

NOISE-SUSTAINED CONVECTIVE INSTABILITY IN A MAGNETIZED TAYLOR–COUETTE FLOW

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ABSTRACT

The helical magnetorotational instability of the magnetized Taylor–Couette flow is studied numerically in a finite cylinder. A distant upstream insulating boundary is shown to stabilize the convective instability entirely while reducing the growth rate of the absolute instability. The reduction is less severe with greater height. After we model the boundary conditions properly, the wave patterns observed in the experiment turn out to be a noise-sustained convective instability. After the source of the noise resulting from unstable Ekman and Stewartson layers is switched off, a slowly decaying inertial oscillation is observed in the simulation. We reach the conclusion that the experiments completed to date have not yet reached the regime of absolute instability.

Key words: accretion, accretion disks – instabilities – methods: numerical – MHD

Online-only material: color figures

1. INTRODUCTION

Magnetorotational instability (MRI) is probably the main source of turbulence and accretion in sufficiently ionized astrophysical disks (Balbus & Hawley 1998). Due to this crucial role in astrophysics, substantial efforts have been made worldwide to observe MRI in a laboratory setting (Ji et al. 2001; Goodman & Ji 2002; Noguchi et al. 2002; Sisan et al. 2004; Velikhov et al. 2006), but MRI has never been conclusively demonstrated in the laboratory.

Most experiments have been done in a cylindrical geometry with a background flow that approximates the ideal Couette rotating profile:

$$\Omega = a + b/r^2, \quad (1)$$

where $a = (\Omega_2 r_2^2 - \Omega_1 r_1^2)/(r_2^2 - r_1^2)$ and $b = r_1^2 r_2^2 (\Omega_1 - \Omega_2)/(r_2^2 - r_1^2)$, Ω_1 and Ω_2 are the rotation speeds of the inner and outer cylinders and r_1 and r_2 are the radii of the inner and outer cylinders, respectively (see Figure 1). For axially periodic or infinite magnetized Taylor–Couette flow, MRI-like modes have been shown theoretically to grow at a much reduced magnetic Reynolds number $Re_m \equiv \Omega_1 r_1 (r_2 - r_1)/\eta$ and Lundquist number $S \equiv V_{A,0} r_1 (r_2 - r_1)/\eta$ in the presence of a combination of axial and current-free toroidal field

$$\mathbf{B}^0 = B_z^0 (\mathbf{e}_z + \beta r_1 / r \mathbf{e}_\varphi) \quad (2)$$

than the standard MRI (SMRI) with purely axial magnetic field (Hollerbach & Rüdiger 2005; Rüdiger et al. 2005). Here the cylindrical coordinates (r, φ, z) are used. B_z^0 and β are constants. The Alfvén speed is defined as $V_{A,0} \equiv B_z^0 / \sqrt{4\pi\rho}$. η and ρ are the magnetic diffusivity and density of the fluid, respectively (see Figure 1).

The Potsdam ROssendorf Magnetic Instability Experiment (PROMISE) group claimed to have observed this kind of helical MRI (HMRI) experimentally (Stefani et al. 2006, 2007; Rüdiger et al. 2006). However, we have shown that the wave pattern observed in PROMISE is not a global instability, but rather a transient disturbance somehow excited by the Ekman circulation and then transiently amplified as it propagates along

the background axial Pointing flux with nonzero group and phase velocities, but is then absorbed once it reaches the jet formed at midheight between two neighboring Ekman cells (Liu et al. 2007). The PROMISE group have accordingly updated the experimental facility to PROMISE II to allow for two split rings at both end caps: an inner ring attached to the inner cylinder and an outer ring attached to the outer cylinder. If the width of the inner ring is chosen appropriately $\sim 0.4(r_2 - r_1)$, the magnetized Ekman circulation could be significantly reduced, therefore removing one of the possible disturbance sources, i.e., the unsteady jet (Szklański 2007).

As with other examples in the literature, such as drifting dynamo waves (Tobias et al. 1998; Proctor et al. 2000), it is of vital importance to distinguish absolute instability from convective instability in a traveling wave experiment such as PROMISE. It is also an essential ingredient of the threshold prediction for the Riga dynamo (Gailitis et al. 2008). For a traveling wave the positivity of the growth rate implies only an amplification of the perturbation as it moves downstream. In one case, despite the movement of the wave packet, the perturbation increases without limit in the course of time at any point fixed in space; this kind of instability with respect to any infinitesimal perturbations will be called *absolute instability*. In the other case, the packet is carried away so swiftly that at any point fixed in space the perturbation tends to zero as $t \rightarrow \infty$; this kind will be called *convective instability* (see the details of Section 2; Landau & Lifshitz 1987). For PROMISE II, it appears that under the experimental conditions the second kind occurs. A recent preprint also highlights the importance of the distinction between absolute and convective instabilities in the context of the HMRI (Priede & Gerbeth 2008).

In a Taylor–Couette experiment bounded by insulating end caps, Tobias et al. (1998) have pointed out that without any external disturbances except a small initial disturbance needed as a seed for the instability, the distant upstream insulating boundary acts as an “absorbing” boundary while the characteristics of the downstream end cap is unimportant. Due to this absorption the convective unstable state cannot be sustained by a uniform driving force, therefore this unstable mode eventually decays (Tobias et al. 1998). This driving force is not the aforementioned noise, but the power to drive the instability, which in the usual Taylor–Couette experiments can be quantified by the

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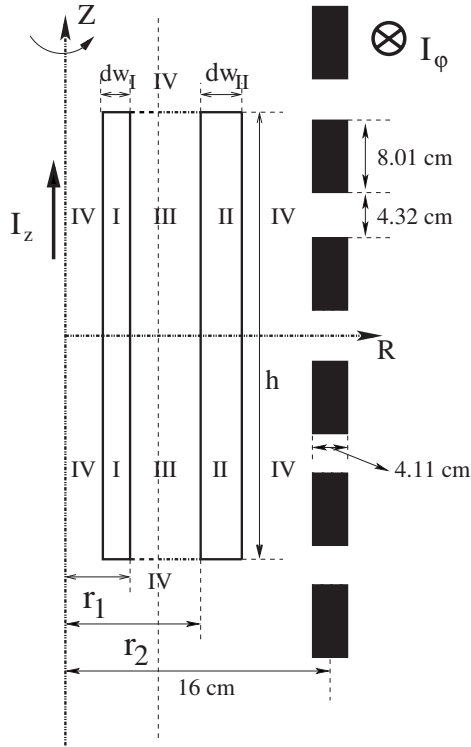


Figure 1. Computational domain for simulations of the PROMISE II experiment. Region (I): inner copper cylinder, angular velocity Ω_1 . (II): outer copper cylinder, Ω_2 . (III): liquid gallium; (IV): vacuum. Thick dashed line: insulating inner ring, corotating with the inner cylinder. Thick dash-dot line: insulating outer ring, corotating with the outer cylinder. The junction of these two rings lies at 40% of the gap ($D = r_2 - r_1$) between the inner and outer cylinder (Szklański 2007). Dimensions: $r_1 = 4.0$ cm; $r_2 = 8.0$ cm; $h = 40.0$ cm; $d_{wI} = 1.0$ cm; $d_{wII} = 1.5$ cm; $\Omega_1/2\pi = 3.6$ rpm; $\Omega_2/2\pi = 0.936$ rpm. Note that $\mu = \Omega_2/\Omega_1 = 0.26$, rather than $\mu = 0.27$ used in previous work (Stefani et al. 2006, 2007; Rüdiger et al. 2006; Liu et al. 2007). The exact configuration of the toroidal coils being unavailable to us, six coils (black rectangles) with dimensions as shown were used, with 67 turns in the two coils nearest the mid-plane and 72 in the rest. Currents I_φ were adjusted to reproduce the reported Hartmann numbers $Ha \equiv B_z^0 r_1 / \sqrt{\rho \mu_0 \eta \bar{v}}$.

magnetic Reynolds number Re_m . This conclusion has been rigorously demonstrated in the very resistive limit in Section II.C of Liu et al. (2006a) using a perturbative approach and Section II.D of Liu et al. (2006a) using a modified WKB analysis, showing that the insulating end cap entirely stabilizes the HMRI mode, which is a convective unstable mode given the parameters of the PROMISE experiment.

The absorbing boundary is essential to the development, regardless of how distant it may be. The larger height only defers the time when we have to wait for the boundary-induced dissipation to dominate (Tobias et al. 1998). On the other hand, if Re_m exceeds a higher threshold $Re_{m,f}$, the driving force of the system overcomes the dissipation and a globally unstable mode appears (Tobias et al. 1998). Therefore in a bounded system the unstable mode appears at $Re_{m,f}$ rather than $Re_{m,c}$, where $Re_{m,c}$ is the critical magnetic Reynolds number for the onset of the convective unstable mode without the “absorbing” boundary. Tobias et al. (1998) has shown that in the presence of an “absorbing” boundary and large h , a global unstable mode appears when

$$Re_m \geq Re_{m,f} \equiv Re_{m,a} + O(h^{-2}),$$

where $Re_{m,a}$ is the critical magnetic Reynolds number corresponding to the onset of the absolute instability without the “absorbing” boundary.

Table 1

Influence of the Height h Upon the Growth Rate γ of the Absolute Instability in a Bounded Cylinder

$\Omega_1 h / V_{A,0}$	20.3	40.6	81.2	Periodic
Growth rate γ s ⁻¹	0.27	0.58	0.82	1.06

Notes. $r_1 = 7.1$ cm, $r_2 = 20.3$ cm, $\Omega_1 = 400$ rpm, $\Omega_2 = 53.3$ rpm, $B_z = 500$ G, $B_\theta(r_1) = 1$ kG, the height $h = 27.9$ cm, 55.8 cm and 111.6 cm; the material properties are based on gallium: $\eta \approx 2000$ cm² s⁻¹ and $\rho \approx 6$ g cm⁻³, which give $Re_m = 2$ and $S = 2.7$, no explicit viscosity present. The simulations are performed using a modified version of the astrophysical code ZEUS2D (Stone & Norman 1992a, 1992b; Liu et al. 2006b). The boundary conditions adopt the one introduced in Section II.D of Liu et al. (2006a). Note that no-slip boundary conditions are employed on all applicable boundaries and ideal Couette state (Equation (1)) is enforced at both end caps in order to remove the Ekman circulation and possible disturbances induced by this boundary layer effect. The one labeled “periodic” uses vertically periodic boundary conditions with periodicity length $h = 27.9$ cm.

This has raised a major obstacle to researchers trying to observe an absolutely unstable HMRI in the laboratory. The advantage of HMRI itself, i.e., unstable with a low critical Reynolds number (three orders lower than the SMRI) conflicts with the necessarily high threshold of the onset of an absolute HMRI mode, i.e., excited at a reasonably high critical magnetic Reynolds number, thus a high Reynolds number, which would result in much more severe end effects than expected. Moreover the fact that the critical Lundquist number must usually increase together with the magnetic Reynolds number and high ratio of toroidal-to-poloidal magnetic field requirement ($\beta > 1$) would even worsen the situation.

We also find that by nonlinear numerical simulation the insulating end cap reduces the growth rate of the absolute instability somewhat. The greater the height h , the less the growth rate is reduced (Table 1).

In a typical experiment, the experiment is, however, highly likely affected by small external noise either from a physical cause or experimental imperfection such as the misalignment of the cylinders. If the system is convectively unstable, i.e., disturbances grow as they move downstream, noise would sustain structures in the system even if no global mode were unstable (Deissler 1987; Proctor et al. 2000). In the present paper, we show by numerical simulations that the perturbations from the unstable magnetized residual Ekman layer and Stewartson layer at the upper end cap would play the role of “noise” generator, though this perturbation level is reduced with increased axial magnetic field (Liu 2008a). What is observed in PROMISE II turns out to be a noise-sustained convective traveling wave, not the absolute unstable mode.

This paper is organized as follows: Section 2 presents the wave packet analysis in an unbounded cylinder, which is the basis of the following sections. We report the nonlinear simulation results with *partially* conducting boundary conditions of PROMISE II experiment in Section 3. The final conclusions and implications to the HMRI experiments are given in Section 4.

2. WAVE PACKET ANALYSIS IN AN UNBOUNDED CYLINDER

Assuming a cylinder of infinite height h , k_z is a continuous variable. Let the gap width be fixed and finite, so $k_r \cong \pi/(r_2 - r_1)$. We define the total wavenumber $K = \sqrt{k_r^2 + k_z^2}$ and the growth rate γ .

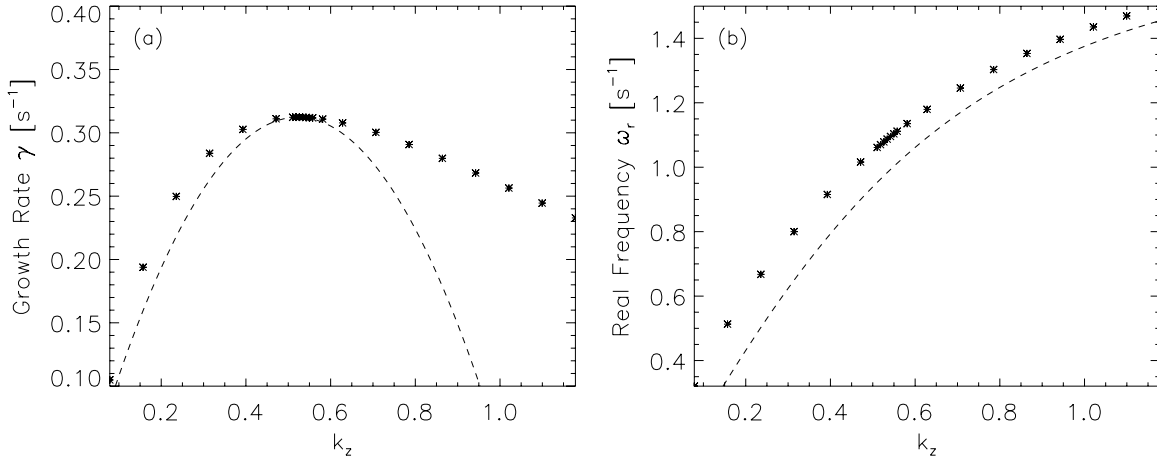


Figure 2. (a) Growth rate γ ; (b) real frequency $\Re\omega$. *: linear calculation approximation by Equations (5) and (6): $r_1 = 4.0$ cm, $r_2 = 8.0$ cm, $\Omega_1 = 101.25$ rpm, $\Omega_2 = 26.325$ rpm, $B_z = 220.5$ G, $\beta = 4.0$; the material properties are based on gallium: $\eta = 2.43 \times 10^3$ cm² s⁻¹, $\nu = 3.4 \times 10^{-3}$ cm² s⁻¹ and $\rho = 6.35$ g cm⁻³. The calculations are performed using a code (Goodman & Ji 2002) adapted to allow for a helical field. Vertical periodicity is assumed, but the radial equations are solved directly by finite differences with perfectly conducting boundary conditions (Section II.B of Liu et al. 2006a).

Since the fast growing mode is the dominant mode, here we focus on waves with vertical wavenumber k_z close to that of the fastest growing mode, k_z^0 . The range of values of k_z lies near the point for which $\gamma(k_z)$ is a maximum, i.e. $d\gamma/dk_z = 0$ at $k_z = k_z^0$ (as seen from Figure 2(a)). Let a slight perturbation occurs near the middle of the flow ($z \sim 0$) in the format of a wave packet as follows:

$$B_r(z, t = 0) = b_0 \exp\left(-\frac{z^2}{2L^2}\right) \exp(ik_z^0 z), \quad (3)$$

where we have used the envelope $\exp(-z^2/2L^2)$ to confine the perturbation around the central part of the cylinder, where $L \sim O(h)$. In the course of time, the components for which $\gamma(k_z) > 0$ will be amplified, while the remainder will be damped. The amplified wave packet thus formed will also be carried downstream with a velocity equal to the group velocity $d\omega/dk_z$ of the packet, where $\omega = \Re\omega + i\gamma$ and $\Re\omega$ is the real part of the frequency; since we are now considering waves whose wave numbers lie in a small range near the point where $d\gamma/dk_z = 0$, the quantity

$$V_g = d\omega/dk_z \cong d(\Re\omega)/dk_z \quad (4)$$

is real, and is therefore the actual propagation velocity of the packet. This downstream displacement of the perturbations is very important, and causes the complications of absolute instability versus convective instability.

We can approximate the dispersion relation like (Figure 2):

$$\Re\omega = \Re\omega(k_z) = \kappa \frac{k_z}{K}; \quad (5)$$

$$\gamma = \gamma(k_z) = \gamma^0 - \frac{\sigma}{2}(k_z - k_z^0)^2, \quad (6)$$

in which $\kappa^2 = (1/r^3)d(r^2\Omega)^2/dr = 4(1 + Ro)\Omega^2$ and $Ro \equiv 1/2d \ln \Omega/d \ln r = a/\Omega - 1$ is the Rossby number. We know $\gamma = 0$ when $k_z = 0$. Thus $\sigma = 2\gamma^0/k_z^0{}^2$. And in order to simplify the derivation, we assume $K \approx \text{constant}$ from now on (though this is not a good approximation, we can get some insightful results from this simple approximation). From Equation (5), we get $V_g = \kappa/K$.

At later time $t > 0$

$$\begin{aligned} \tilde{B}_r(k_z, t) &= \tilde{B}_r(k_z, 0) \exp(\gamma(k_z)t + i\Re\omega(k_z)t) \\ &= \tilde{B}_r(k_z, 0) \exp\left\{\left[\gamma^0 - \frac{\sigma}{2}(k_z - k_z^0)^2\right]t + i\kappa \frac{k_z}{K}t\right\}. \end{aligned} \quad (7)$$

If we define $D = \sqrt{L^2 + \sigma t}$, the result can be expressed as

$$B_r(z, t) = b_0 \frac{L}{D} \exp(\gamma^0 t) \exp\left[-\frac{(z + V_g t)^2}{2D^2}\right] \exp[ik_z^0(z + V_g t)]. \quad (8)$$

In Equation (8), as $t \rightarrow 0$, Equation (8) can be simplified as

$$B_r(z, t) = b_0 \exp(\gamma^0 t) \exp[ik_z^0(z + V_g t)], \quad (9)$$

which is a ‘‘transiently’’ growing phase. As $t \rightarrow \infty$,

$$B_r(z, t) = b_0 \frac{L}{\sqrt{\sigma t}} \exp\left[\left(\gamma^0 - \frac{V_g^2}{2\sigma}\right)t\right] \exp[ik_z^0(z + V_g t)]. \quad (10)$$

Obviously if $\gamma^0 < \gamma_a = V_g^2/2\sigma$, we will get *convective instability*, which starts with a transiently growing phase (Equation (9)) followed by a phase asymptotically decaying to zero (Equation (10)). If $\gamma^0 > \gamma_a$, we will get *absolute instability*.

3. NOISE-SUSTAINED CONVECTIVE INSTABILITY IN THE PROMISE II EXPERIMENT

In order to reduce the undesirable effects induced by the end caps and also the accompanying hydromagnetic asymmetries, Szklarski (2007) has proposed to split both end caps into two rings which are attached to both cylinders and found that if the width of the inner ring is chosen to be $0.4D$ (see Figure 1), where $D = r_2 - r_1$ is the gap between the inner and outer cylinders, the magnetic energy in terms of b_φ , where b_φ is the perturbed azimuthal magnetic field, is minimized. Therefore the magnetized Ekman circulation is significantly reduced, leading to a satisfactory ideal Couette state (Equation (1)) in the bulk flow. PROMISE has accordingly been updated to PROMISE II adopting this idea.

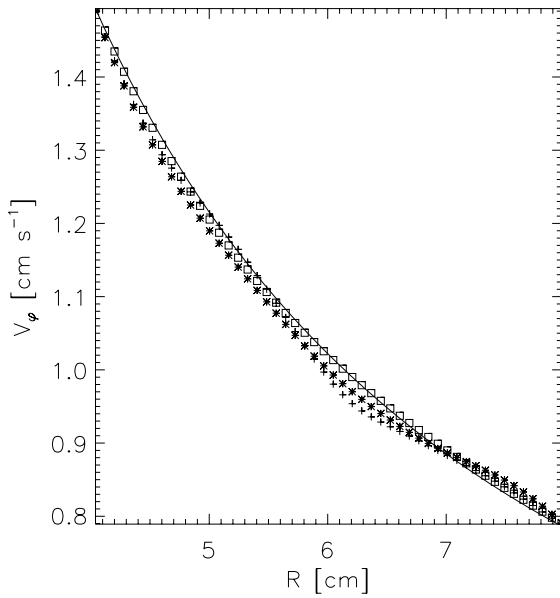


Figure 3. Azimuthal velocity vs. radius r . $Re = 1775$, $\beta = 3.81$ and $I_\phi = 75$ A. Solid line, ideal Couette state; +, 1.31 cm; *, 2.72 cm; □, 13.95 cm.

While we have confirmed their conclusions (Figure 3; please note that in Szklarski 2007, this conclusion is derived with $\beta = 0$, i.e., no background toroidal magnetic field, while our simulation results show that this conclusion is also valid with nonzero β), here we report nonlinear simulations with the ZEUS-MP 2.0 code (Hayes et al. 2006; Liu 2008b), which is a time-explicit, compressible, astrophysical ideal MHD parallel 3D code, to which we have added viscosity, resistivity (with subcycling to reduce the cost of the induction equation), and *partially* conducting boundary conditions (Liu et al. 2007), for axisymmetric flows in cylindrical coordinates (r, ϕ, z) . It has been demonstrated that the finite conductivity ($\eta_{Cu} = 1.335 \times 10^2 \text{ cm}^2 \text{ s}^{-1}$) and thickness of the copper vessel are important, and this noticeably improves agreement with the measurements compared to previous much simplified boundary condition (Liu et al. 2007). Note that in this paper $\mu = \Omega_2/\Omega_1 = 0.26$, rather than $\mu = 0.27$ reported in previous work. The parameters of PROMISE II as reported in or inferred from Stefani et al. (2008) used are gallium density $\rho = 6.35 \text{ g cm}^{-3}$; magnetic diffusivity $\eta = 2.43 \times 10^3 \text{ cm}^2 \text{ s}^{-1}$; magnetic Prandtl number $Pr_m \equiv \nu/\eta = 1.40 \times 10^{-6}$; Reynolds number $Re \equiv \Omega_1 r_1 (r_2 - r_1)/\nu = 1775$; axial current $I_z = 6000$ A; toroidal-coil currents $I_\phi = 0, 50, 75, 120$ A; and dimensions as in Figure 1.

For comparison, we start with purely hydrodynamic (unmagnetized) simulations (Figure 4). From Figure 4(a), after splitting the end caps into two rings, the two big Ekman cells are divided into four smaller cells and localized near the end caps. Compared to the simulation results of PROMISE (Liu et al. 2007), there is no flapping “jet” near the midplane as in the usual Ekman circulations. This removes the possible noise from this unsteadiness. However from Figure 4(b), there are some perturbations near both end caps, which supply the possible sources of noise in the system. These perturbations are resulted from unstable Ekman layer and Stewartson layer (Liu 2008a). The magnitude of this noise is around $\pm 0.2 \text{ mm s}^{-1}$. As we will see later (Figure 5), this unsteadiness is reduced by increasing the axial magnetic field (Gilman 1971; Liu 2008a).

Figure 5 displays vertical velocities near the outer cylinder in simulations corresponding to the experimental runs of Stefani et al. (2008) for several values of the toroidal current, I_ϕ . A wave

pattern very similar to that in the experimental data (Stefani et al. 2008) is seen. Since now there is no jet, the traveling wave propagates to the bottom end cap and absorbed there while in the old PROMISE experiment, the traveling wave disappears at the jet (Liu et al. 2007). We also note that the perturbation near the upper end cap weakens with strong axial magnetic field. This could be explained by a more stable magnetized residual Ekman layer and Stewartson layer (Liu 2008a). Both the weakening of the noise sources and disappearance of the amplifying mechanism leads to a rather steady state with $I_\phi = 120$ A.

It is highly possible that there is much noise in the real experiment due to some experimental imperfection such as misalignment and in the numerical simulation such as numerical noise. Also the noise could result from physical causes such as the unsteady Ekman layer or Stewartson layer. These noises would cause a noise-sustained convective instability in the system as in Proctor et al. (2000). The continuous impulse from the noise sources would have the system always in the state of “transiently growing” phase (Equation (9)). This results in similar wave patterns as the ones from the primary instability without noise, which are observed in PROMISE and PROMISE II experiments and simulations (Liu et al. 2007). The noise-induced wave pattern is always susceptible to noise-induced disruption as discussed by Deissler (1987). That is exactly what we found here and in Liu et al. (2007). We can see this point more clearly by following Liu et al. (2007): performing a simulation that begins with the experimental boundary conditions until the traveling waves are well established, and then switches abruptly to ideal-Couette end caps (Figure 6). After the switch, the traveling waves disappear after one axial propagation time and slowly decaying inertial oscillations (asymptotically to zero) result. The main differences in results between Liu et al. (2007) and the present simulation are (1) there is no jet, thus the traveling waves are absorbed near the bottom end cap both before and after the switch; (2) there is no change of wave speed associated with the switch since the background state does not change much before and after the switch. We reach the conclusion that even after the end caps are split into two rings as in PROMISE II, the wave patterns observed in the experiment are not global instability, but rather noise-sustained convective instability. The similarity between these “inertial-oscillation-induced waves” after the switch and the earlier noise-sustained “MRI waves” in the simulation or “MRI-type waves” observed in the various versions of PROMISE stems from the physical nature of HMRI that HMRI is a weakly destabilized inertial oscillation (Liu et al. 2006a). More importantly, this similarity supports our conclusion in another aspect: the frequency and wavenumber selection mechanism for a noise-sustained structure is determined by a linear mechanism, thus resembling the properties of the primary instability. In contrast, in the globally unstable regime, a nonlinear eigenvalue problem selects the frequency (Proctor et al. 2000).

4. DISCUSSION

In this paper, nonlinear simulations of the helical MRI in a magnetized Taylor–Couette flow are performed. The geometry mimics the PROMISE II experiment with end caps split into two rings. The partially conducting boundary condition introduced in Liu et al. (2007) is used. The wave patterns change with the applied magnetic field as in the experiment. However via numerical tests, we find that the wave patterns observed in the PROMISE II experiment are not due to a global instability, but rather a noise-sustained convective instability.

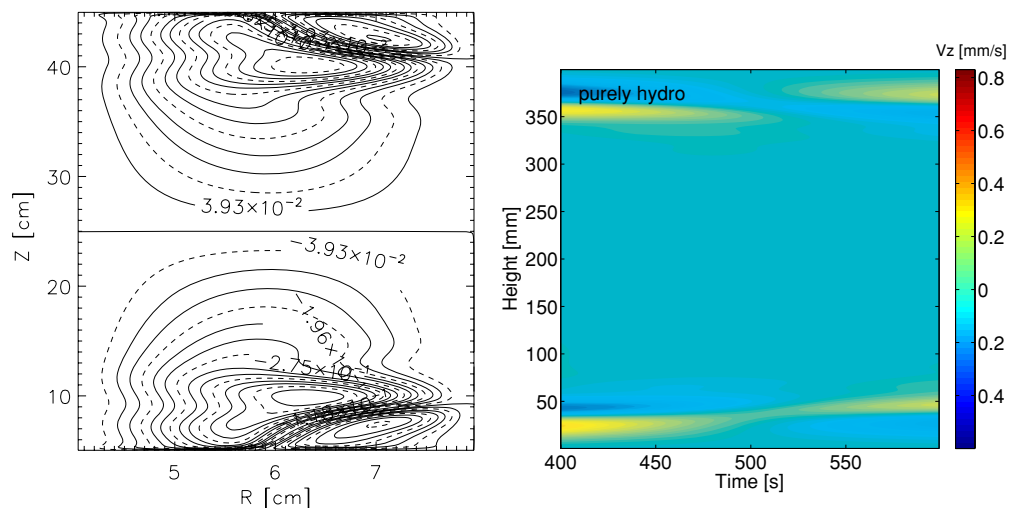


Figure 4. Purely hydrodynamic (unmagnetized) simulations. Left: time-averaged poloidal flow stream function Ψ ; right: axial velocities (mm s^{-1}) vs. time and depth sampled at $r = 6.5$ cm, for the parameters of the PROMISE II experiment without any magnetic field. Height increases upward from the bottom end cap. No-slip velocity boundary conditions are imposed at the rigidly rotating end caps. The steady part of the resulting Ekman circulation is suppressed in right panel by subtracting the time average at each height.

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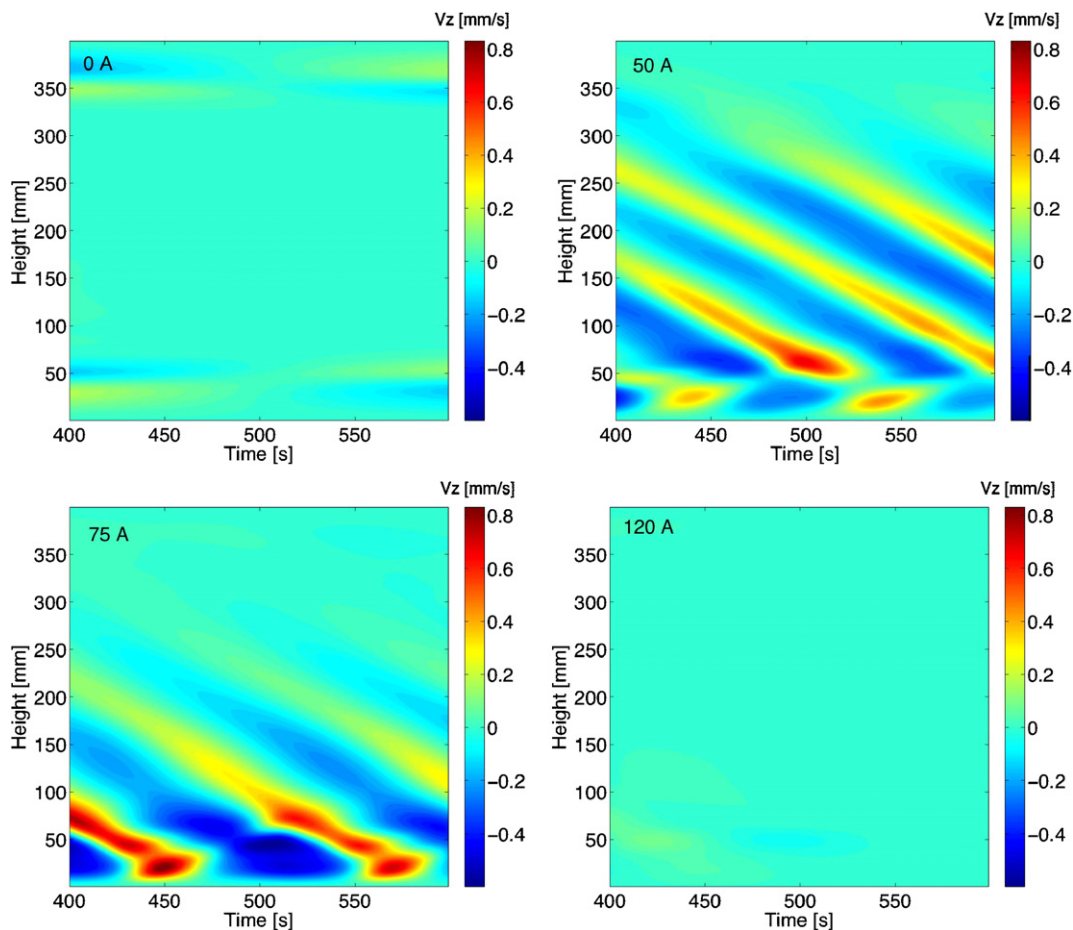


Figure 5. Axial velocities (mm s^{-1}) vs. time and depth sampled at $r = 6.5$ cm, for the parameters of the PROMISE II experiment with toroidal currents I_ϕ as marked. No-slip velocity boundary conditions are imposed at the rigidly rotating end caps. The steady part of the resulting Ekman circulation is suppressed in these plots by subtracting the time average at each height. The waves appear to be absorbed near the bottom end cap.

(A color version of this figure is available in the online journal.)

The importance of the distinction between absolute and convective instability in a bounded system with broken reflection symmetry is discussed. The addition of the toroidal magnetic

field breaks the axial symmetry of the system. In such cases, the effects of distant upstream insulating boundaries on the absolute instability differs remarkably from the ones on the convective

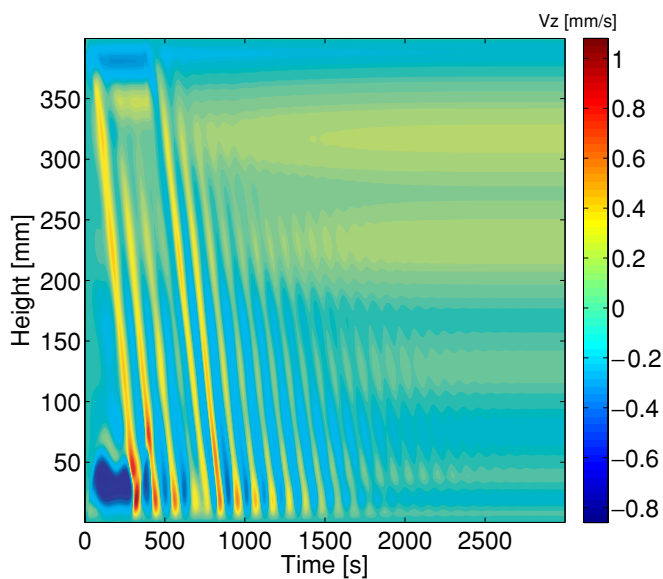


Figure 6. An extended version of the case $I_\phi = 75$ A shown in Figure 5 but without subtraction of the time average. After $t = 360$ s, the no-slip boundary condition at both end caps is switched to an ideal Couette profile (Equation (1)). A slowly decayed inertial oscillation results.

(A color version of this figure is available in the online journal.)

instability. The insulating end cap would only reduce the growth rate of the absolute instability, but would stabilize the convective instability entirely, however distant it may be. For the absolute instability, the more distant insulating end cap would less reduce the growth rate, while for the convective instability the more distant end cap would only have the system wait longer for the dissipation due to the “absorption” boundary to dominate. These discoveries cast great obstacles for people to observe the helical MRI in the laboratory: an absolute HMRI is needed to observe the global unstable mode in the experiment.

Unfortunately, it is not easy to derive the critical magnetic Reynolds number $Re_{m,f}$ of the absolute HMRI analytically in a bounded system. However we can get a rough estimate of $Re_{m,a}$, i.e., the critical magnetic Reynolds number of the absolute HMRI in an unbounded system, by wave packet analysis (Section 2) and the approximate dispersion relation from Figure 2. From Figure 2, we derive the group velocity $V_g \sim 1.08 \text{ cm s}^{-1}$, $\gamma^0 \sim 0.31 \text{ s}^{-1}$, $k_z^0 \sim 0.52 \text{ cm}^{-1}$ and $\sigma \sim 2.29 \text{ cm}^2 \text{ s}^{-1}$. Therefore $\gamma^0 - V_g^2/2\sigma \sim 0.05 \text{ s}^{-1} > 0$, which corresponds to an absolute HMRI instability with $Re_{m,a} \sim 0.07$. We therefore conjecture that $Re_{m,f} \equiv Re_{m,a} + O(h^{-2}) \gtrsim 0.07$ in PROMISE II. The critical magnetic Reynolds number is somehow one order of magnitude lower than the standard MRI, but still requires Reynolds number $Re \sim 10^5$. Therefore we need to rotate the cylinder typically with more than one hundred rpm. Such rotation rates are of course achievable, however with such a Reynolds number the advantage of HMRI with much lower Reynolds number, thus much lower end effects, is not so great as people had expected. Moreover in most HMRI unstable modes $\beta > 1$ is preferred, this suggests a toroidal magnetic field typically at ~ 1000 G, which requires axial currents $> 10^4$ A inside the inner cylinder. This is a major engineering challenge in itself. The technical constraints prevent us to try to find the threshold by nonlinear numerical simulations such as (1) the current code can not afford large Reynolds number ($\sim 10^5$),

which is required for the HMRI to enter the absolutely unstable regime; (2) from the global linear calculation, the HMRI mode is stabilized if an artificially low Reynolds number like $\sim 10^3$, which could be afforded by the current code, is employed.

The nonaxisymmetric $m = 1$ modes are observed in the experiments (Stefani et al. 2006, 2007; Rüdiger et al. 2006). Unfortunately since the simulations presented in this paper are all axisymmetric, the possibility to study this important mode is excluded. The extension of the current work to 3D will be the subject of the future study. Rüdiger et al have already done some excellent work on this issue and found that given PROMISE parameters the nonaxisymmetric HMRI modes are always harder to excite than the symmetric mode (Rüdiger et al. 2005; Rüdiger & Schultz 2008).

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