points,
(3) piecewise-polynomials (splines/DG): don't try to fit full domain with single polynomial…

(1) & (2) don't help with discontinuities, (3) does. Might think discontinuities only relevant to problems with shocks, but small-scale features that cascade to arbitrarily small scales (unless diffusion is important) will be naturally produced in many systems, by turbulence and other nonlinear dynamics.

https://en.wikipedia.org/wiki/Runge's_phenomenon

Discontinuous Galerkin (DG) Combines Attractive Features of Finite-Volume & Finite Element Methods

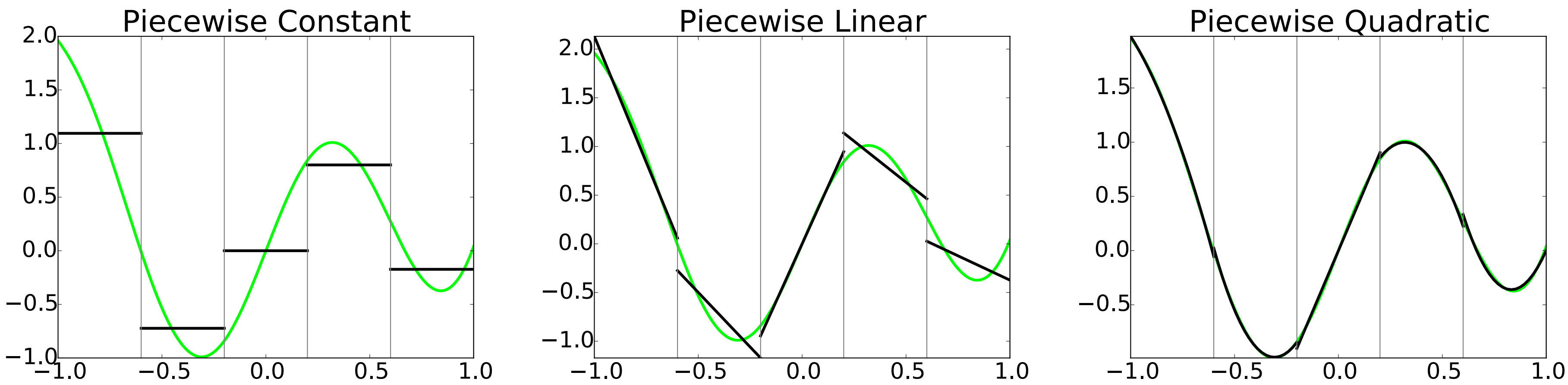


Standard finite-volume (FV) methods evolve cell averages + interpolations.

DG evolves higher-order moments in each cell. I.e. uses higher-order basis functions, like finite-element methods, but, allows discontinuities at boundary like shock-capturing finite-volume methods --> (1) easier flux limiters like shock-capturing finite-volume methods (preserve positivity) (2) calculations local so easier to parallelize.

Hot topic in CFD & Applied Math: >1500 citations to Cockburn & Shu JCP/SIAM 1998

Discontinuous Galerkin (DG) Combines Attractive Features of Finite-Volume & Finite Element Methods



Don't get hung up on the word "discontinuous". Simplest DG is piecewise constant: equivalent to standard finite volume methods that evolve just cell averaged quantities. Can reconstruct smooth interpolations between adjacent cells when needed.

Need at least piecewise linear DG for energy conservation (conserves energy even with upwinding). Standard Finite Volume methods do not conserve energy exactly (except Arakawa, which has overshoots). Unlike Navier-Stokes fluid eqs., energy conservation in kinetic/Vlasov-Boltzmann equations is indirect, involving integration-by-parts and particle-field energy exchange.

Maxwellian-Weighted DG Basis Functions

Standard DG Polynomial Basis Functions:

$$\frac{\partial f(v,t)}{\partial t} = G[f]$$

In each cell Ω_j, expand in basis fcns: $f(v,t) \approx f_h(v,t) = \sum_k f_k(t) b_k(v)$

Choose $\dot{f}_k = df_k/dt$ to minimize error: $\epsilon^2 = \int_{\Omega_j} dv \left(\sum_k \dot{f}_k b_k - G \right)^2$

Error projected into space of $b_k(v)$ is zero: $\int_{\Omega_j} dv\, b_k(v) \left(\dot{f}_h - G \right) = 0$

If $G = -\partial\Gamma/\partial v$, then $b_0(v) = 1$ give density conservation:

$$\int_{\Omega_j} dv\, \dot{f}_h = -\Gamma(v_{j+1/2}) + \Gamma(v_{j-1/2})$$

(This is the essence of DG, combined with efficient evaluation of integrals & Godunov approach of a Riemann solver / upwind fluxes at discontinuous boundaries.)

Standard Maxwellian-Weighted DG Basis Functions:

For many plasma problems of interest, we know Maxwellian-weighted basis functions would be more efficient. Polynomial basis functions are ill-behaved at high v, can't integrate to v = ∞, where asymptotic behavior is Maxwellian (perhaps w/ higher "temperature"). Helps handle moderate collision frequencies of edge region.

$$f(v,t) \approx f_h(v,t) = \sum_k f_k(t) \underbrace{\exp(-\beta v^2/2) b_k(v)}_{\hat{b}_k(v)}$$

$$\text{Minimizing error leads to: } 0 = \int_{\Omega_j} dv\, \hat{b}_k(v) \left(\dot{f}_h - G \right)$$

But now, $\hat{b}_0 = \exp(-\beta v^2/2)$ does *not* lead to standard particle conservation if $G = -\partial\Gamma/\partial v$

$$\int_{\Omega_j} dv\, \hat{b}_0 \dot{f}_h = - \left. \hat{b}_0(v)\Gamma(v) \right|_{v_{j-1/2}}^{v_{j+1/2}} + \int_{\Omega_j} dv \frac{\partial \hat{b}_0}{\partial v} \Gamma(v)$$

Standard energy conservation doesn't hold either.

Conservative Maxwellian-Weighted DG Basis Functions:

The trick for preserving conservation properties of DG with Maxwellian-weighted basis functions, $\hat{b}_k(v) = W(v) b_k(v)$, starts by going back to beginning, to the norm defining the error, and introducing a weighting factor:

$$\epsilon^2 = \int_{\Omega_j} dv\, W^{-1}(v) \left(\sum_k \dot{f}_k \hat{b}_k(v) - G \right)^2$$

Choosing $\dot{f}_k$ to minimize error gives:

$$\int_{\Omega_j} dv\, W^{-1}(v) \hat{b}_m(v) \left(\sum_k \dot{f}_k \hat{b} - G \right) = 0$$

$$\int_{\Omega_j} dv\, b_m(v) \left(\sum_k \dot{f}_k \hat{b}_k - G \right) = 0$$

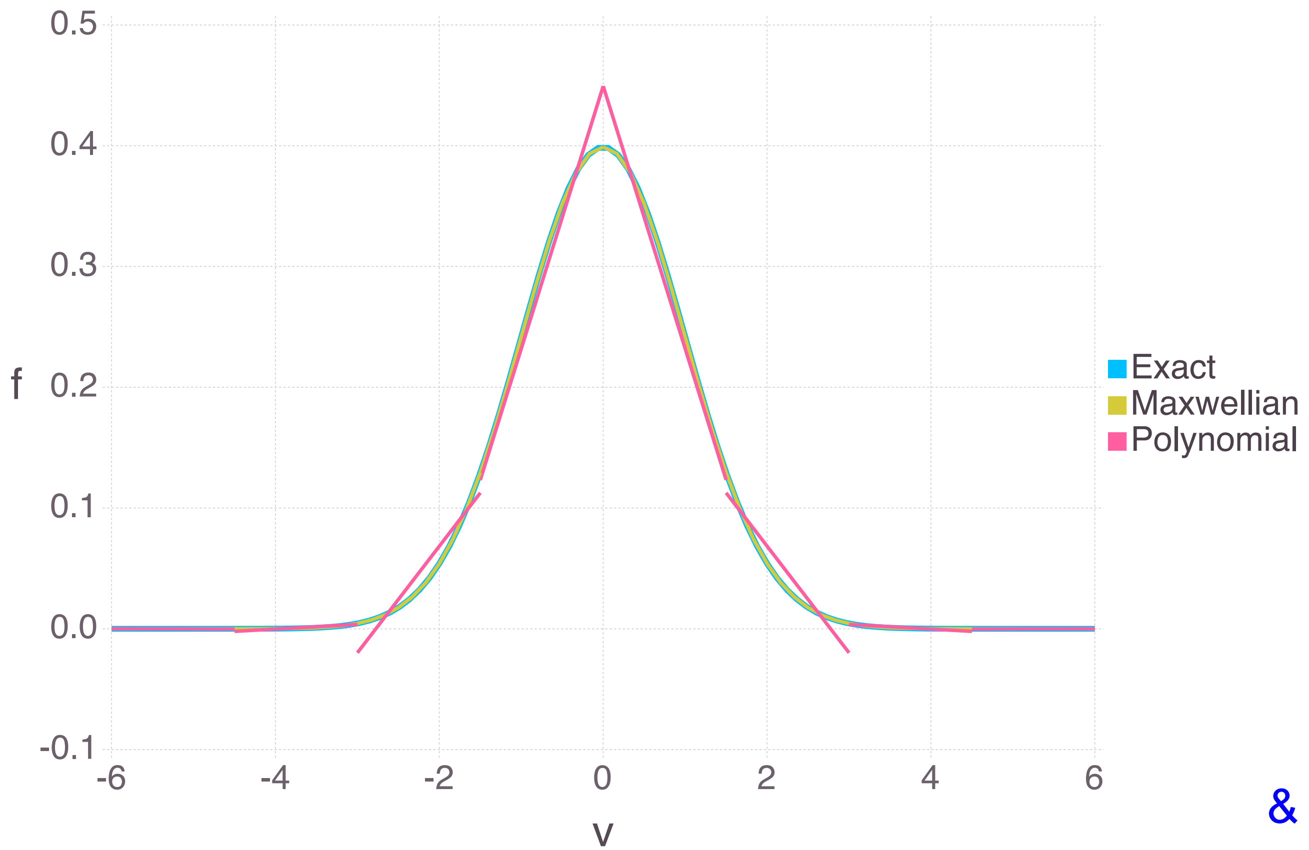
Now $b_0(v) = 1$ gives standard particle conservation. Higher moments give momentum and energy conservation for collision operator (Hamiltonian terms more complicated..., see A. Hakim's poster.)

Weighted DG can be thought of as Petrov-Galerkin, test fncs $\neq$ basis fcns

Collision Operator Benchmark

Compare Maxwellian-weighted and polynomial basis functions by solving the equation (Lenard-Bernstein collision operator)

$$\frac{\partial f}{\partial t} = C[f] = \nu \frac{\partial}{\partial v_{\|}} \left(v_{\|} f + v_T^2 \frac{\partial f}{\partial v_{\|}} \right)$$



Eric Shi
& G. Hammett

Example Using Local Maxwellian Parameters

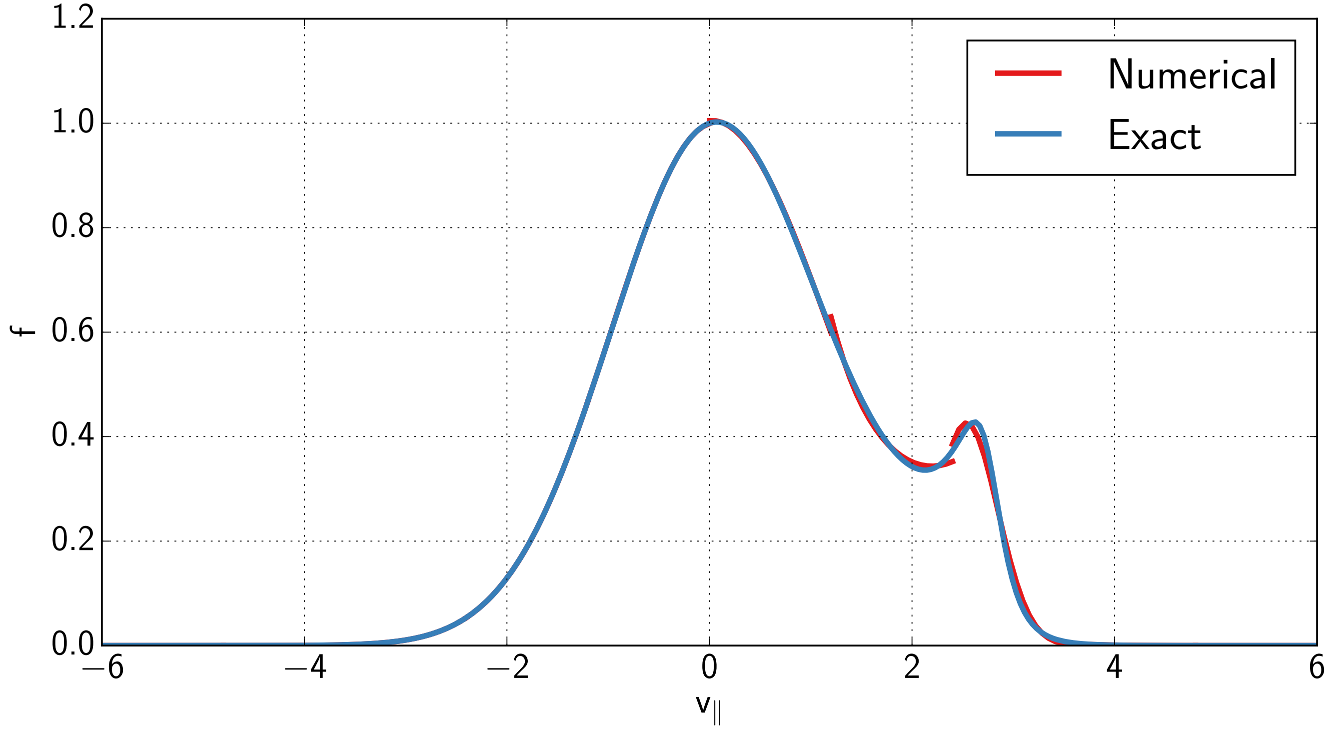


Figure: The local Maxwellian parameter calculation is applied to discretize a function including a non-monotonic bump to demonstrate the ability to handle strongly non-Maxwellian functions.

1D Test problem: Classical Parallel Heat Conduction

$$\frac{\partial f(z, v_{||}, t)}{\partial t} + v_{||}\frac{\partial f}{\partial z} = C[f]$$

Background temperature gradient (w/ force balance), Chapman-Enskog-Braginskii problem locally becomes equivalent to 1D problem:

$$\frac{\partial f(v_{||}, t)}{\partial t} = C[f] + \kappa_T\, v_{||}\left(\frac{1}{2}\frac{v_{||}^2}{v_t^2} - c_1\right) f$$

($\kappa_t \ll 1$. c_1 determined by constraint of no momentum injection.)
Lenard-Bernstein Collision model (much better than Krook model for plasmas):

$$C[f] = \frac{\partial}{\partial v_{||}}\left(\nu v_{||} f + \nu v_t^2 \frac{\partial f}{\partial v_{||}}\right)$$

Solve to steady state, calculate heat flux $= \int dv_{||}(1/2)mv_{||}^3 f$.

Maxwellian-weighted basis functions much more efficient

4x faster in 1v,
16x faster in 2v

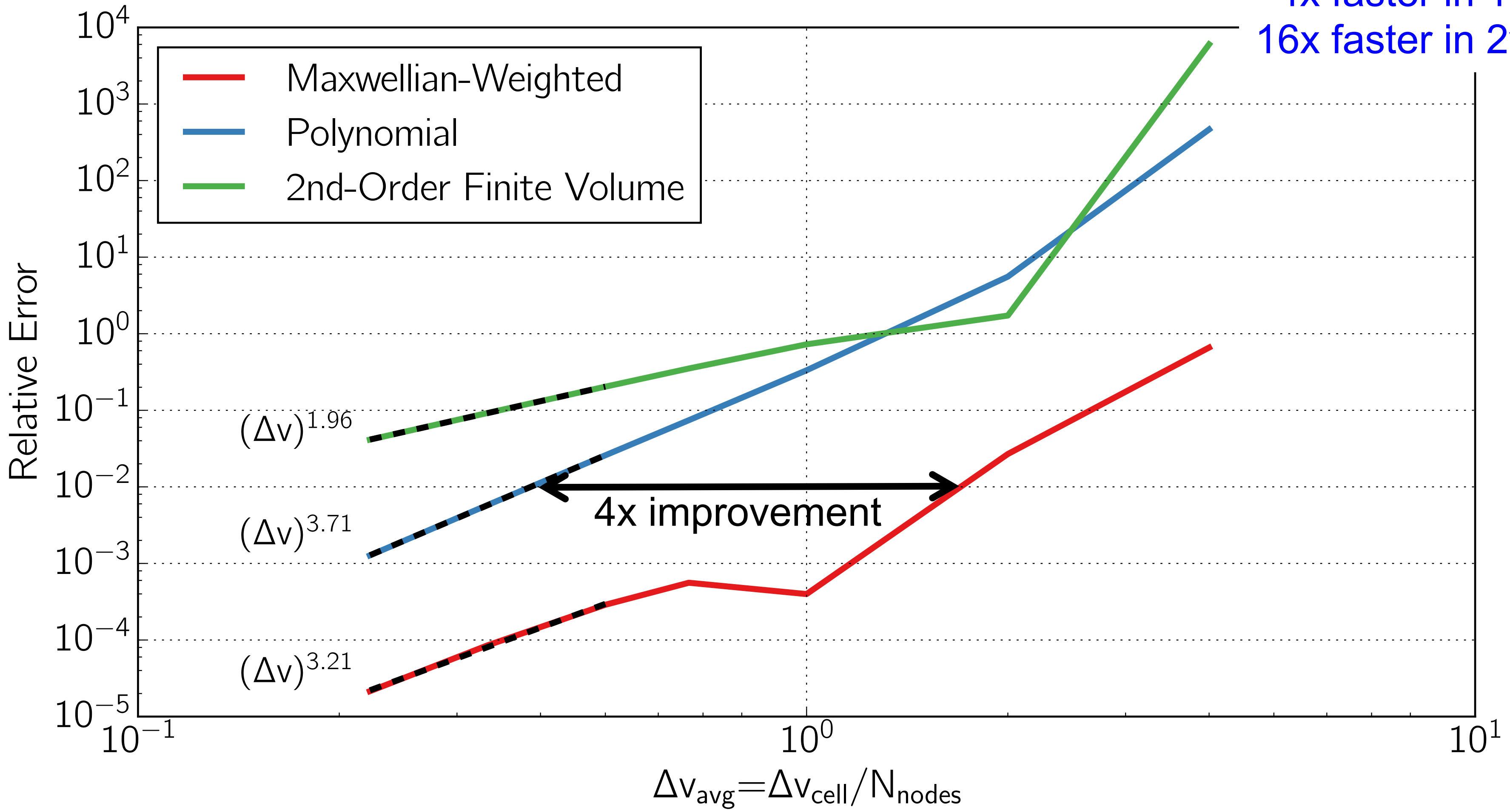


Figure: Relative error in heat flux calculation for cases of varying cell width, keeping $v_{\mathrm{max}} = 8 v_T$.