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The Dynamics of Small-Scale Turbulence-Driven Flows

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Abstract

We investigate the dynamics of small-scale turbulence-driven sheared $\mathbf{E} \times \mathbf{B}$ flows in nonlinear gyrofluid simulations. The importance of these zonal flows in the regulation of the turbulence was shown in our early simulations[1–3] and has been widely confirmed. Most of these flows experience fast collisionless linear damping, but there is a residual non-Maxwellian component of the flow which is undamped[4] and scales with r/R. In our original treatment, we included collisionless damping terms which capture the fast collisionless damping of the damped components, but which do not accommodate the linearly undamped components. Here, we modify the gyrofluid closures to account for Pfirsch-Schlüter heat flows. This modification allows a linearly undamped component of the $\mathbf{E} \times \mathbf{B}$ flows, and here we begin to assess its importance in nonlinear simulations. Our preliminary results indicate very near marginal stability zero-flux states can exist where the undamped zonal flows completely damp the turbulence, as shown in Ref. 5. But away from marginal stability, we find that retaining the undamped component of the flow has very little effect.

I. Introduction

The dynamics of small-scale fluctuation-driven flows are of great interest for micro-instability driven turbulence, since nonlinear simulations of core tokamak turbulence[1–3,6,7] have shown that these flows play an important role in the regulation of the turbulence and transport levels. Earlier simulations of resistive edge turbulence also found that turbulence-generated flows are important[8–10] (see also the discussion in Ref. 1). These "zonal flows" are constant on a flux surface, with radial variations on the scale of the turbulence (a few gyroradii) and hence have also been called "radial modes." The zonal flows are nonlinearly driven by the turbulence and in turn suppress the turbulence through radial shearing of eddies.[11] Thus, a proper treatment of the zonal flows has been of concern for some time, and in particular, the damping of these poloidal flows is a sensitive control of the transport levels. The gyrofluid damping of these flows was shown to be accurate for times shorter than a bounce time.[3] Since the decorrelation times of the turbulence are generally shorter than a bounce time, our original hypothesis was that this description was adequate. Recent work[4] has pointed out possible problems with this hypothesis, emphasizing the existence of a linearly undamped component of the flow which could build up in time and lower the final turbulence level.

The existence of linear collisionless poloidal flow damping is sensitive to initial conditions. If the distribution function at $t = 0$ is Maxwellian, as seems likely for $\mathbf{E} \times \mathbf{B}$ nonlinearly driven flows, most of the flow damps away (due to collisionless Landau damping) at a rate $\sim v_t/qR$. The residual flow correctly calculated in Ref. 4 is a non-Maxwellian component of the flow which is not collisionlessly damped. In our original gyrofluid treatment,[3] we included collisionless damping terms which were appropriate (and essential) to capture the fast collisionless damping of the damped components, but which do not accommodate the linearly undamped components. While gyrofluid and gyrokinetic results agree on the initial collisionless damping, the original closures[12] used in the gyrofluid equations fail to describe the residual non-zero potential at times longer than several bounce times.

Although this residual component of the flow is undamped linearly, it can still be damped nonlinearly by the turbulence itself, i.e., by turbulent viscosity. Thus the nonlinear drive of the poloidal flows can, at least in some cases, be balanced by the nonlinear damping of these flows and a steady state balance can be reached.

In this paper, we extend our gyrofluid equations to include this undamped component of the flow by modifying the closure approximations for the zonal components. Then, we test the nonlinear importance this residual flow by comparing nonlinear simulations with and without the undamped residual component. We find that for strongly turbulent cases, the undamped component of the flow is nonlinearly damped and has little impact on the turbulence levels. Very near marginal stability, however, we find that the system may bifurcate into a state with all flow and no turbulence (and hence no turbulent viscosity to damp the flows), as seen in Ref. 5. It is possible that this state is an artifact due to our purely collisionless approximation and/or the simulation initial conditions.

II. Linear Zonal Flow Damping

In this section we present a brief outline of the derivation of modifications to our gyrofluid closures which retain the undamped residual flow component. The collisionless zonal flow evolution is governed by the linearized toroidal electrostatic gyrokinetic equation:

$$\frac{\partial f}{\partial t} + (v_\parallel \hat{\mathbf{b}} + \mathbf{v}_d) \cdot \nabla f + \bar{\mathbf{v}}_E \cdot \nabla F_0 + \left(\frac{e}{m} \bar{E}_\parallel - \mu \nabla_\parallel B + v_\parallel (\hat{\mathbf{b}} \cdot \nabla \hat{\mathbf{b}}) \cdot \bar{\mathbf{v}}_E \right) \frac{\partial F_0}{\partial v_\parallel} = 0, \tag{1}$$

where $f(v_\parallel, \mu)$ is the perturbed distribution function and $\mathbf{v}_d$ is the combined ∇B and curvature drift. For the zonal flows, where $\mathbf{v}_E \cdot \nabla F_0 = 0$, changing variables to $f(E, \mu)$, this becomes:

$$\frac{\partial f}{\partial t} + v_\parallel \nabla_\parallel f + i\omega_d f + i(eF_0/T)\omega_d \Phi = 0. \tag{2}$$

If we initialize a Maxwellian perturbation in f that is initially constant along a field line but has an $\exp(ik_r r)$ radial variation, then for times shorter than a bounce time the $i\omega_d = \mathbf{v}_d \cdot \nabla = i(k_r \rho_i / v_t R)(v_\parallel^2 + v_\perp^2/2) \sin\theta$ terms introduce variations in θ (along the field line) which are then phase mixed away by parallel free streaming. After a few bounce times this phase mixing stops (the trapped particles don't keep phase mixing), and an equilibrium is reached where parallel variations in f balance the variations induced by the cross field drifts, analogous to the equilibrium Pfirsch-Schlüter flows. Rosenbluth and Hinton[4] showed that this equilibrium could be written:

$$f = -(e\Phi/T)F_0 + h(E,\mu)e^{-ik_r\rho\frac{qB_0 v_\parallel}{\epsilon B v_t}}. \tag{3}$$

where $h(E,\mu)$ is an arbitrary function but is constrained to satisfy $\frac{\partial h}{\partial l} = 0$, and $\epsilon = r/R$. In the notation for the circular model we are using here, $k_r\rho\frac{qB_0}{\epsilon v_t}$ is constant on a flux surface, while B and $v_\parallel$ (at fixed E and μ) introduce variations in f along the field line. This prevents f from being a simple Maxwellian that is constant on a field line, and it is this equilibrium solution that we would like to recover. We can expand this for small banana width:

$$f = -(e\Phi/T)F_0 + h(E,\mu)[1 - ik_r\rho\frac{qB_0 v_\parallel}{\epsilon B v_t} + \cdots], \tag{4}$$

and integrate to find the equilibrium moments. There is some freedom in the choice of h, but generally we see that the $u_\parallel$ moment is supported by the radial gradient of the perturbed pressure p (i.e., the usual balance that leads to the Pfirsch-Schlüter particle flow), p is supported by radial gradients of heat flux moments q, q is supported by gradients of 4th moments (the analogous balance for the Pfirsch-Schlüter heat flow), etc.... In our original closure[12] we closed higher moments to retain collisionless Landau damping, but in such a way that the perturbed moments relaxed to be constant along a field-line, thus missing these $k_r\rho_{\mathrm{banana}}$ corrections. Now we chose closures which damp to the above collisionless equilibrium that includes some parallel variation. We can accomplish this by the following modifications to the equations in Ref. 12:

$$\begin{array}{ll} \sqrt{2}D_{||}|k_\parallel|q_\parallel \to \sqrt{2}D_{||}|k_\parallel|(q_\parallel - q_\parallel^{(0)}) & \text{in Eq. (91)} \\ \sqrt{2}D_\perp|k_\parallel|q_\perp \to \sqrt{2}D_\perp|k_\parallel|(q_\perp - q_\perp^{(0)}) & \text{in Eq. (92)} \end{array}$$

Where the Pfirsch-Schlüter equilibrium heat flows are $q_\parallel^{(0)} = 3ik_r\rho_i\frac{qB_0}{\epsilon B}T_\parallel$ and $q_\perp^{(0)} = ik_r\rho_i\frac{qB_0}{\epsilon B}T_\perp$, for a bi-Maxwellian h. Note that these modifications preserve the fast linear collisionless damping coming from the $|k_\parallel|$ terms, but now damp to a non-zero equilibrium solution. Further, to support this $q_\parallel^{(0)}$ and $q_\perp^{(0)}$, we must close the $i\omega_d(r_{\parallel,\parallel} + r_{\parallel,\perp})$ and $i\omega_d(r_{\parallel,\perp} + r_{\perp,\perp})$ terms in the pressure equations in a manner which preserves this equilibrium. We are presently investigating various forms of these closures, but for the simulations presented in this paper, we chose $\nu_1 = (0, -3)$ $\nu_2 = (0, 1)$, $\nu_3 = (0, 0)$, $\nu_4 = (0, -3/2)$. Consistent with a small banana width limit we dropped all other toroidal phase mixing closure terms ($\nu_5 - \nu_{10} = 0$). Importantly, because our flux-tube simulation is spectral in the perpendicular directions,[13] it was easy to implement these modified closures just for the evolution equations for the $n = 0$ zonal flows without changing the equations for the $n \neq 0$ components.

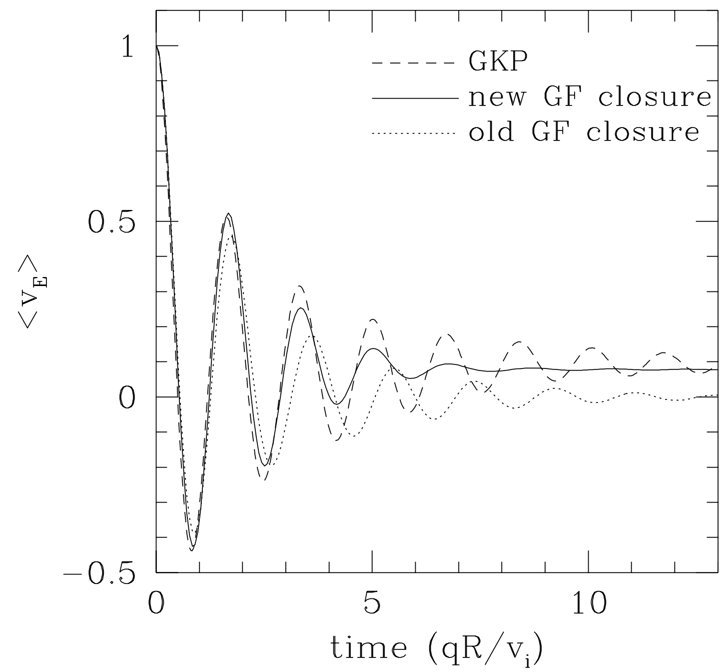

Figure 1: Linear flow damping comparison of the flux surface averaged $\mathbf{E} \times \mathbf{B}$ flow, $\langle \mathbf{v}_E \rangle$, vs. time, showing reasonable agreement between gyrokinetic particle simulation (GKP) and the new gyrofluid (GF) closure which retains the undamped residual component. Also shown is the previous gyrofluid closure which admitted no undamped component.

We now compare our linear zonal flow damping with fully kinetic results. Figure 1 shows a comparison between the modified closure described above, our previous results, and results from Z. Lin's gyrokinetic particle simulation (reported in Ref. 14). The parameters for this comparison correspond to DIII-D shot 81499: $q = 1.4$, $\epsilon = 0.18$, as will be used for the nonlinear results shown below. Also, for this mode

$k_r\rho_i = 0.2$. While our previous closures did not support the undamped component of the flow, our new closure agrees quite well with the kinetic results, and reasonably well with the prediction from Ref. 4, $v_f/v_i = c\sqrt{\epsilon}/q^2/(1 + c\sqrt{\epsilon}/q^2)$, where $c = 0.625$. The agreement is not as good at lower $k_r\rho_i$, and we are presently investigating further modifications to the closures which may improve the comparison. However $k_r\rho_i \sim 0.2$ is the region which contributes most to the effective shearing rate relevant to the turbulence. Furthermore, the existence of this undamped component allows us to test its importance in fully nonlinear simulations.

III. Nonlinear Tests of the Importance of the Undamped Flow Component

We now compare two nonlinear simulations, one using the old closures for the zonal flows, and one using the new closure which retains the undamped residual component. Again, only the dynamics of the zonal flows are changed, all other modes obey the same equations in both simulations. The other relevant parameters for shot 81499 are: $L_{ne}/R = 0.45$, $R/L_{Ti} = 6.9$, $\hat{s} = 0.78$, and we further assume $T_i = T_e$, circular geometry, and neglect impurities. This is the case for which our gyrofluid χ_i is higher than Dimits' gyrokinetic particle (GKP) simulations[5] by a factor of 3. As shown in Figure 2, including the residual flow component lowered our χ_i by at most 15%, clearly not enough to account for the full discrepancy between the gyrofluid and GKP simulations. At these parameters, the turbulence is strong enough that damping by nonlinear viscosity keeps the undamped components of the zonal flows from growing to large amplitudes.

We now look at a case near marginal stability, with all parameters the same as in Fig. 2, but with a weaker ion temperature gradient, $R/L_{Ti} = 4$. In Fig. 3(a) we show the run with the old closures, with no residual flow component. In this case the turbulence exhibits an intermittent behavior where a burst of turbulence (χ_i) drives a burst of zonal flows ($\langle\Phi\rangle_{RMS}$ is the volume averaged zonal potential), which then suppresses the turbulence. Then the flow is damped, and eventually the turbulence can grow again and repeat the cycle. It is interesting to note the similarity with Mazzucato's fluctuation measurements[15] in ERS plasmas, which are very likely near marginal stability (when equilibrium flows are included).

In Fig. 3(b) we repeat the $R/L_{Ti} = 4$ run with the new closures, which retain the linearly undamped

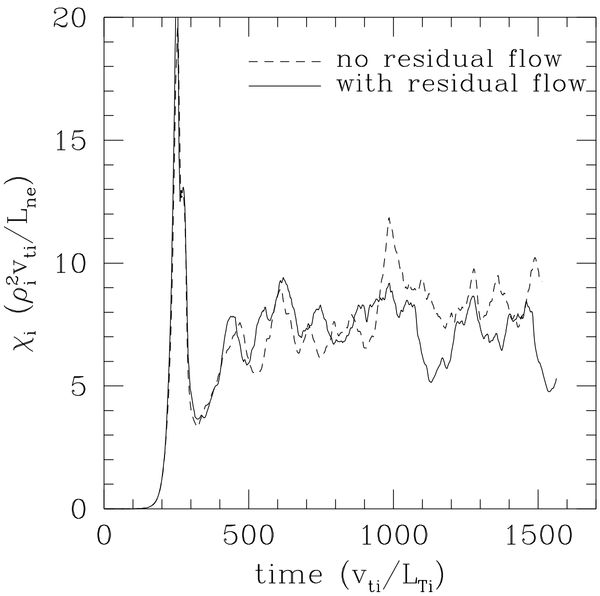

Figure 2: Comparison of χ_i time histories for two nonlinear runs, with the old closure and new closure. In this strong turbulence regime (parameters taken from DIII-D shot 81499), retaining the linearly undamped component of the flow does not significantly reduce the turbulence, indicating that nonlinear damping (turbulent viscosity) keeps the linearly undamped components of the zonal flows from growing to large amplitudes.

residual flow component. In this case the turbulence again drives a burst of flow, which damps the turbulence. But now this flow is linearly undamped, and once the turbulence dies away, there is no nonlinear viscosity to damp the residual flow, unlike the case in Fig. 2, above. The system is then stuck in this all flow, zero flux state forever. This zero flux above but near marginal stability, or nonlinear upshift in the critical gradient, was first seen in Ref. 5. It seems possible that this state is an artifact due to the purely collisionless limit and/or the simulation initial conditions. Perhaps if this run were initialized with a larger amplitude and broader spectrum of modes the flow would not be as strongly driven at the peak in χ_i. Once a linearly undamped zonal flow is permitted, nonlinear simulations can be dependent on initial conditions, since an arbitrarily large flow could be initialized which damps all other modes, thus a zero flux state is possible at any R/L_{Ti}. The collisional flow damping time,[16] $\tau_{00} = \epsilon/(1.5\nu_{ii})$, corresponding to these parameters is

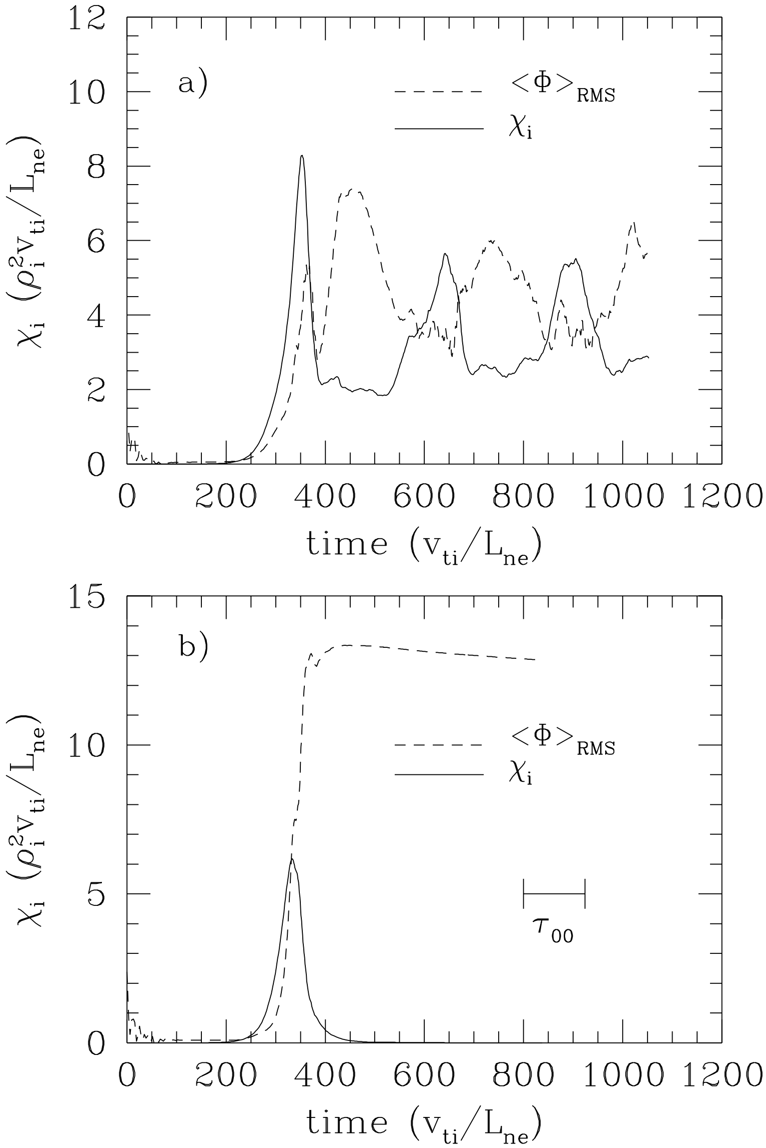

Figure 3: Nonlinear runs close to marginal stability, one with the old closure and no residual component (a), and one with the new closure retaining the residual flow component (b). With the old closure (a), the turbulence drives flow which then quenches the turbulence until the flow is damped away, after which the turbulence grows again, leading to a bursty behavior. With the new closure (b), the undamped component of the flow keeps the turbulence suppressed for the duration of the run. Also noted in (b) is the collisional damping time for the zonal flows (not included in these collisionless simulations). If included, this run might exhibit the bursty behavior shown in (a).

also shown in Fig. 3(b), and if collisions were included, perhaps a turbulent steady state or a bursty state with period on the order of τ_{00} would be reached. In any event, the inclusion of a realistic collisionality would change Fig. 3(b).

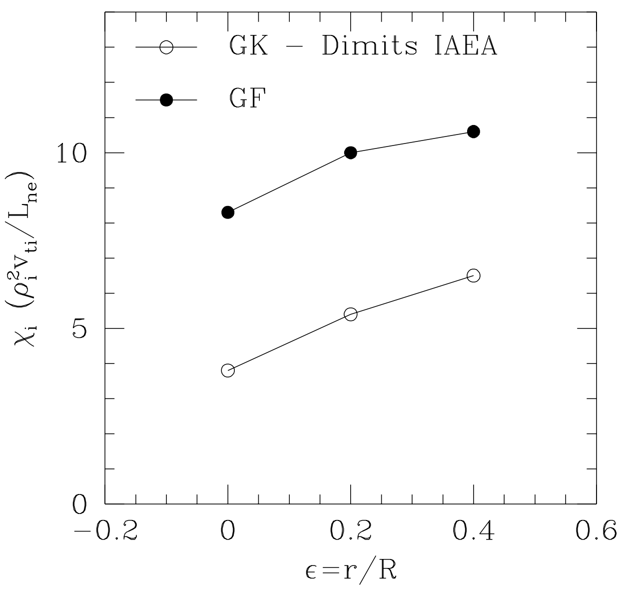

Figure 4: Comparison of gyrokinetic and gyrofluid nonlinear results as $\epsilon = r/R$ is varied (with the old GF closures). Since the residual flow component is proportional to ϵ, if the residual flow component were responsible for the difference in the two predictions, the results would converge at $\epsilon = 0$. The fact that the difference is independent of ϵ implies that for these parameters, turbulent viscosity is sufficiently damping the residual flow components.

Before these recent modifications of the gyrofluid closures, we earlier did a different test of whether or not the residual flows can account for the difference between GF and GKP simulations, by comparing GF simulations without residual flows to GKP simulations with residual flows in an ϵ scan. The size of the residual undamped flow scales with $\sqrt{\epsilon}$.[4] Figure 4 compares GF and GKP simulations[7] varying ϵ, showing similar scalings with ϵ, at least for these parameters, which are not near marginal stability. If the residual flow components were responsible for these differences, one would expect the GF and GKP χ_i to converge as $\epsilon \to 0$ and the residual components are removed. This again suggests that nonlinear damping of the zonal flows is providing a sufficient sink.

IV. Conclusions

We have extended our previous gyrofluid equations[12] to model the undamped residual component of the zonal flows emphasized in Ref. 4. Our preliminary results indicate that for parameters corresponding to DIII-D shot 81499, including the residual component only lowered our predicted χ_i by 15%. Closer to marginal stability, we find that including the residual flows allows the system to bifurcate into a state with all flow and zero flux, confirming the nonlinear upshift in the critical gradient,[5] but also isolating the zonal flows as the mechanism. Future work is needed to investigate the effects of collisions and simulation initial conditions on the results near marginal stability.

There are several possible reasons for the differences seen between gyrofluid and gyrokinetic turbulence simulations. We plan to investigate frequency-dependent closures suggested by N. Mattor and methods to improve resolution (as suggested by A. Dimits or with larger parallel computers). Alternatively, even a weak amount of collisions can significantly affect nonlinear kinetic calculations in some cases. Finally, the addition of trapped electrons and/or collisions can lead to stronger turbulence regimes where the gyrofluid and gyrokinetic simulations may agree better.

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