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Comments on “A nonlinear correction to Landau-fluid closures”, Part II

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I. Weak-turbulence expansion of a 4-moment closure

It may be clearer to show the relationship between our 4-moment closures and Scott's nonlinear correction to a 3-moment closure to do the reverse of my previous note. So here I will start with the 4-moment closure, and do the same weak-turbulence expansion Scott did.

I will write the equation for the parallel heat flux q in the 4-moment Landau-fluid approximation to the nonlinear drift-kinetic equation in a slab as:

$$\partial q/\partial t + \vec{v}_E \cdot \nabla q + c_1 i k_\parallel v_t T + c_2 |k_\parallel| v_t q = 0 \tag{1}$$

where c_1 and c_2 are order unity closure coefficients. Assume the $\vec{v}_E$ is small and expand. The lowest order Eq. is for the linear part of q^L:

$$\partial q^L/\partial t + c_1 i k_\parallel v_t T + c_2 |k_\parallel| v_t q^L = 0 \tag{2}$$

The next order equation determines the nonlinear correction q^{NL}:

$$\partial q^{NL}/\partial t + \vec{v}_E \cdot \nabla q^{NL} + c_2 |k_\parallel| v_t q^{NL} = 0 \tag{3}$$

These are the Landau-fluid analogues of Scott's Eqs. 3 & 4. Comparing with Scott's Eq. 5 brings up an interesting point. It looks to me like there is a term $\dot{\mathbf{z}}^0 \cdot \nabla \delta f^{NL}$ missing from the LHS of Scott's Eq. 5. This is independent of a general weak turbulence expansion. It is only if one does a subsidiary $\omega \gg k_\parallel v_t$ expansion that one can neglect this term. That would be a cold plasma or fluid limit, where usual fluid equations do okay already. We are trying to keep some kinetic effects, so for now I will leave Eq. (3) as it is.

Solving Eq. (2) for q^L gives

$$q^L = \frac{c_1 i k_\parallel v_t}{-i\omega + c_2 |k_\parallel| v_t} T \tag{4}$$

If one was to take the $\omega \ll |k_\parallel| v_t$ limit, then this reduces to a 3-moment closure, Parker's Eq. 6. Inserting Eq. (4) (in its present form) into Eq. (3) gives:

$$q^{NL} = -\frac{1}{-i\omega + c_2 |k_\parallel| v_t} (\vec{v}_E \cdot \nabla) \left(\frac{c_1 i k_\parallel v_t}{-i\omega + c_2 |k_\parallel| v_t} T \right) \tag{5}$$

Here I am using quantum mechanical operator notation, where operators don't commute. Expressing T and Φ in k space:

$$T = \sum_{k'} T_{k'} \; e^{i\vec{k}' \cdot x - i\omega' t} \tag{6}$$

and

$$\Phi = \sum_{k''} \Phi_{k''} \; e^{i\vec{k}'' \cdot x - i\omega'' t} \tag{7}$$

Eq. (5) can be written in the usual convolution form as

$$q^{NL} = \frac{1}{-i\omega + c_2 |k_\parallel| v_t} \sum_{k'} \frac{c}{B} \hat{z} \times \vec{k}'' \cdot \vec{k}' \Phi_{k''} \frac{c_1 i k'_\parallel v_t}{-i\omega' + c_2 |k'_\parallel| v_t} T_{k'} \tag{8}$$

This reduces to Scott's results in the same limits he considered: assuming $\omega \gg |k_\parallel| v_t$ in the first denominator (the approximation Scott made in his Eq. 5, as discussed above after my Eq. 3), while assuming assuming $\omega' \ll |k'_\parallel| v_t$ in the second denominator (equivalent to reducing Eq. (2) to a 3-moment closure), then this exactly reproduces the second half of Parker's Eq. 10 (the first half is just q^L, and the second half is q^{NL}):

$$q^{NL} = -\sum_{k'} \frac{c}{B} \hat{z} \times \vec{k}'' \cdot \vec{k}' \Phi_{k''} \frac{c_1 k'_\parallel v_t}{\omega c_2 |k'_\parallel| v_t} T_{k'} \tag{9}$$

To summarize, it seems to me that the 4-moment Landau fluid closures already contain nonlinear effects on q equivalent to Scott's nonlinear correction to a 3-moment closure in the limit he considers. It seems to me that the 4-moment closure is more general for other limits.

[There are some minor subtletites in the way I treated the ω dependence in Eq. (6) and Eq. (7), where I made an often made but often unjustified approximation that each k' mode has a single frequency ω'. With just a little more algebra, one can keep general frequency dependence and then Eq. (8) will contain a convolution over frequency in addition to the k convolution. But since I was going to make the simplification $\omega' \ll c_2 |k'_\parallel| v_t$ anyway, to recover the limit Scott was considering, the extra algebra didn't seem worth the bother.]

II. strong-turbulence effects

The strong-turbulence regime is very difficult to treat with any rigorous analytic theory. But just to illustrate one kind of qualitative effect the 4-moment closures have, consider the random walk or crude renormalization approximation (which I discussed in my previous notes) applied to Eq. (1). If $\vec{v}_E$ is a randomly fluctuating field (and assuming that q doesn't have strong correlations with it, such as the density would in the adiabatic electron limit), then we might try replacing the $\vec{v}_E \cdot \nabla$ term with $D_{NL} k_\perp^2$. With this approximation, we can now solve Eq. (1) for q to find:

$$q = \frac{c_1 i k_\parallel v_t}{-i\omega + D_{NL} k_\perp^2 + c_2 |k_\parallel| v_t} T$$

Note that in the strong turbulence limit where D_{NL} is very large, the Landau damping will be reduced. This is qualitatively what one expects.

As I've said in the past, the Mattor-Parker PRL is very interesting and contains a lot of innovative ideas. There may still be some ways to use those ideas to develop computationally simple improvements to Landau-fluid closures. Their idea of retaining ω-dependence in the closures as a WKB-type sampling of the frequency domain may help improve the results.

But it seems to me that strong turbulence regimes are actually easier in some sense than weak turbulence regimes where one may not get enough phase-space islands to overlap and flattening near localized resonances can affect nonlinear results and the results might be sensitive to a weak amount of collisions.