Enhanced Self-Focusing of an Ion Beam Pulse Propagating through a Background Plasma along a Solenoidal Magnetic Field

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It is shown that the application of a weak solenoidal magnetic field along the direction of ion beam propagation through a neutralizing background plasma can significantly enhance the beam self-focusing for the case where the beam radius is small compared to the collisionless electron skin depth. The enhanced focusing is provided by a strong radial self-electric field that is generated due to a local polarization of the magnetized plasma background by the moving ion beam. A positive charge of the ion beam pulse becomes overcompensated by the plasma electrons, which results in the radial focusing of the beam ions. The expression for the self-focusing force is derived analytically and compared with the results of numerical simulations.

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Neutralization and focusing of charged particle beam pulses by a background plasma form the basis for a wide range of applications to high energy accelerators and colliders [1], ion-beam-driven high energy density physics and heavy ion fusion [2], and astrophysics [3]. Even for the simple case of charge bunch propagation through a dense neutralizing background plasma, the bunch space-charge is typically better neutralized than its current. As a result, a net focusing (self-pinching) force is produced due to the self-magnetic field [4]. The fundamental effect of self-focusing was discovered over 50 years ago and has been successfully utilized for a wide range of applications involving charged particle beam transport and focusing. For instance, self-focusing can compensate for the transverse spreading of the ion bunch, thus providing self-pinched ion beam transport over long distances [5]. The effects of self-pinching become most pronounced when the beam radius is small compared to the collisionless plasma electron skin depth, $r_b < c/\omega_{pe}$. In this case the beam current is almost unneutralized, and the self-magnetic field is a maximum. Here, ω_{pe} is the electron plasma frequency, and c is the speed of light *in vacuo*. In this Letter we demonstrate that for an ion beam with $r_b < c/\omega_{pe}$ the self-focusing force can be significantly enhanced if a moderately weak solenoidal magnetic field satisfying

$$\omega_{ce} \gg \beta_b \omega_{pe} \tag{1}$$

is applied along the beam propagation direction. Here, $\omega_{ce} = eB_0/m_e c$ is the electron cyclotron frequency, B_0 is the magnitude of the applied magnetic field, $-e$ and m_e are the electron charge and mass, respectively, and $\beta_b = V_b/c$, where V_b is the longitudinal beam velocity. The inequality in Eq. (1) can be expressed as $B_0 \gg \beta_b (n_p[\mathrm{cm}^{-3}]/10^{11})^{1/2}$ kG. For ion beams with $\beta_b \sim 0.1$ propagating through a background plasma with density $n_p \sim 10^{11}\ \mathrm{cm}^{-3}$, this corresponds to a weak magnetic field threshold of the order of 100 G. Although the influence of the external magnetic force acting on the beam ions and plasma ions is negligible in this regime, the plasma electron dynamics is significantly affected by the applied magnetic field. As a result, the moving ion beam polarizes the magnetized plasma background, creating a strong radial self-electric field, which provides the enhanced self-focusing. Note that generation of a focusing radial electric field implies that a positive charge of the ion beam pulse becomes *overcompensated* by the background plasma electrons. The effects of the enhanced self-focusing are of particular importance for the neutralized drift compression experiment (NDCX) and its upgrades, where the ion beam pulse is first compressed ballistically as it propagates through a background plasma, and is then focused on the target by a strong (few Tesla) final focus solenoid [6]. A weak magnetic fringe field (of the order of 100 G) can penetrate far into the long drift section filled with a neutralizing plasma, and can therefore provide conditions for the enhanced focusing to occur, as shown below.

In order to analyze the self-focusing effect, we now derive a general expression for the radial component of the Lorentz force,

$$F_r = Z_b e(E_r - \beta_b B_\varphi), \tag{2}$$

acting on the ion beam pulse propagating through a background plasma along a uniform magnetic field $\mathbf{B_0} = B_0\mathbf{z}$. Here, B_φ and E_r are the azimuthal component of the self-magnetic field, and the radial component of the self-electric field, respectively, and Z_b is the charge state of the beam ions. For simplicity, we assume immobile plasma ions, cold plasma electrons, and investigate the axisymmetric steady-state solution, where all quantities depend on t and z solely through the combination $\xi = z - V_b t$. Assuming that the beam density is small compared to the electron density ($n_b \ll n_e$), we solve for the collisionless linear plasma response, in which the nonrelativistic plasma electron dynamics is governed to leading order by

$$m_e V_b \frac{\partial \mathbf{V_e}}{\partial \xi} = \frac{e}{c}[\mathbf{V_e} \times \mathbf{B}_0] + e\mathbf{E}. \tag{3}$$

Here, $\mathbf{V_e}$ is the electron flow velocity and we have made use of $\partial/\partial t = -V_b \partial/\partial \xi$ for the steady-state electron response. Applying the curl operator to the both sides of Eq. (3) and making use of Faraday law, we readily obtain

$$m_e V_b \frac{\partial}{\partial \xi}\left(\nabla \times \mathbf{V_e} - \frac{e}{m_e c}\mathbf{B}\right) = \frac{e}{c}\nabla \times [\mathbf{V_e} \times \mathbf{B_0}]. \tag{4}$$

In cylindrical coordinates the φ component of Eq. (4) yields

$$m_e V_b \frac{\partial V_{ez}}{\partial r} = -\frac{e}{c}B_0 V_{e\phi} - \frac{e}{c}V_b B_\phi + m_e V_b \frac{\partial V_{er}}{\partial \xi}, \tag{5}$$

and the radial component of Eq. (3) is

$$m_e V_b \frac{\partial V_{er}}{\partial \xi} = \frac{e}{c}V_{e\varphi}B_0 + eE_r. \tag{6}$$

Using Eqs. (5) and (6) to determine $E_r - \beta_b B_\varphi$, we find that the radial component of the Lorentz force in Eq. (2) is given by

$$F_r = Z_b e E_r - \frac{Z_b e}{c}V_b B_\phi = Z_b m_e V_b \frac{\partial V_{ez}}{\partial r}. \tag{7}$$

For the case where the beam current is fully neutralized, i.e., $n_e V_{ez} = Z_b n_b V_b$, Eq. (7) takes on the simple form

$$F_r = Z_b^2 m_e V_b^2 \frac{1}{n_e}\frac{dn_b}{dr}. \tag{8}$$

Equation (8) describes the total focusing force acting on the beam ions, provided the beam current is neutralized. Note that the same expression has been previously derived in [5], for the special case where a magnetic field is not applied ($B_0 = 0$), and the beam radius is large compared to the electrons skin depth, i.e., $r_b \gg c/\omega_{pe}$. For this case, the beam current is well neutralized [7], and therefore the result for the total self-focusing force obtained in [5] is consistent with the general analysis presented in this Letter. If there is no externally applied magnetic field ($B_0 = 0$) the beam current becomes unneutralized when the beam radius is small compared to the electron skin depth [7] $r_b < c/\omega_{pe}$, and Eq. (8) is not valid in that case. However, if a weak magnetic field is applied, the beam current can be effectively neutralized even in the regime $r_b < c/\omega_{pe}$. In this Letter we show that in the presence of an applied magnetic field, the condition for current neutralization becomes

$$r_b \gg r_{ge} \equiv \frac{V_b}{\omega_{ce}}(1 + \omega_{ce}^2/\omega_{pe}^2)^{1/2}; \tag{9}$$

i.e., the beam radius should be large compared to the effective electron gyroradius r_{ge} defined in Eq. (9). Note that condition in Eq. (9) can be satisfied even in the limit $r_b \ll c/\omega_{pe}$ provided $\omega_{ce} \gg \beta_b \omega_{pe}$.

It is of particular importance to compare the self-pinching force, F_0, acting on the beam ions in the absence of an applied magnetic field to the collective self-focusing force, F_r, given by Eq. (8) for the case where the beam radius is small compared to the electron skin depth. In the absence of an applied magnetic field, in the regime where $r_b \ll c/\omega_{pe}$, the beam current is not neutralized, Eq. (8) is no longer a valid expression, and the self-magnetic field is now given by

$$B_\varphi = \frac{4\pi}{cr}Z_b e V_b \int_0^r r' n_b dr'. \tag{10}$$

The beam space-charge is well neutralized provided the beam pulse duration is much longer than the plasma period, $\omega_{pe}\tau_b \gg 1$, and therefore, the electric component of the Lorentz force is small compared to the magnetic force [7]. Substituting Eq. (10) into Eq. (2), we readily obtain for the radial component of the self-pinching force

$$F_0 = -\frac{4\pi Z_b^2 e^2 V_b^2}{c^2 r}\int_0^r r' n_b dr'. \tag{11}$$

The ratio of the collective self-focusing force in the presence of an applied magnetic field [Eq. (8)] to the self-pinching force for $\omega_{ce} = 0$ [Eq. (11)] can be estimated as $F_r/F_0 \sim (c/r_b\omega_{pe})^2 \gg 1$. That is, the self-focusing of the ion beam when $r_b \ll c/\omega_{pe}$ is greatly enhanced by application of a solenoidal magnetic field. Note that for a typical ion beam injector aperture of the order of 1 cm, the beam radius (~1 cm) is small compared to the electron skin depth provided the beam and plasma density are in the range of $n_b < n_p < 2.8 \times 10^{11}/(r_b[\mathrm{cm}])^2\ \mathrm{cm}^{-3}$, which are typical parameters for several beam transport applications [2,6].

A significant increase in the self-focusing force in the presence of a weak applied magnetic field has been observed in electromagnetic particle-in-cell simulations performed using the 2D cylindrical version of the LSP code [8]. As an illustrative example, we consider a Gaussian ion beam pulse with density profile $n_b = 0.14 n_p \exp[-r^2/r_b^2 - (z - vt)^2/l_b^2]$ with effective beam radius, $r_b = 0.55c/\omega_{pe}$, and beam pulse half-length, $l_b = 1.875c/\omega_{pe}$ (beam pulse duration $\tau_b = 75/\omega_{pe}$), propagating with velocity $V_b = 0.05c$ through a background plasma with density $n_p = 10^{10}\ \mathrm{cm}^{-3}$. The simulations in Fig. 1 show the significant enhancement of the radial component of the Lorentz force due to an applied magnetic field of $B_0 = 300$ G. Furthermore, the results of the numerical simulations are found to be in good agreement with the analytical predictions given in Eq. (8) (solid pink curve in Fig. 1). Note that the total normalized radial self-focusing force (i.e., the sum of the electric and magnetic components of the Lorentz force), $F_r/Z_b e$, is plotted in Fig. 1, and the units of electric field, V/cm, are chosen for practical representation of its numerical value.

We now demonstrate that the beam current is indeed effectively neutralized provided the condition in Eq. (9) is

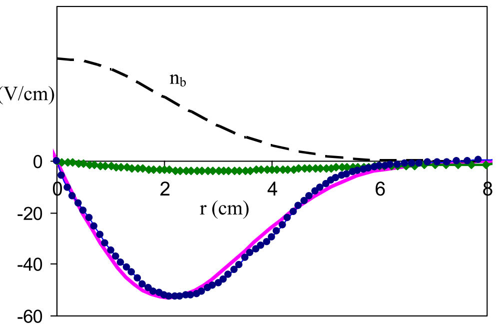

FIG. 1 (color online). Radial dependence of the normalized focusing force at the beam center. The results of the numerical simulations correspond to $B_0 = 300$ G and $\omega_{ce}/\beta_b\omega_{pe} = 18.7$ (blue circles), and $B_0 = 0$ (green diamonds). The analytical results in Eq. (8) are shown by the solid pink curve. The beam-plasma parameters correspond to $Z_b = 1$, $r_b = 0.55c/\omega_{pe}$, $\tau_b = 75/\omega_{pe}$, $\beta_b = 0.05$, and $n_p = 10^{10}$ cm^{-3}. The dashed black curve corresponds to the beam radial density profile.

satisfied. Following Ref. [9], we analyze the reduced nonlinear equations governing the evolution of the electromagnetic field and the nonrelativistic particles dynamics. We express the induced magnetic field as $\mathbf{B} = V \times \mathbf{A}$ and make use of the transverse Coulomb gauge, $\nabla_\perp \cdot \mathbf{A} = 0$. Assuming a long beam pulse with $l_b \gg r_b$ and $\omega_{pe}\tau_b \gg 1$, the displacement current can be neglected compared to the electron current [7], and Ampere's equations can be expressed as [9]

$$-\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right) = \frac{4\pi e}{c}(Z_b n_b V_b - n_e V_{ez}), \tag{12}$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (rA_\phi)}{\partial r}\right) = \frac{4\pi e}{c} n_e V_{e\phi}. \tag{13}$$

Here, $V_{e\varphi}$ and V_{ez} are the azimuthal and longitudinal components of the electron flow velocity, respectively. The electron flow velocity can be calculated making use of the conservation of generalized vorticity [7,10]

$$\left(\frac{\partial}{\partial t} + \mathbf{V_e}\cdot\nabla\right)\left(\frac{\mathbf{\Omega}}{n_e}\right) = \left(\frac{\mathbf{\Omega}}{n_e}\cdot\nabla\right)\mathbf{V_e}, \tag{14}$$

where the generalized vorticity is defined as $\mathbf{\Omega} = \nabla \times (m_e \mathbf{V_e} - e\mathbf{A}/c)$, and $\mathbf{V_e}$ is the electron flow velocity. Projecting out the longitudinal and azimuthal components of Eq. (14), we obtain [9]

$$V_{ez} = \frac{e}{m_e c} A_z - \frac{B_0}{4\pi m_e V_b n_e}\frac{1}{r}\frac{\partial (rA_\phi)}{\partial r}, \tag{15}$$

$$V_{e\varphi}\left(1 + \frac{\omega_{ce}^2}{\omega_{pe}^2}\right) = \frac{e}{m_e c} A_\varphi + \frac{B_0}{4\pi m_e V_b n_e}\frac{\partial A_z}{\partial r}. \tag{16}$$

In deriving Eqs. (15) and (16) we have taken into account, for $n_b \ll n_e$, that the radial component of the electron force balance equation gives $E_r = -V_{e\varphi}B_0/c$, where Poisson's equation can be used to determine the radial electric field. Eqs. (12), (13), (15), and (16) can then be used to calculate the self-magnetic field of the beam pulse. Taking the radial derivative of Eq. (12) and making use of Eqs. (13), (15), and (16) we obtain

$$-\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_z}{\partial r}\right)\right) = \frac{4\pi e}{c} Z_b V_b \frac{dn_b}{dr} + \frac{1}{r_{ge}^2}\frac{\partial A_z}{\partial r} + \frac{V_b\omega_{pe}^2}{c^2\omega_{ce} r_{ge}^2} A_\varphi - \frac{\partial}{\partial r}\left(\frac{\omega_{pe}^2}{c^2} A_z\right). \tag{17}$$

It now follows that the left-hand side of Eq. (17) is small compared to the term $r_{ge}^{-2}(\partial A_z/\partial r)$ on the right-hand side, and therefore the beam current is neutralized provided the condition in Eq. (9) holds, i.e., provided $r_b \gg r_{ge}$.

We emphasize that the nature of the self-focusing effect is different for the cases where the external magnetic field is zero or not. In the absence of an applied magnetic field, the self-focusing force is due to the self-magnetic field of the beam pulse. In contrast, if an external solenoidal magnetic field is applied, the beam current becomes well-neutralized and the self-magnetic field is significantly suppressed, provided the conditions in Eqs. (1) and (9) are satisfied. Nevertheless, the total self-focusing force is increased for the case where $r_b < c/\omega_{pe}$. Since the magnetic component of the Lorentz force is suppressed, the main focusing contribution comes from the strong radial electric field. Figure 2(a) illustrates the radial component of the self-electric field generated by an ion beam pulse propagating through a magnetized background plasma. The system parameters assumed in this simulation are the same as for Fig. 1. The results of the numerical simulations show that the contribution of the electric component to the total Lorentz force (Fig. 1) constitutes more than 99%. It should be noted that appearance of a sizable self-electric field created by an ion beam propagating through a background plasma along a solenoidal magnetic field has been recently investigated by Kaganovich *et al.* [9] for the case

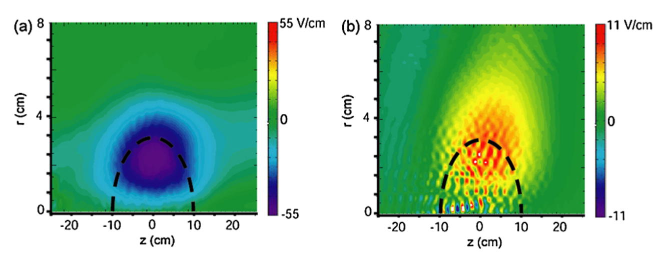

FIG. 2 (color online). Plots of the radial self-electric field corresponding to (a) $B_0 = 300$ G ($\omega_{ce}/\beta_b\omega_{pe} = 18.7$) and (b) $B_0 = 25$ G ($\omega_{ce}/\beta_b\omega_{pe} = 1.56$). Other parameters are the same as in Fig. 1. Zero value of the axial coordinate corresponds to the beam center. Dashed lines correspond to the contour of constant beam density corresponding to the effective beam radius.

where $\omega_{ce} < 2\beta_b\omega_{pe}$. However, for that regime, the radial electric field provides a defocusing of the ion beam [9]. Moreover, the magnitude of the self-electric field increases significantly as the value of the external magnetic field increases from $\omega_{ce} < 2\beta_b\omega_{pe}$ to $\omega_{ce} \gg \beta_b\omega_{pe}$. A detailed analysis of the plasma response will be presented in a follow-up publication [11]. Here we present the results of numerical simulations demonstrating the polarity change and the significant increase in the magnitude of the radial electric field as the applied magnetic field increases from $B_0 = 25$ G corresponding to $\omega_{ce}/\beta_b\omega_{pe} = 1.56$ [Fig. 2(b)] to $B_0 = 300$ G corresponding to $\omega_{ce}/\beta_b\omega_{pe} = 18.7$ [Fig. 2(a)]. Finally, it should be mentioned that the effect of the enhanced self-focusing is robust and does not depend explicitly on the value of the applied magnetic field, as can be seen from Eq. (8).

Note that the maximum value of the self-focusing force can be achieved when $Z_bn_b \sim n_e$ and $r_b \sim r_{ge}$, and is given by $F_r \sim m_ir_b\omega_{ce}\omega_{ci}$, provided $\omega_{ce} \ll \omega_{pe}$, where m_i and ω_{ci} are the mass and cyclotron frequency of the beam ions, respectively. This value of the self-focusing force is of the same order as the value of the collective focusing force obtained by Robertson [12], for the case where an ion beam propagates through a magnetic solenoidal lens carrying an equal amount of neutralizing electrons. In this case, the neutralizing electrons entering the lens experience much stronger magnetic focusing than the beam ions and tend to build up a negative charge around the lens axis. As a result, an electrostatic ambipolar electric field develops that significantly increases the total focusing force acting on the beam ions. Note that the neutralizing electrons should enter the lens from a region of zero magnetic field in order to acquire the azimuthal angular momentum necessary for radial focusing inside the lens. By contrast, for the case of ion beam propagation through a background plasma along a *uniform* magnetic field considered in this Letter, the collective response of the plasma electrons to the ion beam pulse provides the enhanced self-focusing force. Furthermore, it has been demonstrated [13] that if background plasma (or background electrons) is present inside the magnetic lens, then the collective focusing mechanism described by Robertson is absent, since the neutralizing rotating electrons are replaced by the background plasma electrons inside the solenoid.

In conclusion, it should be emphasized that the enhanced ion beam self-focusing can be of considerable importance for the proposed neutralized drift compression experiment (NDCX-II), which is designed to study energy deposition by a highly compressed intense ion beam pulse onto a target for warm dense matter physics studies [6]. To obtain a short high-current ion beam pulse, a long, singly charged lithium ion bunch carrying a current of $I_b \sim 2$ A is initially accelerated to $V_b \sim 0.032c$ as it is transported through a set of transport magnets. Leaving the transport section, a radially converging beam pulse (with beam radius, $r_b \sim 1$ cm) acquires a head-to-tail velocity tilt and enters a long drift section ($L_d \sim 2$ m) filled with a background plasma ($n_p \sim 10^{10}$–10^{11} cm^{-3}), which neutralizes the beam space charge and facilities the ballistic beam compression. To provide the final transverse focusing, a strong magnetic lens with magnetic field of $B_L \sim 8$ T and length, $l \sim 10$ cm, is placed downstream of the beam line after the drift section. For the parameters characteristic of NDCX-II ($\beta_b \sim 0.032$, $n_p \sim 10^{11}$ cm^{-3}, $r_b \sim 1$ cm), the value of the magnetic field determined from $\omega_{ce} = 2\beta_b\omega_{pe}$ corresponds to a weak magnetic field of $B_c \sim 65$ G. The fringe magnetic field of the lens penetrates deep into the neutralizing plasma at a magnitude significantly larger than B_c, thereby providing conditions for the enhanced beam self-focusing. Note that for the NDCX-II parameters considered here, the beam radius is small compared to the electron skin depth, and therefore the self-focusing force is large compared to the self-pinching force corresponding to the case of zero applied magnetic field. Moreover, the self-focusing effect during beam pulse propagation through the neutralizing background plasma in the drift section can become comparable to the applied focusing effect provided by the 8 Tesla magnetic lens. Introducing the dimensionless parameter $\delta = F_{sf}L_d/(F_Ll)$, where $F_L \sim m_i\omega_{ci}^2r_b/4$ is the magnetic focusing force acting on the beam ions inside the lens, and $F_{sf} \sim m_eV_b^2/r_b$ is the self-focusing force ($n_b \sim n_e$ is assumed), we readily obtain that $\delta \sim 0.5$.

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