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# The small magnetic Prandtl number approximation suppresses magnetorotational instability

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**Abstract.** Axisymmetric stability of viscous resistive magnetized Couette flow is re-examined, with emphasis on flows that would be hydrodynamically stable according Rayleigh's criterion: opposing gradients of angular velocity and specific angular momentum. In this regime, magnetorotational instability (MRI) may occur. The governing system in cylindrical coordinates is of tenth order. It is proved, by methods based on those of Synge and Chandrasekhar, that by dropping one term from the system, MRI is suppressed, in fact no instability at all occurs, with insulating boundary conditions. This term is often neglected because it has the magnetic Prandtl number, which is very small, as a factor; nevertheless it is crucially important.

**Mathematics Subject Classification (2000).** 76E25.

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## 1. Introduction

Magnetorotational instability (MRI) is important to theoretical astrophysics because it is the only linear instability known to grow robustly under the conditions prevailing in most accretion disks: an electrically conducting fluid; a positive gradient of specific angular momentum, $\partial(r^2\Omega)^2/\partial r > 0$; and a negative gradient of angular velocity, $\partial\Omega^2/\partial r < 0$ ([1]). Some believe that purely hydrodynamic nonlinear or non-modal mechanisms may drive turbulence in such disks, but this point is controversial ([14], [9]). Turbulence, though not directly observable, is required to explain the luminosity of the disk by dissipation of orbital energy.

MRI was originally conceived as an ideal-MHD instability ([23]; [2]) but liquid metals are far from ideal on laboratory scales, especially in their magnetic diffusivity ($\eta$), which is typically $\sim 10^6$ times larger than their kinematic viscosity ($\nu$). This makes MRI experimentally challenging. Until recently, the literature on liquid-metal Couette flow has treated magnetic effects as modifications to the Taylor instability, in which viscous and inertial forces are comparable and $\partial(r^2\Omega)^2/\partial r < 0$. In this regime, as first shown by Chandrasekhar, the equations of motion can be scaled so that terms proportional to the magnetic Prandtl number $P_{\rm m} \equiv \nu/\eta \sim 10^{-6}$ are manifestly negligible, permitting a reduction of the axisymmetric stability analysis from tenth to eighth order in radial derivatives ([3]). Chandrasekhar's "small$-P_{\rm m}$" approximation governed essentially all analyses of magnetized Couette flow for the following forty years; none of these works predicted MRI, nor did contemporary experiments observe it (*e.g.*, [4]; [8]; [24]; [22];[20]; [5]). Yet recently, several groups have embarked on experiments to demonstrate MRI in liquid-metal flows ([12]; [13]), and one has even claimed success ([19]).

More recent theoretical analyses of magnetized Couette flow do predict MRI ([16]; [7], henceforth (GJ); [17]; [18]). These studies do not use Chandrasekhar's small$-P_{\rm m}$ approximation, but they tend to express their results in terms of $P_{\rm m}$ , even though $P_{\rm m} \sim 10^{-6}$ in the liquid metals that are to be used in experiments. GJ argued that it is more natural to describe the onset of MRI in terms of dimensionless parameters that do not involve $\nu$. The authors of [17] confirmed that the critical values of such parameters for MRI are insensitive to $P_{\rm m}$ when it is sufficiently small. It remains interesting to understand why the standard small$-P_{\rm m}$ approximation is inappropriate given that $P_{\rm m}$ is so very small in experiments. An attempt to clarify this was made by GJ, who argued that MRI cannot occur without one of the terms that Chandrasekhar dropped from his dimensionless equations on the grounds that it is proportional to $P_{\rm m}$. However, GJ proved their assertion only in the limit of a narrow gap between the cylinders—a regime that is entirely impractical for laboratory MRI.

The purpose of the present paper is to extend GJ's results to wide-gap Couette flows. That is, we prove that Chandrasekhar's reduced system of equations predict stability when $\partial(r^2\Omega)^2/\partial r > 0$, at least for insulating magnetic boundary conditions, which are particularly relevant to experiments. Our proof makes use of insights and techniques developed by Herron and Ali ([11]) (see also [10]).

## 2. The small magnetic Prandtl number equations and proof of stability

The background flow is $\mathbf{v} = r\Omega\mathbf{e}_\theta$ (in cylindrical coordinates $r, \theta, z$) between two cylinders of radii $r_1, r_2$ and angular velocities $\Omega_1, \Omega_2$:

$$\Omega(r) = a + \frac{b}{r^2}\,, \quad a = \frac{\Omega_2 r_2^2 - \Omega_1 r_1^2}{r_2^2 - r_1^2}\,, \quad b = \frac{\Omega_1 - \Omega_2}{r_2^{-2} - r_1^{-2}}. \tag{2.1}$$

The assumption that $\partial(r^2\Omega)^2/\partial r > 0$ implies $ab > 0$, and we take $a, b > 0$, so that $\Omega(r) > 0$, without loss of generality. The profile (2.1) supports a radially constant viscous torque $4\pi\rho\nu b$ per unit height $dz$, in which $\rho$ is the density of the fluid and $\nu$ the kinematic viscosity. A uniform magnetic field $\mathbf{B} = B_0\mathbf{e}_z$ permeates the fluid.

### 2.1. The governing equations

Following ([3]), linear perturbations are taken to be sinusoidal in $z$. In the conventions of GJ ([7]),

$$\begin{aligned} \delta v_r &= \varphi_r(r,t)\sin kz, \qquad & \delta B_r/\sqrt{\mu\rho} &= \beta_r \cos kz,\\ \delta v_\theta &= \varphi_\theta(r,t)\sin kz, \qquad & \delta B_\theta/\sqrt{\mu\rho} &= \beta_\theta \cos kz,\\ \delta v_z &= \varphi_z(r,t)\cos kz, \qquad & \delta B_z/\sqrt{\mu\rho} &= \beta_z \sin kz. \end{aligned}$$

Notice that the magnetic components have been scaled so as to have dimensions of Alfvén velocity; it is convenient to express the background field similarly, $V_A \equiv B_0/\sqrt{\mu\rho}$. The linearized equations of motion become (GJ)

$$\dot\beta_\theta = \eta(DD_* - k^2)\beta_\theta + kV_A\varphi_\theta + \underline{r\Omega'\beta_r}, \tag{2.2}$$

$$\dot\varphi_\theta = \nu(DD_* - k^2)\varphi_\theta - kV_A\beta_\theta - r^{-1}(r^2\Omega)'\varphi_r, \tag{2.3}$$

$$\dot\beta_r = \eta(DD_* - k^2)\beta_r + kV_A\varphi_r, \tag{2.4}$$

$$(DD_* - k^2)\dot\varphi_r = \nu(DD_* - k^2)^2\varphi_r - kV_A(DD_* - k^2)\beta_r - 2\Omega k^2\varphi_\theta. \tag{2.5}$$

Primes denote radial derivatives of background quantities. For perturbations, we follow Chandrasekhar's notation $Df \equiv \partial f/\partial r$, $D_*f \equiv r^{-1}D(rf)$, and use dots for time derivatives. The vertical components $\varphi_z$ and $\beta_z$ have been eliminated from eqs. (2.2)-(2.5) using $\nabla\cdot\mathbf{v} = \nabla\cdot\mathbf{B} = 0$.

The underlined term in (2.2) is the one which Chandrasekhar ([3]) argued was negligible on the grounds that $P_\mathrm{m} \ll 1$. This term represents the twisting of radial magnetic components into azimuthal ones by the background shear. One can see why it might be thought to be unimportant: near marginal instability, where the dotted terms vanish, eqs. (2.2) and (2.4) suggest that the magnetic perturbations are $\sim O(kV_AL^2/\eta)$ compared to the velocity perturbations, where $L$ is a characteristic length such as $r_2$, $r_2 - r_1$, or $k^{-1}$. Since $\eta$ is large, the underlined term would seem to be small compared to the two terms preceding it in eq. (2.2). But the importance of the underlined term cannot depend upon $P_\mathrm{m}$ alone, since the viscosity does not appear in eq. (2.2), and yet $P_\mathrm{m} \to 0$ as $\nu \to 0$ at fixed $\eta$. In fact, when $P_\mathrm{m}$ is small, the relative importance of the magnetic perturbations depends upon dimensionless ratios such as the magnetic Reynolds number $R_\mathrm{m} \equiv L^2\Omega/\eta$ and Lundquist number $S \equiv LV_A/\eta$ that do not involve $\nu$. MRI is possible when $R_\mathrm{m}$ and $S$ are $\gtrsim O(1)$ (GJ).

The boundaries are impenetrable and "no-slip", so that

$$\begin{aligned} \varphi_r &= \varphi_r' = 0, & (2.6)\\ \varphi_\theta &= 0, \text{ at } \; r = r_1, r_2. & (2.7) \end{aligned}$$

The condition on $\varphi_r'$ derives from the continuity equation $D_*\varphi_r = -k\varphi_z$ since $\varphi_z = 0$. We take the cylinders to be perfectly insulating, and the magnetic perturbations to match onto exterior solutions of $\nabla \times \delta\mathbf{B} = 0$ that are well-behaved as $r \to 0$ and as $r \to \infty$:

$$\frac{\partial}{\partial r}(r\beta_r) = \beta_r \frac{[krI_0(kr)]}{I_1(kr)} \text{ at } r = r_1, \tag{2.8}$$

$$\frac{\partial}{\partial r}(r\beta_r) = -\beta_r \frac{[krK_0(kr)]}{K_1(kr)} \text{ at } r = r_2, \tag{2.9}$$

$$\beta_\theta = 0, \text{ at } r = r_1, r_2, \tag{2.10}$$

where $I_n(kr)$ and $K_n(kr)$ are the modified Bessel functions (of orders $n = 0, 1$ in this work). Eqs. (2.6)-(2.10) impose ten boundary conditions on the tenth-order differential system (2.2)-(2.5).

To proceed with the analysis, we assume a mode with a growth rate $s$. Equations (2.2)-(2.5) become, with $\omega_A \equiv kV_A$,

$$s\beta_\theta = \eta(DD_* - k^2)\beta_\theta + \omega_A\varphi_\theta, \tag{2.11}$$

$$s\varphi_\theta = \nu(DD_* - k^2)\varphi_\theta - \omega_A\beta_\theta - r^{-1}(r^2\Omega)'\varphi_r, \tag{2.12}$$

$$s\beta_r = \eta(DD_* - k^2)\beta_r + \omega_A\varphi_r, \tag{2.13}$$

$$s(DD_* - k^2)\varphi_r = \nu(DD_* - k^2)^2\varphi_r - \omega_A(DD_* - k^2)\beta_r - 2\Omega k^2\varphi_\theta, \tag{2.14}$$

where the underlined term in (2.2) was dropped from (2.11). Note that for marginal modes, where $s = 0$, $\beta_r$ appears in the combination $(DD_* - k^2)\beta_r$ only, which can be eliminated between eqs. (2.13) and (2.14) to yield a reduced system of eighth order in radial derivatives. Chandrasekhar exploited this simplification. A peculiar feature of this system is that the radial magnetic boundary condition is irrelevant to the marginal mode, in the sense that it does not enter the relation between wave number $k$ and the parameters of the background flow. Still for the eighth order system, in the *Rayleigh-unstable* case, Herron ([10]) was able to establish the principle of exchange of stabilities (PES).

### 2.2. Abstract formulation

In order to simplify the proof, we make an abstract formulation. An operator notation is introduced, which clarifies the nature of the analysis. The system thereby becomes

$$-sM\varphi_r = \nu M^* M\varphi_r + \omega_A M_1\beta_r - 2\Omega k^2\varphi_\theta. \tag{2.15}$$

$$s\beta_\theta = -\eta M_0\beta_\theta + \omega_A\varphi_\theta, \tag{2.16}$$

$$s\varphi_\theta = -\nu M_0\varphi_\theta - \omega_A\beta_\theta - r^{-1}(r^2\Omega)'\varphi_r, \tag{2.17}$$

$$s\beta_r = -\eta M_1\beta_r + \omega_A\varphi_r. \tag{2.18}$$

In this notation, $M$, $M^*$, $M_0$, and $M_1$ all denote $-DD_* + k^2$, but are considered different operators because of the distinct boundary conditions satisfied by the functions on which they act, while $M^*M$ denotes $(-DD_* + k^2)^2$ ([10]). That is, $M$ acts on functions that have the same boundary conditions as $\varphi_r$ [eq. (2.6)], $M^*$ assumes that the functions satisfy no particular boundary condition, whereas $M_0$ uses the boundary conditions of $\varphi_\theta$ and $\beta_\theta$ [eqs. (2.7), (2.10)] and $M_1$ those of $\beta_r$ [eqs. (2.8), (2.9)].

Introduce an inner product,

$$\langle f, g\rangle = \int_{r_1}^{r_2} r f(r)\bar{g}(r)dr, \tag{2.19}$$

in which the overbar denotes complex conjugation. The differential operators $M$, $M^*M$, $M_0$, and $M_1$ all have the property of being *positive definite* in this inner product. For example, let us show that $\langle M\varphi_r, \varphi_r\rangle > 0$. This follows quite readily by defining

$$\begin{aligned}\langle M\varphi_r, \varphi_r\rangle &= \int_{r_1}^{r_2} r\bar{\varphi}_r(r)(-DD_* + k^2)\varphi_r(r)dr \\ &= \int_{r_1}^{r_2} \left\{-r\bar{\varphi}_r \frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(r\varphi_r)\right] + rk^2 \left|\varphi_r\right|^2\right\} dr\end{aligned} \tag{2.20}$$

(and integrating by parts to obtain)

$$= \int_{r_1}^{r_2} r\left(\left|D_*\varphi_r\right|^2 + k^2\left|\varphi_r\right|^2\right) dr > 0.$$

The boundary conditions (2.6), $\varphi_r = \varphi_r' = 0$ at $r = r_1, r_2$ were applied.

The calculations for $\langle M_0\varphi_\theta, \varphi_\theta\rangle$ and $\langle M_0\beta_\theta, \beta_\theta\rangle$ are similar to those for $\langle M\varphi_r, \varphi_r\rangle$, except that $\varphi_\theta = \beta_\theta = 0$, (2.7), (2.10), at $r_1, r_2$ are applied. Likewise, it may be shown that $\langle M^*M\varphi_r, \varphi_r\rangle = \langle M\varphi_r, M\varphi_r\rangle \equiv \left\|M\varphi_r\right\|^2 > 0$. However, for $\langle M_1\beta_r, \beta_r\rangle$ one notes that since $\beta_r$ satisfies (2.8, 2.9), the boundary terms do not vanish after integration by parts; instead

$$\begin{aligned}\langle M_1\beta_r, \beta_r\rangle &= \int_{r_1}^{r_2} r\bar{\beta}_r\left(-DD_* + k^2\right)\beta_r(r)dr \\ &= \left[-rD_*\beta_r(r)\bar{\beta}_r(r)\right]_{r_1}^{r_2} + \int_{r_1}^{r_2}\left[D_*\beta_r(r)\frac{d}{dr}\left(r\bar{\beta}_r(r)\right) + k^2 r\left|\beta_r\right|^2\right] dr \\ &= \int_{r_1}^{r_2}\left(r\left|D_*\beta_r\right|^2 + k^2 r\left|\beta_r\right|^2\right) dr \\ &\quad + \frac{kK_0(kr_2)}{K_1(kr_2)} r_2\left|\beta_r(r_2)\right|^2 + \frac{kI_0(kr_1)}{I_1(kr_1)} r_1\left|\beta_r(r_1)\right|^2 \\ &> 0.\end{aligned}$$

### 2.3. Proof of stability

**Theorem 2.1.** *MRI is suppressed, in fact no instability at all occurs with insulating boundary conditions, if the underlined magnetic shear term in (2.2) is ignored. We also conclude that there are no marginal modes.*

*Proof:* Make use of the inner product (2.19) as in (2.20), forming that of (2.15) with $\varphi_r$ to obtain

$$\langle(\nu M^*M + sM)\,\varphi_r, \varphi_r\rangle + \omega_A\langle M_1\beta_r, \varphi_r\rangle - \langle 2\Omega k^2\varphi_\theta, \varphi_r\rangle = 0. \tag{2.21}$$

From (2.18) this may be re-written as

$$\langle(\nu M^*M + sM)\,\varphi_r, \varphi_r\rangle + \langle M_1\beta_r, (\eta M_1 + s)\beta_r\rangle - \langle 2\Omega k^2\varphi_\theta, \varphi_r\rangle = 0. \tag{2.22}$$

The last term on the left of (2.22) can be found by making use of (2.17), by taking its inner product with $2\Omega k^2\varphi_\theta$. This gives

$$\langle r^{-1}(r^2\Omega)'\varphi_r, 2\Omega k^2\varphi_\theta\rangle + \langle(\nu M_0 + s)\,\varphi_\theta, 2\Omega k^2\varphi_\theta\rangle + \omega_A\left\langle\beta_\theta, 2\Omega k^2\varphi_\theta\right\rangle = 0. \tag{2.23}$$

Since $r^{-1}(r^2\Omega)' = 2a$ is a positive constant, the first term on the left of (2.23) is the complex conjugate of what is sought for (2.22) except that now, the last term of (2.23) must be managed. So, take the inner product of (2.16) with $2\Omega k^2\beta_\theta$. The result is

$$\langle(\eta M_0 + s)\beta_\theta, 2\Omega k^2\beta_\theta\rangle - \omega_A\langle\varphi_\theta, 2\Omega k^2\beta_\theta\rangle = 0. \tag{2.24}$$

Combining (2.22), (2.23) and (2.24) gives

$$\begin{aligned}&\langle(\nu M^*M + sM)\,\varphi_r, \varphi_r\rangle + \langle M_1\beta_r, (\eta M_1 + s)\beta_r\rangle +\\ &(2a)^{-1}\left\{\langle(\nu M_0 + \bar{s})\,\varphi_\theta, 2\Omega k^2\varphi_\theta\rangle + \langle(\eta M_0 + s)\beta_\theta, 2\Omega k^2\beta_\theta\rangle\right\} = 0.\end{aligned} \tag{2.25}$$

It was established by Synge ([21]) and by Chandrasekhar ([3]), that for the azimuthal velocity function $\varphi_\theta$, the following is true:

$$\mathrm{Re}\langle(-DD_* + k^2)\varphi_\theta, 2\Omega k^2\varphi_\theta\rangle > 0, \tag{2.26}$$

by virtue of (2.7). Likewise, for the azimuthal component of the magnetic field $\beta_\theta$,

$$\mathrm{Re}\langle(-DD_* + k^2)\beta_\theta, 2\Omega k^2\beta_\theta\rangle > 0, \tag{2.27}$$

using (2.10). Hence, taking the real part of (2.25), combining terms having the growth rate $s$, the result is

$$\begin{aligned}&\mathrm{R}e(s)\left\{\langle M\varphi_r, \varphi_r\rangle + \langle M_1\beta_r, \beta_r\rangle + (2a)^{-1}\langle 2\Omega k^2\varphi_\theta, \varphi_\theta\rangle\right.\\ &\left.+(2a)^{-1}\langle 2\Omega k^2\beta_\theta, \beta_\theta\rangle\right\}\\ = \;&-\nu\left\|M\varphi_r\right\|^2 - \eta\left\|M_1\beta_r\right\|^2\\ &-(2a)^{-1}\left\{\nu\mathrm{Re}\left(\langle M_0\varphi_\theta, 2\Omega k^2\varphi_\theta\rangle\right) + \eta\mathrm{Re}\left(\langle M_0\beta_\theta, 2\Omega k^2\beta_\theta\rangle\right)\right\}\\ < \;&0.\end{aligned} \tag{2.28}$$

An immediate consequence of this is $\mathrm{R}e(s) < 0$, and hence stability. Since $\mathrm{R}e(s)$ *is* strictly negative, there are no marginal modes when the magnetic shear term is ignored, with insulating boundary conditions. □

The counterpart to (2.27) for perfectly conducting boundaries, that is for $D_* b_\theta(1) = 0$ and $D_* b_\theta(\eta) = 0$, does not readily follow and indeed may not be true. Thus, we cannot exclude the possibility of MRI in wide gaps under the small-$P_{\rm m}$ approximation if the boundaries are conducting. It would be of interest then to see if the analysis may be extended to the conducting case. However, much like the dilemma which Chandrasekhar ([3], p.298) experienced in his attempt to deduce the PES, a lack of positive definiteness is a major hindrance.

## 3. Concluding comments

In summary, when using dimensionless equations of motion scaled by viscosity, as is traditional in analyses of Taylor-Couette flow, one must be cautious about discarding terms proportional to the magnetic Prandtl number even when $P_{\rm m} \ll 1$. Whether these terms matter depends upon the type of disturbance envisaged. In particular, we have proved that neglect of one such term predicts axisymmetric stability when the angular-momentum gradient is positive; in fact, however, such states may be subject to axisymmetric magnetorotational instability if magnetic Reynolds number and Lundquist number are sufficiently large (GJ, [7]). For analyses of MRI in liquid-metal experiments, one would do better to scale the equations of motion by the diffusivity rather than the viscosity, as one finds that viscosity has rather little influence on MRI growth rates if $P_{\rm m} \ll 1$ (GJ). This is not to say that $P_{\rm m}$ is unimportant: for example, when $P_{\rm m} \sim O(1)$, non-axisymmetric MRI modes may be preferred ([18]). Also, the smallness of $P_{\rm m}$ has practical implications for experiments if, as is usual, the background flow is established or maintained by viscous stresses at the boundaries. In that case, the Ekman time-scale for spin-up is longer than the magnetic diffusion time, and hence for marginal MRI also longer than the reciprocal of the final rotation rate, by a factor $\sim P_{\rm m}^{-1/2} \sim 10^3$.

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