A Renormalized Theory of Noise & Its Effect on Particle-In-Cell Simulations of Plasma Turbulence

G.W. Hammett (PPPL)
W.M. Nevins, A.M. Dimits, D.E. Shumaker (LLNL)
W. Dorland (U. Maryland)

Dec. 2, 2005 Columbia Plasma Physics Colloquium

http://w3.pppl.gov/~hammett/papers, Phys. Plasmas, Dec. 2005

PPPL PRINCETON PLASMA PHYSICS LABORATORY

UNIVERSITY OF MARYLAND

Acknowledgments: S. Cowley, B. Cohen, F. Jenko, J. Krommes, M. Kotschenreuther

Outline

- Motivation: Jenko and Dorland continuum gyrokinetic simulations found that ETG turbulence could be large in some regimes. Recent PIC simulations by Z. Lin (IAEA 2004, APS 2004) claimed much lower ETG turbulence. Investigate particle noise as possible explanation of differences between the two simulations.

- Simple estimate of the spectrum of potential fluctuations due to a discrete number of gyrokinetic particles, shielded by an adiabatic species. Within a factor of 2 of the more detailed calculation based on an extension of Krommes' 1993 using the classic fluctuation-dissipation theorem or test-particle superposition principle.

- Noise spectrum agrees very well with Dimit's gyrokinetic PIC ETG simulations (with no free parameters!)

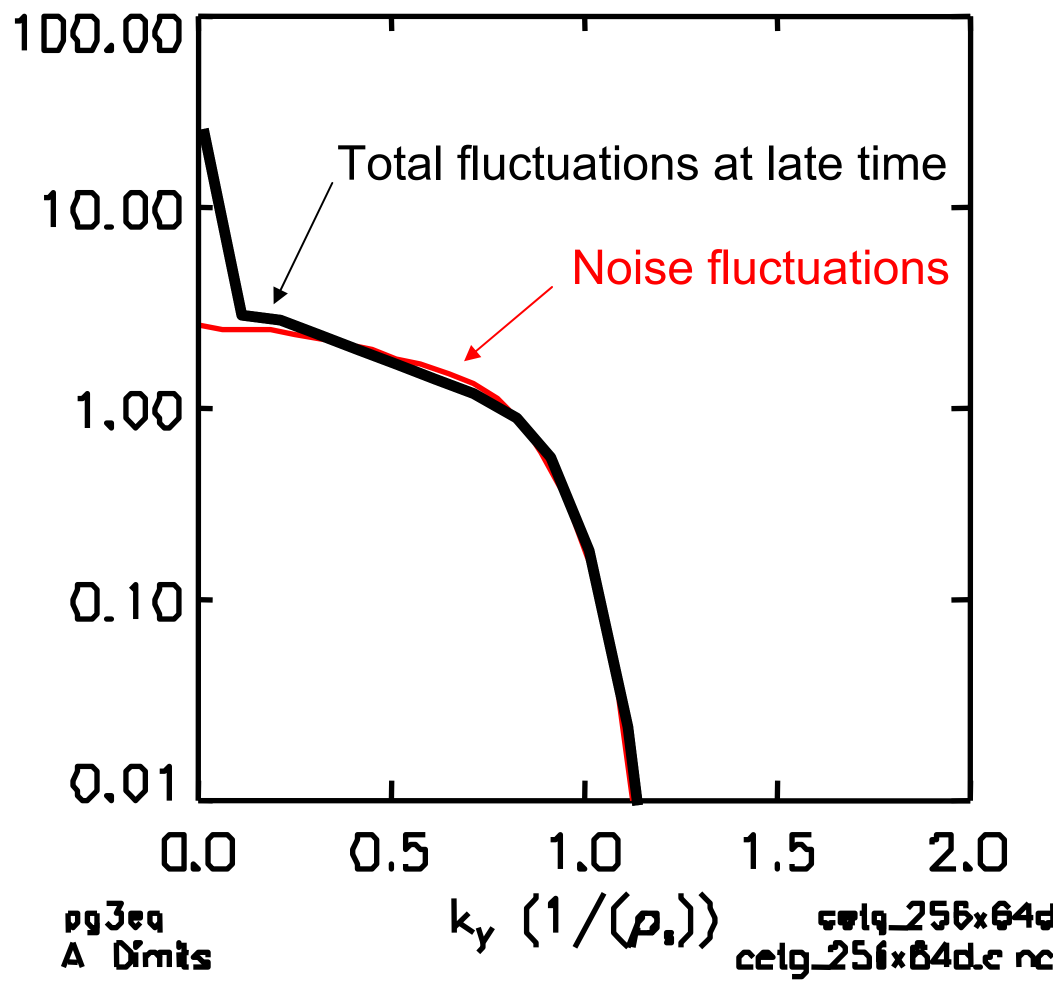
- Renormalized calculation of noise-induced ExB diffusion, D_{noise}, for a test-particle in this spectrum of random potential fluctuations. Agrees very well with observed χ at late times when noise dominates (again with no free parameters).

Predicted Noise Spectrum Agrees Well with Dimits PIC ETG Simulation at Late Times

- No Free Parameters in Theory!

- Discrete particle noise in PIC codes is quantifiable — well studied in past:
  - Langdon '79 – Birdsall&Langdon '83, Krommes '93

⇒ Useful code verification tool. We can develop objective criteria to determining when discrete particle noise is a problem



Isn't Electron Temperature Gradient (ETG) Turbulence Too Weak?

- ◘ ETG modes with $k_\perp \rho_e \sim 1$ with adiabatic ion response are (nearly) isomorphic to
  ITG modes with $k_\perp \rho_i \sim 1$ with adiabatic electron response
  therefore 60× smaller ?

$$\chi_i = \chi_{i0}\left(\frac{v_{ti}}{L_T}\right)\rho_i^2$$

$$\chi_e = \chi_{e0}\left(\frac{v_{te}}{L_T}\right)\rho_e^2 = \sqrt{\frac{m_e}{M_i}}\chi_i$$

- But Jenko & Dorland (PRL 2002, PoP 2000) found that isomorphism broken by nonlinearly by difference in ion/electron adiabatic response:
  zonal flows reduced in ETG -->
  In some regimes ETG turbulence increases: $\chi_{e0} >> \chi_{i0}$ so that $\chi_e \sim \chi_i$

Stiff critical gradients can cause $\chi_e \sim \chi_i$ even if χ_{e0} isn't as big as 60 χ_{i0}.

$$\chi_i = \chi_{i0}\left(\frac{v_{ti}}{R}\right)\rho_i^2\left(\frac{R}{L_{Ti}} - \frac{R}{L_{Ti,crit}}\right)$$

$$\chi_e = \chi_{e0}\left(\frac{v_{te}}{R}\right)\rho_e^2\left(\frac{R}{L_{Te}} - \frac{R}{L_{Te,crit}}\right)$$

ITG threshold improved by high Ti/Te, impurity dilution
Ions may be close to critical gradient,
Electrons may be further above critical gradient

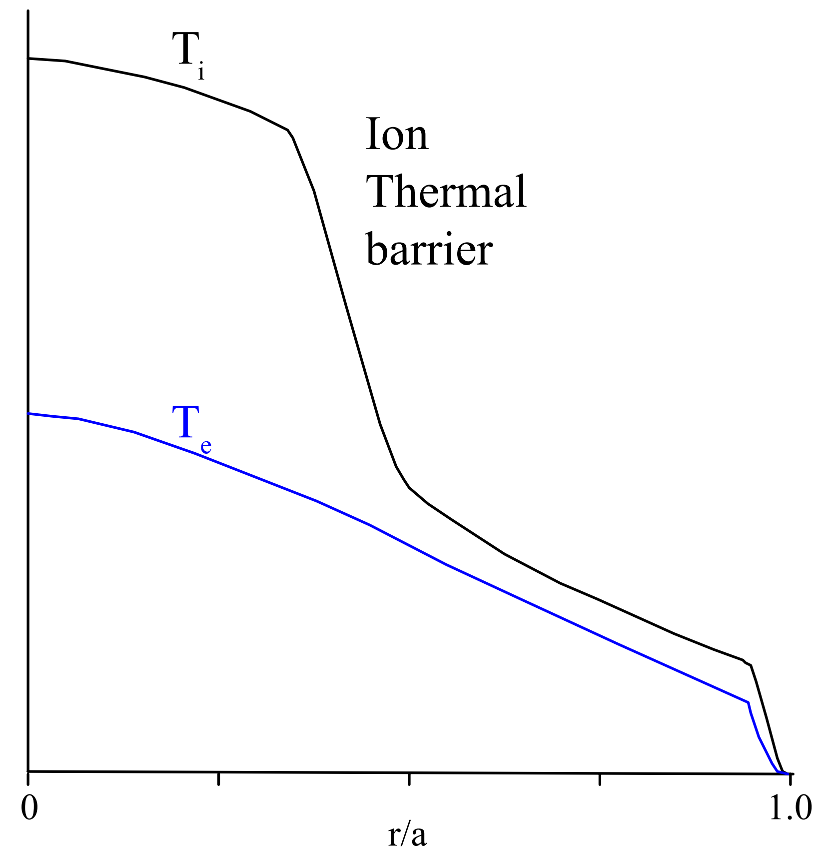
Why do we care about ETG modes?

- Ion Thermal barriers w/o corresponding electron thermal barrier
- Electron thermal transport doesn't always turn off with ion transport

  ⇒ Mechanisms which transports electrons only:

  - Broken flux surfaces

  ⇒ Instabilities with $\lambda \sim \rho_e$



Jenko & Dorland found ETG turbulence >> ITG turbulence (in Gyro-Bohm units)

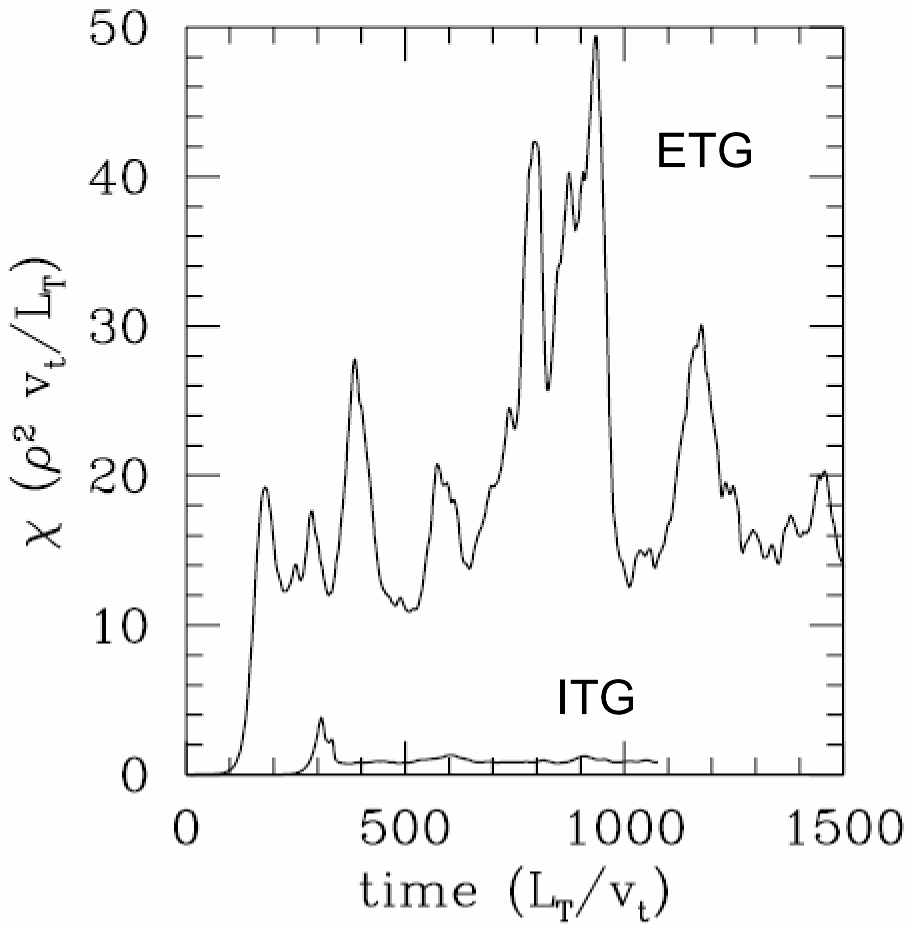


FIG. 1. χ_e^{ETG} (upper curve) and χ_i^{ITG} (lower curve) for similar parameters.

(Dorland & Jenko 2000, see also Jenko & Dorland 2002: with larger box, Lx=512 ρ, report χ_e = 13)

ETG eddies are radially extended streamers

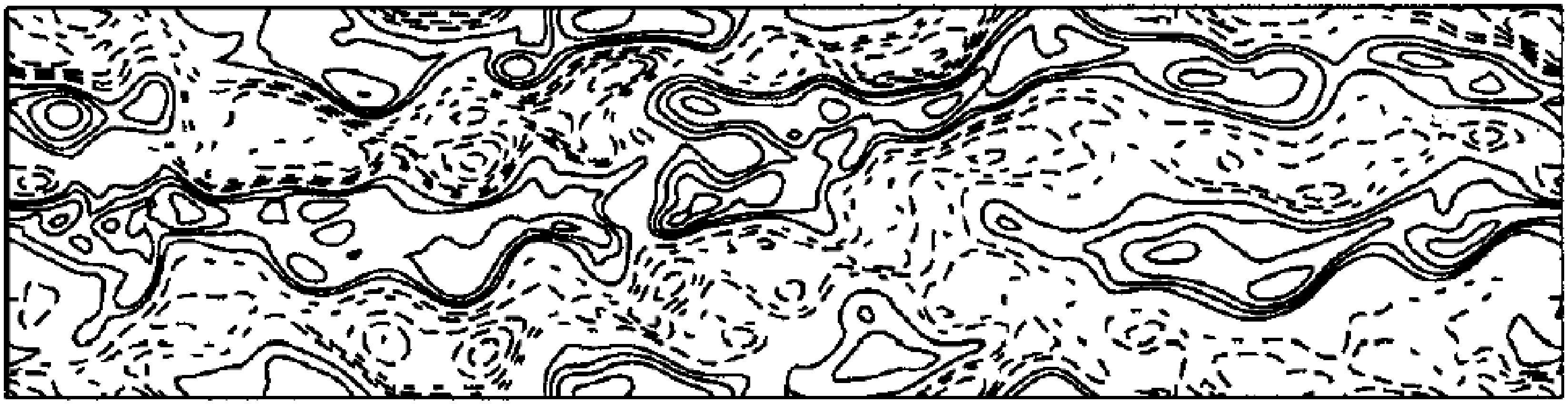


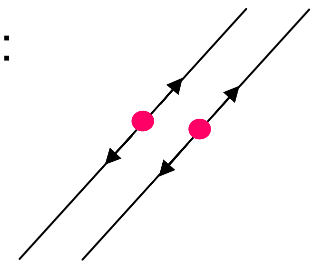
FIG. 2. Characteristic ϕ contours in the outboard x-y plane. This snapshot was taken at the end of the ETG run shown in Fig. 1. The figure is $256\rho_e \times 64\rho_e$.

High ETG transport relative to ITG transport theoretically understood as due to difference in adiabatic response for ions vs. electrons ==> reduces ETG zonal flows ==> ETG streamers get to higher velocity and are more elongated. (Rogers & Dorland, Jenko & Dorland 2000, 2002, etc.)

(Jenko & Dorland 2000)

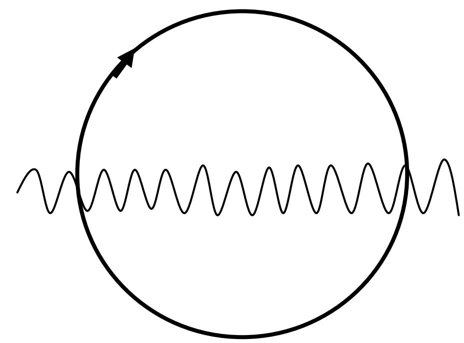
Key ITG/ETG Difference:
different adiabatic response to zonal flows

ITG turbulence, adiabatic electron response:



electrons ***don't*** respond to zonal flows ($k_{||}$=0, pure E_r).
since electrons are adiabatic because $k_{||} v_{te} >> \omega$

ETG turbulence, adiabatic ion response:



Ions do shield zonal flows for ETG
Since ions are adiabatic because $k_{\perp}\rho_i >> 1$.
↓ zonal flows --> streamers elongate --> transport ↑

Detailed secondary/tertiary instability analysis includes this, explains ITG/ETG saturation level differences, scalings (Rogers, Dorland, Jenko papers)

Key ITG/ETG Difference: different adiabatic response to zonal flows

ITG turbulence, adiabatic electron response:

$$n_e = n_i$$

$$n_{e0} \frac{e}{T}\left(\Phi - \left\langle \Phi \right\rangle\right) = \int d^3v\, J_0 f_i - n_{i0}\left(1 - \Gamma_0(k_\perp \rho_i)\right)\frac{e}{T}\Phi$$

Flux-surface averaged potential, electrons adiabatic because $k_{||}=v_{te} >> \omega$ don't respond to zonal flows ($k_{||}=0$, pure E_r).

ETG turbulence, adiabatic ion response:

$$n_i = n_e$$

$$n_{i0} \frac{e}{T}\Phi = \int d^3v\, J_0 f_e + n_{ei0}\left(1 - \Gamma_0(k_\perp \rho e)\right)\frac{e}{T}\Phi$$

Ions adiabiatic because $k_\perp \rho_i >> 1$. Ions CAN shield zonal flows.
↓ zonal flows --> streamers elongate --> transport ↑

Detailed secondary/tertiary instability analysis includes this, explains ITG/ETG saturation level differences, scalings (Rogers, Dorland, Jenko papers)

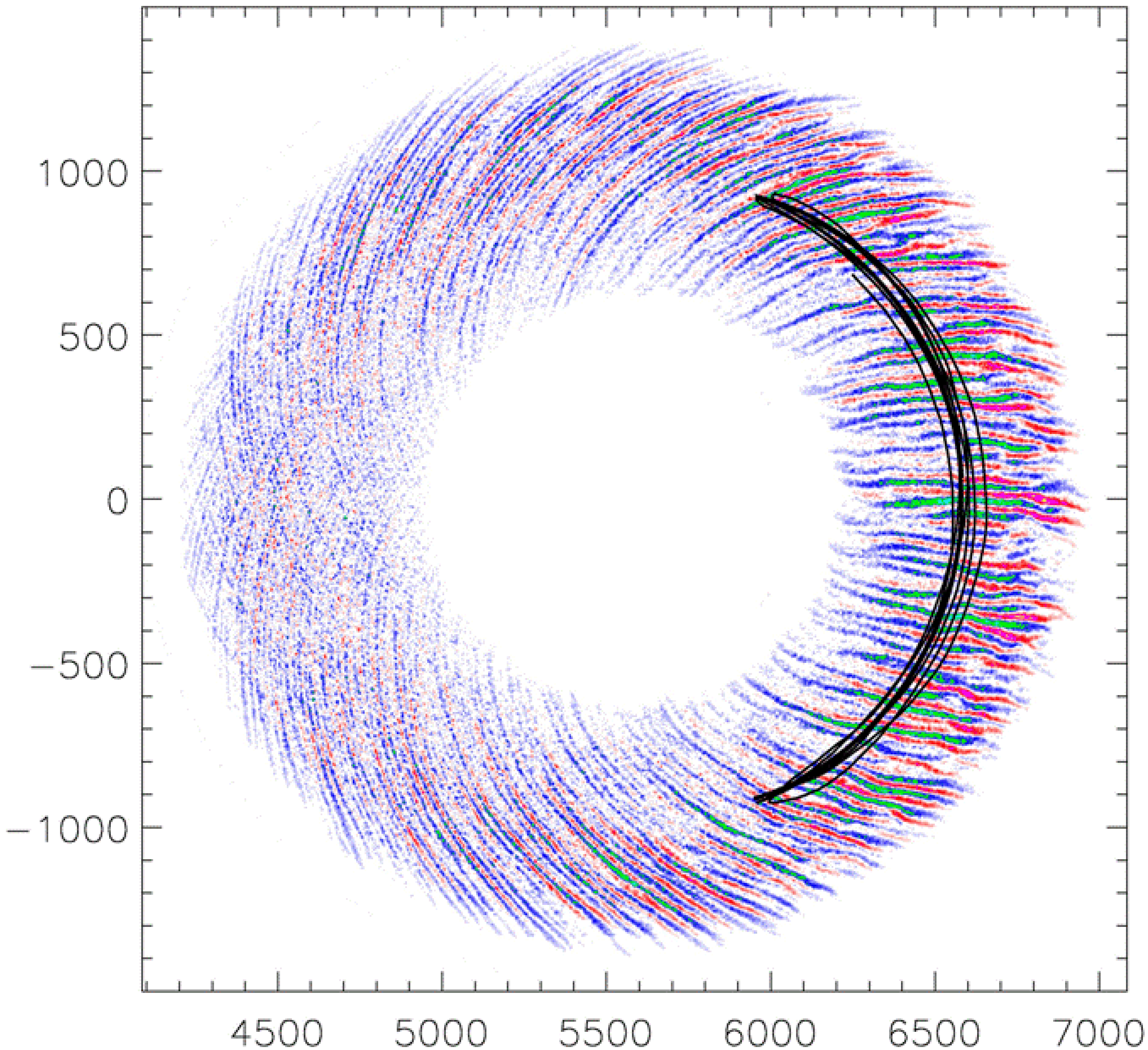
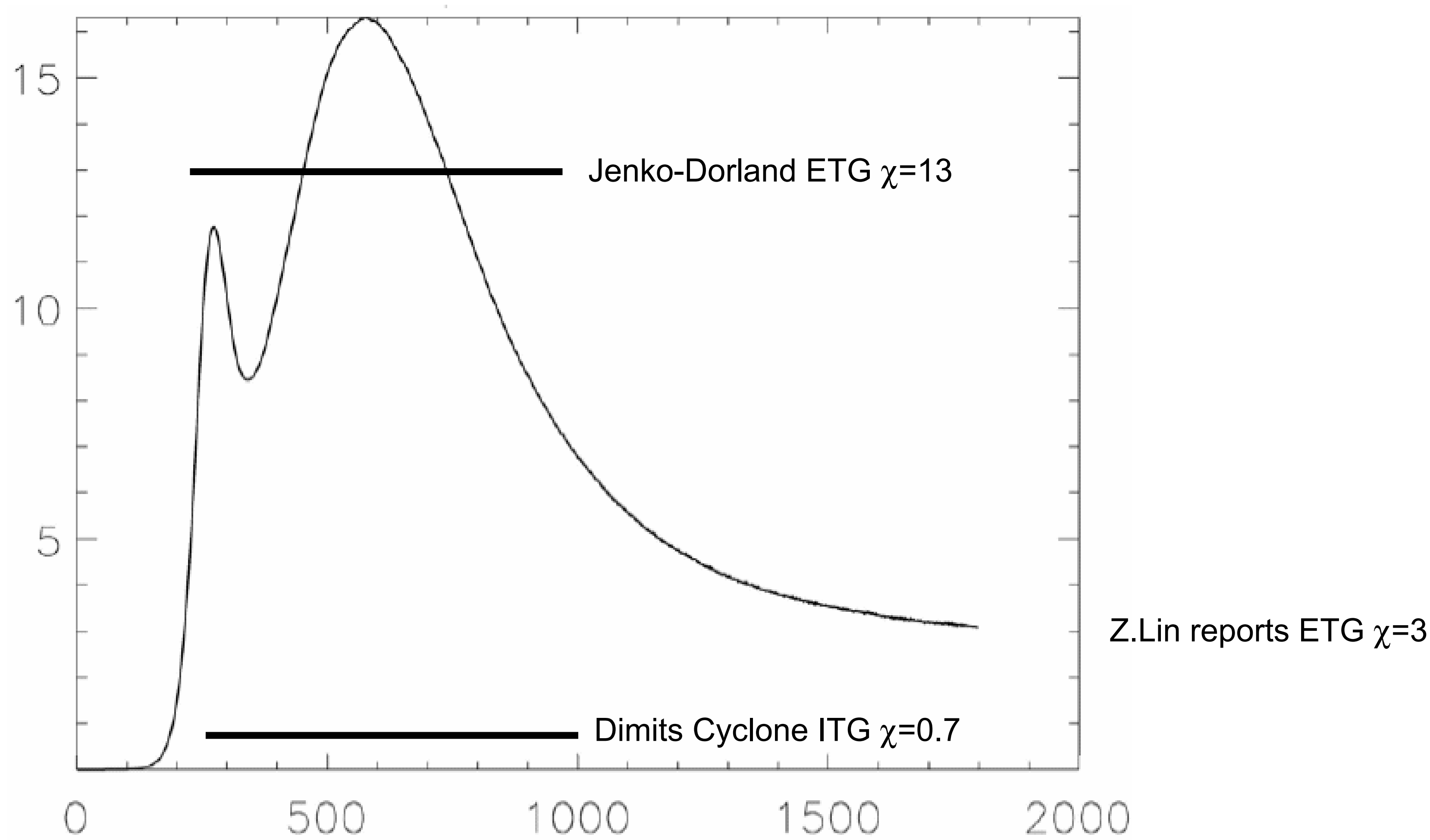


Fig. of etg streamers
from Z. Lin global PIC
simulations IAEA 2004



From Z.Lin's IAEA 2004 slides (at URL below). Believed to be $\chi_{etg}(t)$. Initial large values of χ_{etg} comparable to Jenko-Dorland 2002 $\chi_{etg} \sim 13$. Ignoring initial "transient", reported result is $\chi_{etg} \sim 3$. Scanned 5 to 20 particles/cell.

http://www.cfn.ist.utl.pt/20IAEAConf/presentations/T5/2T/5_H_8_4/Talk_TH_8_4.pdf

Standard approach to discrete particle noise

Particle discreteness ==> Fluctuations ==> Collision operator

Klimontovich Eq. ==> Vlasov Eq. + Collisions C(f)

diffusion in velocity
from $<\delta E^2>$ fluctuations
$\propto 1/(n\lambda_D^3)$

Gyrokinetic Klimontovich Eq. ==> Gyrokinetic Eq. + $C_{GK}(f)$

diffusion in g.c. position
from $<(\delta ExB)^2>$ fluctuations
$\propto 1/(nV_{smooth})$

Various standard approaches to calculating C(f): binary collision operator cut off at Debye shielding scale, BBGKY hierarchy, etc.

Krommes' Calculation of Gyrokinetic Noise Spectrum

Krommes' 1993 calculation of the gyrokinetic noise spectrum uses the classic fluctuation-dissipation theorem, and shows equivalent results from the test-particle superposition principle (shielded test particles can be treated as independent). (see also W.W. Lee 1987, classic paper by A.B. Langdon 1979)

Krommes' calculation used shielding by linear dielectric from gyrokinetic equation in a slab, uniform plasma. Hu & Krommes 94 extended to δf.

We have extended Krommes' test-particle superposition calculation to:

- Treat one species as adiabatic instead of with particles.

- Include factors for finite-size particle shape S (accounts for interpolation of particle charge to grid, and forces from grid to particles) & S_{filt} factor for explicit filtering of Φ. Important for quantitative comparisons.

- Use a renormalized dielectric, including a $k_\perp^2 D_{NL}$ term on the non-adiabatic part of the shielding cloud, and including random walks in the test particle trajectories instead of assuming straight-line trajectories. Affects frequency spectrum of fluctuations, but not the frequency-integrated k spectrum.

Applying thermal noise to non-equilibrum systems

Can't directly apply Fluctuation-Dissipation Theorem or Test-Particle Superposition Principle to a linearly unstable plasma, because they use the dielectric response to calculate particle correlations and shielding, and rely on all poles being in lower half ω plane. In an unstable plasma, the linear dielectric leads to amplification of noise, not shielding.

Standard approach is indirect: use FDT or TPSP to calculate spectrum of fluctuations and the resulting collision operator C(f) in thermal equilibrium, then use that C(f) to study collisional effects on non-equlibrium problems (e.g., effects of collisions on trapped particle modes or reconnection). Good approximation if sufficient separation of time/space scales.

Our calculation of $|\Phi_{noise}(k,\omega)|^2$ spectrum and D_{noise} follow similar approach. D_{noise} dominated by high $k_\perp$ & high $k_{||}$, so a self-consistent regime exists where

$$\nu_{noise} \sim |k_{||}|\, v_t + k_\perp^2 D_{noise} >> \gamma, \omega_* \Gamma_0$$

Alternatively, as suggested in Hu & Krommes '94, etc., one could use a renormalized nonlinear dielectric to model nonlinearly saturated turbulent system. (All perturbations decay in nonlinearly saturated system. Future work…)

Simple Estimate of Noise: Randomly Positioned Particles

Within a factor of 2 of more detailed theory because one species is adiabatic, which provides half of the shielding in thermal equilibrium.

Fourier conventions:
$$\Phi(\vec{x}) = \sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}} \tilde{\Phi}_{\vec{k}}$$

$$\tilde{\Phi}_{\vec{k}} = \frac{1}{V}\int_V d^3x e^{-i\vec{k}\cdot\vec{x}} \Phi(\vec{x})$$

Quasineutrality: Adiabatic species + polarization density = “bare” guiding center contribution

Gyrokinetic Poisson Eq:
(W.W. Lee, Phys. Fluids ’83)
$$n_0 \frac{e\Phi}{T} + n_0 k_\perp^2 \rho^2 \frac{e\Phi}{T} = S_{filt}\int d^3\mathrm{v} J_0 \delta f$$
$$= S_{filt}\sum_i w_i J_{0i}\delta(\vec{x}-\vec{x}_i)$$

Fourier transform:
$$n_0\left(2-\Gamma_0\right)\frac{e\tilde{\Phi}_k}{T} = \frac{S_{filt}}{V}\sum_i w_i J_{0i} e^{-i\vec{k}\cdot\vec{x}_i}$$

$$\left|\frac{e\tilde{\Phi}_k}{T}\right|^2 = \frac{S_{filt}^2}{n_0^2 V^2 (2-\Gamma_0)^2}\sum_i\sum_j w_i w_j J_{0i} J_{0j} e^{-i\vec{k}\cdot\left(\vec{x}_i-\vec{x}_j\right)}$$

Averages to zero unless i=j

Simple Estimate of Noise: Randomly Positioned Particles (II)

Average over uncorrelated random particles:

$$\left\langle \left| \frac{e\tilde{\Phi}_k}{T} \right|^2 \right\rangle_N = \frac{S_{filt}^2}{n_0^2 V^2 (2-\Gamma_0)^2} \sum_i w_i^2 J_{0i}^2$$

$$= \frac{S_{filt}^2(\vec{k})}{(n_0 V)^2 (2-\Gamma_0)^2} n_0 V \left\langle w_i^2 \right\rangle \Gamma_0$$

$$\left\langle \left| \frac{e\Phi(\vec{x})}{T} \right|^2 \right\rangle = \sum_k \left\langle \left| \frac{e\tilde{\Phi}_k}{T} \right|^2 \right\rangle_N$$

$$= \frac{\left\langle w_i^2 \right\rangle}{n_0 V} \sum_k \frac{S_{filt}^2}{(2-\Gamma_0)^2} \Gamma_0 = \frac{\left\langle w_i^2 \right\rangle}{n_0 V_{smooth,N}}$$

Noise power scales with 1/(Number of particles per smoothing volume)
V_{smooth} ~ 150 cells ~ $(5.3)^3$ cells for Dimits' smoothing parameters

Quantifying Particle Discreteness (2)

(a partially correlated fluctuation spectrum)

- More detailed calculation following Krommes93 gyrokinetic test-particle superposition calculation, including dielectric shielding in kinetic response, numerical filtering/interpolation factors, resonance broadening renormalization:

$$\left\langle \left| \frac{e\phi_k}{T} \right|^2 \right\rangle_H = \frac{\left\langle w_i^2 \right\rangle S_{filter}^2(k) S^2(k) \Gamma_0(k_\perp^2 \rho_{th}^2)}{N_p \left[2-\Gamma_0(k_\perp^2 \rho_{th}^2)\right]\left[2-\left(1-S_{filter} S^2 d_\parallel(k)\right)\Gamma_0(k_\perp^2 \rho_{th}^2)\right]} \xrightarrow{k\to 0} \frac{\left\langle w_i^2 \right\rangle}{2N_p}$$

- The fully uncorrelated spectrum (for comparison), equivalent at high k, only a factor of 2 larger at small k:

$$\left\langle \left| \frac{e\phi_k}{T} \right|^2 \right\rangle_N = \frac{\left\langle w_i^2 \right\rangle S_{filter}^2(k) S^2(k) \Gamma_0(k_\perp^2 \rho_{th}^2)}{N_p \left[2-\Gamma_0(k_\perp^2 \rho_{th}^2)\right]^2} \xrightarrow{k\to 0} \frac{\left\langle w_i^2 \right\rangle}{N_p}$$

Test Particle Superposition with Renormalized Trajectories

Test Particle Superposition Principle: Trajectories of "dressed" test particles can be treated as statistically independent. Dominant correlations included by a shielding cloud (calculated using the plasma dielectric response) that follows each moving test particle.

Intuitive approach, usually agrees with the rigorous Fluctuation-Dissipation Theorem in thermal equilibium (Krommes'93, Rostoker, …).

Include resonance-broadening type of renormalization of test particle trajectory:

$$\left\langle \exp(-i\mathbf{k}\cdot\left(\mathbf{x}_i(t)-\mathbf{x}_i(t_2)\right)\right\rangle = \exp(-ik_\parallel \mathrm{v}_{\parallel,i}(t-t_2) - D_{t.p.}k_\perp^2|t-t_2|)$$

Renormalization important for consistently handling both
weak noise limit, where $D_{noise} \propto \Phi^2_{noise}$, and
strong noise limit, where $D_{noise} \propto \Phi_{noise}$.

Renormalized Dielectric Shielding

Nonlinear gyrokinetic Eq. (uniform slab, electrostatic):

$$\frac{\partial \delta f}{\partial t} + v_{\parallel} \hat{b} \cdot \nabla \delta f + \frac{c}{B} \hat{b} \times \nabla J_0 \Phi \cdot \nabla \delta f = -v_{\parallel} \left(\hat{b} \cdot \nabla J_0 \frac{q\Phi}{T} \right) F_{Max,0}$$

If ExB velocity is small-scale random fluctuations, treat as random walk diffusion:

$$\frac{\partial \delta f}{\partial t} + v_{\parallel} \hat{b} \cdot \nabla \delta f \qquad - D_{NL} \nabla_{\perp}^2 \delta f \qquad = -v_{\parallel} \left(\hat{b} \cdot \nabla J_0 \frac{q\Phi}{T} \right) F_{Max,0}$$

Better renormalization (Catto 78): nonlinearity affects only non-adiabatic part of δf:

$$\frac{\partial \delta f}{\partial t} + v_{\parallel} \hat{b} \cdot \nabla \delta f - D_{NL} \nabla_{\perp}^2 \left(\delta f + F_{Max.0} J_0 \frac{q\Phi}{T} \right) = -v_{\parallel} \left(\hat{b} \cdot \nabla J_0 \frac{q\Phi}{T} \right) F_{Max,0}$$

Insures no nonlinear damping of a thermal equilibrium solution (the adiabatic solution) (Catto78, Krommes81, Krommes02). Combined with using the same D_{NL} for the shielding cloud as the $D_{t.p.}$ for the test-particle trajectory, preserves the form of the Fluctuation-Dissipation Theorem.

Detailed Calculation of Noise-Spectrum Incl. Self-Shielding

Potential induced by shielded test particle density ρ_{ext}: $\Phi = \frac{4\pi q \rho_{ext}}{k^2 \varepsilon(\vec{k},\omega)}$

$$\varepsilon(\vec{k},\omega) = \frac{k_D^2}{k^2}\left[\frac{T}{T_a}\left(1-\delta_{k_\parallel}\right) + 1 - \Gamma_0 + S_{filt} d_\parallel S^2 \left\langle J_0^2 \right\rangle \left(1 + \zeta Z(\zeta + i\zeta_D)\right)\right]$$

Gyrokinetic dielectric shielding including simple renormalized D_{NL} model of nonlinear effects on shielding cloud and test-particle random walk trajectory, $\zeta_D = k_\perp^2 D_{NL}/(|k_\parallel|\, v_t\, 2^{1/2})$. Integrating $\langle|\Phi_k|^2\rangle(\omega)$ over all ω gives a result independent of D_{NL} (but D_{nl} important for getting frequency spectrum right to calculate test-particle diffusion). Resulting k spectrum:

$$\left\langle \left| \frac{e\tilde{\Phi}_{noise,k}}{T} \right|^2 \right\rangle = \frac{V^2 \left\langle w^2 \right\rangle}{N} \frac{S_{filt}^2 S^2 \left\langle J_0^2 \right\rangle}{\left[\frac{T}{T_a}\left(1-\delta_{k_\parallel}\right) + 1 - \Gamma_0 + \underbrace{S_{filt} d_\parallel S^2 \left\langle J_0^2 \right\rangle}\right]\left[\frac{T}{T_a}\left(1-\delta_{k_\parallel}\right) + 1 - \Gamma_0\right]}$$

Only difference from simple random-particle spectrum. $\langle \Phi_k^2 \rangle$ only 50% lower at low $k_\perp$ (adiabatic electrons already got half of shielding), equal at high $k_\perp$ (ion shielding vanishes)

Why Particle Weights Grow in Time

$$\frac{\partial f}{\partial t} + \left(\mathrm{v}_{\parallel} + \mathrm{v}_{ExB}\right)\cdot \nabla f + \frac{q}{m} E_{\parallel} \frac{\partial f}{\partial \mathrm{v}_{\parallel}} = 0$$

$$\frac{Df}{Dt} = 0$$

Clever δf algorithm to reduce noise: f = smooth f_0 + particles δf

$$\frac{D}{Dt}\delta f = -\frac{D}{Dt} f_0 \approx -\mathrm{v}_{ExB} \cdot \nabla f_0$$

$$\frac{D}{Dt}\delta f = -\frac{Dx}{Dt}\frac{df_0}{dx}$$

$$\delta f = \left(x - x_0\right)\frac{df_0}{dx}$$

$$\delta f = \sum_i w_i(t)\delta\left(x - x_i(t)\right)\delta\left(\mathrm{v} - \mathrm{v}_i(t)\right)$$

f = constant along particle's trajectory. But as particle moves to position where local f_0 is different than the f where particle started, weight grows to represent difference.

$$\frac{dw_{rms}^2}{dt} = \frac{d}{dt}\frac{\left\langle \left(\delta f\right)^2 \right\rangle}{f^2} \approx \frac{2\chi_{tot}}{L_T^2}$$

entropy balance in steady state
W.W. Lee & W. Tang 88

Note: In the following plots comparing the noise formula on the last page with <$\Phi^2(k_x,k_y)$> spectra from Dimits gyrokinetic PIC simulations, there are no free parameters.

Detail: Nevin's GKV diagnostic is the $k_\perp$ spectrum at z=0, so have to sum the expression for the noise $\Phi_k{}^2$ spectrum over all k_z:

$$\left\langle \left| \frac{e\widetilde{\Phi}_{k_x,k_y}(z=0)}{T} \right|^2 \right\rangle = \frac{1}{L_z^2} \sum_{k_z} \left\langle \left| \frac{e\widetilde{\Phi}_k}{T} \right|^2 \right\rangle$$

Simulation Verification (1)
The Transverse (to ***B***) Fluctuation Spectrum

Requires:

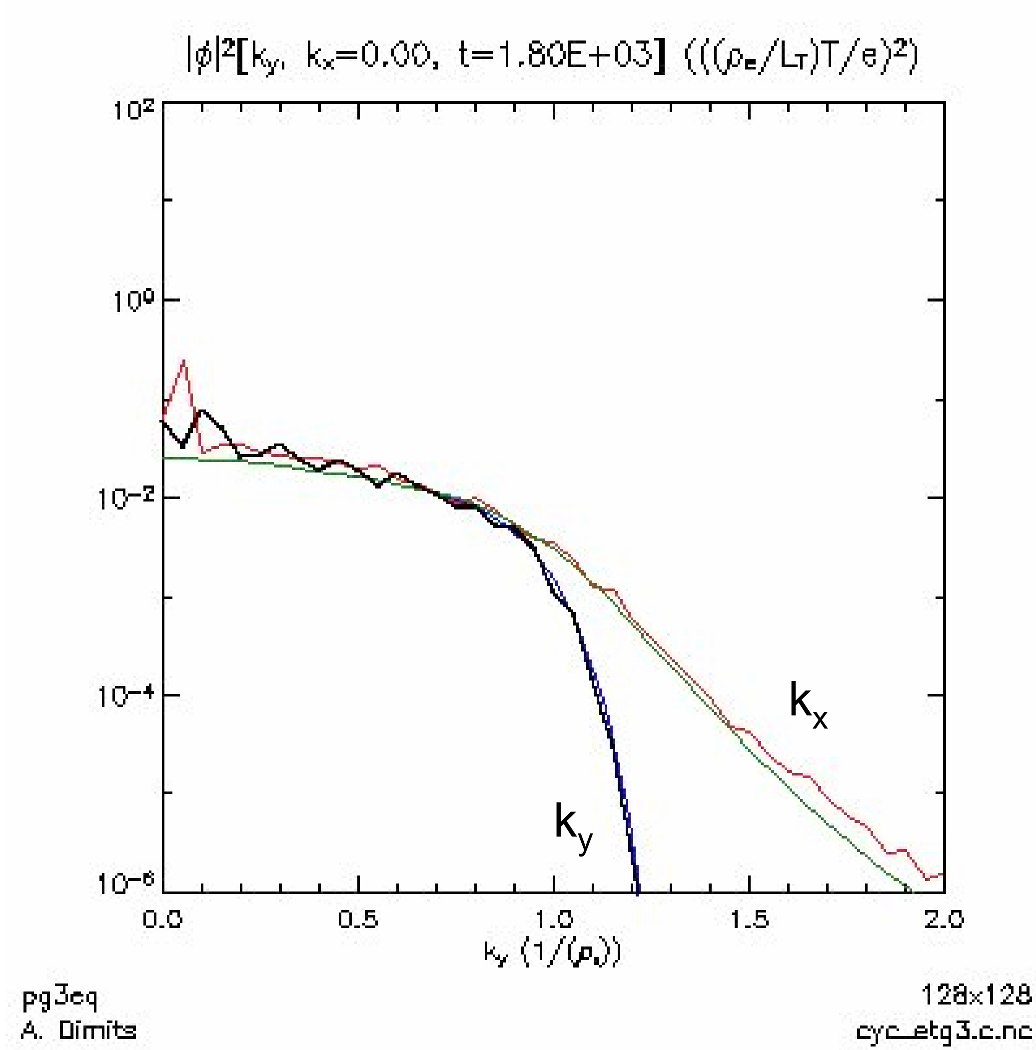
- From Simulation,
  - Fluctuation data in plane $\perp$ to ***B***
  - The time-series $\langle w^2 \rangle(t)$
  - Numerical details about the field-solve
- A mixed representation, $\langle |\phi|^2 \rangle(k_x,k_y,z)$

$$\left\langle \left| \frac{e\phi_{k_x,k_y}(z)}{T} \right|^2 \right\rangle = \frac{1}{L_z^2} \sum_{k_z} \left\langle \left| \frac{e\phi_{k_x,k_y,k_z}}{T} \right|^2 \right\rangle =$$

$$\approx \frac{\langle w^2 \rangle}{n_p (L_x L_y \Delta z)} \left\{ \frac{\Delta z}{2\pi} \int_{-\pi/\Delta z}^{\pi/\Delta z} \frac{S_{filter}^2 \Gamma_0}{[2-\Gamma_0][2-(1-S_{filter} d_{\|})\Gamma_0]} dk_z \right\}$$

$\Rightarrow$ Predicted noise spectrum fits the data.
No Free Parameters!

$\Rightarrow$ This simulation has a noise problem.



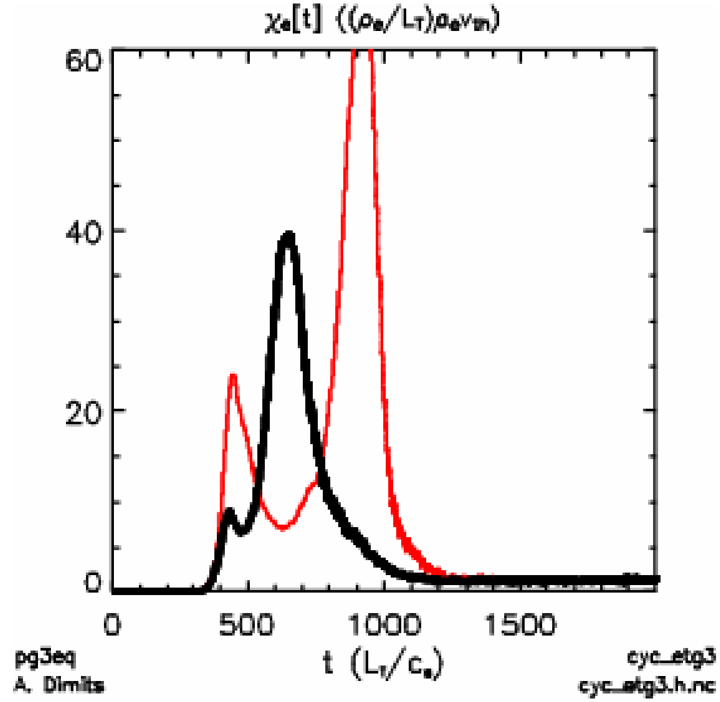
Discrete Particle Noise can sometimes be an issue for Particle-in-cell simulations

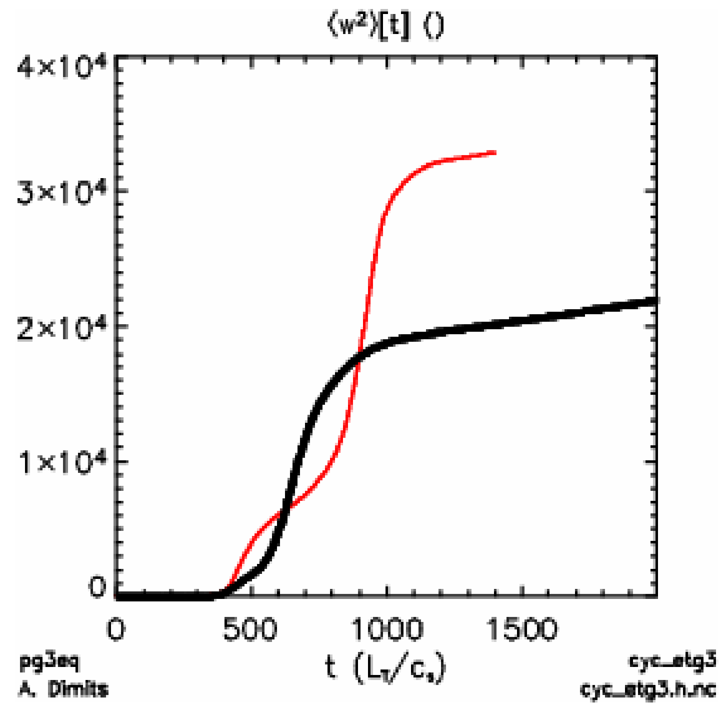
- Red curve: 16 particles/cell, 256 x 64 x 32
- Black curve: 8 particles/cell 128 x 128 x 64
- The mean square particle weight $<w^2>(t)$ measures discrete particle noise

- Entropy Theorem of Lee & Tang:

- χ_e is larger (in GK-units, $\rho^2 v_{th}/L_T$) Discrete particle noise a greater issue for ETG that ITG?

- Discrete particle noise looks important for *t > 700* L_T/v_{th} in PG3EQ simulations





Total & Noise part of ϕ spectrum $S_\phi(k)$ at t=

$S_\phi(k)$ from simulation:

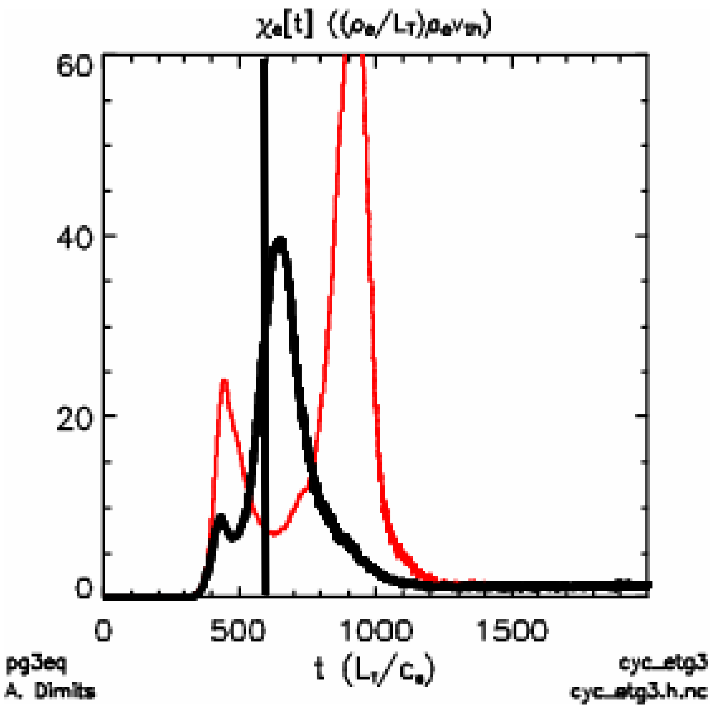
$S_\phi(k_x=0, k_y\ \ )$ —black

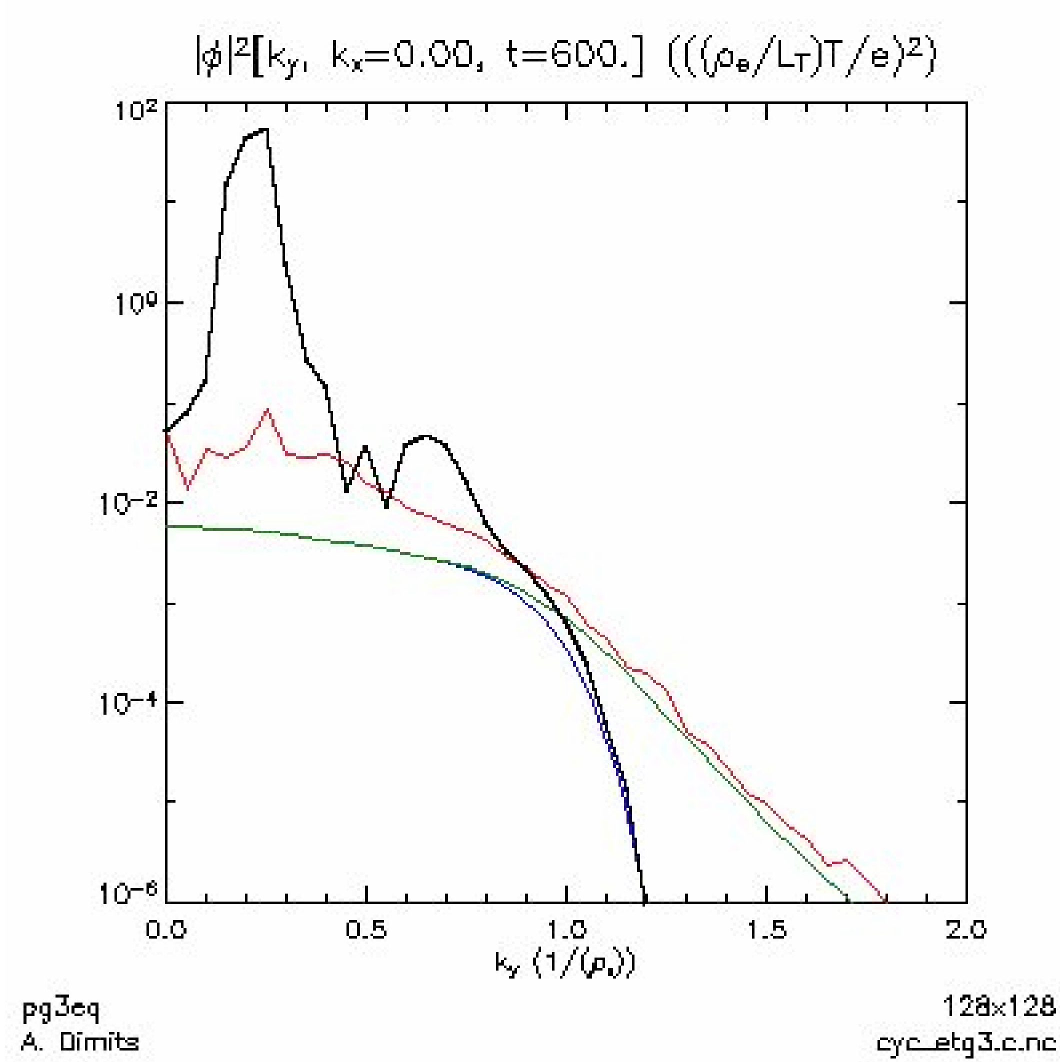
$S_\phi(k_x\ \ , k_y=0)$ —red/brown

$S_\phi(k)$ from noise:

$S_{noise}(k_x=0, k_y\ \ )$ —blue

$S_{noise}(k_x\ \ , k_y=0)$ —green





Total & Noise part of ϕ spectrum $S_\phi(k)$ at t=

$S_\phi(k)$ from simulation:

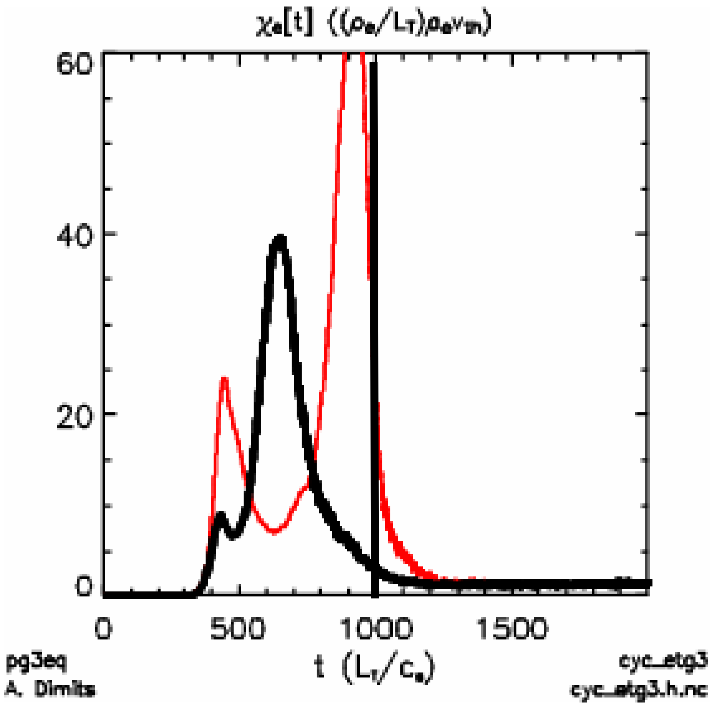
$S_\phi(k_x=0, k_y\ \ )$ —black

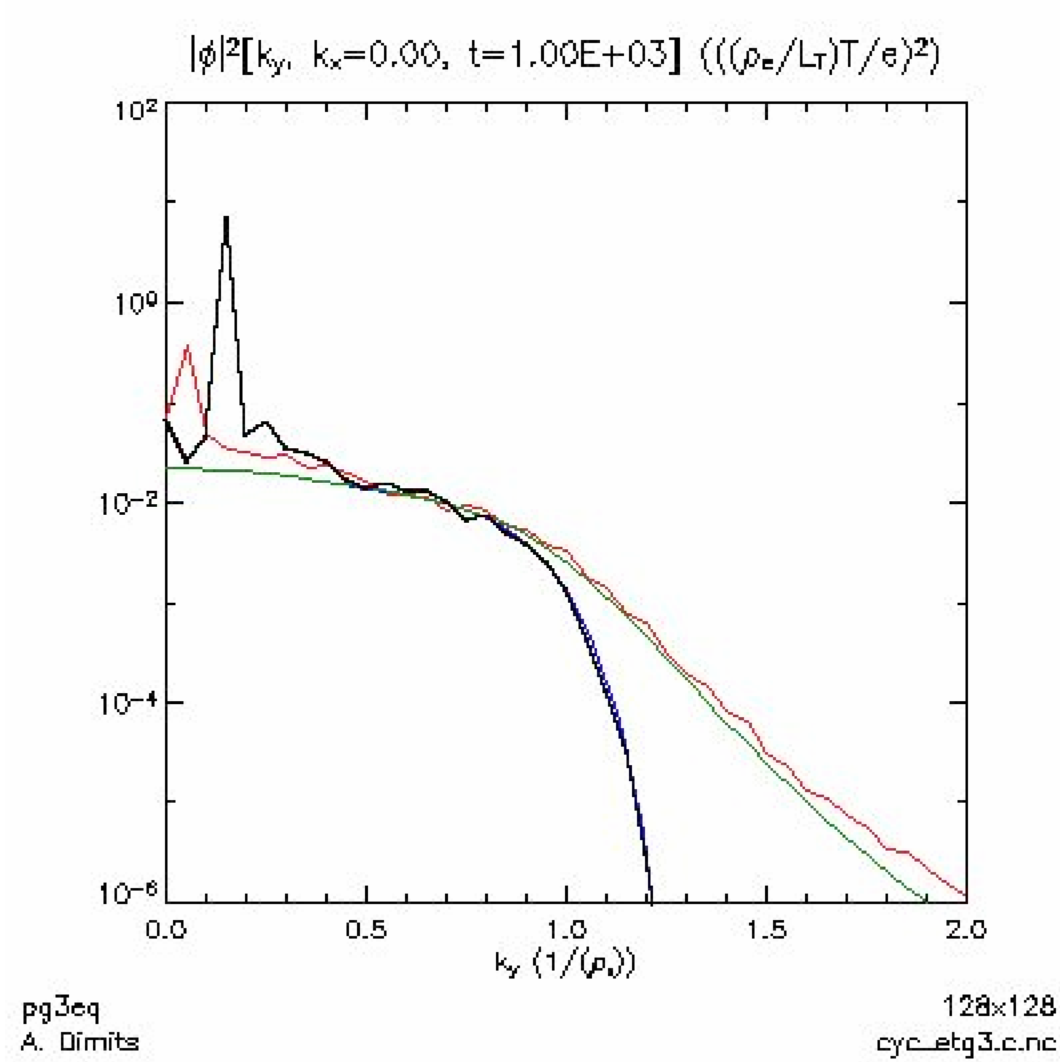
$S_\phi(k_x\ \ , k_y=0)$ —red/brown

$S_\phi(k)$ from noise:

$S_{noise}(k_x=0, k_y\ \ )$ —blue

$S_{noise}(k_x\ \ , k_y=0)$ —green





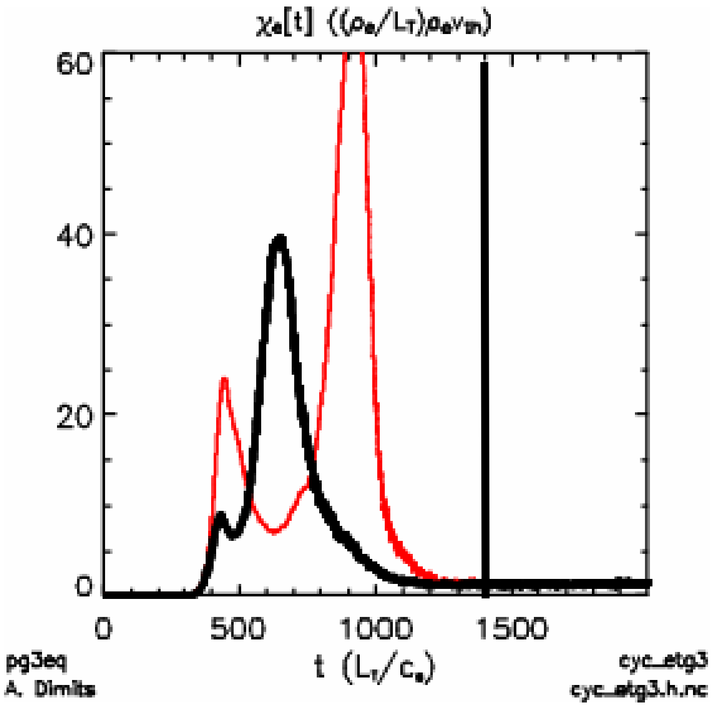
Total & Noise part of ϕ spectrum $S_\phi(k)$ at t=

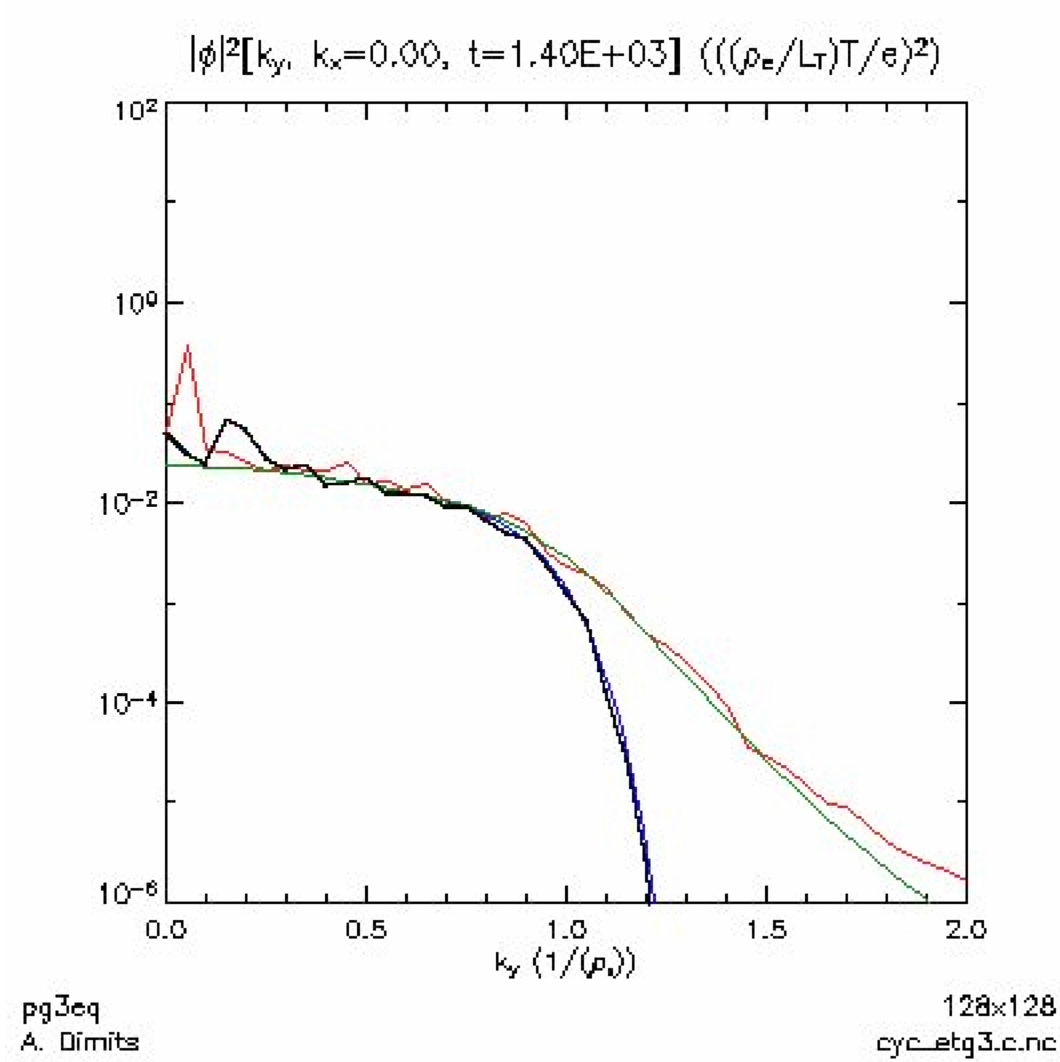
$S_\phi(k)$ from simulation:

- $S_\phi(k_x=0, k_y\ \ )$ —black
- $S_\phi(k_x\ \ , k_y=0)$ —red/brown

$S_\phi(k)$ from noise:

- $S_{noise}(k_x=0, k_y\ \ )$ —blue
- $S_{noise}(k_x\ \ , k_y=0)$ —green





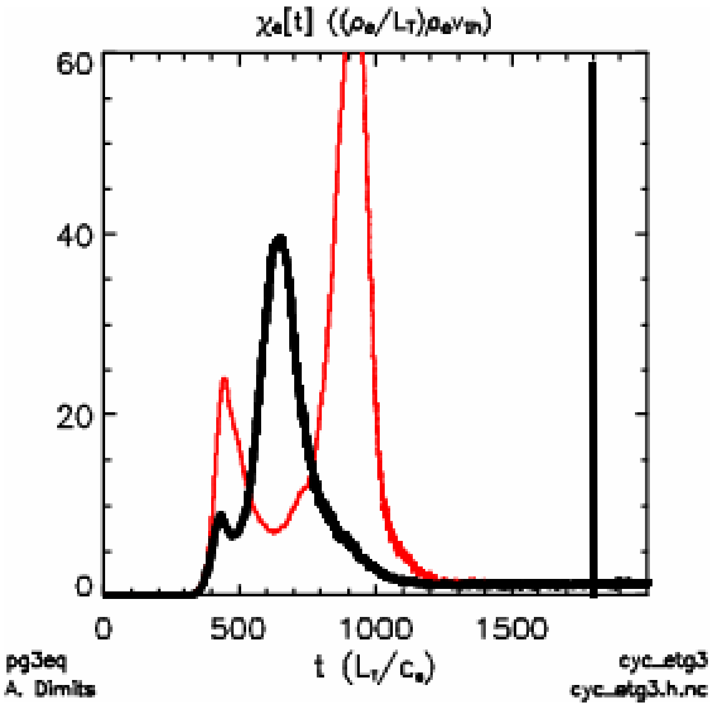
Total & Noise part of ϕ spectrum $S_\phi(k)$ at t=

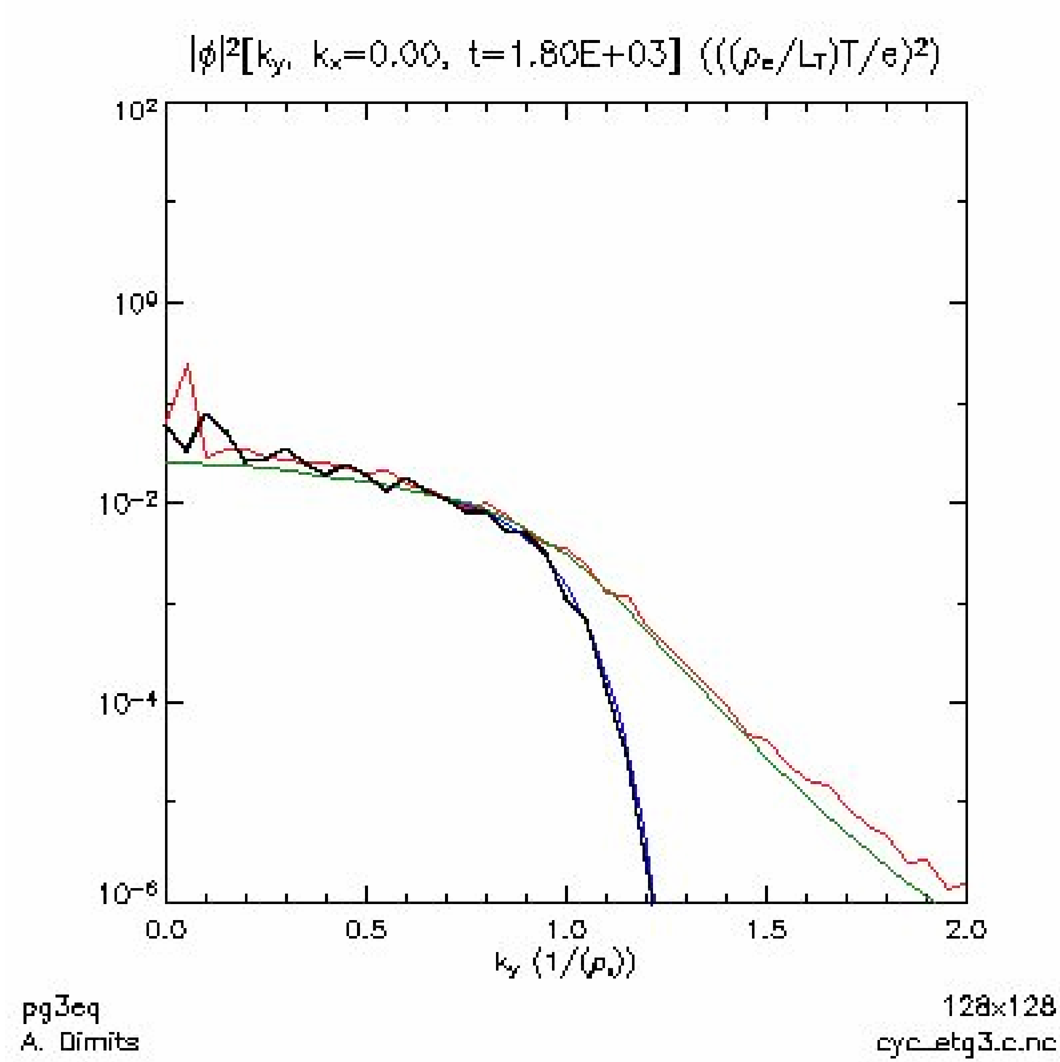
$S_\phi(k)$ from simulation:

- $S_\phi(k_x=0, k_y\ \ )$ —black
- $S_\phi(k_x\ \ , k_y=0)$ —red/brown

$S_\phi(k)$ from noise:

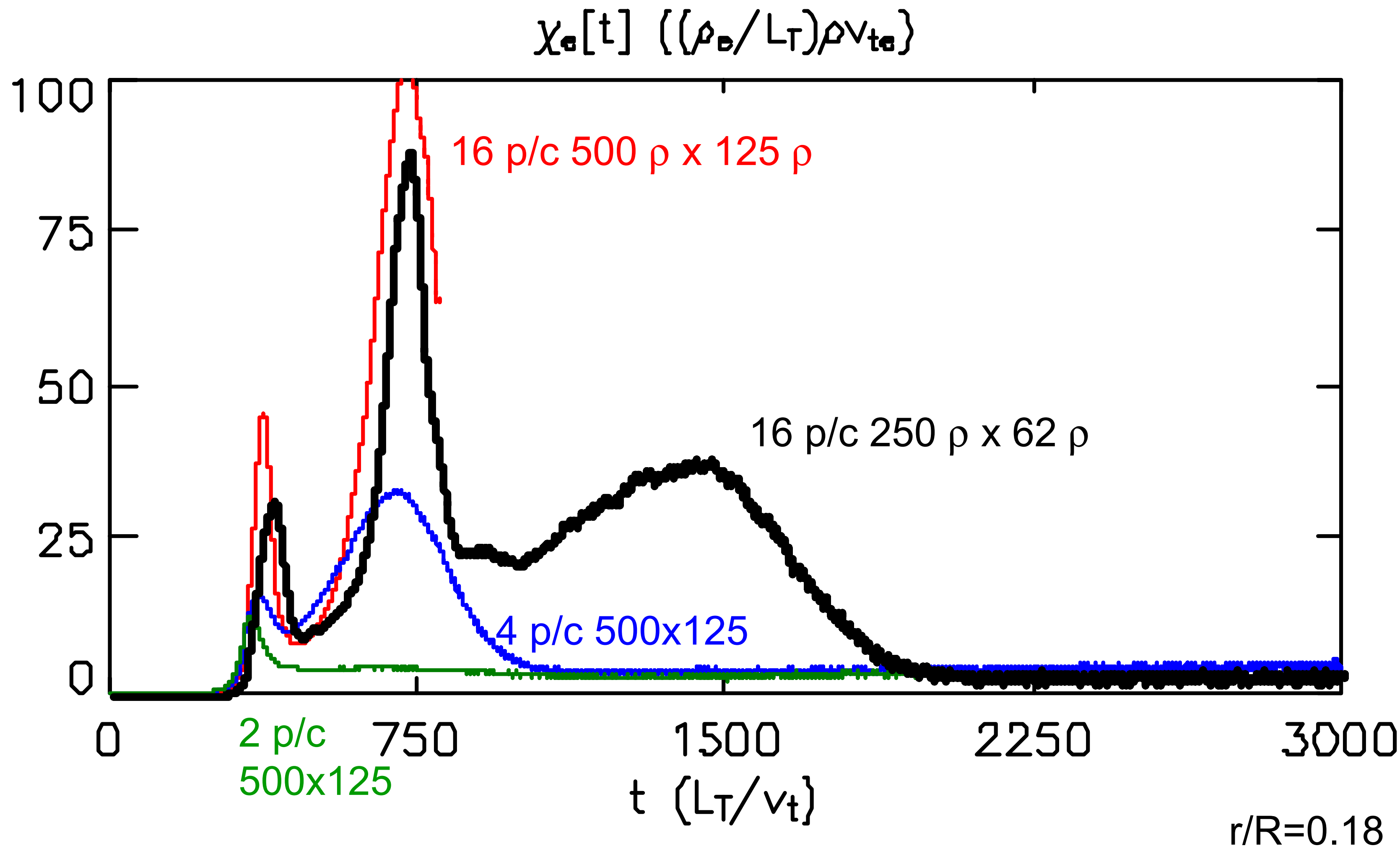
- $S_{noise}(k_x=0, k_y\ \ )$ —blue
- $S_{noise}(k_x\ \ , k_y=0)$ —green



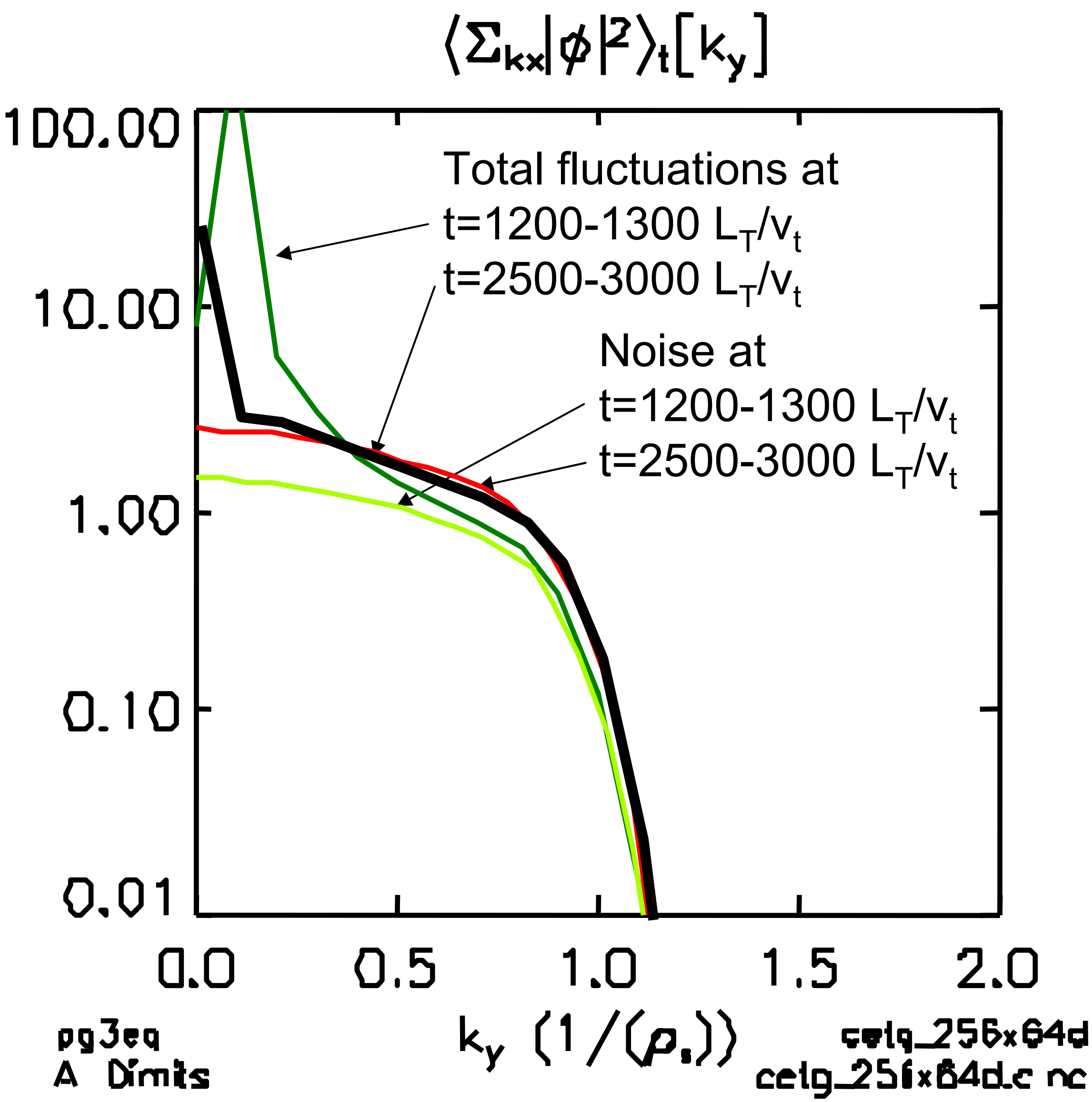


$1/(D_{noise}\, k_\perp^2) \sim 1/((2/3)\ 2\ 0.05^2) \sim 300$

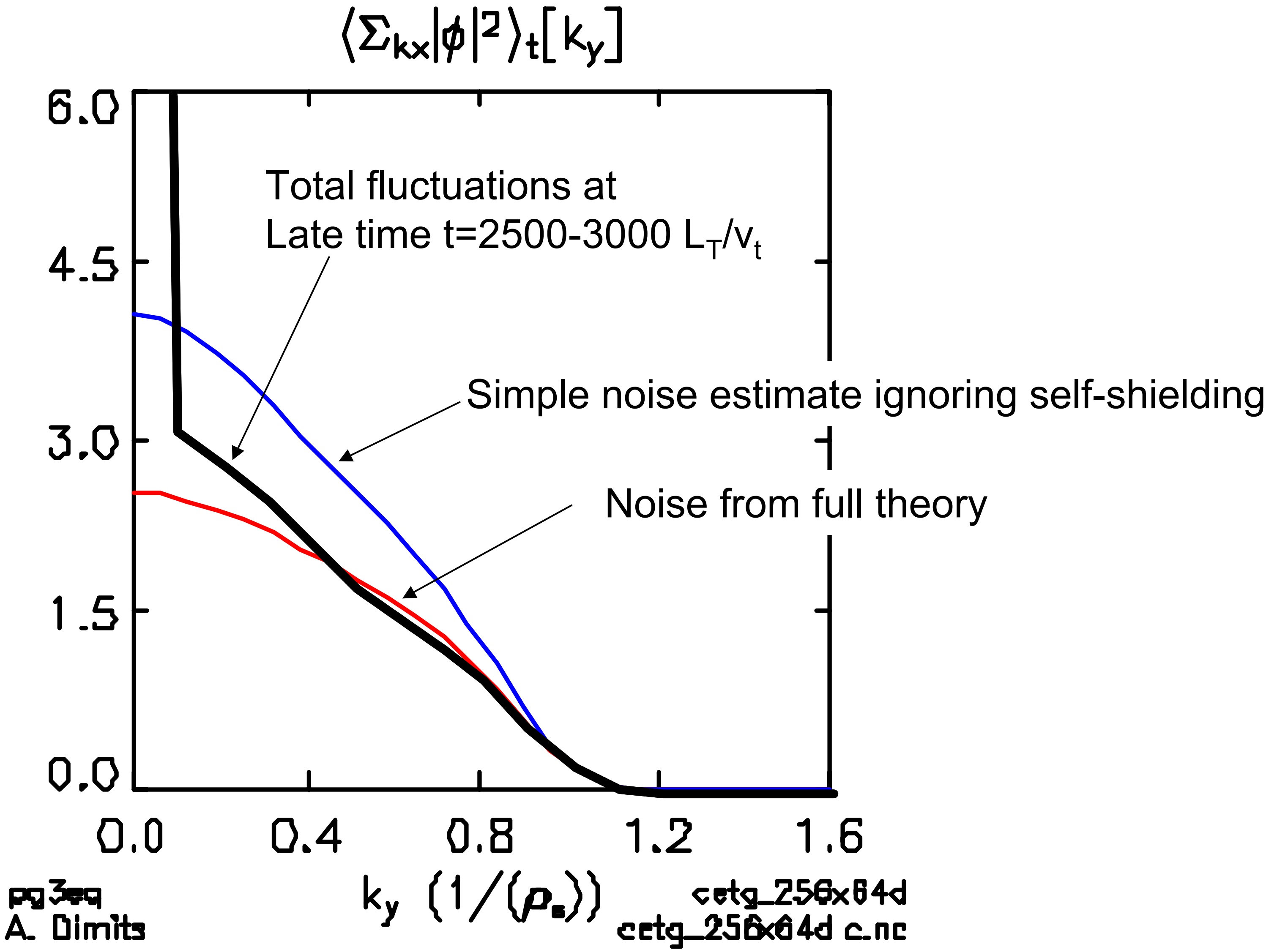
As number of particles ↑, the duration of strong turbulence ↑, but eventually the noise grows to the same level needed to suppress turublence.



Noise Theory Agrees Very Well With Simulation
ϕ Spectrum At Late Times



Noise Theory Agrees Very Well With Simulation ϕ Spectrum At Late Times



Same late-time data as previous slide, on a linear scale

Simulation Verification (2)
The Fluctuation Intensity

A less computationally intensive diagnostic

$$\left\langle \left| \frac{e\phi}{T} \right|^2 \right\rangle = \frac{1}{V^2} \sum_k \left\langle \left| \frac{e\phi_k}{T} \right|^2 \right\rangle = \frac{\langle w^2 \rangle}{n_p V_{shield}}$$

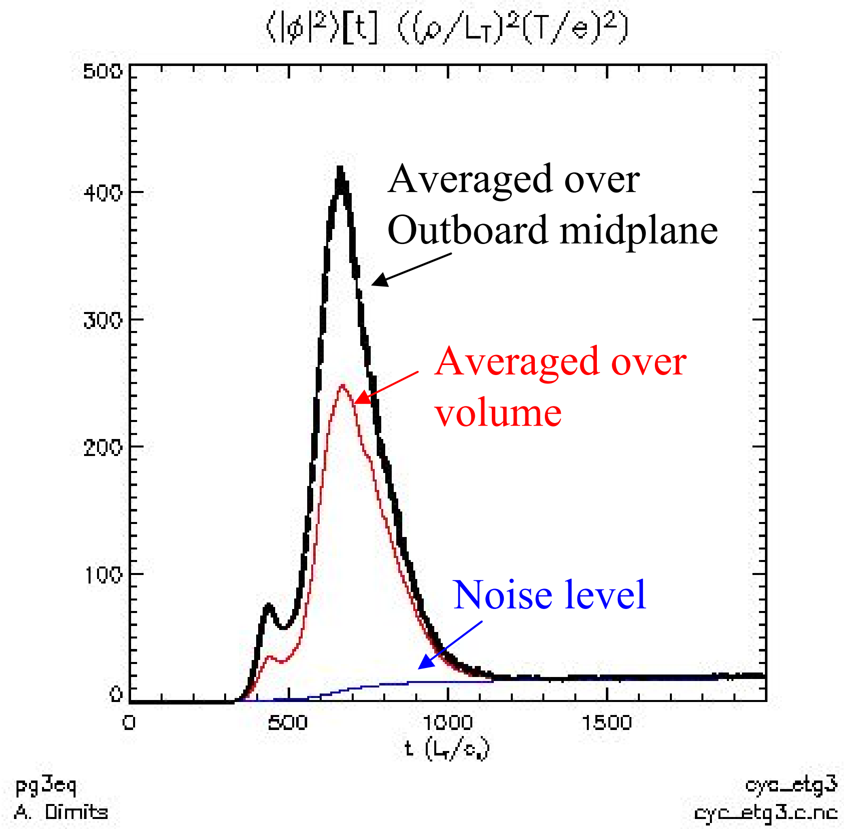
where

$$V_{shield}^{(H)} \equiv \left\{ \frac{1}{(2\pi)^3} \int d^3k \frac{S_{filter}^2 \Gamma_0(k_\perp^2 \rho_{th}^2)}{[2-\Gamma_0][2-(1-S_{filter} d_\|)\Gamma_0]} \right\}^{-1}$$

$$V_{shield}^{(N)} \equiv \left\{ \frac{1}{(2\pi)^3} \int d^3k \frac{S_{filter}^2 \Gamma_0(k_\perp^2 \rho_{th}^2)}{[2-\Gamma_0(k_\perp^2 \rho_{th}^2)]^2} \right\}^{-1}$$

Typically $V_{shield} \approx 30\ \Delta x \Delta y \Delta z$

Cyclone base-case-like ETG
Fluctuation Intensity



Simulation Verification (3)
The Fluctuation Energy Density

Fluctuation energy density may be a more relevant diagnostic:

- Has direct physical significance (energy associated with ExB motion)
- Closely related to transport coefficient $D \approx \langle V_{ExB}{}^2 \rangle \tau_{corr}$

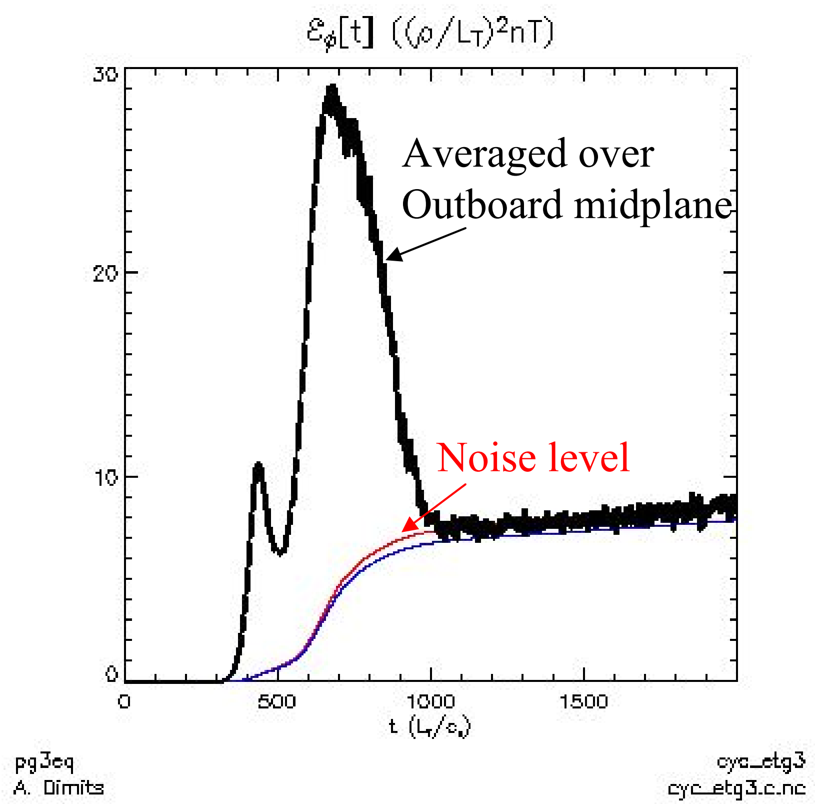
$$-\frac{\omega_p^2}{\Omega_c^2}\left\langle \frac{\phi \nabla_\perp^2 \phi}{4\pi} \right\rangle = nT \frac{\langle w^2 \rangle}{n_p V_{shield}} \left\langle K_\perp^2 \rho^2 \right\rangle_{noise}$$

where

$$\left\langle K_\perp^2 \rho^2 \right\rangle_{noise}^{(H)} \equiv \left\{ \frac{V_{shield}}{(2\pi)^3} \int d^3k \frac{K_\perp^2(k)\rho^2 S_{filter}^2(k)\Gamma_0(k_\perp^2 \rho_{th}^2)}{[2-\Gamma_0][2-(1-S_{filter}d_\parallel)\Gamma_0]} \right\}$$

$$\left\langle K_\perp^2 \rho^2 \right\rangle_{noise}^{(N)} \equiv \left\{ \frac{V_{shield}}{(2\pi)^3} \int d^3k \frac{K_\perp^2(k)\rho^2 S_{filter}^2(k)\Gamma_0(k_\perp^2 \rho_{th}^2)}{[2-\Gamma_0(k_\perp^2 \rho_{th}^2)]^2} \right\}$$

Cyclone base-case-like ETG
Fluctuation Energy



Renormalization of Noise-induced test-particle diffusion

$$D_{noise} = \int_{-\infty}^{t} dt' \left\langle \mathrm{v}(x(t),t)\,\mathrm{v}(x(t'),t') \right\rangle$$

$$= \left\langle \mathrm{v}_{ExB}^2 \right\rangle \tau_c \quad \propto \sum_{\vec{k}} k_y^2 J_0^2 \,|\,\phi_{noise,k}\,|^2 \,\frac{1}{|\,k_{||}\,|\,\mathrm{v}_t}$$

(in simple limits)

$$\rightarrow \sum_{\vec{k}} k_y^2 J_0^2 \,|\,\phi_{noise,k}\,|^2 \,\frac{1}{|\,k_{||}\,|\,\mathrm{v}_t + k_\perp^2 D_{noise} + \nu_{turb}}$$

$$\propto L_z \sum_{k_x,k_y} k_y^2 J_0^2 \,|\,\phi_{noise,k}\,|^2 \log\left(\frac{|\,k_{||,\max}\,|\,\mathrm{v}_t + k_\perp^2 D_{noise} + \nu_{turb}}{k_\perp^2 D_{noise} + \nu_{turb}} \right)$$

Test-particle diffusion coefficient has a logarithmic divergence in the correlation time if integration is over straight-line trajectories. Use standard trick of treating trajectories as stochastic random walks consistent with diffusion. Include also model of effect of larger-scale turbulence on smaller scales as turbulent shearing rate ν_{turb} (but results insensitive to this at late times when turbulence is small).

Noise-induced test-particle diffusion

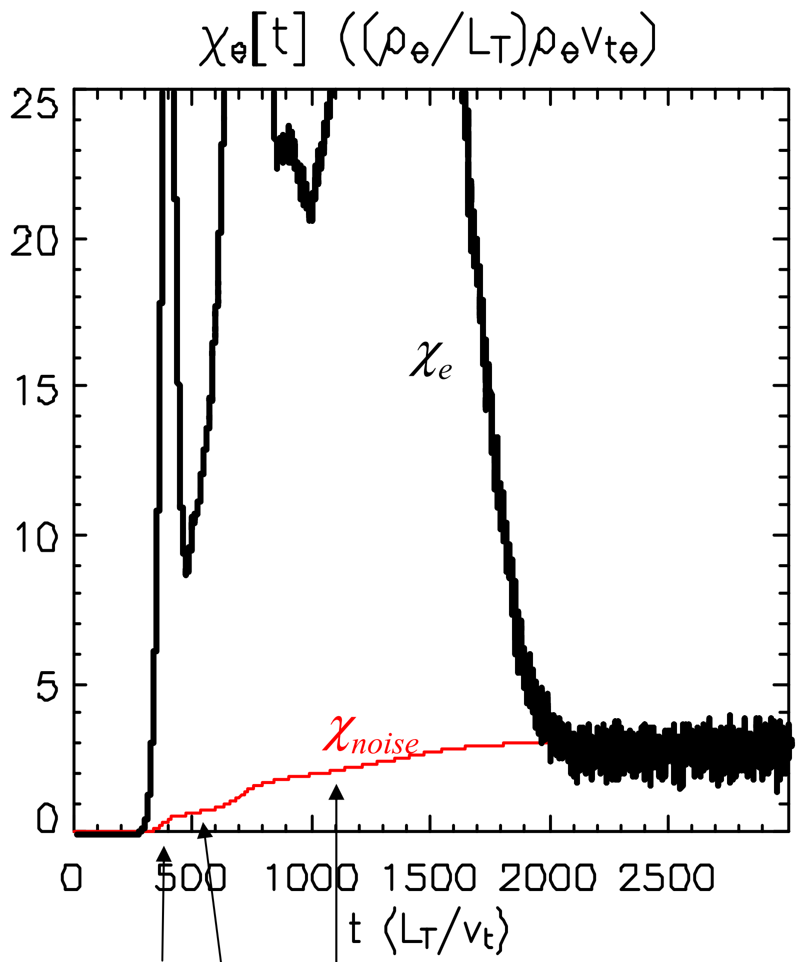
Integrate over properly weighted (ω,k) spectrum of noise fluctuations to find test-particle diffusion coefficient. Used a renormalized propagator to resolve a logarithmic divergence in the correlation time. (Have also included est. of turbulent shearing decorrelation, etc.)

$$D_{noise} = \frac{1}{12}\frac{V_{Shield}^{(H)}}{V_{Sh,2}} \underbrace{k_{\perp N}^2 \left(\frac{cT}{eB}\right)^2 \left\langle \left|\frac{e\phi}{T}\right|^2 \right\rangle_{noise}}_{V_{ExB}^2} \underbrace{\frac{3.05}{k_{\| \max} v_{te}\sqrt{2}}}_{\text{Decorrelation rate } \nu_{\|}} \underbrace{\log\left[1 + \frac{k_{\| \max} v_{te}\sqrt{2}}{3.05\left(D_{noise}k_{\perp N}^2 + \nu_*\right)}\right]}_{\text{Renormalized decorrelation rate depends on } D_{noise}}$$

$$\chi_{noise} \equiv \tfrac{3}{2} D_{noise}$$

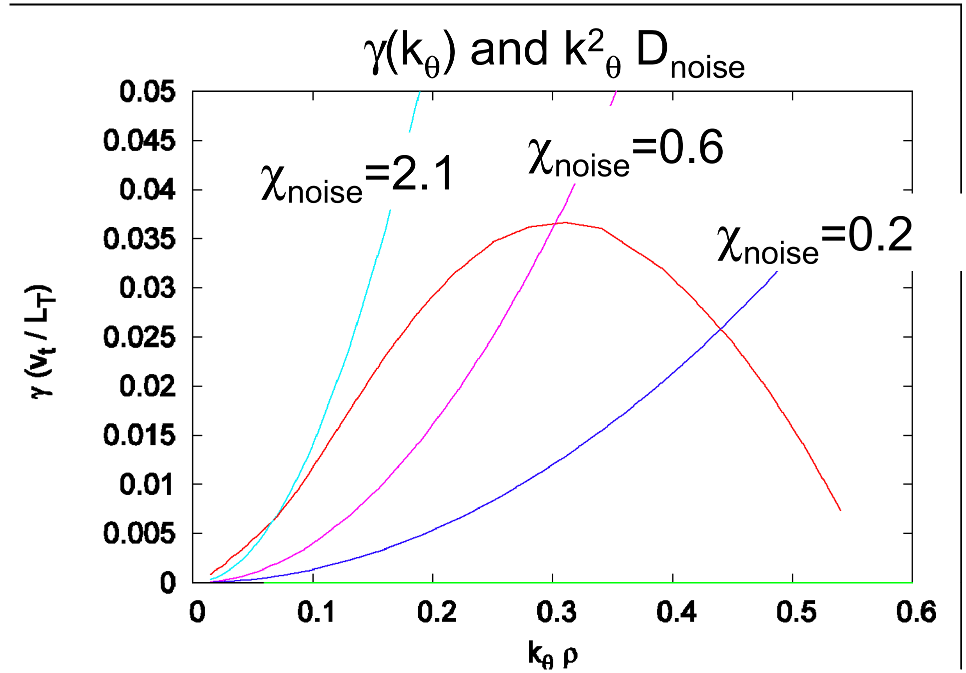
For details see
http://www.mfescience.org/mfedocs/ucrl-jrnl-212536.pdf

Dimits GK ETG simulations demonstrate χ_e falls to χ_{noise} by end of run, when $D_{noise}\, k_\perp^2 > \gamma$



χ_{noise}=0.2, 0.6, 2.1

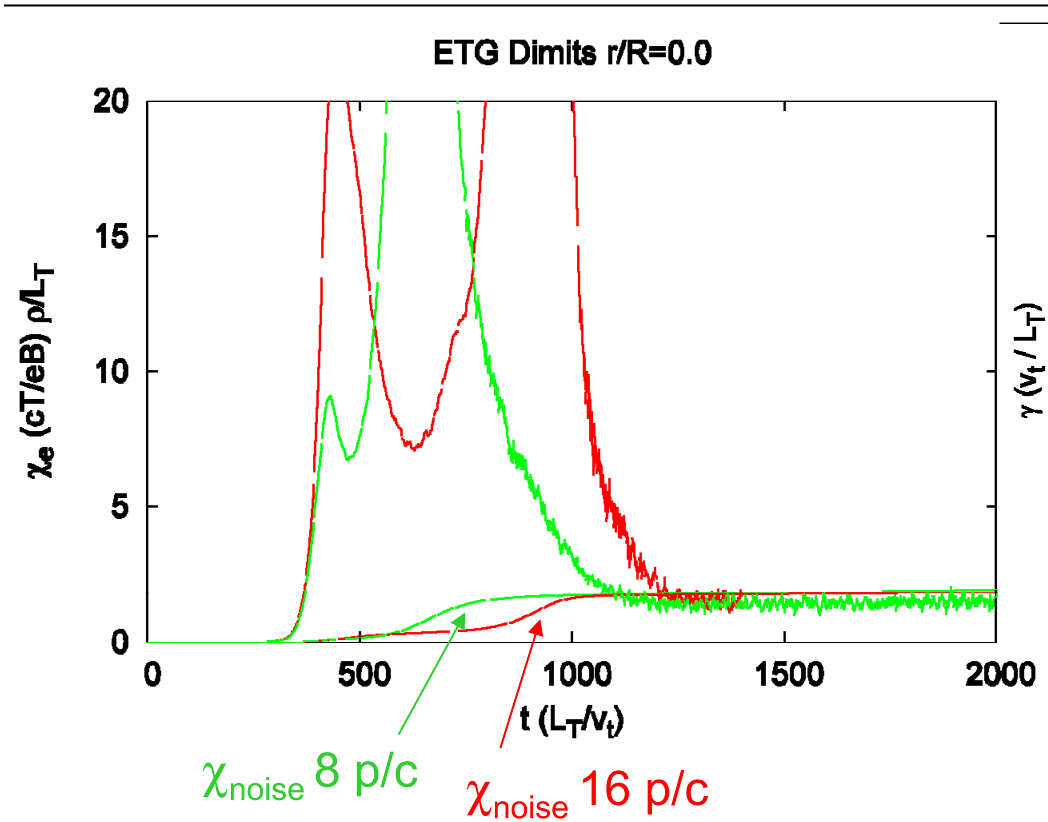
Although $\chi_{noise} \ll$ max $\chi_{turbulence}$, this χ_{noise} is still large enough to give significant damping, $\chi_{noise}\, k_\perp^2 \gg \gamma_{linear}$

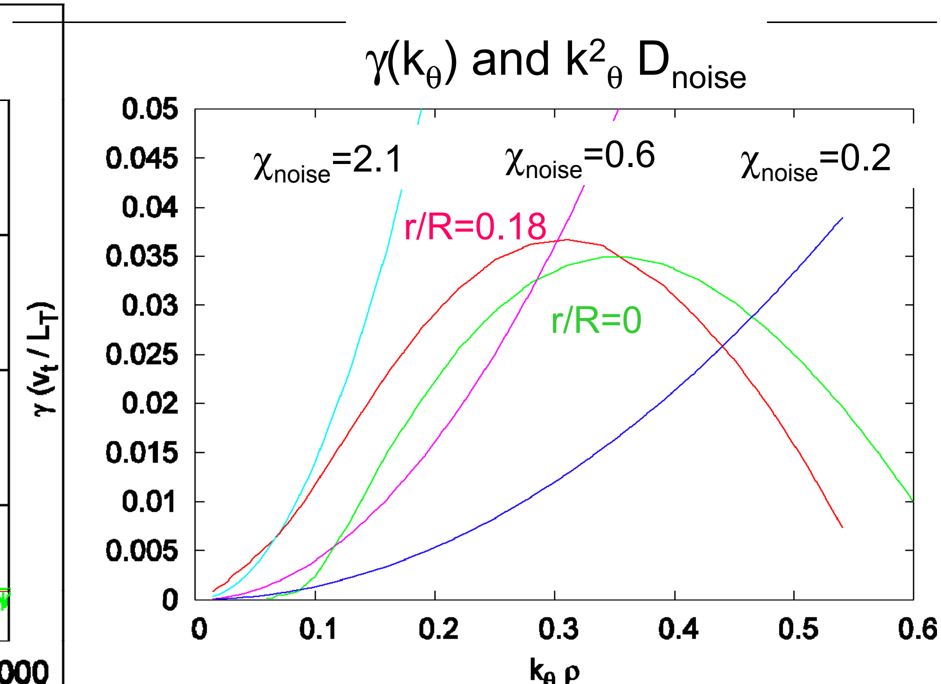


Growth rate for Cyclone base case with s-alpha geometry.

Final χ_{noise} (with no free parameters) agrees well with simulation at late times

Noise follows expected trends as particle number varied & trapping turned off



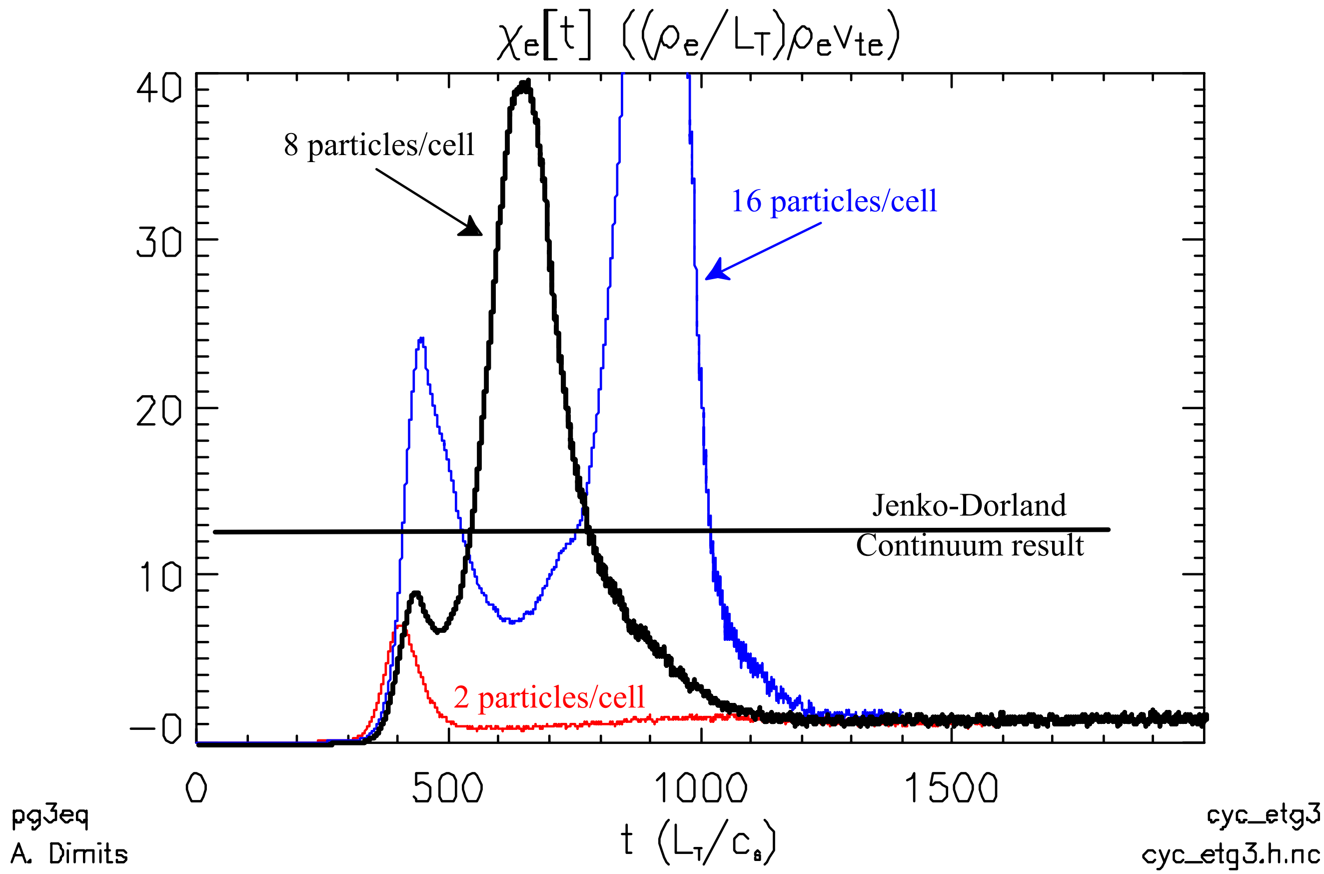


Turning off particle trapping (*r/R*=0) significantly reduces γ at low *k*

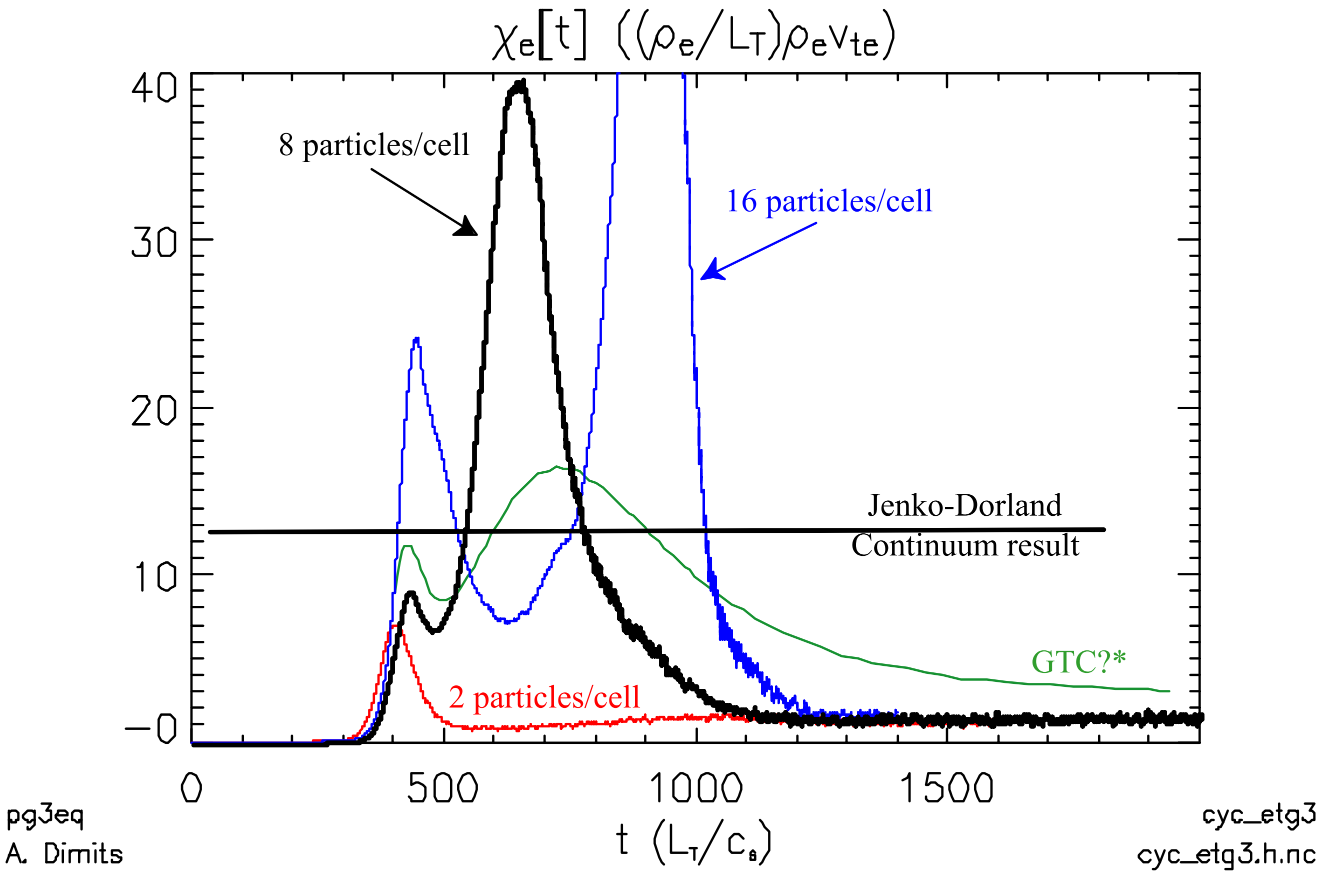
$$\chi_{noise} \propto \frac{w^2}{N} \propto \frac{\int^t dt'\chi(t)}{N}$$

Increasing # particles just leads to longer initial period of high χ_{tot}, so final χ_{noise} is not sensitive to N.

Discrete Particle Noise Suppresses Transport In Cyclone-base-case like ETG Simulations



Discrete Particle Noise Suppresses Transport In Cyclone-base-case like ETG Simulations



*GTC curve from Slide #13 of Z. Lin's IAEA presentation, which can be found at: http://www.cfn.ist.utl.pt/20IAEAConf/presentations/T5/2T/5_H_8_4/Talk_TH_8_4.pdf

The Bolton/Lin "Noise Test"

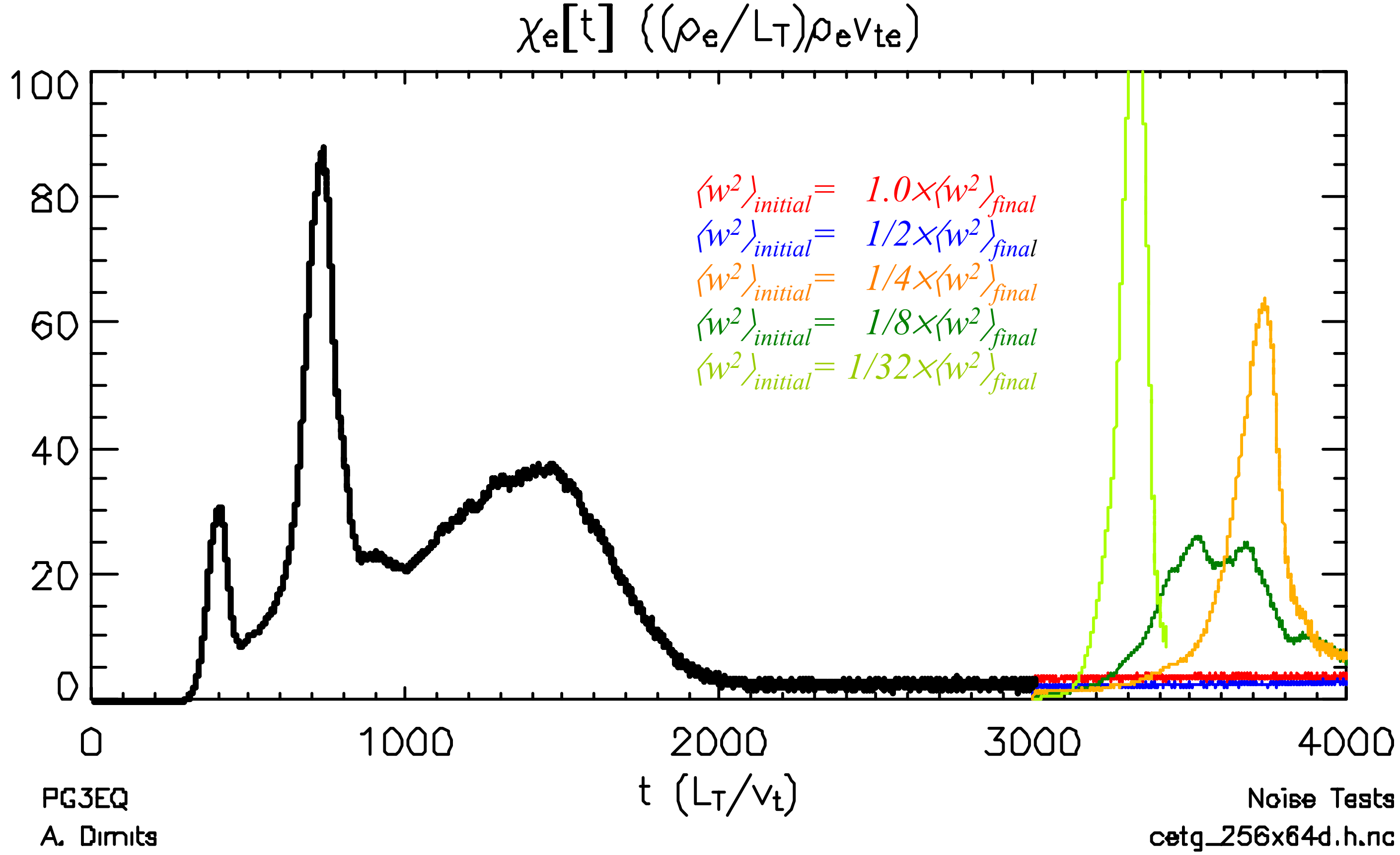
- Select reference simulation:
  - $r/R_0=0.18$
  - $250\rho_e \times 62.5\ \rho_e$
  - 16 particles/cell
- Determine $\langle w^2 \rangle$ at end of simulation ($\langle w^2 \rangle_{final} = 7.8 \times 10^4$)
- Restart simulation with:
  - Same physics operating point
  - Same simulation parameters
  - New particle positions
  - New particle weights, $\{w_i\}$ chosen by random number generator such that new $\langle w^2 \rangle_{initial}$ proportional to old $\langle w^2 \rangle_{final}$

$\Rightarrow$ Only "memory" in GK simulations encoded in particle weights/positions

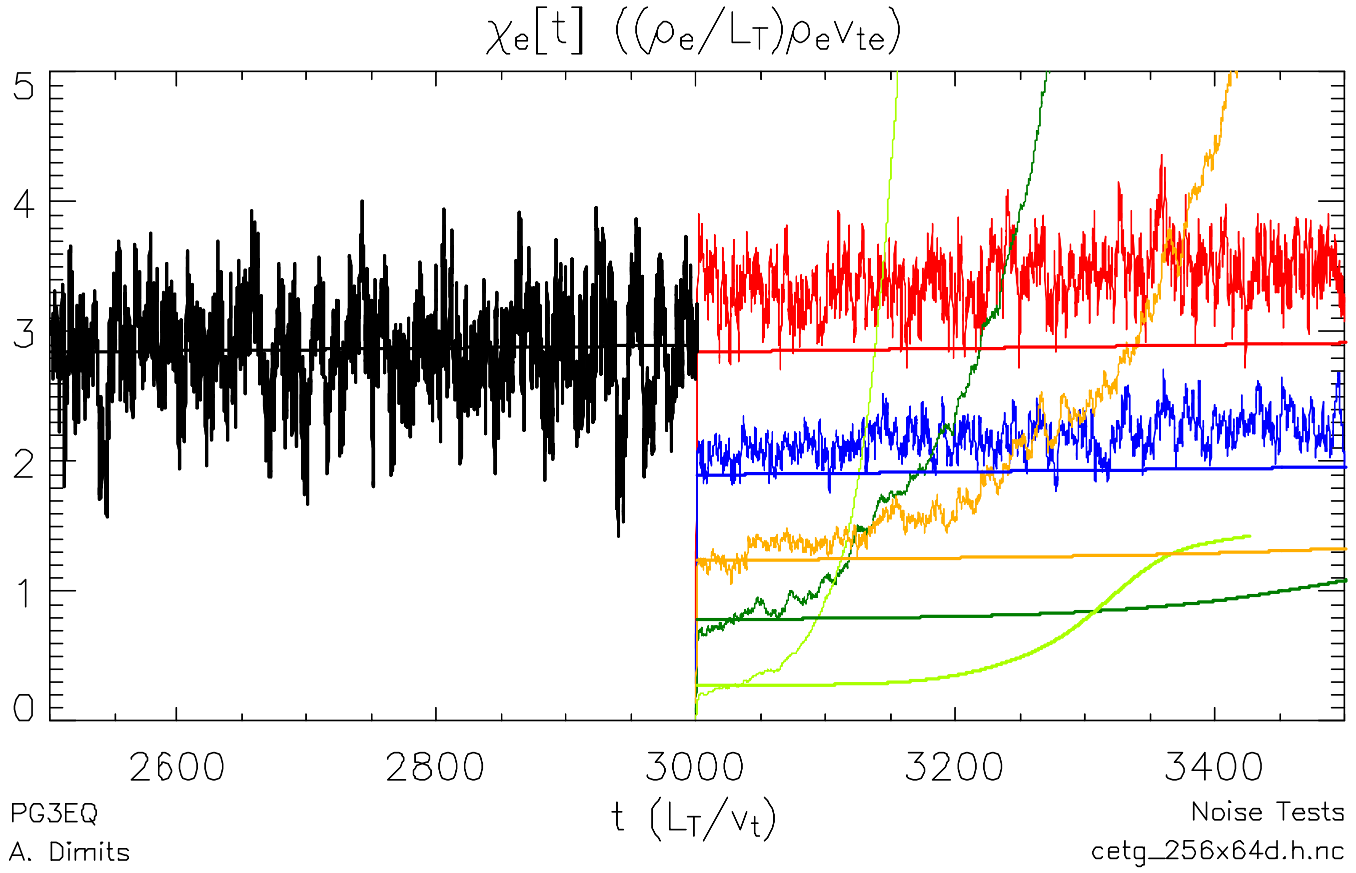
- If noise suppresses of ETG:
  - $\langle w^2 \rangle_{initial} = \langle w^2 \rangle_{final}$
    - $\langle \phi^2 \rangle \approx$ constant
    - $\chi_e \approx$ constant
  - $\langle w^2 \rangle_{initial} < \langle w^2 \rangle_{final}$
    - Exponential growth of $\langle \phi^2 \rangle$
    - γ increases as $\langle w^2 \rangle_{initial}$ decreases
    - χ_e starts low, grows with $\langle \phi^2 \rangle$
- If noise does not suppress ETG:
  - No dependence on $\langle w^2 \rangle_{initial}$
  - New runs similar to previous run:
    - Bust of ETG turbulence
    - χ_e independent of $\langle w^2 \rangle_{initial}$

The Bolton/Lin "Noise Test":

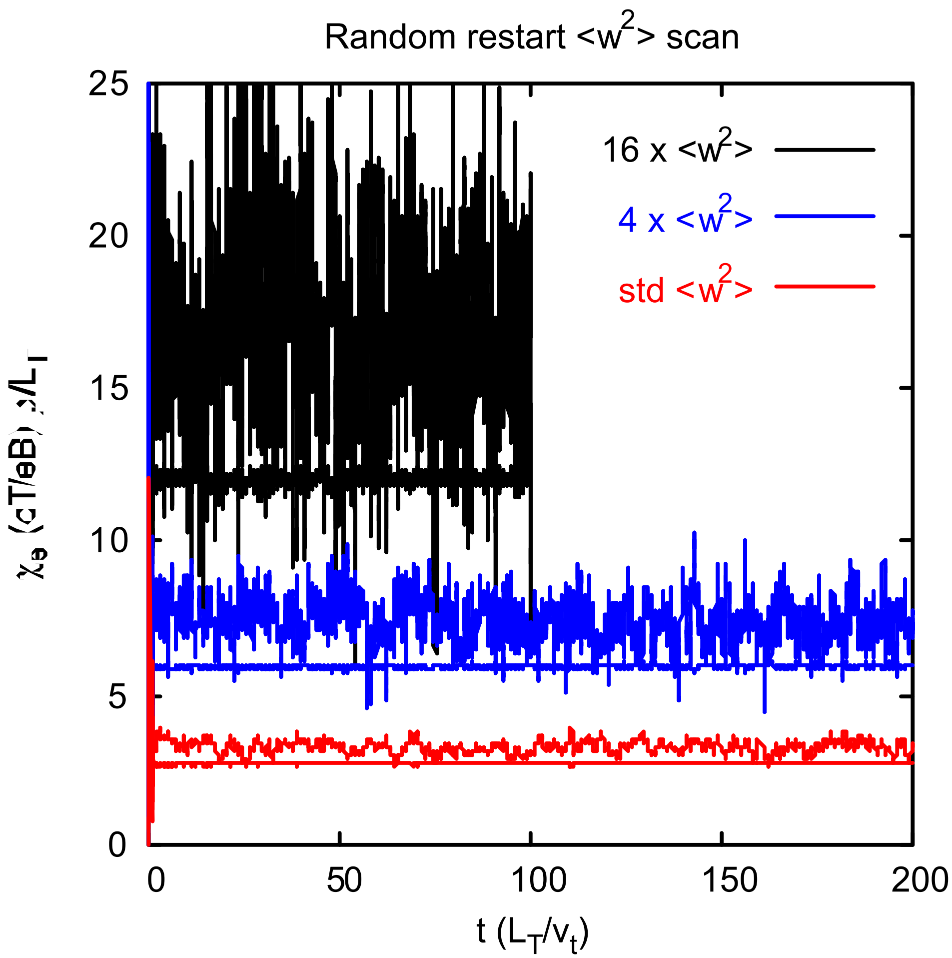
Discrete particle noise suppresses ETG transport



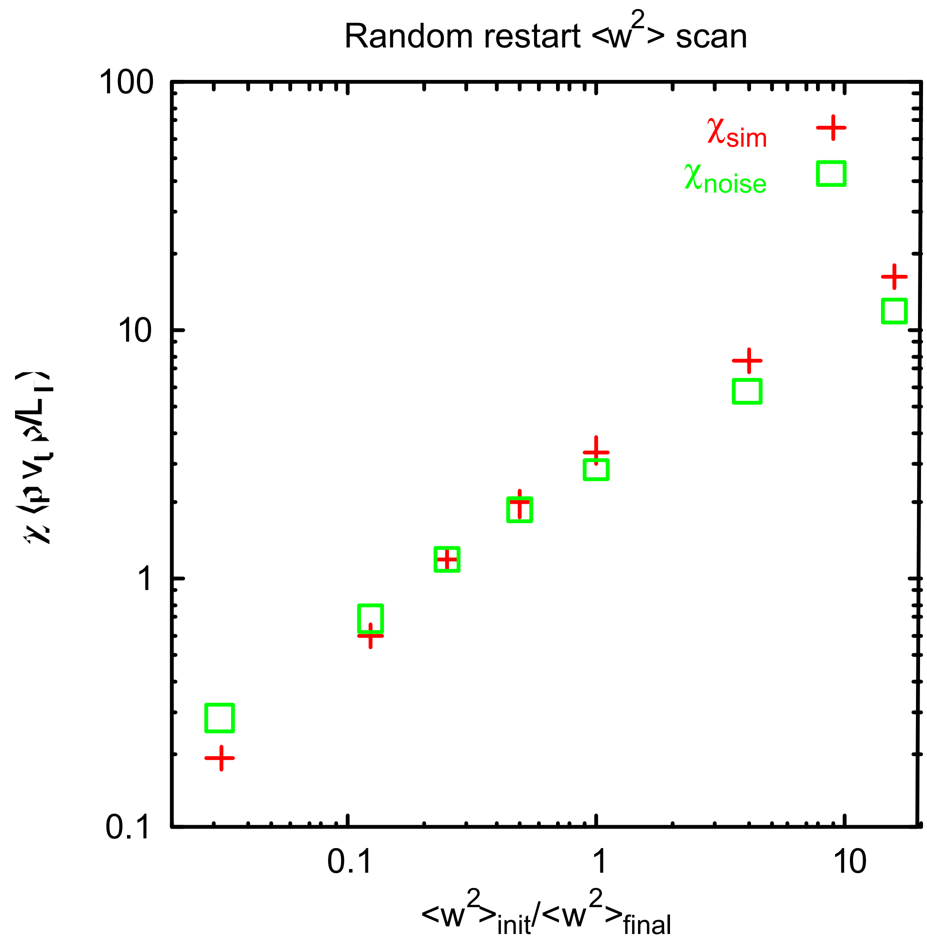
Predicted $\chi_{noise} \approx$ measured χ_e for every $\langle w^2 \rangle_{initial}$ in noise test

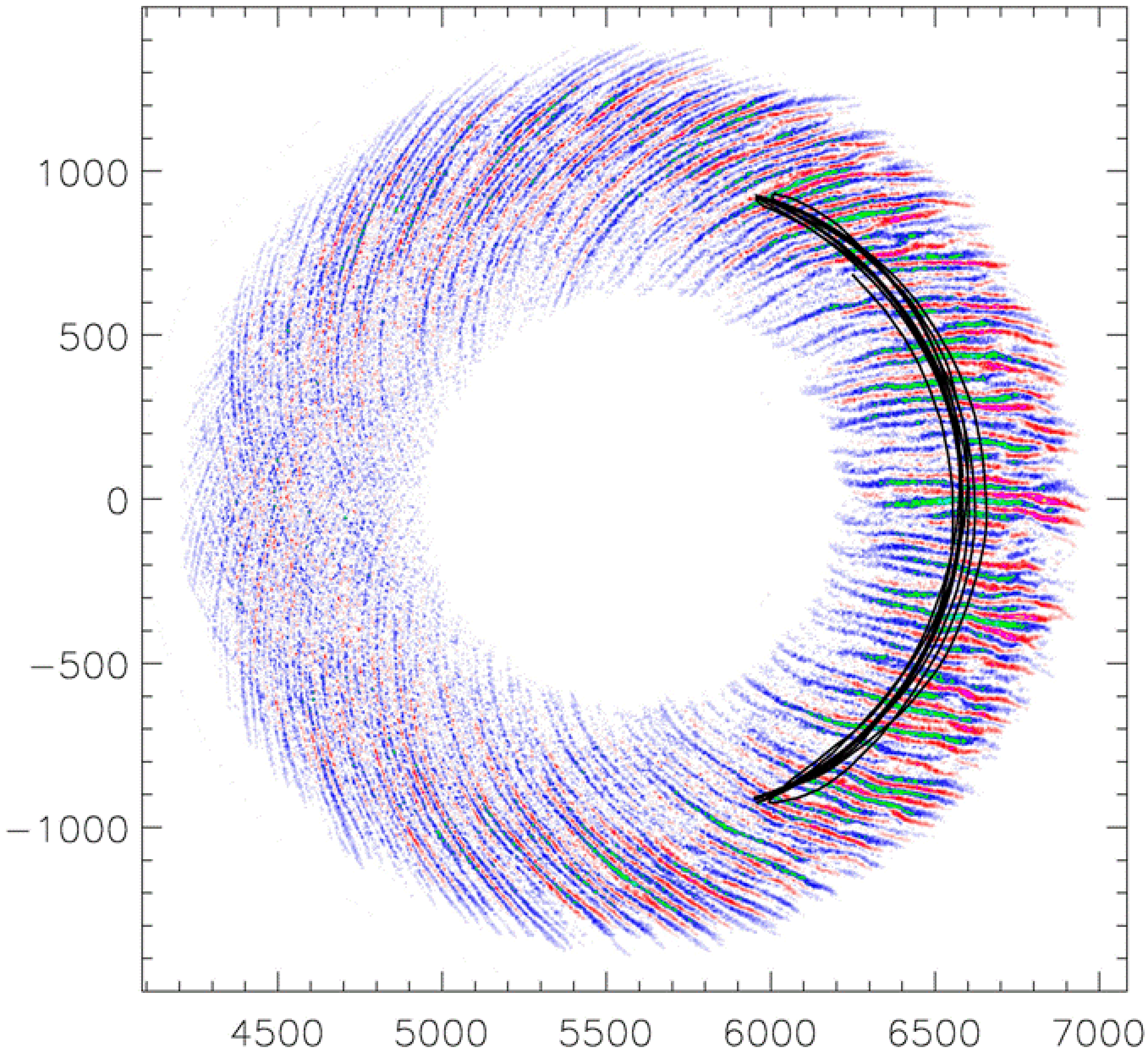


Predicted $\chi_{noise} \approx$ measured χ_e as $\langle w^2 \rangle_{initial}$ varies by factor of 512 in noise test



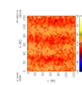
Scan to larger random weights

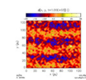




Dimits contour plots at
Same scale as Z. Lin's

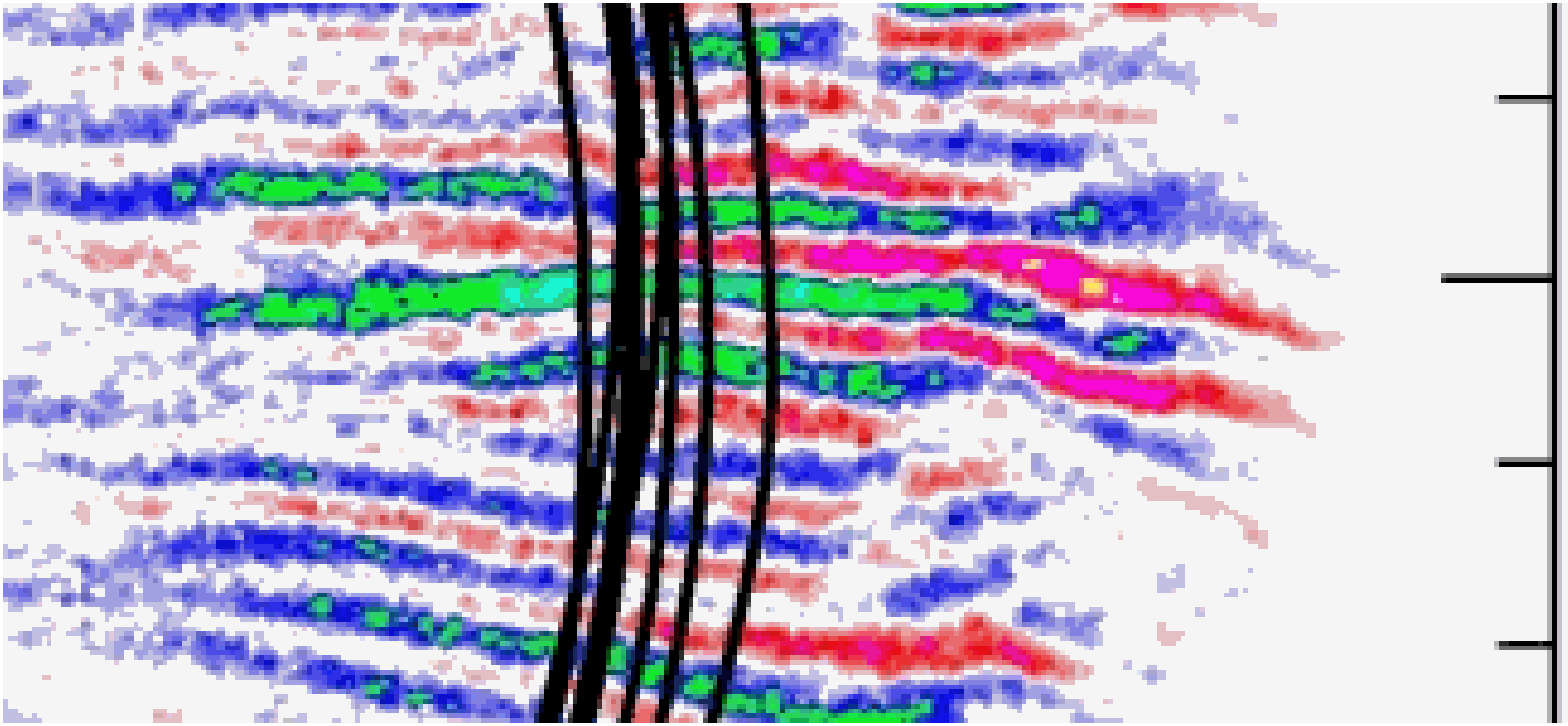
Dimits contour plot with
Re-scaled color bar

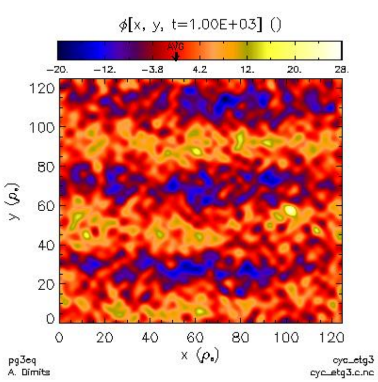


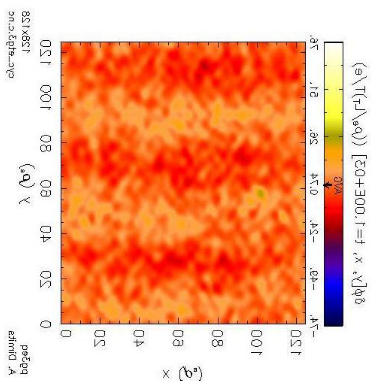


Dimits contour plot at t=1000, when χ_e ~2 x final χ_{noise}. This is when noise effects are strong enough to reduce χ_e to ~1/4th of Jenko-Dorland result, but ETG mode is still apparent.

But if one shrinks the contour plot to the scale used in Z. Lin's plots, then the eye (and the finite resolution of the computer screen) will average out the noise to make it less apparent.







If we blow up Z. Lin's contour plot, then we can see the noise at small scales more easily. It looks roughly comparable to Dimits' contour plot at t=1000 (when $\chi_e \sim 2$ x final $\chi_{noise} \sim 1/4^{th}\ \chi_{Jenko\text{-}Dorland}$.

Eyeball comparisons depend on choice of color table, smoothing in graphics, etc. as illustrated by two versions of Dimits' contour plot to left which differ only in the color table employed. Hence, we need more quantitative measures of noise than the "eyeball test".

Discussion of results

- Large initial transients in $\chi_{etg}(t)$ seen by Dimits & by Z. Lin are larger than or comparable to Jenko & Dorland χ_{etg}. But this high χ_{etg} quickly drives weights so large that $k_\perp^2 D_{noise} \sim \gamma_{lin}$ and the turbulence is suppressed or significantly reduced.

- Scanning from 5-20 particles/cell appears to be converged (but isn't). Just wait longer and the weights build up to give the same noise level:

$$D_{noise} \propto \left[\frac{\langle w^2 \rangle}{n V_{smooth}} \right]^{1/2} \propto \left[\frac{\int_0^t dt' \, \chi(t')}{n V_{smooth}} \right]^{1/2}$$

- ETG eddies are radially very extended but still short scale in poloidal direction, so only takes a little bit of D_{noise} << Jenko-Dorland χ_{etg} to suppress or significantly reduce the turbulence. Radially extended ETG eddies more sensitive to noise than ITG is, requires many more particles to converge:

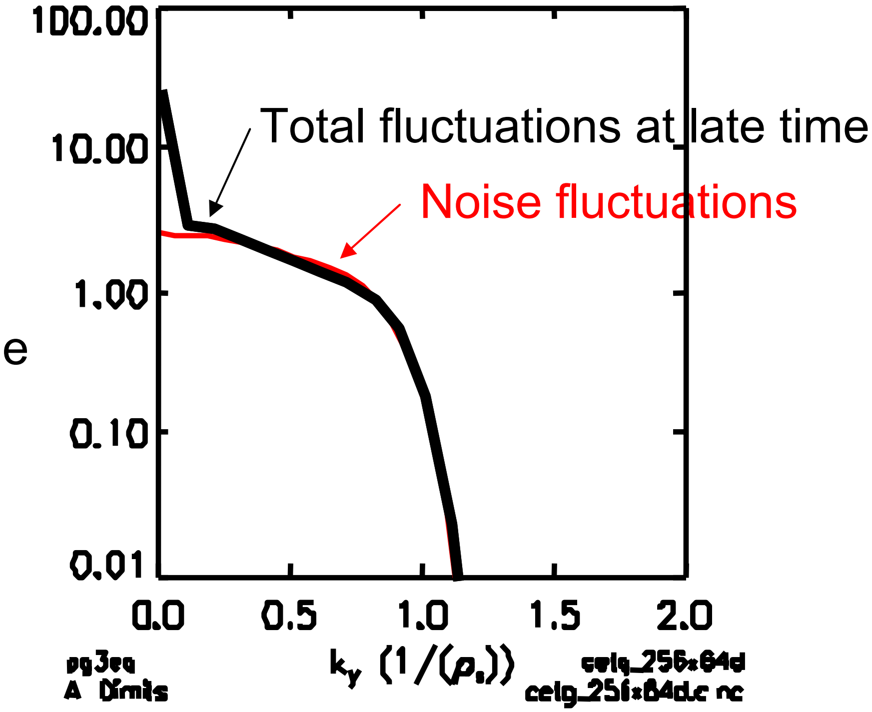
$$\boxed{D_{total} = \frac{\gamma - k_\perp^2 D_{noise}}{k_r^2} + D_{noise}}$$

Caveats

- Used guestimates of filtering/numerical parameters for Z. Lin's ETG simulations, based on Lin's previous ITG spectra.
- Neglected differences in zonal components of noise due to differences in ITG/ETG zonal flow dynamics.
- Fluctuation-dissipation theorem used uniform plasma dielectric in unsheared slab geometry. Probably good approximation at late times when noise dominates and turbulence is suppressed, but would be interesting to try a renormalized model of turbulent dielectric.
- Long-time scale variability often seen in $\chi(t)$ makes detection of trends harder
- At present have used just test-particle diffusion coefficients, assuming $\chi = (3/2)\, D_{test}$. Should calculate energy weighted thermal diffusion more consistently, including adiabatic constraints that allow heat transport but no net particle transport.
- Transition from turbulence-dominated state to noise-dominated state expected to be slower in Z. Lin's simulations then in Dimits' initial simulations (because of box size differences).

Conclusions

- Simple calculation of spectrum of noise fluctuations due to random uncorrelated particles, agrees within a factor of 2 of more complicated derivation.



- Detailed calculation of noise spectrum (extending Krommes 93 calculation to include filters, etc.) agrees very well (no free parameters) with observed spectrum at late times in Dimits' gyrokinetic ETG simulations (chosen with parameters similar to Z. Lin's simulations), confirming that noise grows to dominate those ETG results.

- Resolves discrepancy, supports Jenko-Dorland result that ETG can give large transport.

- Renormalized calculation of χ_{noise} (also with no free parameters) agrees very well with PIC simulations.

- ETG simulations require many more particles for convergence than ITG. Motivates search for additional methods of reducing noise (such as the Vadlamani-Parker weight resetting algorithm). Have to be careful that the artificial dissipation introduced by these methods isn't too big…

References

- Nevins, Hammett, Dimits, Dorland, Shumaker,"Discrete Particle Noise in Particle-in-Cell Simulations of Plasma Turbulence", UCRL-212536, August, 2005, to be published in November Physics of Plasmas. http://w3.pppl.gov/~hammett/papers

- Hammett, Nevins, Dimits, Sherwood 2005 Invited Talk "Particle Noise-Induced Diffusion & Its Effect on ETG Simulations", http://w3.pppl.gov/~hammett/talks

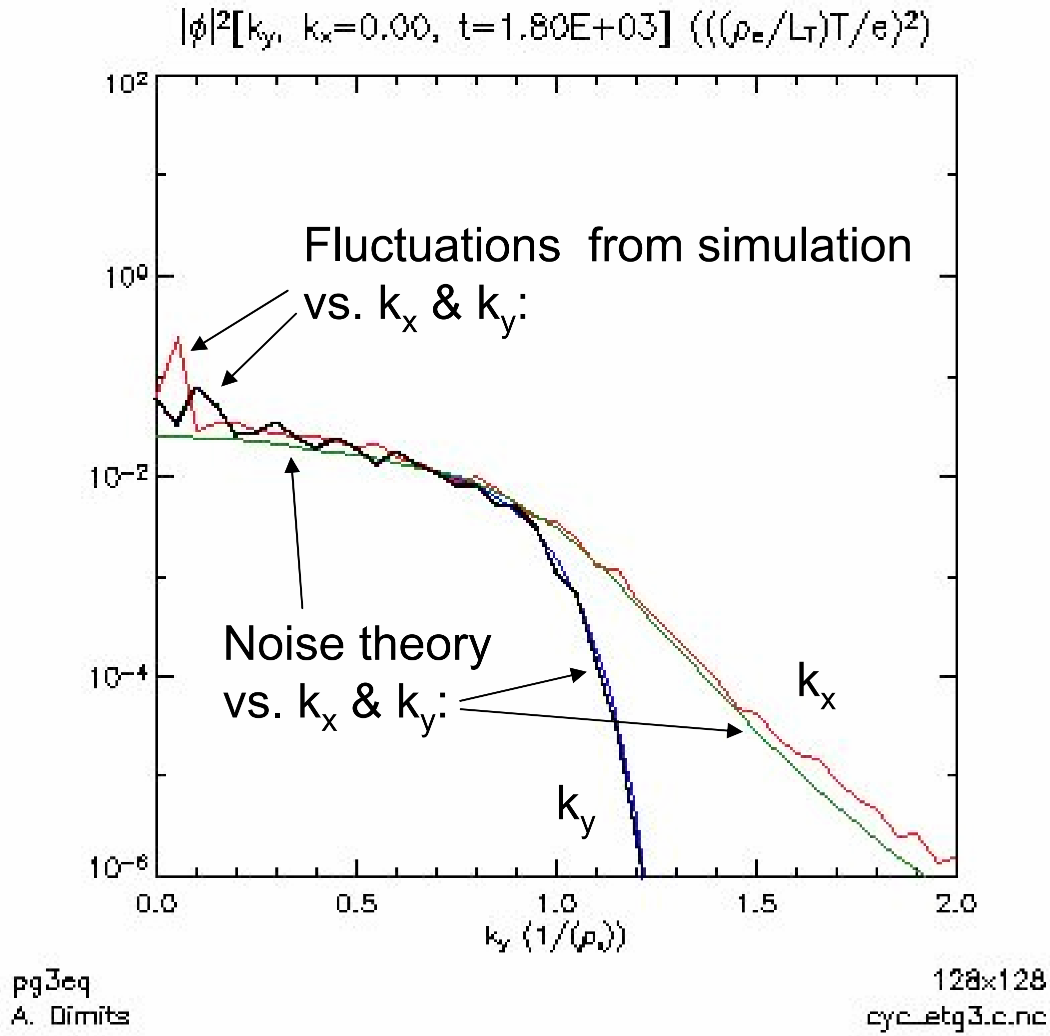
- Krommes 1993, Phys. Fluids B5, 1066
- Hu & Krommes 1994, Phys. Plasmas 1, 863.
- Dimits et al. 2000 Phys. Plasmas "cyclone" paper
- Jenko & Dorland et al. 2000
- Dorland & Jenko et al. 2000
- Jenko and Dorland, PRL <89>, 225001 (25 Nov 2002)
- Rogers, Dorland, Kotschenreuther 2000, PRL <85>, 5336.
- Z. Lin, L. Chen, Y. Nishimura, et al. IAEA 2004
- W.W. Lee & W.M. Tang 1988, entropy balance, Phys. Fluids 31, 612.
- W.W. Lee et al. on comparing PIC with fluctuation-dissipation spectra (incl. finite beta)
- Krommes, paper on entropy paradox resolved
- W.W. Lee 1987 JCP
- Catto 1978, "Adiabatic Modifications to Plasma Turbulence Theories", Phys. Fluids 21, 147.
- A.B. Langdon, 1979, Phys. Fluids 22, 163 (1979)
- Birdsall & Langdon, Plasma Physics Via Computer Simulation (1997).

BACKUP SLIDES

Theoretical Noise Spectrum Agrees Well with Dimits PIC ETG Simulation at Late Times

- Discrete particle noise in PIC codes is quantifiable — well studied in past:
  - Langdon '79 – Birdsall&Langdon '83, Krommes '93

⇒ Useful code verification tool. We can develop objective criteria to determining when discrete particle noise is a problem



Electron Temperature Gradient (ETG) Turbulence has $\lambda \sim \rho_e << \rho_i$

- Electron Heat Transport through ion thermal barriers
  - Need
- Isn't ETG transport too weak?
  - (nearly) Isomorphic to ITG but 60× smaller …

$$\chi_e = \chi_{e0}\left(\frac{\rho_e}{L_T}\right)\rho_e v_{te} = \chi_{e0}\sqrt{\frac{m_e}{M_i}}\left(\frac{\rho_i}{L_T}\right)\rho_i v_{ti}$$

$$\left(\frac{\rho_e}{L_T}\right)\rho_e v_{te} \approx 0.075\left[\frac{T_e}{1\,keV}\right]^{1.5}\left[\frac{B}{1\,Tesla}\right]^{-2}\left[\frac{L_T}{1\,m}\right]^{-1} m^2/s$$

- For ITG, typically $\chi_{i0} < 1$

Can $\chi_{e0} >> 1$ for ETG???
weak zonal flows → strong turbulence?

- Previous simulations:
  - ⇒ Jenko & Dorland, PRL **89**, 225001 (2002)
    flux-tube continuum GK-simulation
    (nearly) Cyclone base-case-like ETG
    $\chi_{e0} \approx 13 \rightarrow \chi_e \approx 1\ m^2/s$
    increases with s, ∇T …
  - – Labit & Ottaviani, Phys. Plasmas **10**, 126 (2003)
    "global" simulations, but $a/\rho_i \sim$ 1-2 dominated by profile variations; model eqs. not full gyro-fluid eqs.
    $\chi_{e0} >> \chi_{i0}$ (but "small")
  - – Li & Kishimoto, Phys. Plasmas **11**, 1493 (2004)
    slab and flux-tube gyro-fluid simulation
    model eqs. not full gyro-fluid eqs
    χ_{e0} increases with s, ∇T
  - ⇒ Lin *et al*, 2004 IAEA Mtg. (for example)
    http://www.cfn.ist.utl.pt/20IAEAConf/presentations/T5/2T/5_H_8_4/Talk_TH_8_4.pdf
    global PIC GK simulation
    Cyclone base-case-like ETG
    $\chi_{e0} \approx 3 \rightarrow \chi_e \approx 0.2\ m^2/s$