Dielectric Permittivity and Nonlocal Electron Transport in Weakly Collisional Plasma.

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Outline

Nonlocal linear hydrodynamics (NLH) – theory of electron transport valid for small perturbations and arbitrary collisionality

- Electron dielectric permittivity

- Classical ⟷ nonlocal transport
  Temperature relaxation. Nonstationary effect.

- Nonlocal nonlinear transport model

- Nonlocal heating of an overdense plasma

Kinetic model

In all studies it is the electron Fokker-Planck equation

$$\Im f = C[f] \longleftrightarrow \boxed{\frac{\partial f}{\partial t} + \mathrm{v}_z \frac{\partial f}{\partial z} - \frac{e}{m} E \frac{\partial f}{\partial \mathrm{v}_z} = C_{ei}[f] + C_{ee}[f, f]}$$

where E is an ambipolar electric field,

$$C_{ei}[f] = \frac{1}{2}\nu_{ei}(\mathrm{v}) \frac{\partial}{\partial \cos\theta} \sin\theta \frac{\partial f}{\partial \cos\theta}, \qquad \nu_{ei}(\mathrm{v}) = \frac{4\pi Z n_0 e^4 \Lambda}{m_e^2 \mathrm{v}^3}$$

e-e collisions are described by C_{ee} – nonlinear integral operator (Landau).

Local Maxwellian is a stationary solution

$$C_{ee}[F_0] = 0 \qquad F_0 = \frac{n_0}{(2\pi)^{3/2} \mathrm{v}_{Te}^3} \exp\left(-\frac{\mathrm{v}^2}{2\mathrm{v}_{Te}^2}\right)$$

$$\mathrm{v}_{Te} = \sqrt{\frac{T_e}{m_e}} \qquad \lambda_{ei} = 3\sqrt{\frac{\pi}{2}} \frac{\mathrm{v}_{Te}}{\nu_{ei}(\mathrm{v}_{Te})}$$

Electron fluid equations

Fluid variables:

$$n_e(\vec{r},t) = \int d^3\mathrm{v}\, f(\vec{r},\vec{\mathrm{v}},t); \;\; n_e\vec{u}_e = \int d^3\mathrm{v}\, \vec{\mathrm{v}} f;$$

$$P_e = n_e T_e = (m_e/3)\int d^3\mathrm{v} (\vec{\mathrm{v}} - \vec{u}_e)^2 f$$

$$\frac{\partial n_e}{\partial t} + \vec{\nabla}\cdot(n_e\vec{u}_e) = 0$$

$$\left(\frac{\partial}{\partial t} + \vec{u}_e\cdot\vec{\nabla}\right)\vec{u}_e = -\frac{1}{m_e n_e}\vec{\nabla}\cdot P_e + \frac{1}{m_e n_e}\vec{\nabla}\cdot\hat{\Pi} - \frac{e}{m_e}\vec{E} - \frac{1}{m_e n_e}\vec{R}_{ie}$$

$$\left(\frac{\partial}{\partial t} + \vec{u}_e\cdot\vec{\nabla}\right)T_e + \frac{2}{3}T_e\vec{\nabla}\cdot\vec{u}_e + \frac{2}{3n_e}\vec{\nabla}\cdot\vec{q}_e - \frac{2}{3n_e}\hat{\Pi}\cdot\vec{\nabla}\vec{u}_e = \frac{2}{3n_e}\vec{u}_e\cdot\vec{R}_{ie}$$

Need closure relations for higher order moments:

$$\vec{q}_e = (m_e/2)\int d^3\mathrm{v} (\vec{\mathrm{v}} - \vec{u}_e)(\vec{\mathrm{v}} - \vec{u}_e)^2 f$$

$$\Pi_{ij} = P_e\delta_{ij} - m_e\int d^3\mathrm{v} (\mathrm{v} - u_e)_i(\mathrm{v} - u_e)_j f$$

$$\vec{R}_{ie} = m_e\int d^3\mathrm{v} (\vec{\mathrm{v}} - \vec{u}_i)\nu_{ei}(|\vec{\mathrm{v}} - \vec{u}_i|) f$$

Classical collision theory

$$f_e = F_0 + f_1$$

Spitzer, Härm[1953]

$$\cancel{\frac{\partial f_1}{\partial t}} + \mathrm{v}\frac{\partial F_0}{\partial x} - \frac{eE}{m}\frac{\partial F_0}{\partial \mathrm{v}} = -\nu_{ei} f_1 \quad \Rightarrow \quad f_1 = -\frac{1}{\nu_{ei}}\left(\mathrm{v}\frac{\partial F_0}{\partial x} - \frac{eE}{m}\frac{\partial F_0}{\partial \mathrm{v}}\right)$$

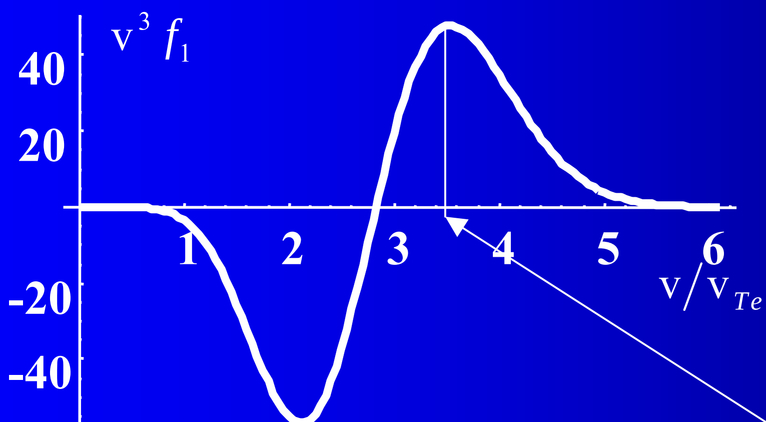
$$j = -e\frac{4\pi}{3}\int d\mathrm{v}\mathrm{v}^3 f_1 \qquad \frac{\partial F_0}{\partial x} = \left(\frac{m_e \mathrm{v}^2}{2T_e} - \frac{3}{2}\right)\frac{1}{T_e}\frac{\partial T_e}{\partial x}F_0$$

$$q = m_e\frac{2\pi}{3}\int d\mathrm{v}\mathrm{v}^3\left(\mathrm{v}^2 - 5\mathrm{v}_{Te}^2\right) f_1$$

$$j = \sigma E + \alpha \nabla T_e$$

$$q = -\alpha T_e E - \chi \nabla \delta T_e$$

Validity of SH approach



$$\lambda_{ei}(\mathrm{v})\frac{\partial \mathrm{Ln}\left[T_e\right]}{\partial x} \ll 1$$

$$\lambda_{ei}(\mathrm{v}) \approx 200\lambda_{ei}(\mathrm{v}_{Te})$$

Effect of e-e collisions

$$f_e = F_0 + f_0 + f_1$$

$$\begin{cases} \dfrac{\partial f_0}{\partial t} + \dfrac{\mathrm{v}}{3}\dfrac{\partial f_1}{\partial x} = C_{ee}\left(F_0, f_0\right) \sim \nu_{ee}\left(f_0 - F_0\right) \\ \cancel{\dfrac{\partial f_1}{\partial t}} + \mathrm{v}\dfrac{\partial f_0}{\partial x} - \dfrac{eE}{m}\dfrac{\partial F_0}{\partial \mathrm{v}} = -\nu_{ei} f_1 \end{cases}$$

$$f_1 = -\frac{1}{\nu_{ei}}\left(\mathrm{v}\frac{\partial f_0}{\partial x} - \frac{eE}{m}\frac{\partial F_0}{\partial \mathrm{v}}\right)$$

$$\frac{\partial f_0}{\partial t} - \frac{\mathrm{v}^2}{3}\frac{\partial}{\partial x}\left(\frac{1}{\nu_{ei}}\frac{\partial f_0}{\partial x}\right) + \frac{\mathrm{v}}{3}\frac{eE}{m}\frac{\partial}{\partial x}\left(\frac{1}{\nu_{ei}}\frac{\partial F_0}{\partial \mathrm{v}}\right) = \nu_{ee}\left(f_0 - F_0\right)$$

$$\frac{\mathrm{v}^2}{\nu_{ei} L^2} f_0 \ll \nu_{ee} f_0$$

$$\omega \ll \max\left[\nu_{ee}, \frac{\mathrm{v}^2}{\nu_{ei} L^2}\right]$$

Electron energy delocalization length

$$\frac{\lambda_\varepsilon^2}{L^2} \ll 1 \qquad \lambda_\varepsilon = \sqrt{\lambda_{ei}\lambda_{ee}} = \sqrt{Z}\,\lambda_{ei}$$

$$\lambda_{ei}\frac{\partial\,\mathrm{Ln}\left[T\right]}{\partial x} \ll \frac{0.05}{\sqrt{Z}}$$

NLH – nonlocal linear hydrodynamics

Bychenkov, Rozmus, Tikhonchuk, Brantov, Phys. Rev. Lett. 75, 4405 (1995); Brantov, Bychenkov, Rozmus, Capjack, Phys. Rev. Lett. 93, 125002 (2004);

- Self-consistent closure of linearized fluid equations valid for the arbitrary ratio λ_{ei} / L
- Transport in the strong collision regime not applicable even for moderate gradients $\lambda_{ei} / L \sim 0.1 / \sqrt{Z}$
- Nonstationary effect on transport coefficients
- Transport occurs as the plasma response to initial pertubations (or alternatively to a source term)

$$\Im f - C(f) = 0, \quad f(t=0) = F_0(\mathrm{v}, z, t=0)$$

- NLH – solution to the initial value problem for the linear FP equation with the local Maxwellian at t=0 in Fourier space.

Derivation of NLH

- Linearize FP equation with respect to small perturbations $\delta f(\mathbf{r},\mathbf{v},t) = f - F_0(\mathrm{v})$ around homogeneous Maxwellian F_0
- Expansion in a series of Legendre polynomials

$$\delta f(k,\mathrm{v},\mu,t) = \sum_{l=0}^{\infty} f_l(k,\mathrm{v}) P_l(\mu)$$

- Account for all $(l>1)$ harmonics by renormalizing e-i collision frequency $(Z>>1)$

$$\nu_l = -i\omega + \frac{1}{2} l(l+1)\nu_{ei} + \frac{(l+1)^2}{4(l+1)^2-1}\frac{k^2\mathrm{v}^2}{\nu_{l+1}}$$

$$-i\omega f_0 + \frac{i}{3} k\,\mathrm{v}\, f_1 = C_{ee}[f_0] + f_0(\mathrm{v}, t=0)$$

$$ik\,\mathrm{v}\, f_0 - \frac{eE}{m}\frac{\partial F_0}{\partial \mathrm{v}} + \nu_{ei} u_i \frac{\partial F_0}{\partial \mathrm{v}} = -\nu_1 f_1$$

- Initial conditions: $f_0(\mathrm{v}, t=0) = \left[\frac{\delta n(0)}{n_0} + \frac{\delta T(0)}{T_0}\left(\frac{\mathrm{v}^2}{2\mathrm{v}_{Te}^2} - \frac{3}{2}\right)\right] F_0(\mathrm{v})$

Initial value problem for f_0

$$\left(\frac{k^2 v^2}{3\nu_1} + i\omega\right)\left(f_0 - \frac{ieE}{kT_0}F_0\right) = -\omega\frac{eE}{kT_0}F_0 - iku_i\frac{v^2}{3v_{Te}^2}\frac{\nu_{ei}}{\nu_1}F_0 + C_{ee}[f_0] + f_0(v,0)$$

- Linear combination of three base functions – response to initial perturbations $\delta n(0), \delta T(0), u_i, E$

$$f_0(v,\omega) = \frac{ieE}{kT_0}F_0 + \left(\frac{\delta n(0)}{n_0} - \omega\frac{eE}{kT_0}\right)\psi^N F_0 + \frac{3}{2}\frac{\delta T(0)}{T_0}\psi^T F_0 - iku_i\psi^R F_0$$

$$\boxed{\left(-i\omega + \frac{k^2 v^2}{3\nu_1}\right)\psi^A - \frac{1}{F_0}C_{ee}\left[\psi^A, F_0\right] = S^A}$$

$$S^N = 1 \qquad S^R = \frac{v^2}{3v_{Te}^2}\frac{\nu_{ei}}{\nu_1}$$

$$S^T = \frac{v^2}{3v_{Te}^2} - 1$$

- To invert C_{ee} we use Sonine-Laguerre polynomial expansion, up to 50^{th} order to describe weakly collisional plasma
- Eliminate $\delta n(0), \delta T(0)$, by taking first two moments and calculate closure relations from higher order moments

Transport relations in NLH

$$\mathbf{j} = \sigma \mathbf{E}^* + \alpha i\mathbf{k}\delta T_e + \beta_j e n \mathbf{u}_i$$

$$\mathbf{q}_e = -\alpha T_0 \mathbf{E}^* - \chi i\mathbf{k}\delta T_e - \beta_q n_0 T_0 \mathbf{u}_i$$

$$\mathbf{E}^* = \mathbf{E} + (i\mathbf{k}/en_0)(\delta n T_0 + \delta T_e n_0)$$

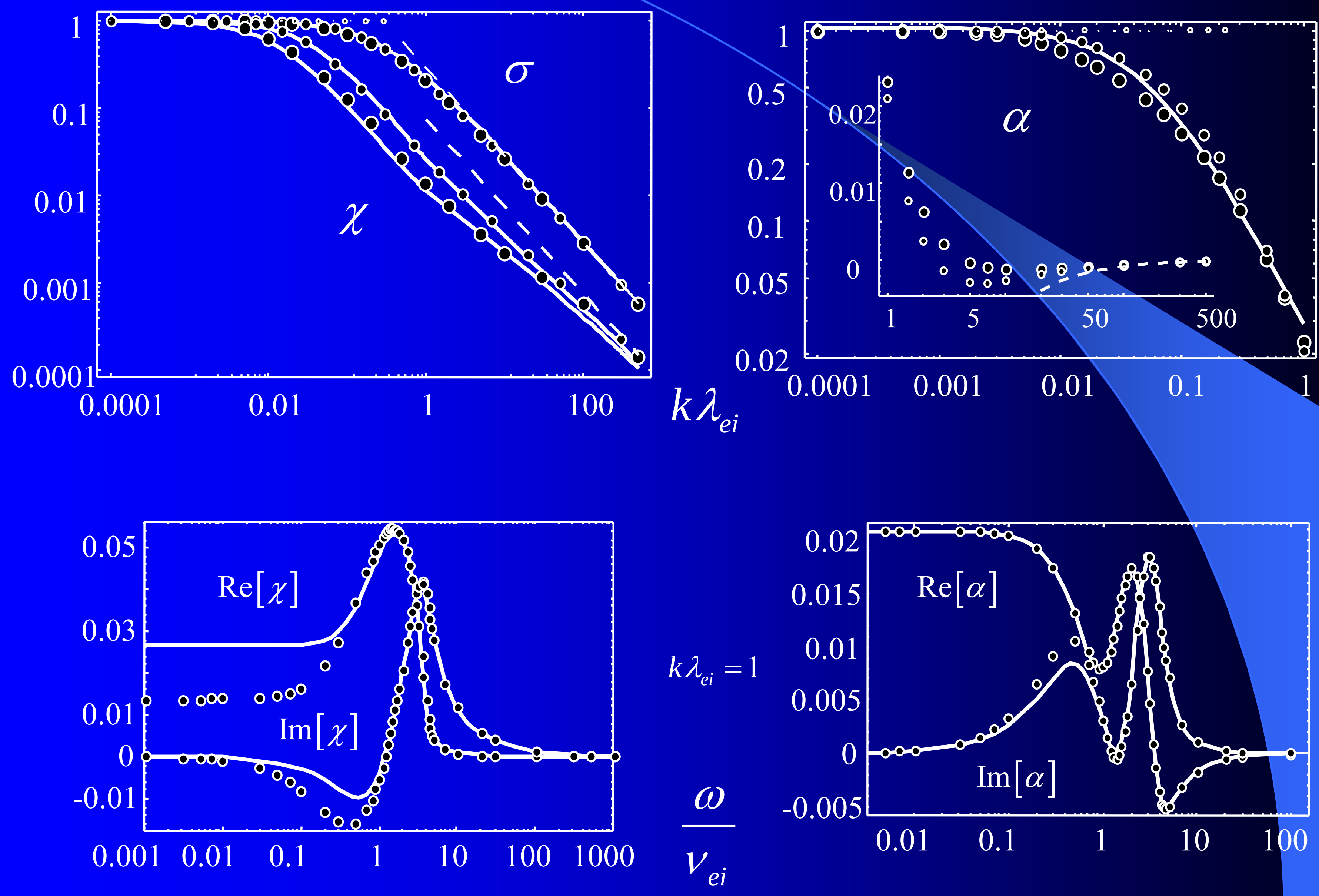
$$i\mathbf{k}\mathbf{\Pi}_e = \mathbf{R}_{ie} + n_0 e \mathbf{E}^* - i\omega m_e n_0 \mathbf{u}_e$$

$$\mathbf{R}_{ie} = -(1-\beta_j) n_0 e \mathbf{E}^* + \beta_q n_0 i\mathbf{k}\delta T_e - \beta_r m_e n_0 \mathbf{u}_i \, \mathrm{v}_{Te}/\lambda_{ei}$$

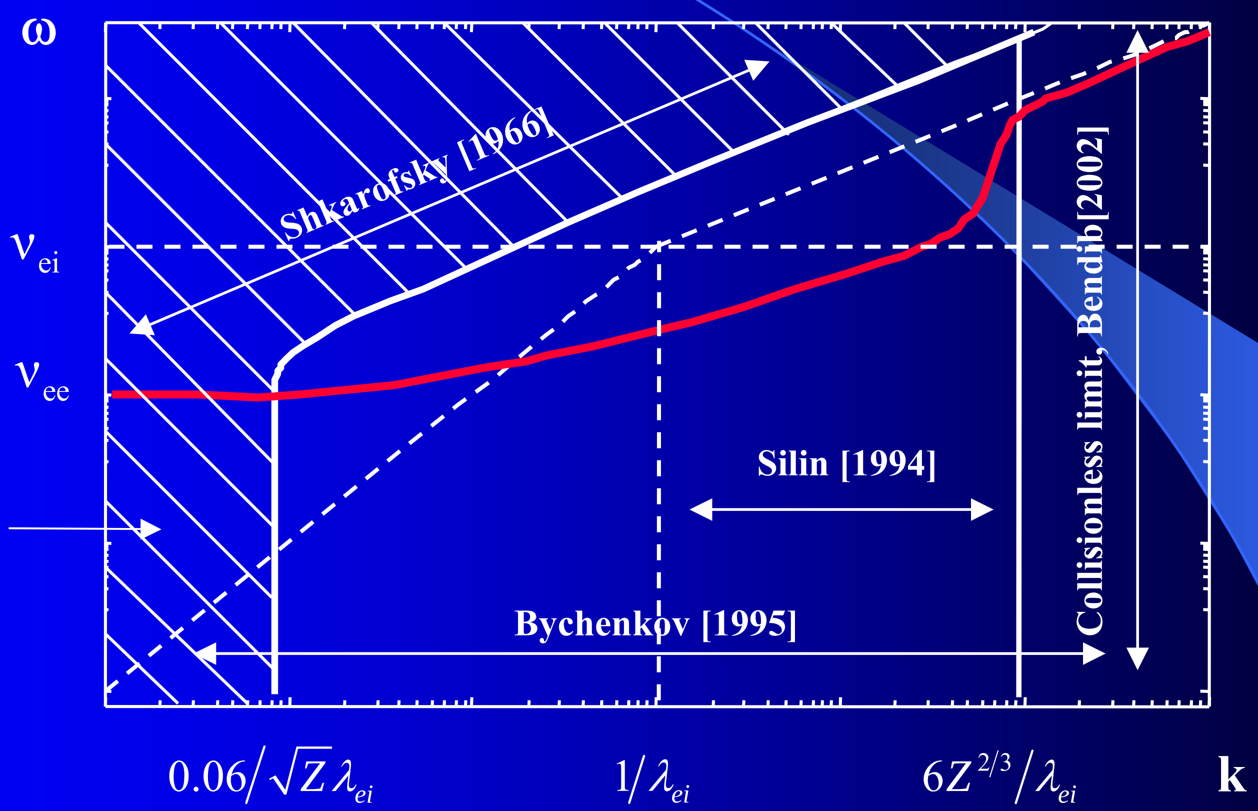
Transport coefficients $\sigma, \alpha, \beta_j, \chi, \beta_q, \beta_r$ are *k*, ω - dependent functions with correct asymptotics in Chapman-Enskog and Vlasov limits.

In quasistatic limit all coefficients are real and k- dependent

Transport coefficients



Electron transport coefficients in (ω, k) plane



Under red curve $\omega = \begin{cases} \max\left[\nu_{ee}, (k\lambda_{ei})^{4/7}/Z^{5/7}\right] & k\lambda_{ei} < Z^{2/3} \\ k v_{Te} & k\lambda_{ei} > Z^{2/3} \end{cases}$ **- quasistatic approximation**

Electron dielectric permittivity for arbitrary collisions

- NLH are equivalent to a kinetic description and completely determine the linear response of a plasma - dielectric permittivity

$$\varepsilon^{l} = 1 + \frac{1}{k^2\lambda_{De}^2}\left[1 - i\omega\left(\frac{e^2 n_e}{k^2 T_e^2 \sigma} + \frac{2n_e(\sigma + e\alpha)^2}{\sigma^2\left(2k^2\kappa - 3i\omega n_e\right)}\right)\right]^{-1} = 1 + \frac{1 + i\omega J_N^N}{k^2\lambda_{De}^2} \qquad J_N^N = \frac{4\pi}{n_0}\int \mathrm{v}^2 \psi^N F_0 d\mathrm{v}$$

Without e-e collisions, only e-i collisions

Bychenkov, Plasma Phys.Rep. 24, 801 (1998)

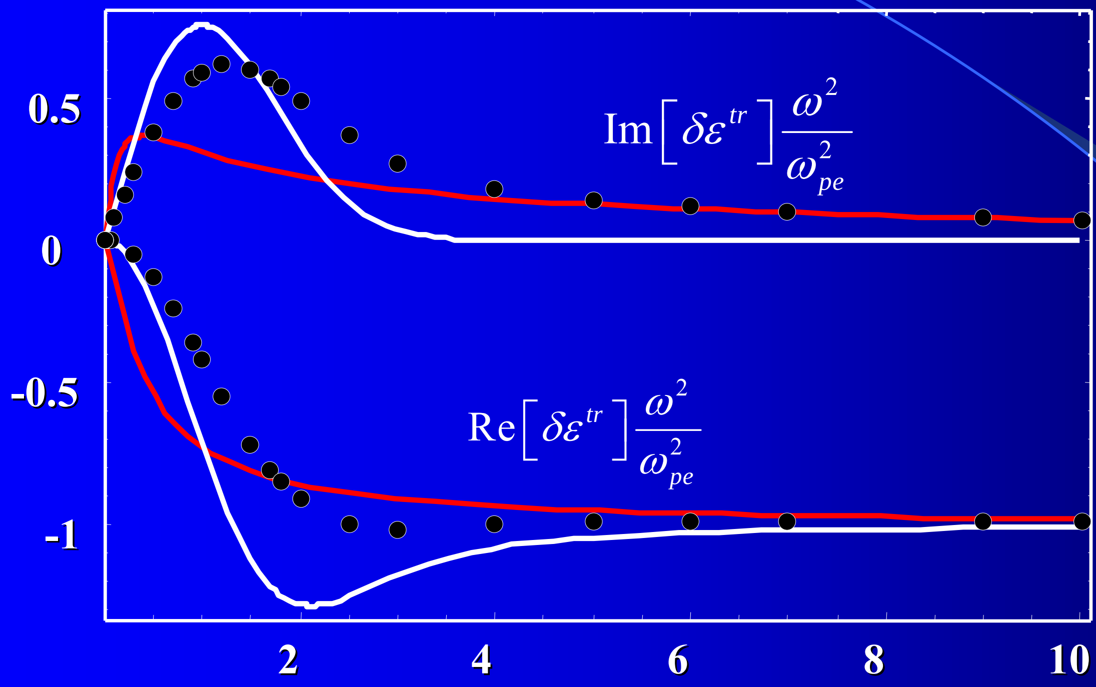
$$\varepsilon^{l} = 1 + \omega_{pe}^2\sqrt{\frac{2}{\pi}}\int dx \frac{x^4 \exp\left(-x^2/2\right)}{k^2 \mathrm{v}_{Te}^2 x^2 - 3i\nu_1\omega}$$

Transverse electron permittivity

Bychenkov et.al., Phys. Plasmas 4, 4205 (1997)

$$\varepsilon^{tr} = 1 + \frac{4\pi i\sigma_{\perp}}{\omega} = 1 + i\omega_{pe}^2\sqrt{\frac{2}{\pi}}\int_0^{\infty} dx \frac{x^4 \exp\left(-x^2/2\right)}{3\nu_{1,1}\omega} \qquad \nu_{l,1} = -i\omega + \frac{1}{2}l(l+1)\nu_{ei} + \frac{(l+1)^2 - 1}{4(l+1)^2 - 1}\frac{k^2\mathrm{v}^2}{\nu_{l+1,1}}$$

Electron transverse susceptibility



$$k\lambda_{ei} = 1$$

$$\omega / k\mathrm{v}_{Te}$$

Drude model $\varepsilon^{tr} = 1 + i\omega_{pe}^2\sqrt{\frac{2}{\pi}}\int_0^\infty dx\,\frac{x^4\exp\left(-x^2/2\right)}{3\left(\nu_{ei}\left(x\right)-i\omega\right)\omega}$

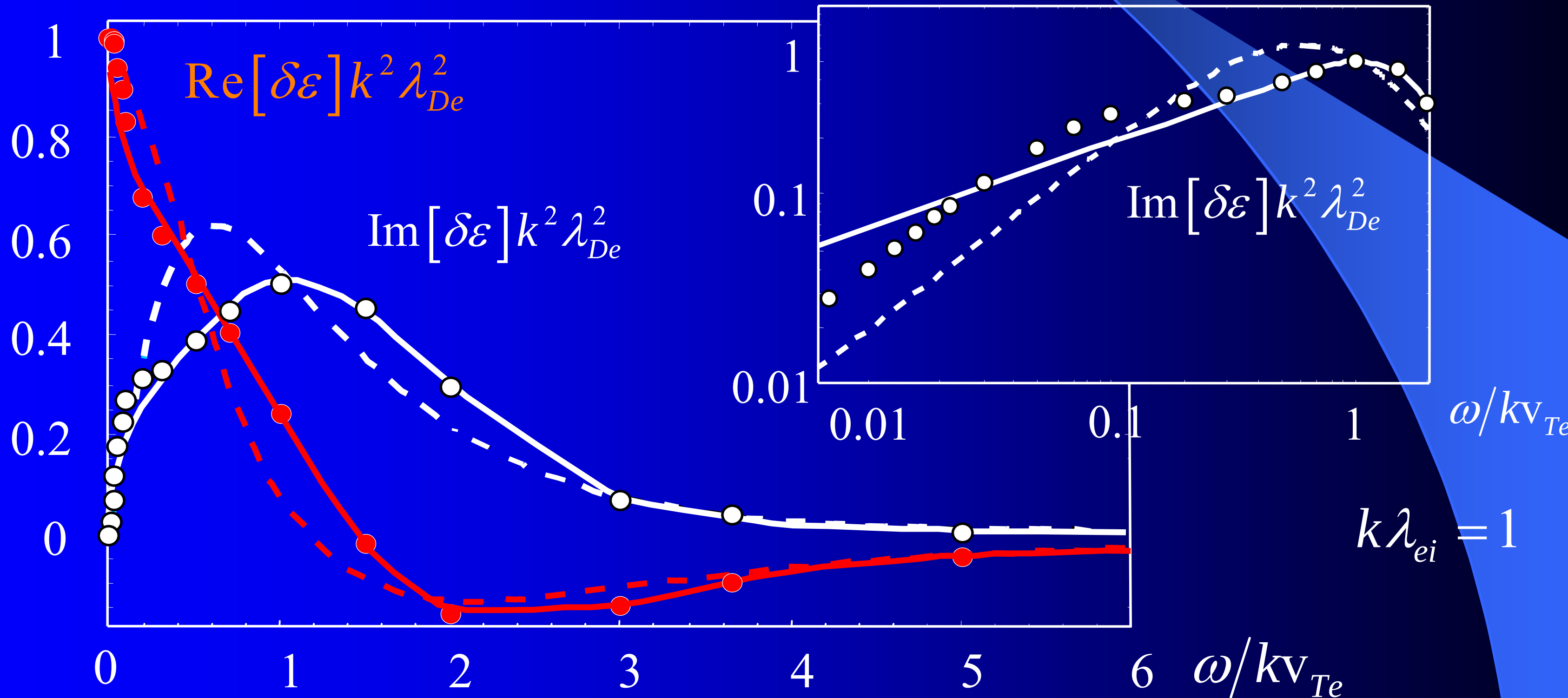
dots
non-stationary
nonlocal theory

Collisionless plasma $\varepsilon^{tr} = 1 - \frac{\omega_{pe}^2}{\omega^2}J_+\left(\frac{\omega}{k\mathrm{v}_{Te}}\right)$

Electron longitudinal susceptibility

BGK approximation for collisions (dashed lines)

$$\varepsilon = 1 + \frac{1}{k^2 \lambda_{De}^2} \frac{1 - J_+\left(\frac{\omega + i\nu_{ei}}{k\,\mathrm{v}_{Te}}\right)}{1 - \frac{i\nu_{ei}}{\omega + i\nu_{ei}} J_+\left(\frac{\omega + i\nu_{ei}}{k\,\mathrm{v}_{Te}}\right)}$$



dots • non-stationary nonlocal theory

solid lines - without e-e collisions

Applications of nonlocal hydrodynamics

- **Transport in magnetized plasma** Brantov et al. Phys. Plasmas 10, 4633 (2003)
- **Ponderomotive effects and inverse bremsstrahlung heating:** Brantov et al. , Phys. Plasmas 5, 2742 (1998)
- **Theory of plasma fluctuations and calculation of Thomson scattering (TS) cross-section:** Myatt et al. , Phys. Rev. E 57, 3383 (1998); Brantov et al, Phys. Plasmas 57, 978 (1998)
- **Ion acoustic dispersion relation:** Bychenkov et al., Phys. Rev. E 52, 6759 (1995)
- **Filamentation instability and stimulated Brillouin scattering:** Bychenkov et al., Phys. Plasmas 7, 1511 (2000)
- **Return current instability:** Brantov, Bychenkov, Rozmus, Phys. Plasmas 8, 3558 (2001)
- **Relaxation of temperature perturbation** Brantov et al. Plasma Phys. Rep. In press(2005) Senecha, Brantov et al., Phys.Rev E., v. 57, p. 978 (1998).
- **Nonlocal nonlinear transport model. Heat wave propogation:** Brantov et al. , Comp.Phys. Comm. 164, 67 (2004)
- **Experimental temperature profile:** Gregory et al. , Phys. Rev. Lett. 92, 205006 (2004)

Heat flux limitation

Strong collisions theory

$$\vec{q} = -\kappa_{SH} \frac{\partial T}{\partial \vec{x}}$$

$$\kappa_{SH} = \frac{128}{3\pi} n_e \mathrm{v}_{Te} \lambda_{ei} = \alpha T_e^{5/2}$$

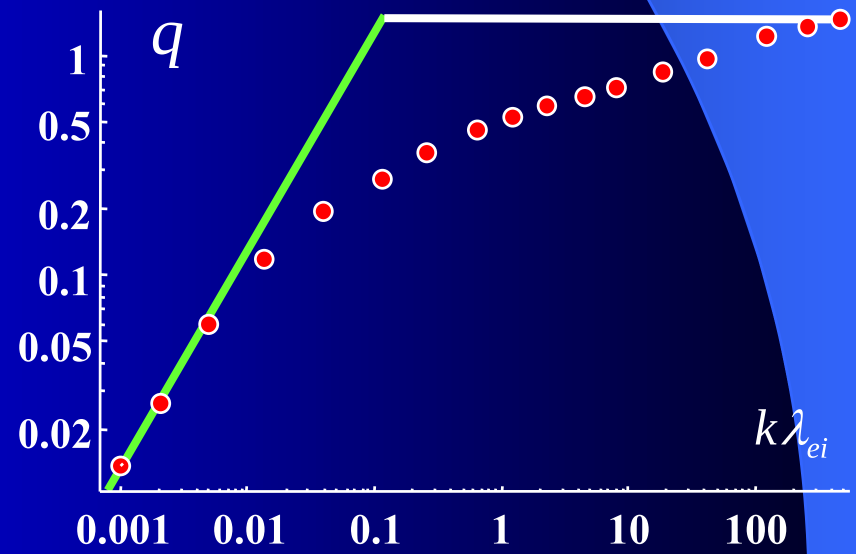
$$q_e = \min \begin{cases} -\kappa_{SH} \dfrac{\partial T_e}{\partial x} \\ -f_{\lim} n_e T_e \mathrm{v}_{Te} \end{cases}$$

$$f_{\lim} \sim (5-10)\%$$

Maximum heat flux without collisions

$$\vec{q}_{\max} = -\int d\vec{\mathrm{v}} \frac{m_e \mathrm{v}^2}{2} \vec{\mathrm{v}} \cdot \vec{n} \ f_e$$

$$q_{\max} = -\sqrt{\frac{2}{\pi}} n_e T_e \mathrm{v}_{Te}$$



Nonlocal transport

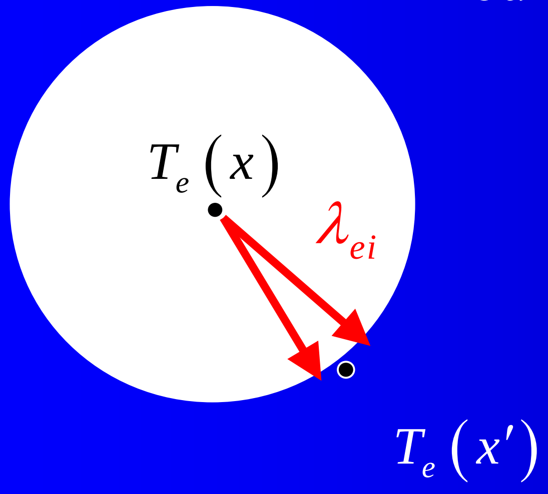
$$\vec{q} = -\kappa(x)\frac{\partial T}{\partial \vec{x}}$$

Local heat flux

$$\vec{q} = -\int Q(\vec{x}-\vec{x}')\kappa(x')\frac{\partial T}{\partial \vec{x}'}$$

Nonlocal heat flux

Heuristic model - delocalization with the scale $\sim \lambda_\varepsilon$



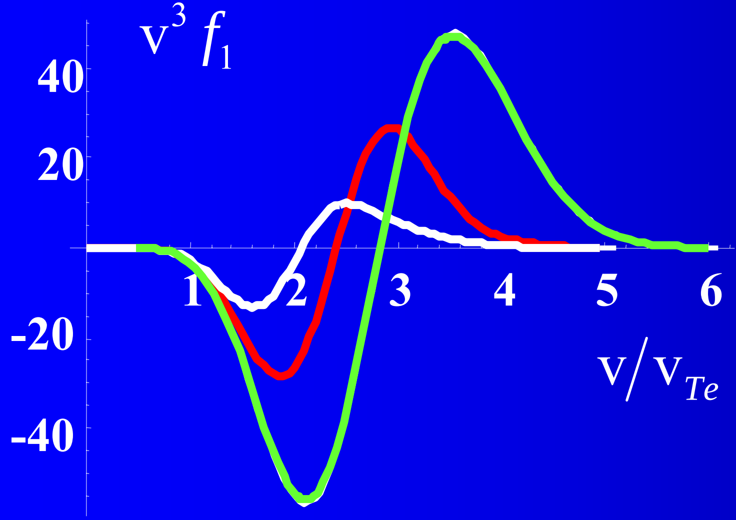
$$Q(\vec{x}-\vec{x}') = \frac{1}{\lambda_\varepsilon} Exp\left[-a\frac{|\vec{x}-\vec{x}'|}{\lambda_\varepsilon}\right]$$

Nonlocal transport for small perturbation in Fourier space

$$\vec{j} = \sigma\vec{E} + i\vec{k}\alpha\delta T_k$$

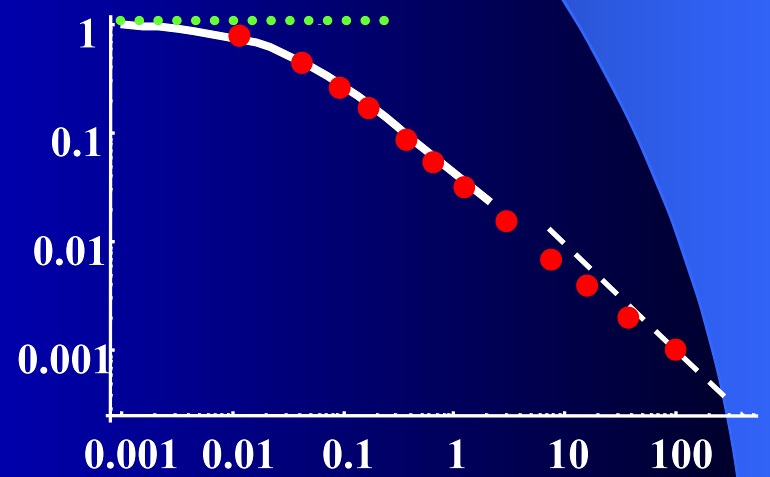
$$\vec{q} = -\alpha T_e\vec{E} - i\vec{k}\chi\delta T_k$$

Property of nonlocal transport



$$\vec{q} = -i\vec{k}\kappa(k)\delta T_k$$

$$\kappa_{nl}(k) = \frac{\kappa_{SH}}{1+\left(10\sqrt{Z}k\lambda_{ei}\right)^{0.9}}$$



Contribution to the heat flux
for the velocity > 4 thermal velocity

SH case $k\lambda_{ei} < 0.01$	40%
$k\lambda_{ei} = 0.1$	8%
$k\lambda_{ei} = 1$	4%

Temperature relaxation in a current free plasma

$$\frac{\partial \delta T_k}{\partial t} + \frac{2}{3n_e} i\vec{k}\cdot\vec{q} = 0$$

$$\vec{q} = -i\vec{k}\,\kappa\,\delta T_k$$

$$\delta T_k = \delta T(0)\int_{-\infty}^{+\infty} d\omega \frac{\exp(-i\omega t)}{-i\omega + 2k^2\kappa/3n_e}$$

Quasistationary approximation :

$$\delta T_k = \delta T_k(0)\exp\left[-\frac{2k^2\kappa}{3n_e}t\right] \quad , \quad \nu_{relax} = \frac{2k^2\kappa}{3n_e}$$

Collision Spitzer Härm (SH) limit

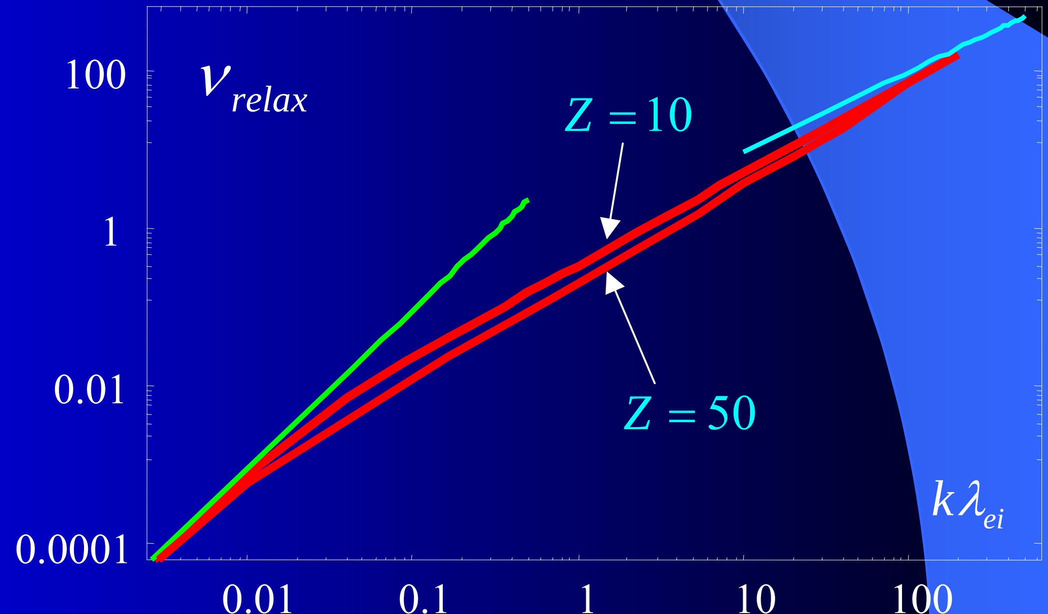
$$\kappa_0 = \frac{128}{3\pi} n_e \mathrm{v}_{Te}\lambda_{ei} \qquad k\lambda_{ei} \ll 1$$

Nonlocal model

$$\kappa_{nl}(k) = \frac{\kappa_0}{1+\left(10\sqrt{Z}\,k\lambda_{ei}\right)^{0.9}} \qquad k\lambda_{ei} \le 1$$

Kinetic, collisionless transport

$$\kappa = \frac{18}{5\sqrt{2\pi}}\frac{n_e \mathrm{v}_{Te}}{k} \qquad k\lambda_{ei} \gg 1$$



Temperature relaxation in collisionless plasma

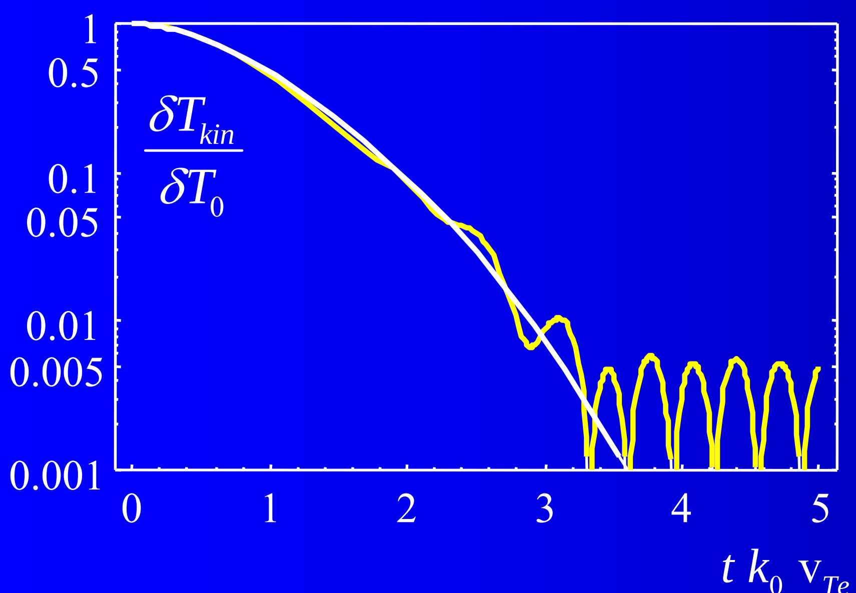
$$\delta T(x,t) = \frac{3}{8\pi^2}\int_{-\infty}^{+\infty} dk\,\delta T_k(0)\int_{-\infty}^{+\infty} d\omega\, e^{ikx-i\omega t}\left(J_T^T - \frac{i\omega J_N^T J_T^N}{k^2\lambda_{De}^2\varepsilon^l}\right) \qquad p = \frac{\omega}{k\,\mathrm{v}_{Te}}$$

$$J_N^N = \frac{i}{\omega}J_+(p) \qquad J_T^T = \frac{i}{9\omega}\left(\left(p^4 - 2p^2 + 5\right)J_+(p) - p^4 + p^2\right)$$

$$J_+(p) = p\exp\left\{-p^2/2\right\}\int_{i\infty}^{p} dt\exp\left\{t^2/2\right\}$$

$$J_T^N = \frac{i}{3\omega}\left(\left(p^2 - 1\right)J_+(p) - p^2\right)$$

Periodic temperature perturbation $\delta T(0) = \delta T_0 \sin\left(k_0 x\right)$

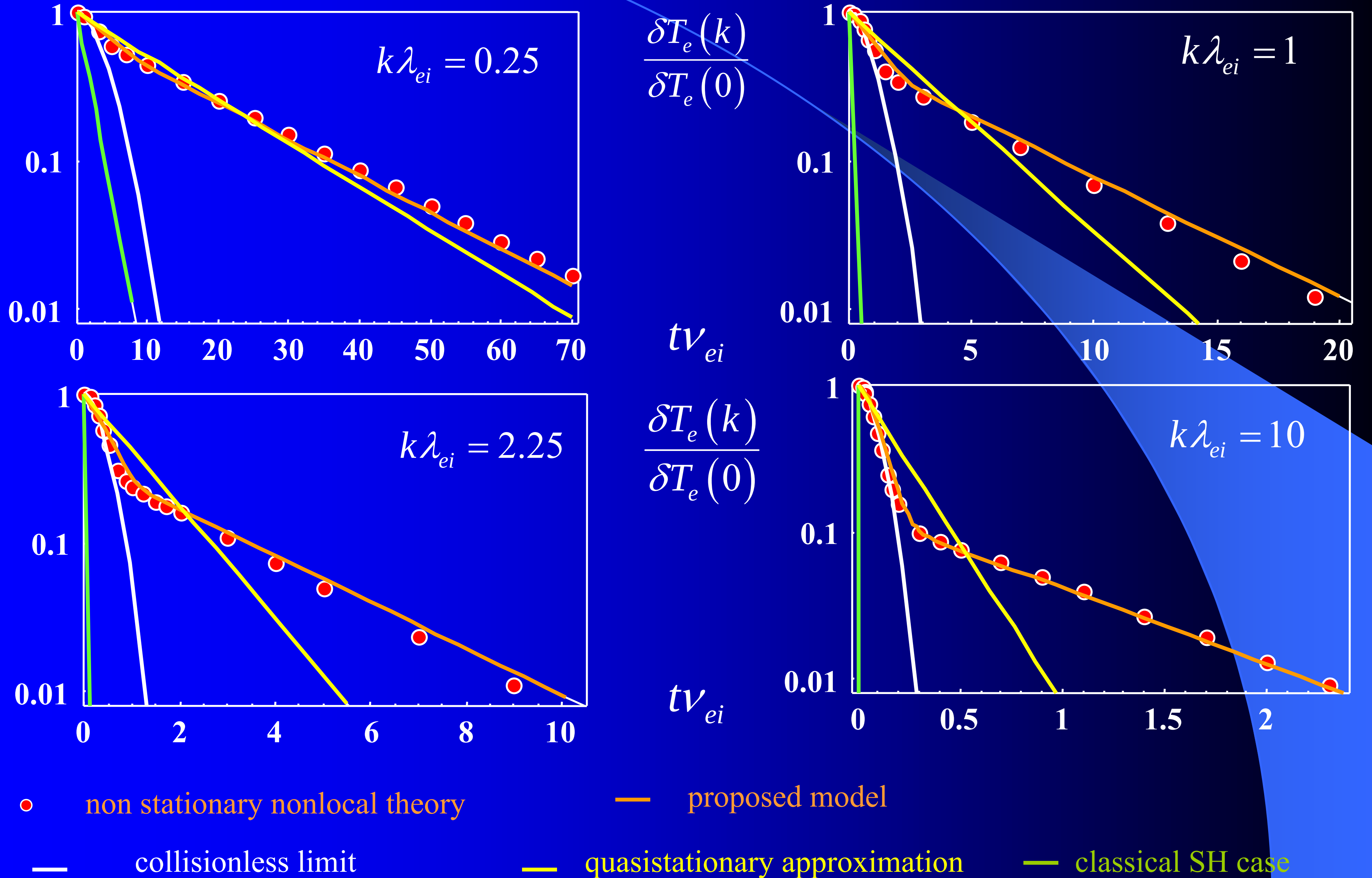


$$\delta T_{kin}(t) \approx \frac{\delta T_0}{3}\left(2\exp\left(-\frac{k_0^2\,\mathrm{v}_{Te}^2\,t^2}{2}\right) + \exp\left(-\frac{3k_0^2\,\mathrm{v}_{Te}^2\,t^2}{2}\right)\right)$$

$$+\frac{2}{3}\delta T_0\left(k^2\lambda_{De}^2\exp\left[-\gamma t\right]\cos\left[\omega_{pe}t\right]\right)$$

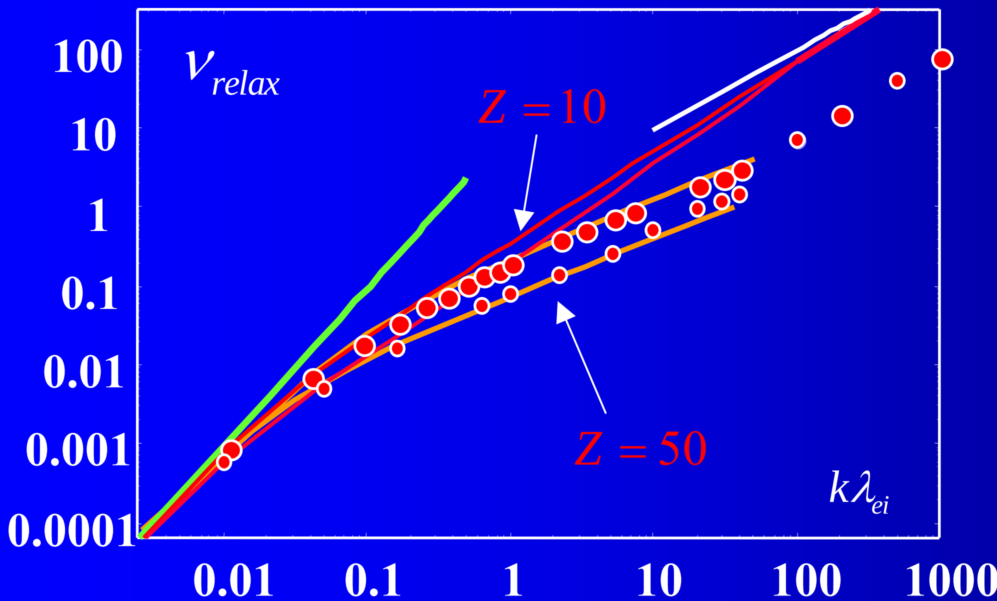
Plasma wave effect $k\lambda_{De} = 0.1$

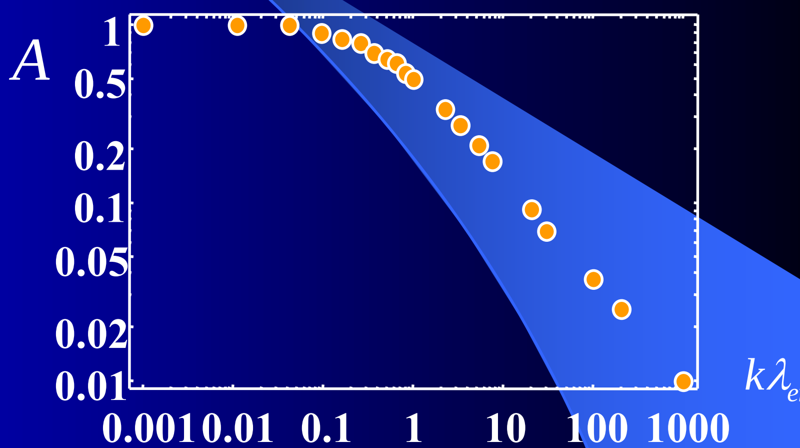
Temperature relaxation for periodic perturbation



A model of the temperature relaxation

$$\delta T(x,t) = \left(A\delta T_{hydro}(t) + (1-A)\,\delta T_{kin}(t) \right)\cos\left[k_0 x\right]$$





$$\delta T_{hydro}(t) = \delta T_0 \exp\left(-t/\tau\right)$$

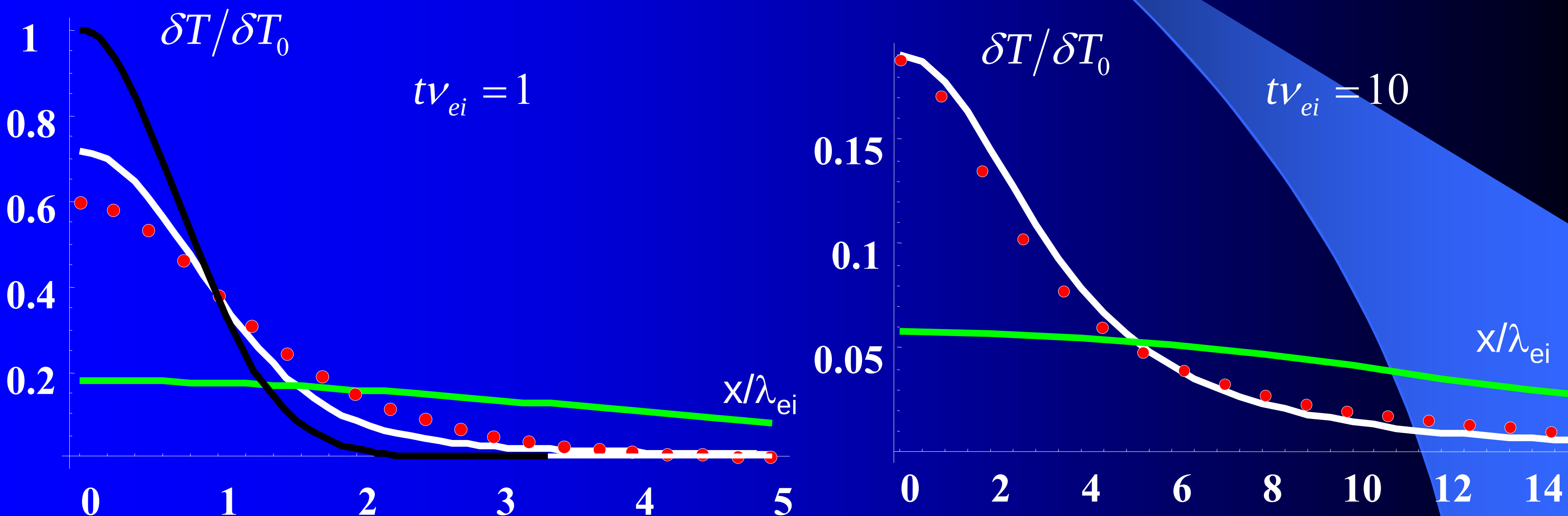
$$\tau = 9\pi n_e \left(1 + 12\left(\sqrt{Z}k_0\lambda_{ei}\right)^{1.2}\right) \Big/ 256\, \mathrm{v}_{Te}\, \lambda_{ei} k_0^2$$

$$A = \left(1 + \left(k_0\lambda_{ei}\right)^{0.8}\right)^{-1}$$

Hot spot relaxation for small perturbations

Gaussian hot spot $$T_e(x) = T_0(x) + \delta T_0 \exp\left(-\frac{x^2}{L^2}\right)$$

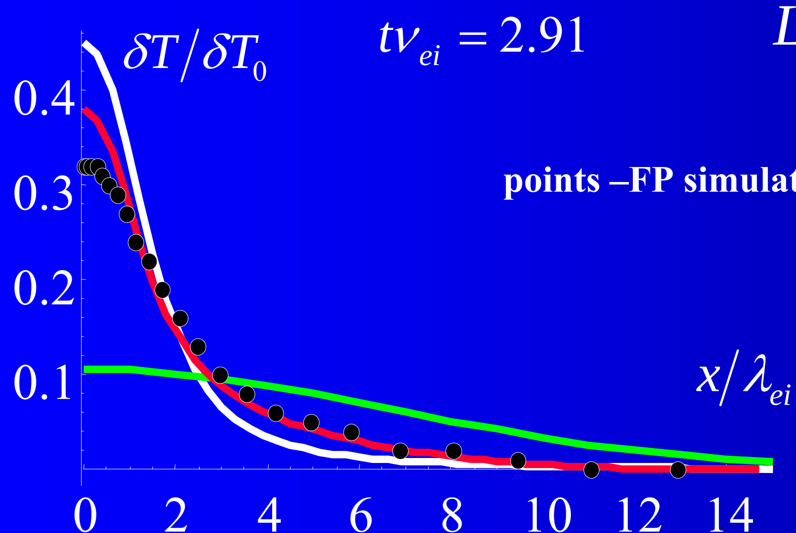
$$L = \lambda_{ei}$$

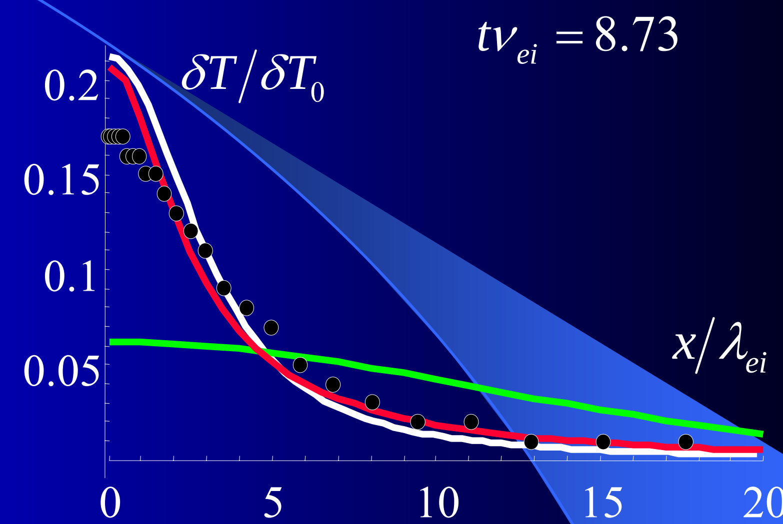


— Classical SH case • non stationary nonlocal theory — quasistationary approximation

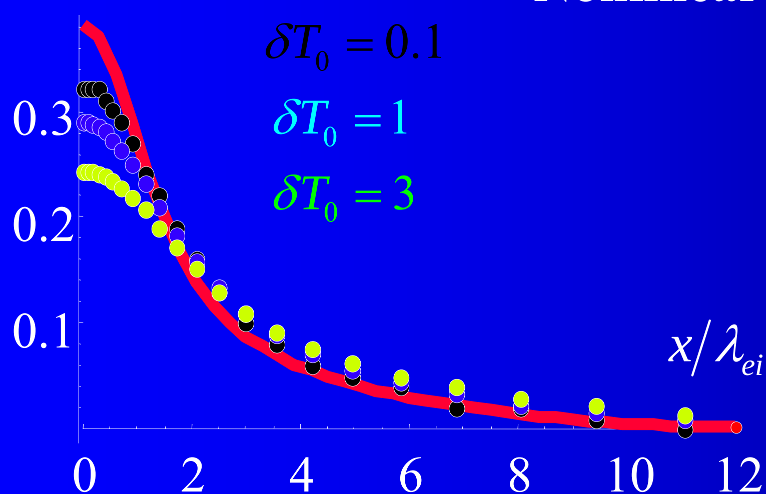
Hot spot relaxation for small perturbation: Comparison with Fokker Plank simulation

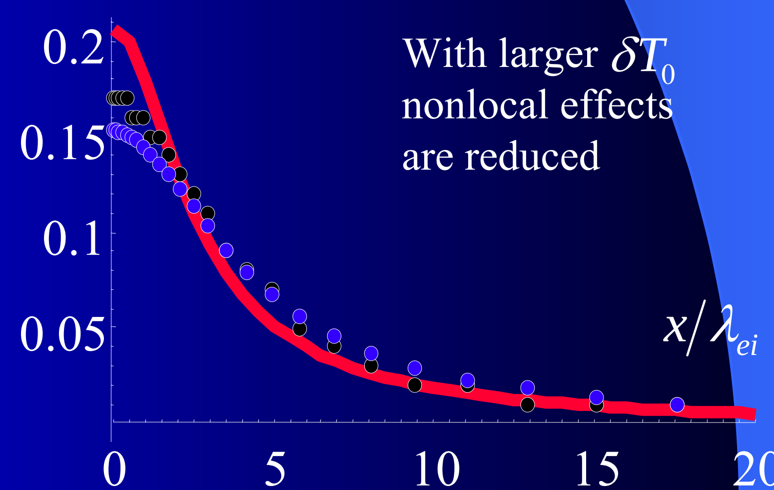
$L = \lambda_{ei}$





Nonlinear effect





Nonlocal nonlinear model

$$q_{SH} = -\kappa_{SH}\frac{\partial T_e}{\partial x} \qquad \kappa_{SH} = \frac{128}{3\pi}\gamma_z n_e v_{Te}\lambda_{ei} \qquad \gamma_z = \frac{0.24+Z}{4.2+Z}$$

$$q(x) = \int_{-\infty}^{+\infty} q_{SH}(x')G(x,x',T(x'))dx'$$

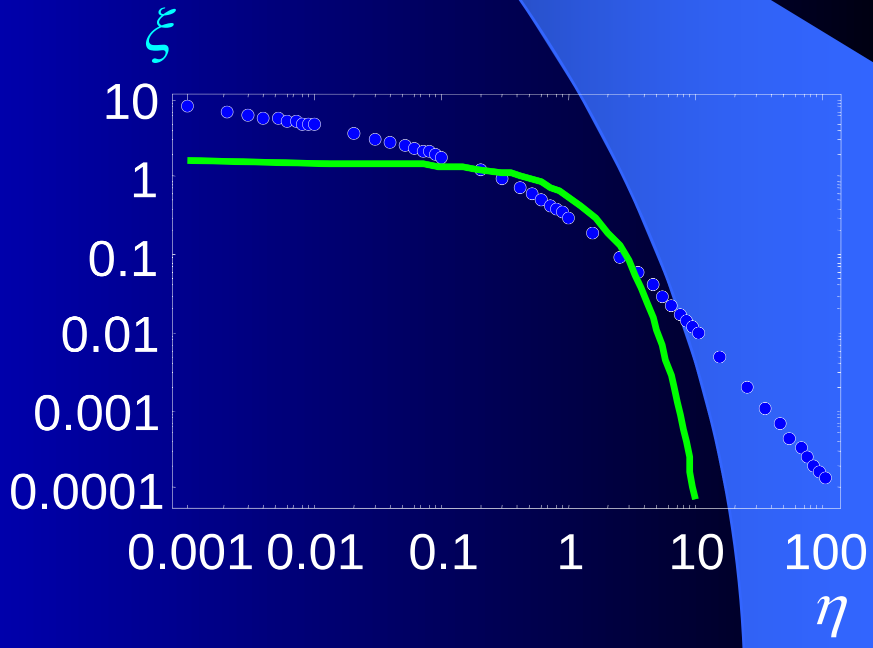
$$G^{LM}(x,x') = \frac{\exp(-\eta_e)}{2a\lambda_e(x')} \qquad a = 30$$

Luciani-Mora (LM) nonlocal model, PRL 51,1664, 1983

$$\lambda_e = \frac{T_e^2}{4\pi\sqrt{Z+1}n_e e^4\Lambda} \qquad \eta_{(e,ei)} = \frac{|x-x'|}{a\lambda_{(e,ei)}}$$

$$G(x,x') = \frac{\xi(\eta)}{\pi a\lambda_{ei}} \qquad \xi = \int_0^\infty \frac{dp\cos\eta_{ei}p}{1+p^{0.9}}$$

$$\lambda_{ei} = \frac{3T_e^2}{4\sqrt{2\pi}Zn_e e^4\Lambda} \qquad a = 10\sqrt{Z}\frac{Z+5}{Z+12}$$



PRL 75,4405, 1995

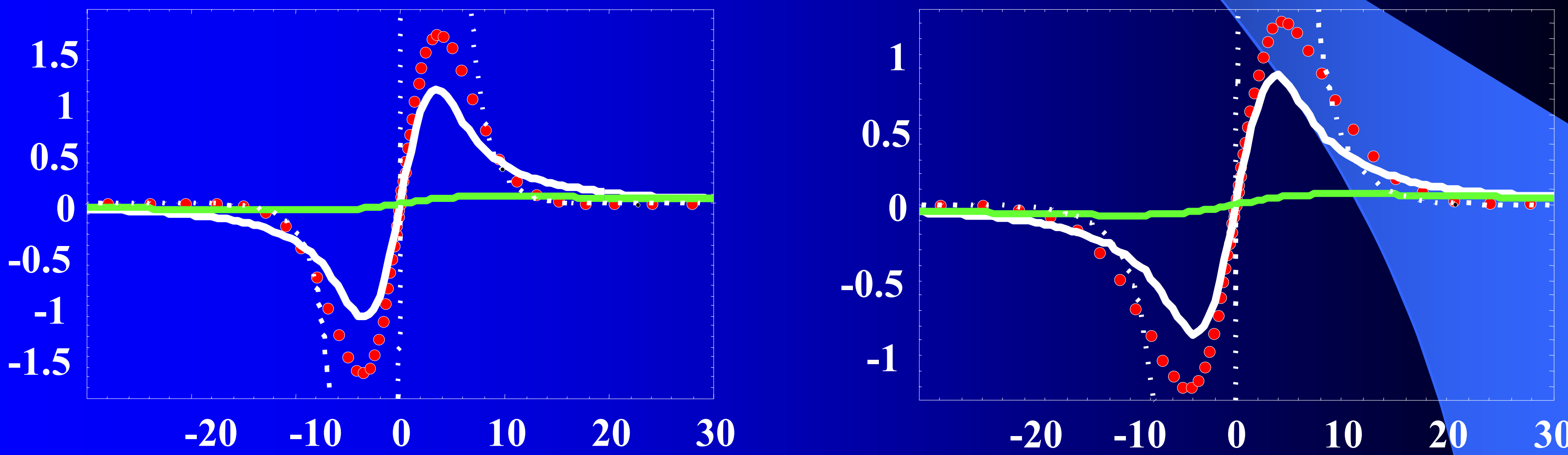
Hot spot relaxation

Initial temperature profile $T_e(x,t=0) = T_0 + T_1 \exp\left(-x^2/L^2\right)$

$$\frac{\partial f_e}{\partial t} + \mathrm{v}_x \frac{\partial f_e}{\partial x} - \frac{e}{m} E \frac{\partial f_e}{\partial \mathrm{v}_x} = C_{ee}\left(f_e, f_e\right) + C_{ei}\left(f_e\right)$$

$$\frac{\partial T}{\partial t} + \frac{2}{3n_e}\frac{\partial}{\partial x} q(x) = 0$$

Heat flux profile for $L/\lambda_{ei} = 3$



$t\,\nu_{ei} = 1.45$ $t\,\nu_{ei} = 2.91$

------ **SH theory**

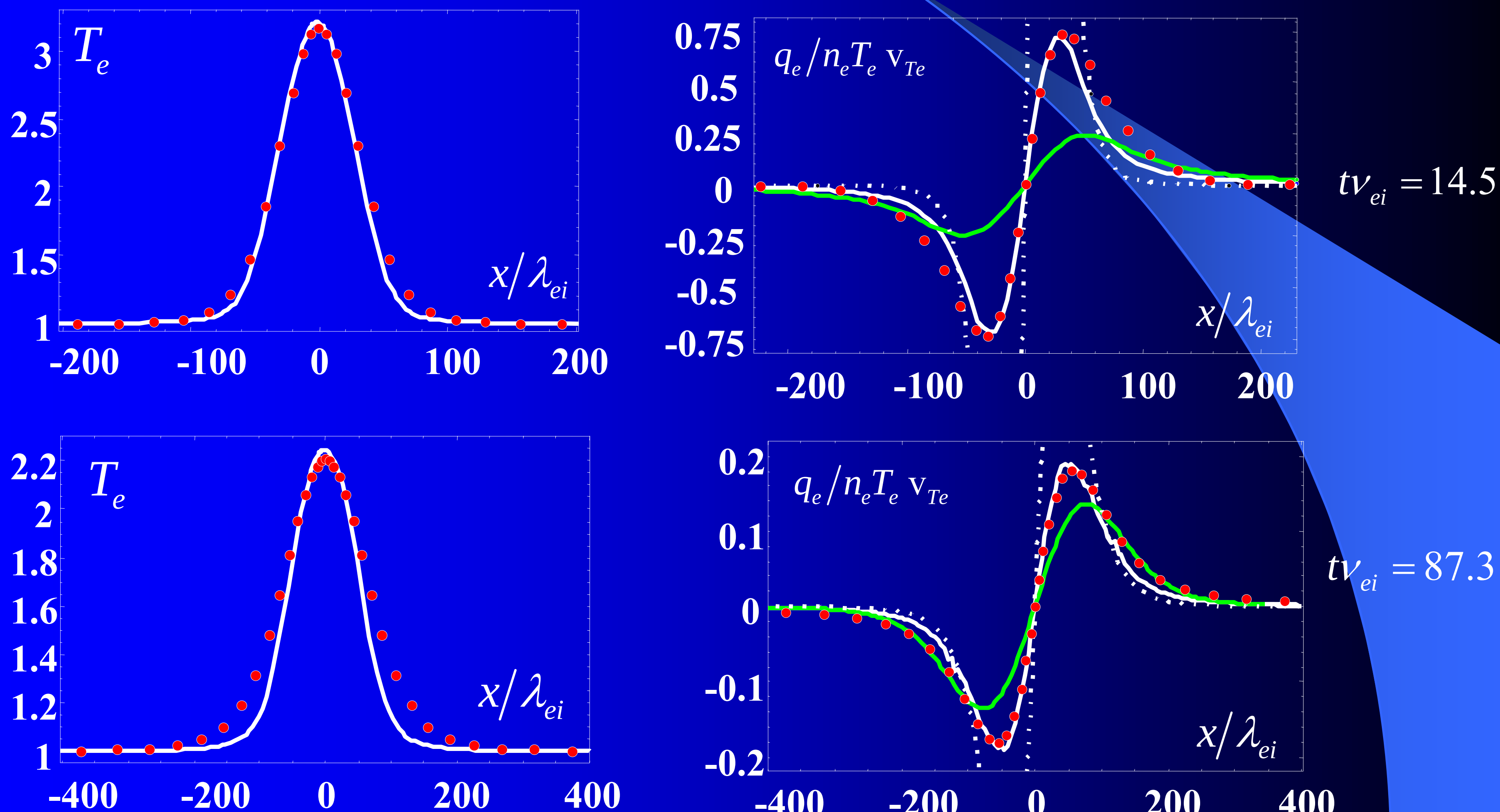
• • • • **FP simulation**

—— **our nonlocal model**

—— **LM nonlocal model**

Hot spot relaxation : comparison nonlocal model with FP simulation.

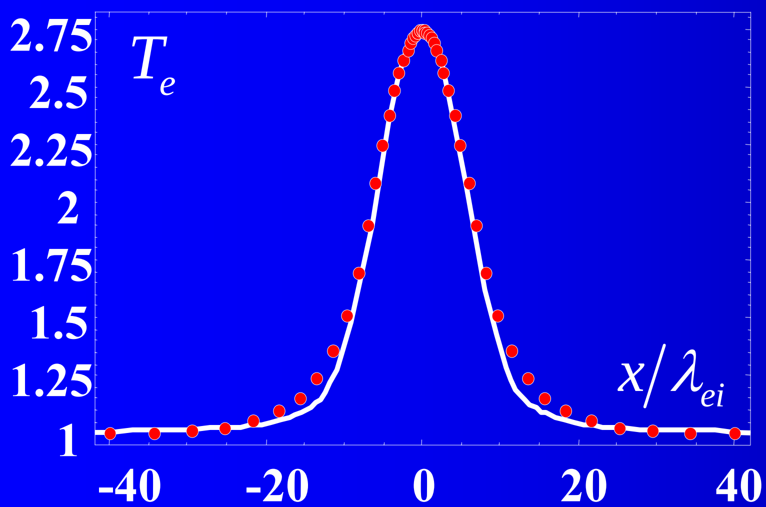
Temperature profile for $L/\lambda_{ei} = 30$ **Heat flux profile for** $L/\lambda_{ei} = 30$

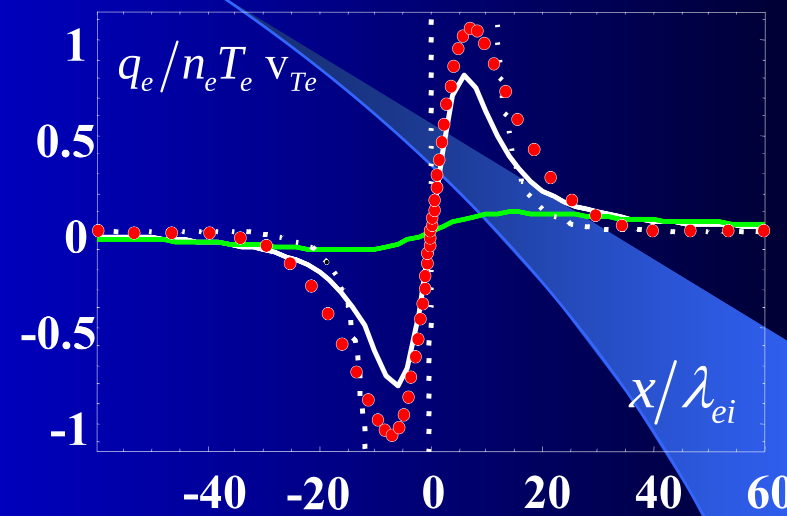


Hot spot relaxation : comparison nonlocal model with FP simulation.

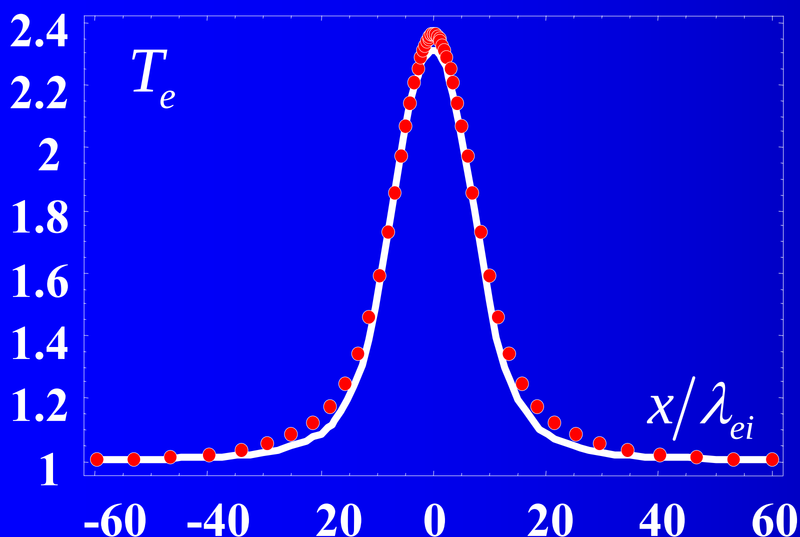
Temperature profile for $L/\lambda_{ei} = 5$

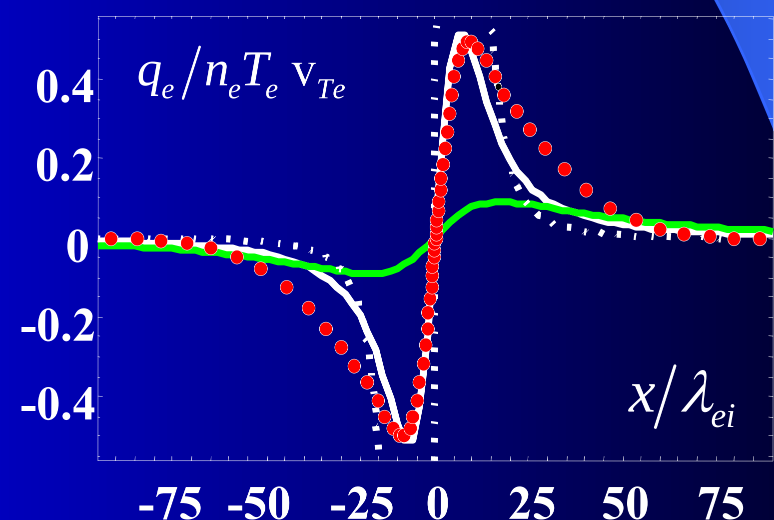
Heat flux profile for $L/\lambda_{ei} = 5$





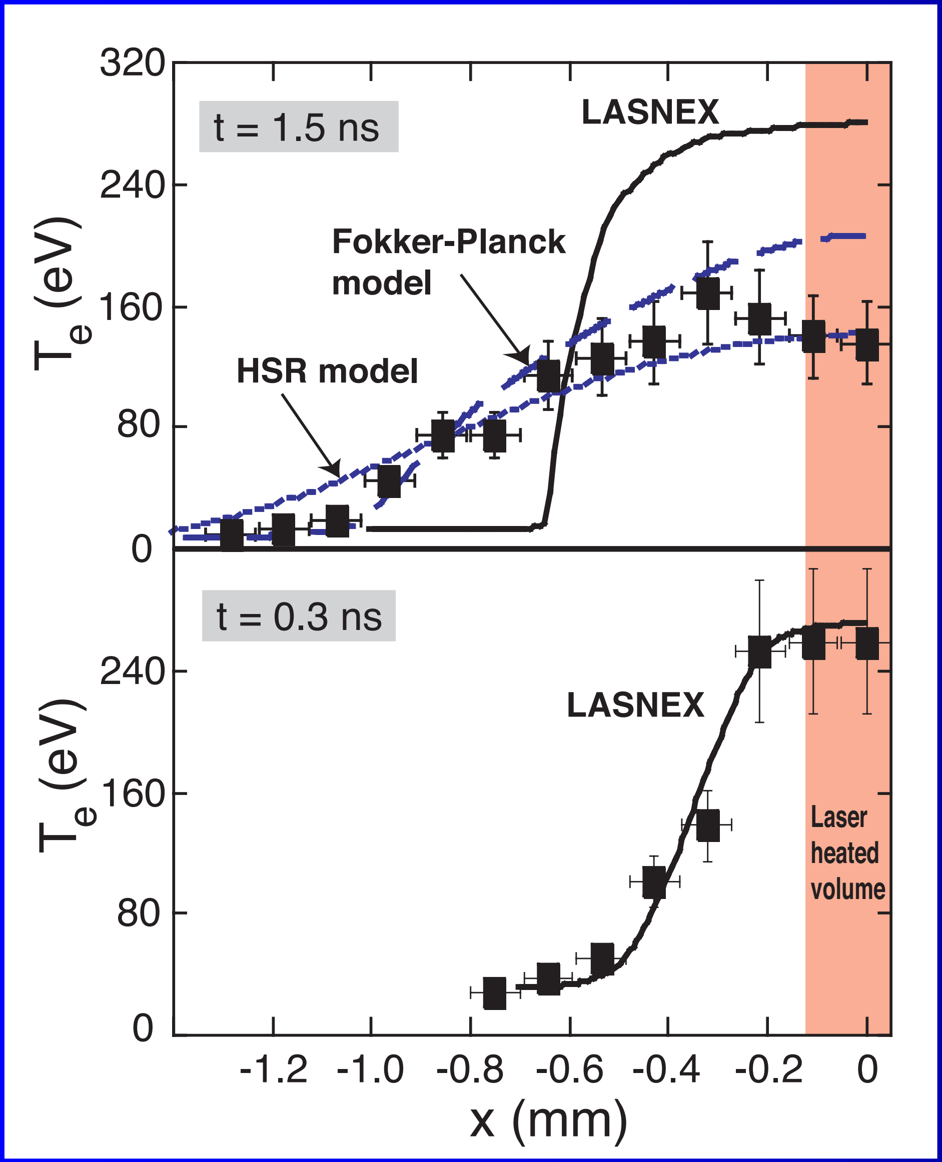
$t\nu_{ei} = 3.65$





$t\nu_{ei} = 7.3$

Hot spot relaxation : comparison with experiment.



Phys. Rev. Letters 92, 205006 (2004)

HSR - nonlinear nonlocal model for the hot spot relaxation problem

Heat wave propagation : theoretical model

$$\frac{\partial}{\partial t}\frac{3}{2}n_e T_e + \frac{\partial}{\partial x}q(x) = 0$$

$$\frac{\partial}{\partial t}\frac{\varepsilon \boldsymbol{E}^2 + \boldsymbol{B}^2}{8\pi} + \frac{c}{4\pi}\frac{\partial}{\partial x}[\boldsymbol{EB}] = 0$$

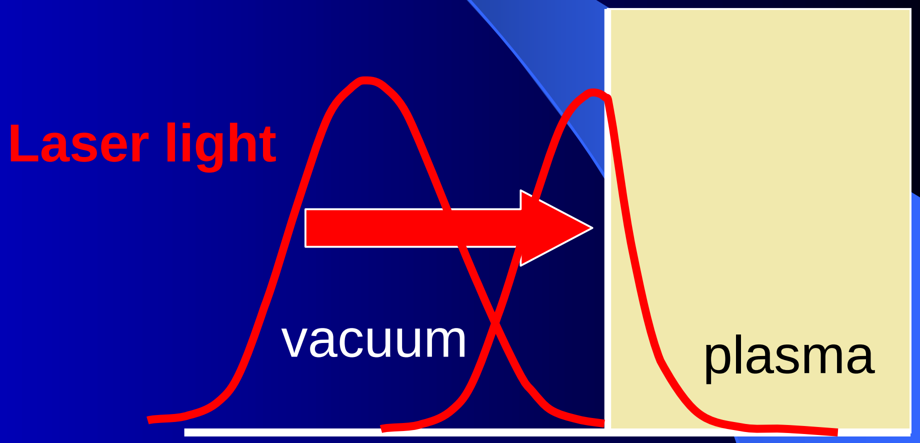
$$\frac{\partial T}{\partial t} + \frac{2}{3n_e}\frac{\partial}{\partial x}\left(q(x) + I(x)\right) = 0$$

$$E_z(x) = E_z(0)\exp\left(-\frac{x}{\lambda_{sk}}\right)$$

λ_{sk} - skin depth

$$I(x) = c\frac{\left|E(x)\right|^2}{8\pi} = I_0 \exp\left(-\frac{2x}{\lambda_{sk}}\right)$$

$$\frac{\partial T}{\partial t} + \frac{2}{3n_e}\frac{\partial}{\partial x}q(x) = \frac{4I_0}{3n_e\lambda_{sk}}\exp\left(-\frac{2x}{\lambda_{sk}}\right)$$



Nonlocal, frequency dependent skin effect

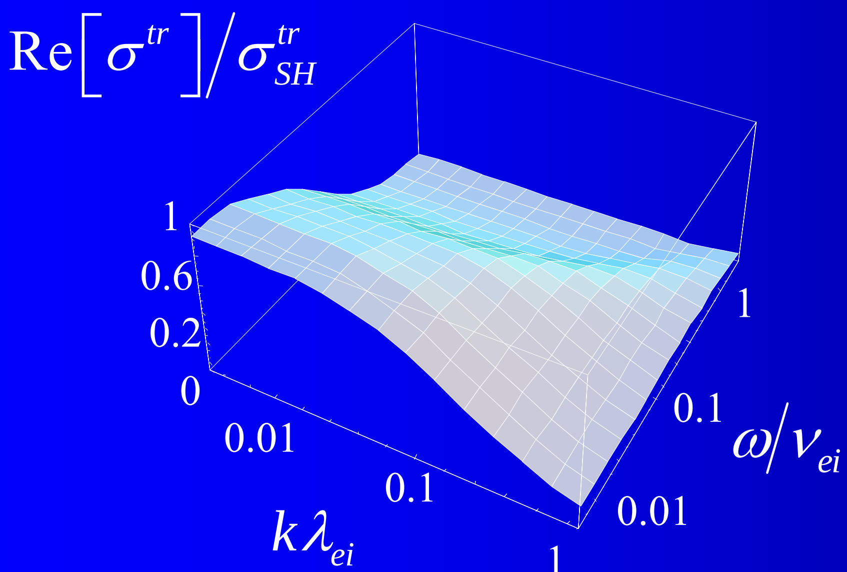
$$\lambda_{sk} = 1/\,\mathrm{Im}[k] \qquad k^2 = \frac{\omega^2}{c^2}\varepsilon^{tr}(k,\omega) \qquad \varepsilon^{tr} = 1 + i\frac{4\pi\sigma^{tr}}{\omega}$$

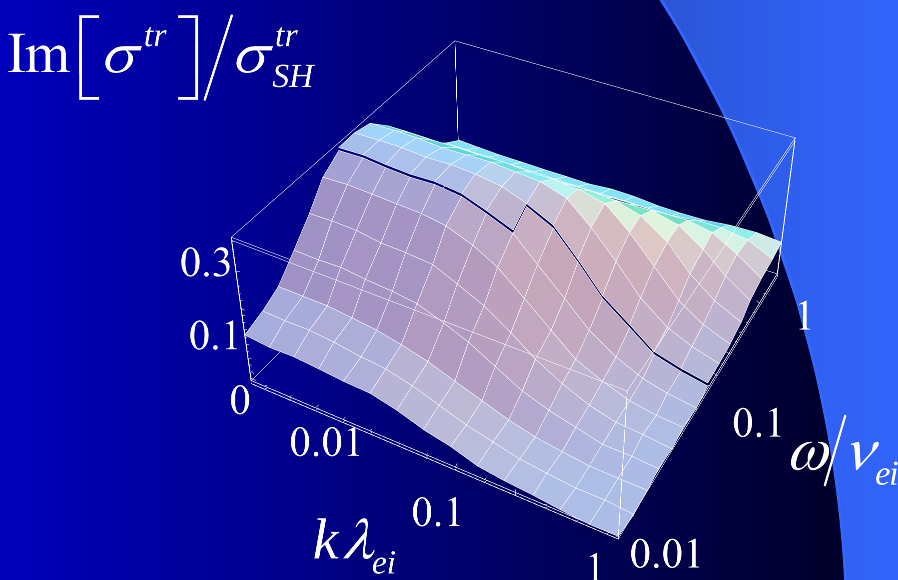
Exact solution (JETP v. 84, p.716, 1996, Phys. Plasmas v.4, p.4205, 1997)

$$\sigma^{tr} = \frac{4\pi e^2}{3T_e}\int dv v^4 \frac{F_M}{\nu_{ei} h_1}$$

$$\sigma_{SH}^{tr} = \frac{32}{3\pi}\frac{n_e e^2}{m_e \nu_{ei}^T}$$

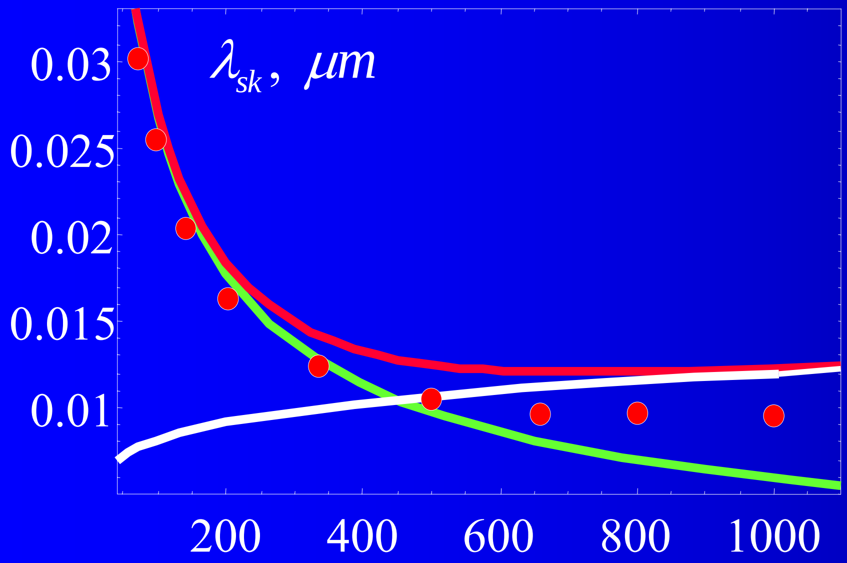
$$h_l = -i\frac{\omega}{\nu_{ei}^F} + \frac{1}{2}l(l+1) + \frac{(l+1)^2 - 1}{4(l+1)^2 - 1}\left(\frac{kv_{Te}}{\nu_{ei}^F}\right)^2 \frac{1}{h_{l+1}}$$



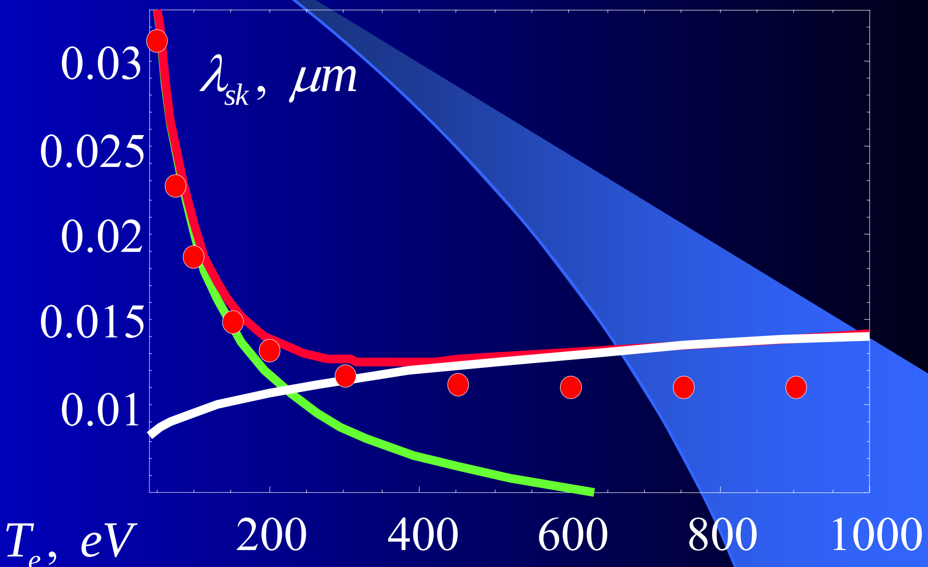


Nonlocal, frequency dependent skin effect

Skin depth for Aluminum Z=Z(T), n_i=6 10^{22}) for 1μm laser



Skin depth for Beryllium (Z=4, n_i=12 10^{22}) for 1μm laser



- **Nonlocal, frequency dependent skin-effect**

Nonlocal skin-effect in low frequency limit smooth transition from normal to anomalous skin-effects

$$\lambda_{sk} = 2\left(\sqrt{\frac{2}{\pi}}\frac{c^2 v_{Te}}{\omega_{pe}^2 \omega}\right)^{1/3}$$ **Anomalous skin effect**

$$\lambda_{sk} = \left(\frac{3\pi\nu_{ei}c^2}{16\omega_{pe}^2\omega}\right)^{1/2}$$ Normal skin effect

Heat wave propagation : nonlocal theory for small perturbation

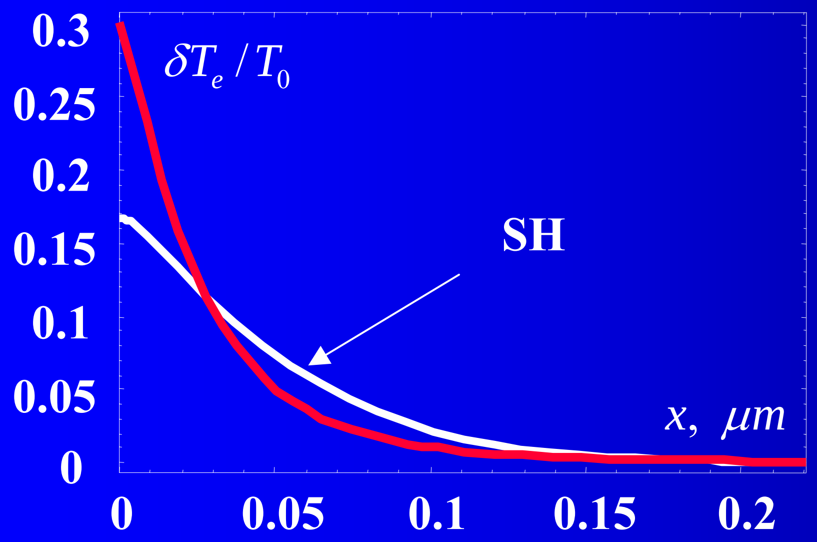
$$\frac{\partial \delta T}{\partial t} + \frac{2}{3n_e} k^2 \kappa \delta T = \frac{16 I_0}{3n_e \left(4 + k^2 \lambda_{sk}^2\right)}$$

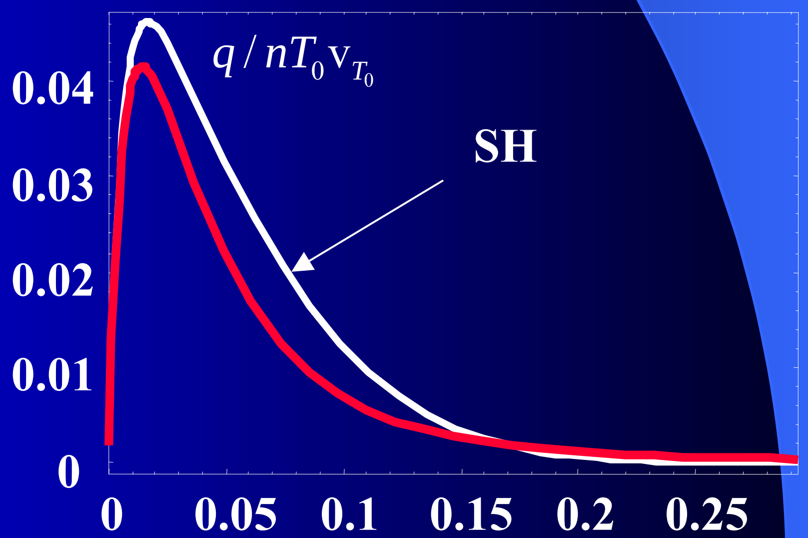
Equation for small temperature perturbation in Fourier space

$$\kappa_k = \frac{\kappa_{SH}}{1 + \left(a\, k \lambda_{ei}\right)^{0.9}}$$

$$\delta T_e = \frac{4 I_0}{\pi} \int_{-\infty}^{+\infty} \frac{dk \exp\left(-ikx\right)}{k^2 \kappa_k (4 + k^2 \lambda_{sk}^2)} \left(1 - \exp\left(-\frac{2k^2 \kappa_k}{3n_e} t\right)\right)$$

Temperature and heat flux profile at time t= 100 ν_{ei}^{-1} (30 fs) for 1 μm laser wavelength, Z=4 (Be), I_0 =10^{15} W/cm^2, T=300eV.

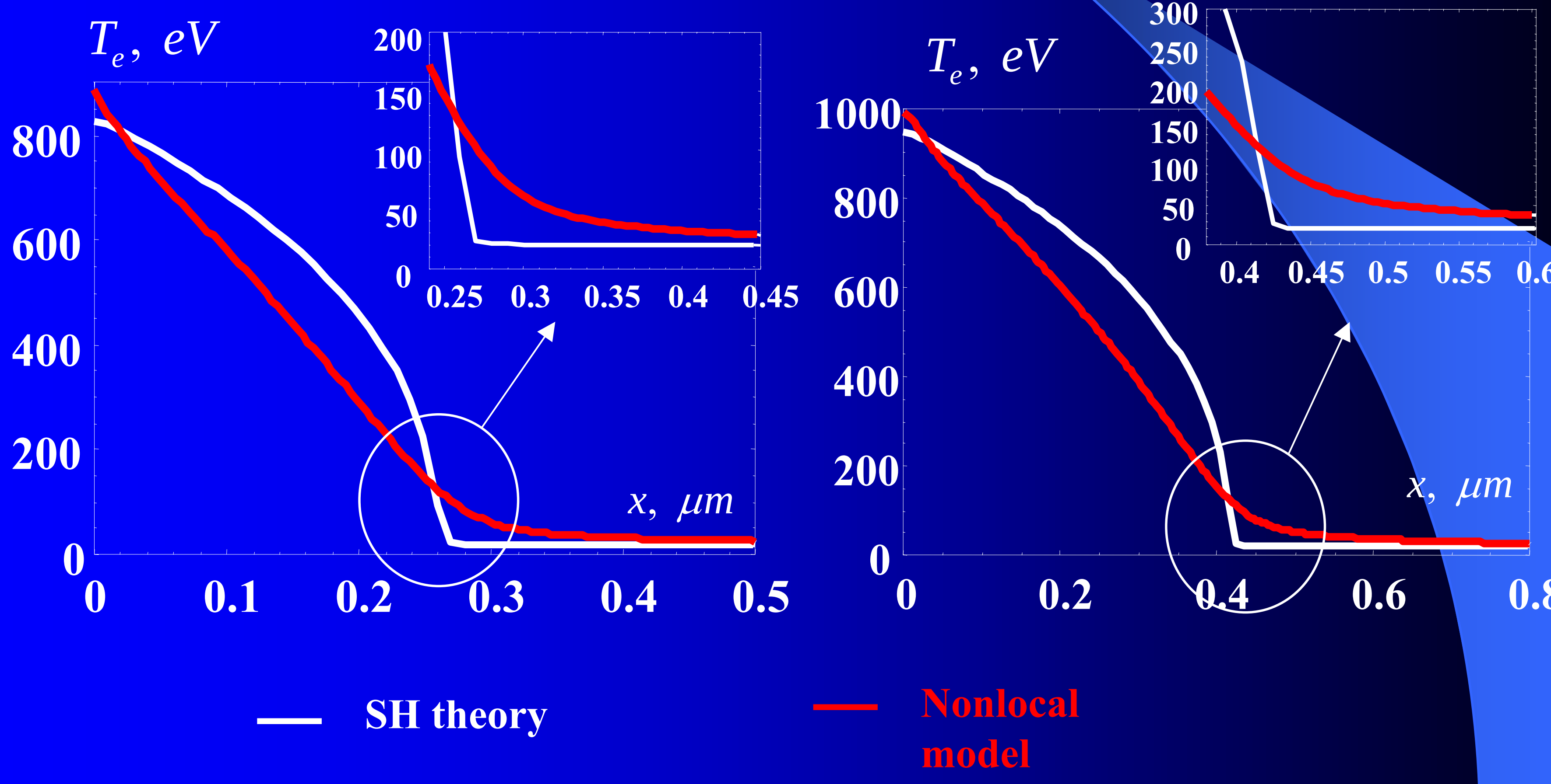




Propagation of nonlocal nonlinear heat wave

Temperature profile for 1 μm laser wavelength, Z=4 (Be), $I_0 = 10^{16}$ W/cm^2 at time

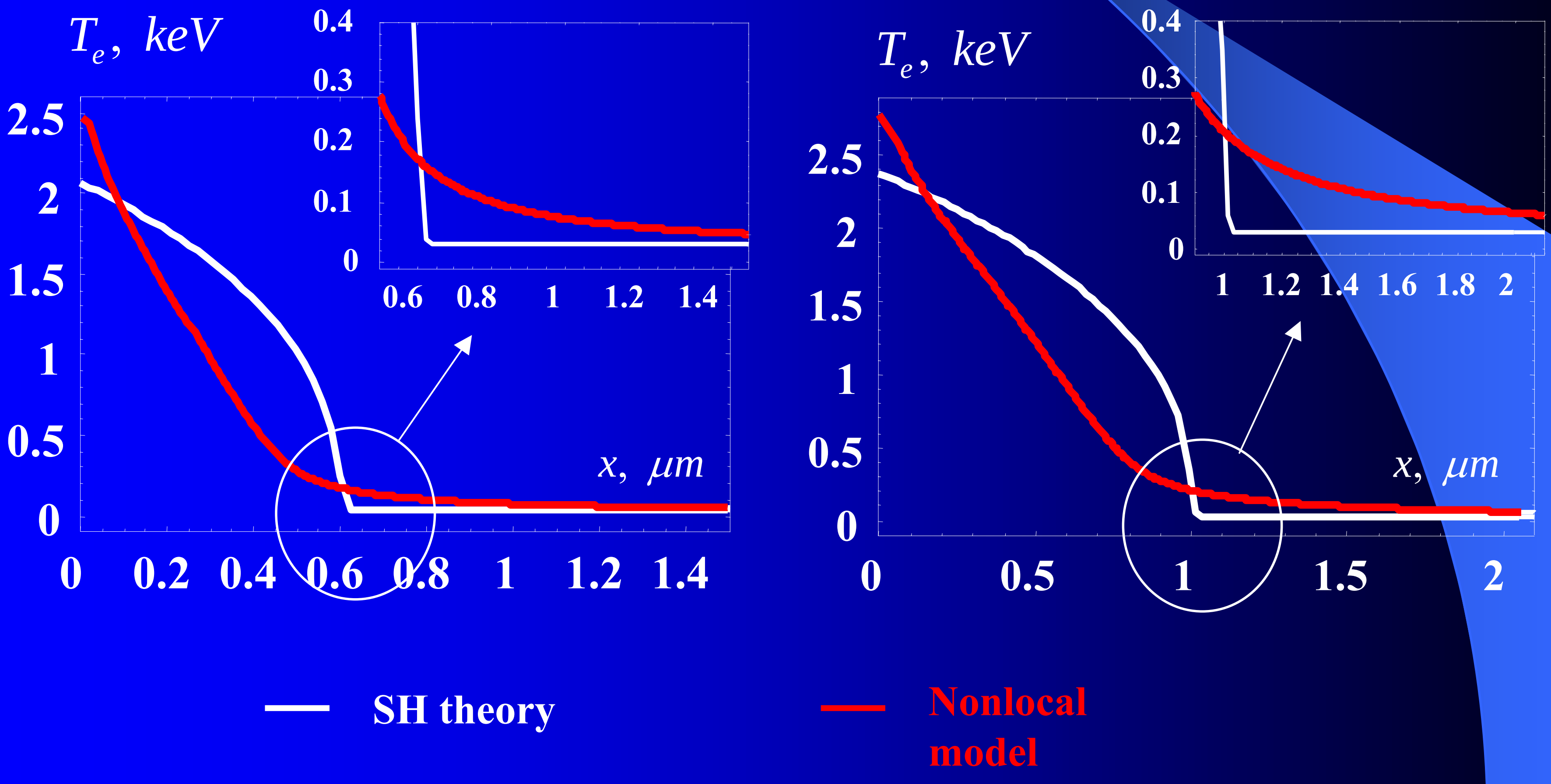
t=0.12 psec and t=0.24 psec



Propagation of nonlocal nonlinear heat wave

Temperature profile for 1 μm laser wavelength, Z=4 (Be), I_0 =10^{17} W/cm^2 at time

t=0.08 psec and t=0.16 psec



Conclusions

Theory of nonlocal nonstationary transport for small perturbations has been developed.

The electron plasma permittivity has been calculated for the entire range of frequencies and wave numbers and for arbitrary particle collisionality.

Different time scales have been identified in temperature relaxation kinetic problem.

Practical, easy to use, formula for the nonlocal thermal conductivity was proposed, was tested in the FP simulation and was compared to the experimental results of temperature profiles.

Heat wave penetration into overdense plasma was studied. Its profile demonstrates different shape compared to the classical theory. The heat flux inhibition is responsible for overheating the plasma near the plasma surface.