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Comments on "A nonlinear correction to Landau-fluid closures"

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Scott, I appreciate your taking the time to think about these problems constructively. I looked over your notes, and it seems to me that our 4-moment closures already include the nonlinear corrections you are attempting to add to a 3-moment closure, at least to the same order. There are some subtleties involved, and there is a slight difference between the way you and we include nonlinearities in the closures, so I want to continue thinking about this a little more.

But first I wanted to show that in the limit you are considering, our 4 moment closure and your approach of adding a nonlinear correction to the 3 moment closure are equivalent.

The one change I will make is to your Eq. 6. At present it is:

$$\delta q_{\parallel}^{L} = -i\sqrt{\frac{8}{\pi}}\frac{k_{\parallel}v_{ti}}{|k_{\parallel}|}\delta T_{\parallel}$$

Allow me to use a linear closure which is 1 order more accurate:

$$\frac{\partial}{\partial t}q_{\parallel}^{L} = G \qquad (1)$$

where G represents the standard linear terms in the 4-moment closure. In slab geometry we got

$$G = -(3+\beta_{\parallel})\nabla_{\parallel}\delta T_{\parallel} - \sqrt{2}|k_{\parallel}|v_{ti}D_{\parallel}\delta q_{\parallel}^{L}$$

In the low frequency limit $\omega \ll |k_{\parallel}|v_{ti}$ this reduces the 3-moment result in your Eq. 6.

Your Eq. 8 for the nonlinear part is:

$$\frac{\partial}{\partial t}\delta q_{\parallel}^{NL} = -\vec{v}_E\cdot\nabla\delta q_{\parallel}^{L}$$

And your Eq. 9 says we should use $\delta q_\parallel = \delta q_\parallel^L + \delta q_\parallel^{NL}$. We can thus sum Eq. (1) and your Eq. 8 to get:

$$\frac{\partial}{\partial t}\delta q_\parallel = \frac{\partial}{\partial t}(\delta q_\parallel^L + \delta q_\parallel^{NL})$$

or

$$\frac{\partial}{\partial t}\delta q_\parallel = G - \vec{v}_E \cdot \nabla \delta q_\parallel^L$$

Since you are doing an expansion with $\delta q_\parallel^{NL} \ll \delta q_\parallel^L$, then to the same order this is equivalent to:

$$\frac{\partial}{\partial t}\delta q_\parallel + \vec{v}_E \cdot \nabla \delta q_\parallel = G$$

This is exactly the 4-moment closure we are already using, and includes a nonlinear term on the LHS.

Thus our closures do include some nonlinear effects. Going to higher order in our closures brings in more nonlinear terms as well as linear terms. N Fluid Eqs. have N ExB nonlinear terms.... As Mattor and you have shown, there are some nonlinear effects in certain limits (such as $\gamma \ll \omega \ll |k_\parallel| v_{ti}$) which are not well represented by low order closures. But we do have some nonlinear effects, and I think that our order N+1 closures have nonlinearities to the same order as your attempt to add a nonlinear correction to an order N closure.

You did a weak turbulence expansion, and in stronger turbulence regimes there would be a difference between our two approaches. But it seems to me that our approach preserves a certain structure in the equations that may be important in strong turbulence limits. Consider the nonlinear gyrokinetic Eq:

$$\partial f_1/\partial t + v_\parallel \partial f_1/\partial z + \vec{v}_E \cdot \nabla f_1 = S$$

If one is in a strong turbulence regime where $\vec{v}_E$ can be considered a rapidly fluctuating velocity, then one can consider a random walk approximation where the ExB nonlinearity is replaced by a diffusive term $D_{NL}\nabla_\perp^2$. This is essentially a very rough form of renormalization, with a resonance broadening $\Delta\omega_{NL} \sim D_{NL}k_\perp^2 \sim \vec{v}_E \cdot \nabla$. Then the kinetic equation becomes:

$$(-i\omega + ik_\parallel v_\parallel + D_{NL}k_\perp^2) f_1 = S$$

As you know, this is sometimes used to determine a renormalized dispersion relation, which will be equivalent to the linear dispersion relation with ω replaced by $\omega + iD_{NL}k_\perp^2$. This is sometimes used to determine a saturation level for the turbulence, and the resulting turbulent diffusion, by requiring that $D_{NL}k_\perp^2$ balance the linear growth rate. Because of the way we do our nonlinear Landau-fluid closures, every $\partial/\partial t$ term is accompanied by a nonlinear $\vec{v}_E \cdot \nabla$ term, and so the structure of the equations is preserved. Thus making the identical (if somewhat crude) approximations of replacing all nonlinear terms with diffusion terms, in both the original kinetic equation and in the Landau-fluid equations, will lead to equivalent results for the renormalized dispersion relation. The Landau-fluid equations provide an N-pole approximation to the linear dispersion relation, and will continue to provide an

N-pole approximation to the renormalized dispersion relation, with ω everywhere replaced by $\omega + iD_{NL}k_\perp^2$.

Of course such a simple D_{NL} substitution procedure is an oversimplification, as $\vec{v}_E$ and f_1 will have some correlations which are lost by this approximation. But the nonlinear Landau-fluid equations attempt to capture this. For example, in the case of adiabatic electrons, the $\vec{v}_E \cdot \nabla n$ nonlinearity vanishes at long wavelengths just as it should because $n = \Phi$. The real situation can be very complicated. In the DIA or RMC strong turbulence theories, there are not just nonlinear damping terms but also nonlinear noise terms. In a kinetic strong-turbulence theory, there should be some echo-like un-phase-mixing occurring, though the net energy flux in velocity space will still be to short scales in velocity (except in some weak turbulence collisionless cases where trapping in isolated islands might occur). It might be useful to try to come up with some kind of model which would inject some noise in the closures to represent some of this backscattering.