Euler and Infinity

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In Paul Hoffman's biography of the prolific and eccentric mathematician Paul Erdös (whose Hungarian name is pronounced "air-dish"), he describes one of the interesting contributions of the famous mathematician Leonhard Euler, stating that Euler is responsible for "perhaps the most concentrated and famous formula in all of mathematics"[1]:

$$e^{i\pi} + 1 = 0, \tag{1}$$

which brings together 5 of the most important numbers in mathematics: 1, 0, π, e, i, in the order they are usually learned in school. [More info on Euler and this equation can be found at *http://mathforum.org/dr.math/faq/faq.euler.equation.html*.]

While sitting in a little restaurant in Turin, Italy one night in the spring of 2002, I was briefly discussing this with some colleagues from a plasma physics conference. One of them, Emilia Solano, pointed out that this equation neglected to include one of the most important and famous of all numbers: infinity. After a moment's reflection we realized that this could be achieved with a simple modification of the equation:

$$\frac{1}{e^{-i\pi} + 2^0} = \infty, \tag{2}$$

which extends the original equation to include not only ∞, but also 2, arguably a significant step beyond the concept of 1, and brings in the concept of negative numbers as well as all of the basic operations of arithmetic: addition, subtraction (implied by using negative numbers), multiplication and division.

Some one else has probably pointed this out before, but I just thought it was a fun little trivial observation to post on my web page.

Acknowledgments

I first learned of Eq.[1] from Prof. Bill Dorland, and thank Mr. Geoffrey Plowden for recommending Ref.[1].

References

[1] Paul Hoffman, *The Man Who Loved Only Numbers*, Hyperion (1998), p.212.