Anomalous Skin Effect Revisited

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Motivation

- **A scientific theory should be as simple as possible, but no simpler. – Albert Einstein**
- **How to explain “simply” anomalous skin effect without abusing physics.**

Outline

- **Skin effect** (Inductively Coupled Plasmas/ Lasers)
  - Normal skin effect
  - Concept of phase-mixing and scale V_T / ω
  - Anomalous skin effect
  - Features of the electric field profile $\int_0^\infty E dx \to 0$
  - Redefinition of penetration width
  - Separating the electric field into an exponent and a tail
  - The Anomalous skin effect in a plasma with a highly anisotropic EVDF

- **Capacitive Sheath** (Capacitive Coupled Plasmas/ Lasers)
  - Landau's linear solution
  - Nonlinear solution

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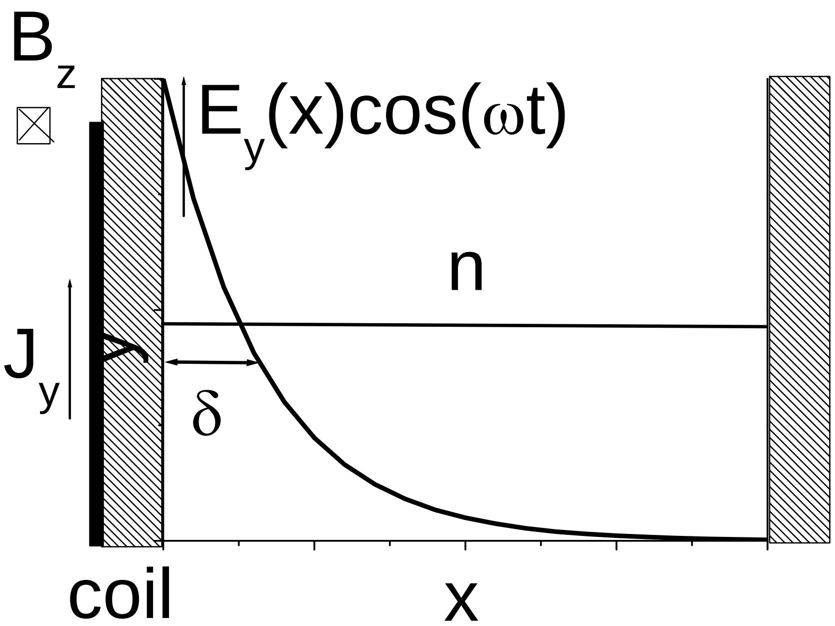
Normal Skin Effect (1/2)

Schematic of skin effect for $\nu \ll \omega \ll \omega_p$



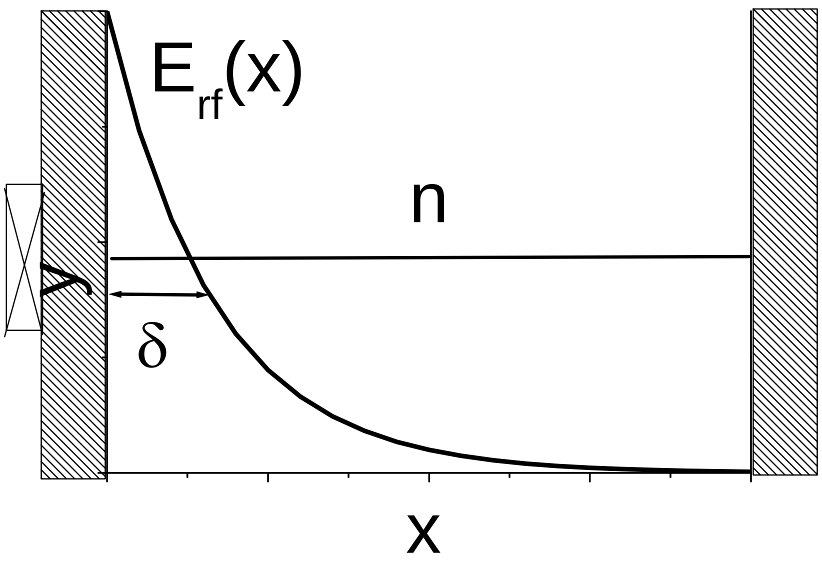
$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{j}$$

$$\vec{E} = E_y(x)e^{-i\omega t} \quad \vec{j} = j_y(x)e^{-i\omega t}$$

$$\frac{d^2 E_y}{dx^2} + \frac{\omega^2}{c^2}E_y = -\frac{4\pi i\omega}{c^2} j_y$$

Normal Skin Effect (2/2)



$$\nu << \omega << \omega_p$$

$$\frac{d^2E_y}{dx^2} + \frac{\omega^2}{c^2}E_y = -\frac{4\pi i\omega}{c^2} j_y$$

$$-i\omega m v_y = -eE_y$$

$$j_y = -en_e v_y = \frac{e^2 n_e}{m}\frac{E_y}{-i\omega}$$

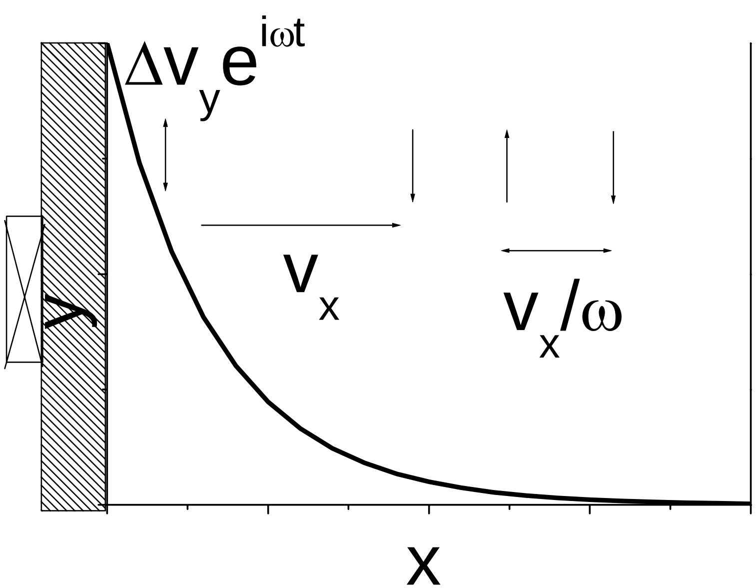
$$\frac{d^2E_y}{dx^2} + \cancelto{0}{\frac{\omega^2}{c^2}E_y} = \frac{4\pi e^2 n}{mc^2}E_y$$

$$E_y = E_{y0}e^{-x/\delta_0}$$

$$\delta_0 = c/\omega_p$$

Anomalous Skin Effect

- **Normal skin: electron thermal velocity is neglected.**
- **Anomalous skin: electrons transport velocity kicks and rf current inside bulk of the plasma.**



$$j_y = en\Delta v_y \cos(\omega x / v_x - \omega t)$$

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$$V_T / \omega$$

$$\int_0^\infty E dx \to 0$$

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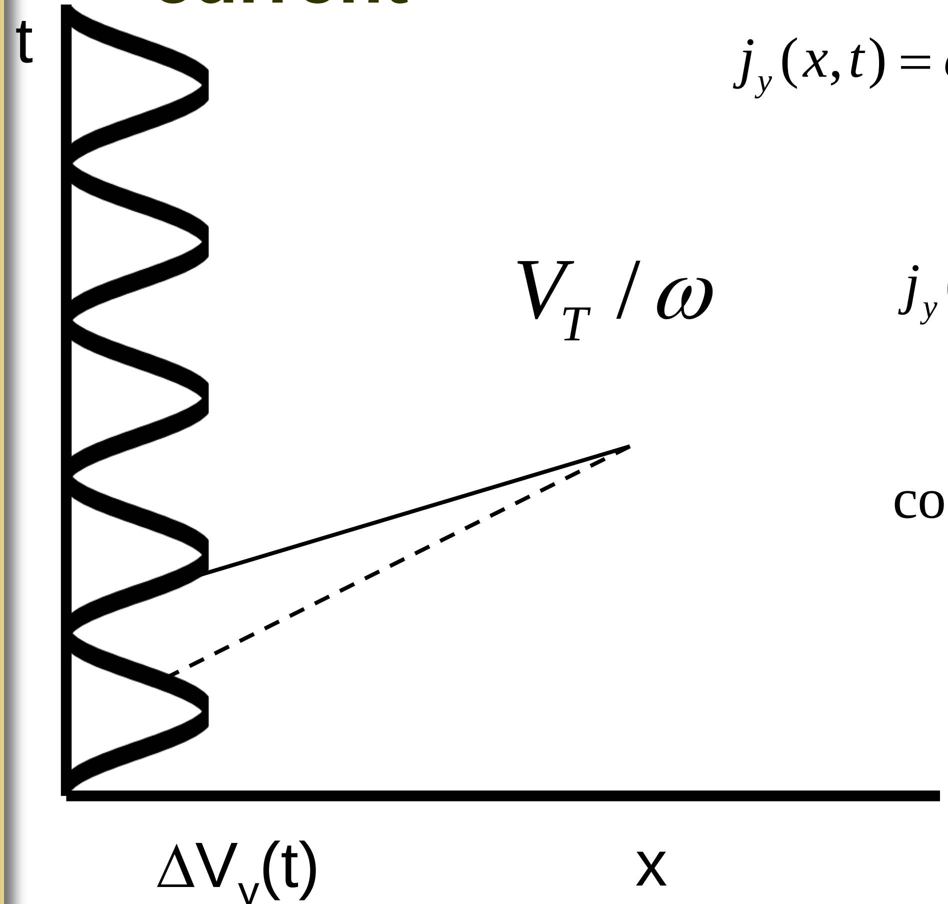
Phase-Mixing

- **Electrons with different v_x phase-mix the current**



$$j_y(x,t) = en\Delta v_y \,\mathrm{Re}\left\{ \int_0^\infty \frac{dv_x}{v_T\sqrt{\pi}} e^{-i\omega x/v_x - i\omega t - v_x^2/v_T^2} \right\}$$

$$j_y(x,t) = \frac{en\Delta v_y}{\sqrt{3}} \exp\left[-\left(\frac{x}{1.1 v_T/\omega} \right)^{2/3} \right] \times$$

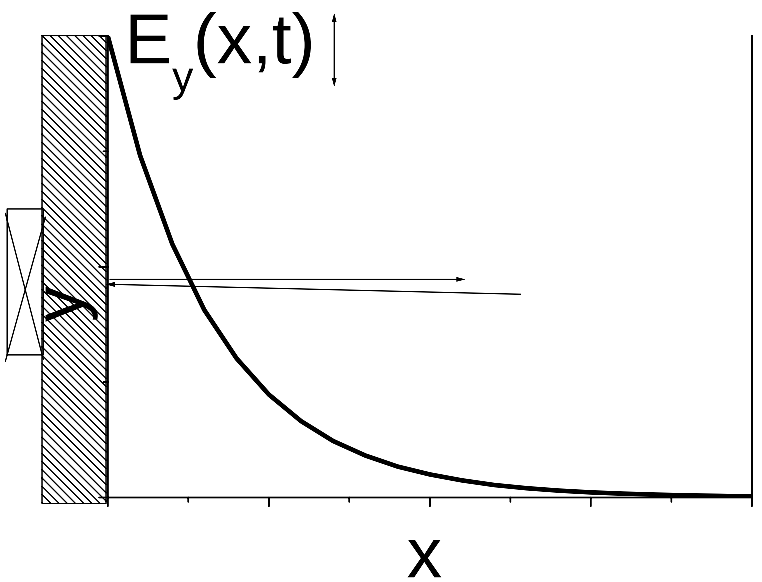
$$\cos\left[\omega t - \left(\frac{x}{0.48 v_T/\omega} \right)^{2/3} \right]$$

Nonlocal Conductivity

Electron velocity is an integral over electric field profile

$$\frac{d}{dt} v_y = -\frac{e}{m} E_y$$

$$v_y = \frac{e}{m} \int_{-\infty}^{t} E_y[x(\tau), \tau] d\tau$$



$$J_y(x) = \frac{e^2 n_{e0}}{m} \left(\int_0^x G(x, x') E_y(x') dx' + \int_x^L G(x', x) E_y(x') dx' \right)$$

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Anomalous Skin Effect, limit $\frac{v_T}{\omega} \gg \frac{c}{\omega_p}$

Estimate for skin layer width

$$\frac{d^2 E_y}{dx^2} = -\frac{4\pi i\omega}{c^2} j_y \qquad j_y = \frac{e^2 n}{m}\left(\int_{-\infty}^{0} dv_x f(v_x) \int_x^{\infty} \frac{dx'}{v_x} E(x') + \ldots\right)$$

$$j_y \sim \frac{e^2 n\delta E}{m v_T} \qquad \frac{E_y}{\delta^2} \sim \frac{4\pi\omega e^2 n\delta E}{c^2 m v_T}$$

$$\delta_a \sim \left(\frac{c^2}{\omega_p^2}\frac{v_T}{\omega}\right)^{1/3} \qquad \frac{\delta_a}{\delta_0} \sim \left(\frac{v_T}{\omega}\frac{\omega_p}{c}\right)^{1/3}$$

Skin layer width does not increase much ~2!

Why in the limit $\frac{v_T}{\omega} \gg \frac{c}{\omega_p}$ $\delta_a \ll \frac{v_T}{\omega}$?

Electric field is tied to the current and "wants" to spread to V_T / ω **=>**

$$\frac{d^2 E_y}{dx^2} = -\frac{4\pi i\omega}{c^2} j_y$$

However, $\delta_a \ll v_T / \omega$ **i,.e., the main part of the electric field can not propagate on distances of** V_T / ω **.**

What is the solution?

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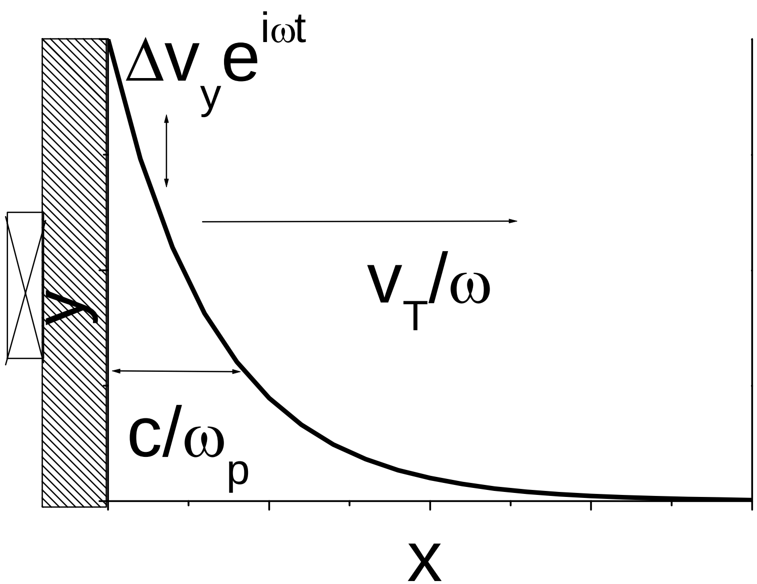
How in the limit $\frac{v_T}{\omega} \gg \frac{c}{\omega_p}$ $\delta_a \ll \frac{v_T}{\omega}$.



Current tends to spread on distances of order $\frac{v_T}{\omega}$

Maxwell's equation give skin layer width $\delta_a \ll \frac{v_T}{\omega}$

How?

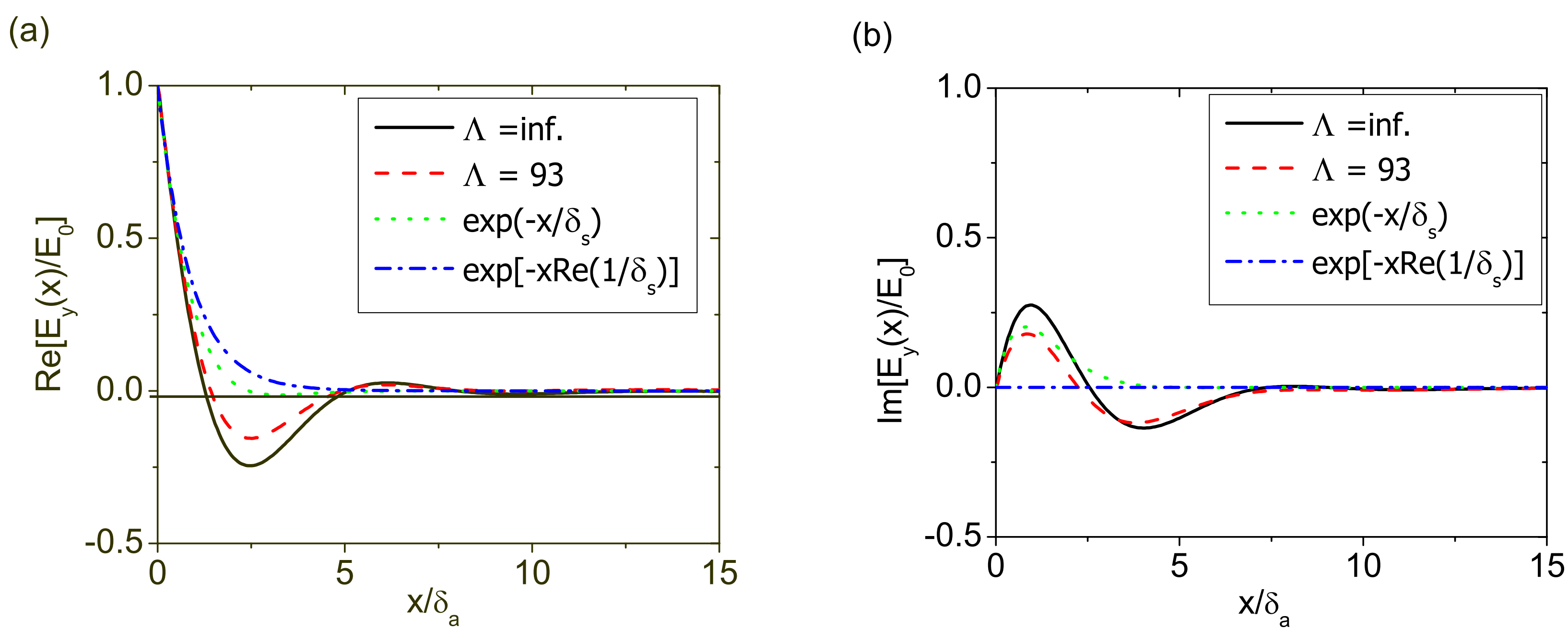
$$j_y = \frac{e^2 m}{m}\left(\int_{-\infty}^{0} dv_x f(v_x) \int_x^{\infty} \frac{dx'}{v_x} E(x') + \ldots\right)$$

$$\int_0^{\infty} E dx \to 0$$

For more info: Y. Aliev, I. Kaganovich, H. Shluter, PoP, 1997.

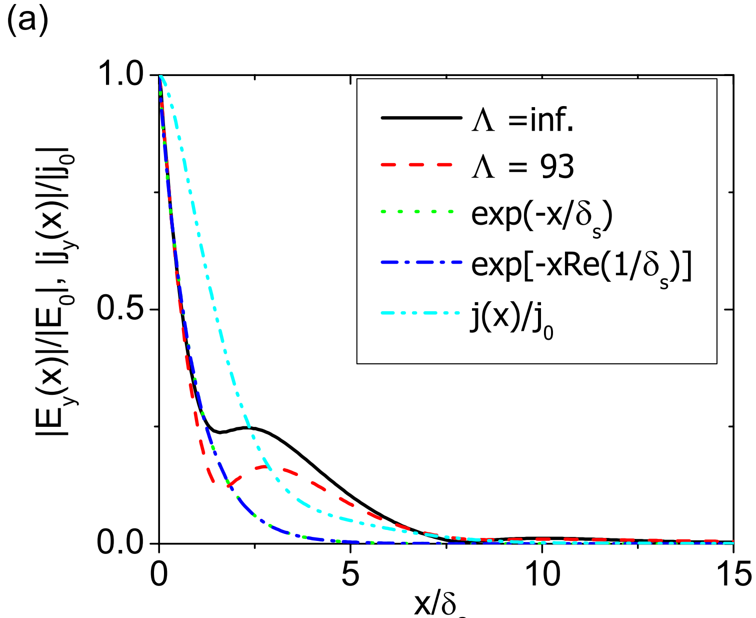
Profile of the Electric Field $\Lambda = \frac{v_T}{\omega}\frac{\omega_p}{c} \gg 1$

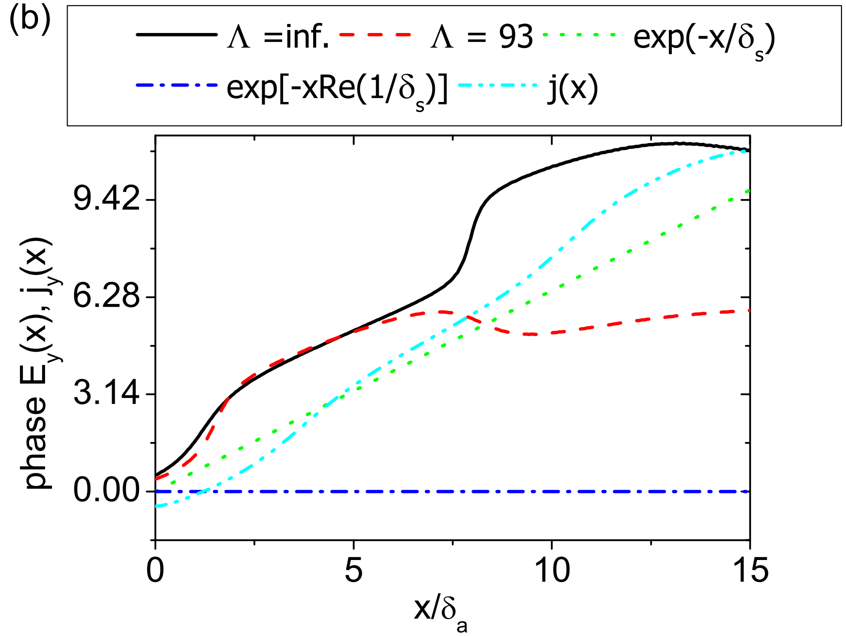
Plot of the rf electric field as a function of the normalized coordinate x/δ_a. The solid curve corresponds to the solution in the limit =∞; dashed line - Λ=93 (plasma parameters $n=10^{11}cm^{-3}$, T_e=3eV, f=1MHz). The dotted and dash-dotted lines show the skin approximation and impedance approximation (a) real, and (b) imaginary part of the electric field.



Profile of the Electric Field $\Lambda = \frac{v_T}{\omega}\frac{\omega_p}{c} \gg 1$

Plot of the rf electric field and *electron current* as a function of the normalized coordinate x/δ_a. The same profiles as before shown are (a) amplitude, and (b) phase with respect to the phase of the electric field generated by the field in vacuum.





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$$V_T / \omega$$

$$\int_0^{\infty} E dx \to 0$$

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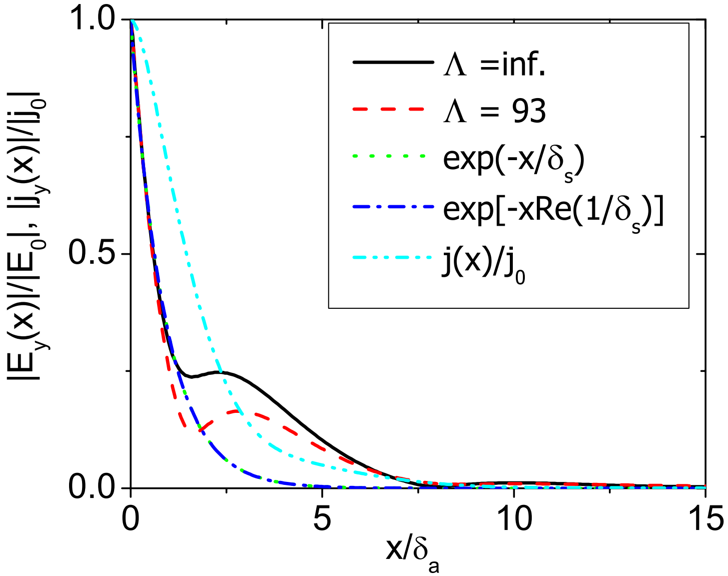
"Definition" of the Penetration Width

$$\lambda_E = \int_0^\infty E dx / E_0 << \delta_a \qquad \int_0^\infty E dx \to 0$$

$$\lambda_j = \int_0^\infty j dx / j_0 \qquad \lambda_E << \lambda_j$$

(a)



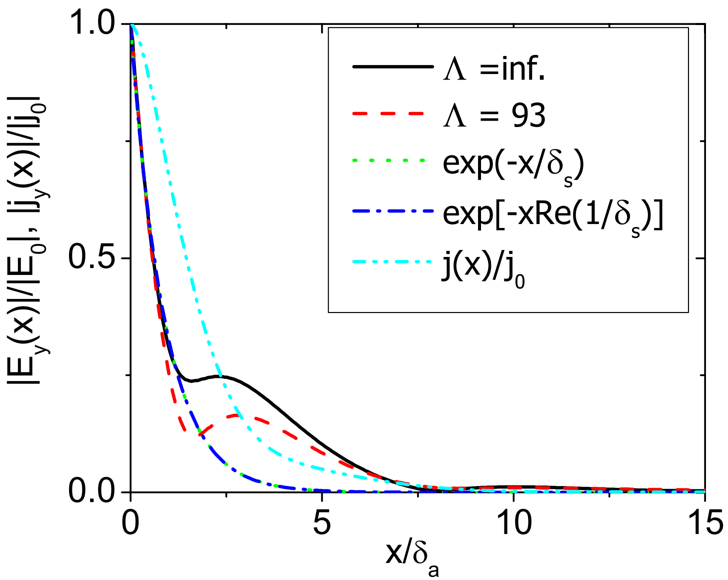
Two books claim that current penetrates much deeper than the electric field?

Redefinition of the Penetration Width

$$\lambda_{|E|} = \int_0^\infty |E|\,dx / |E_0| \qquad \lambda_{|j|} = \int_0^\infty |j|\,dx / |j_0|$$

$$\lambda_{|E|} = 1.64\delta_a \qquad \lambda_{|j|} = 1.78\delta_a$$

(a)



Moral:

"Skepticism is a good guarding dog, if the owner knows when to let it loose." R. Stout

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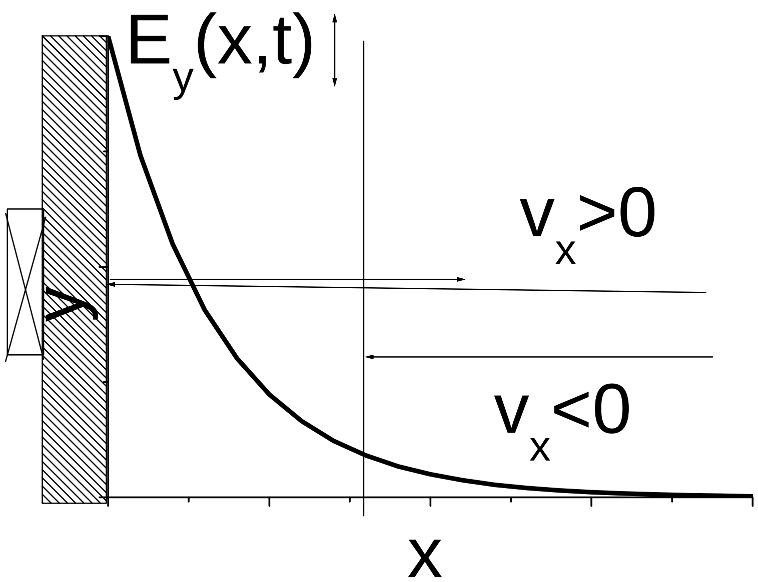
Separating the Electric Field into an Exponent and a Tail (1/3)

- **$E = E_0 \exp(-x/\delta) + \ldots.?$**
- **$J = j_0 \exp(-x/\delta) + \ldots.?$**



$$\Delta v_y = -\frac{e}{m}\int_{-\infty}^{t} d\tau E[x(\tau),\tau]$$

$$v_x < 0 \qquad \Delta v_y = \frac{eE_0}{m}\frac{\exp(-x/\delta - i\omega t)}{v_x/\delta + i\omega}$$

$$v_x > 0$$

Not an exponent, what to do?

Separating the Electric Field into an Exponent and a Tail (2/3)

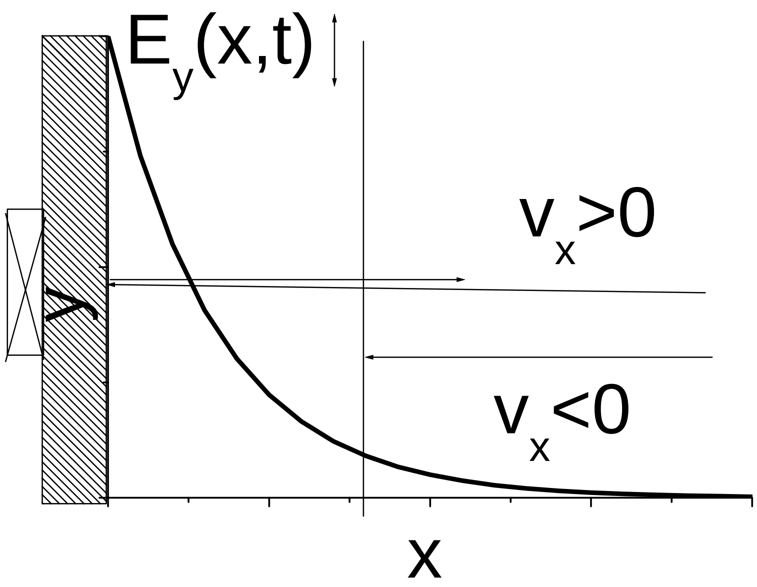
- **$E = E_0 \exp(-x/\delta) + \ldots.?$**
- **$J = j_0 \exp(-x/\delta) + \ldots.?$**

$$v_x > 0$$

$$\Delta v_y = -\frac{e}{m}\int_{-\infty}^{t} d\tau E[x(\tau),\tau]$$

$$\Delta v_y = -\frac{e}{m}\left\{\int_{-\infty}^{\infty} - \int_{t}^{\infty} d\tau E[x(\tau),\tau]\right\}$$

$$\Delta v_y = \Delta v_y^{\infty} e^{-i\omega(t - x/v_x)} + \frac{eE_0}{m}\frac{\exp(-x/\delta - i\omega t)}{v_x/\delta + i\omega}$$



Separating the Electric Field into an Exponent and a Tail (3/3)

$$E_y(x) = \frac{2i\omega I}{c^2} \int_{-\infty}^{\infty} \frac{e^{ikx}}{k^2 - \omega^2 \varepsilon_t(\omega, |k|)/c^2}$$

Typical error loose |k|

$$\int_{-\infty}^{\infty} dk \frac{e^{ikx}}{D(|k|)} = \int_{-\infty}^{\infty} dk \frac{e^{ikx}}{D(-k)} + \int_{0}^{\infty} dk e^{ikx} \left(\frac{1}{D(k)} - \frac{1}{D(-k)} \right)$$

$$D(-k_p) = 0 \qquad \frac{e^{ik_p x}}{D'(-k_p)}$$

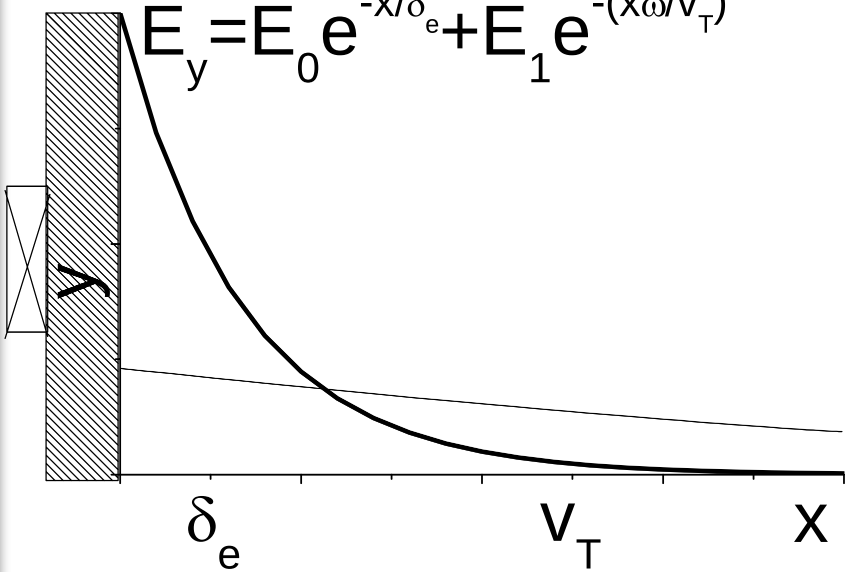
Non-exponential part

Exponential Part of the Electric Field Profile

$$\delta_e^{-2} = \frac{\omega_p^2}{c^2} f\left(\frac{\omega \delta_e}{v_T}\right)$$

$$f(s) = isZ(is)^* = s\sqrt{\pi} e^{s^2} erfc(s) \rightarrow \begin{cases} \sqrt{\pi} s & s \ll 1 \\ 1 & s \gg 1 \end{cases}$$

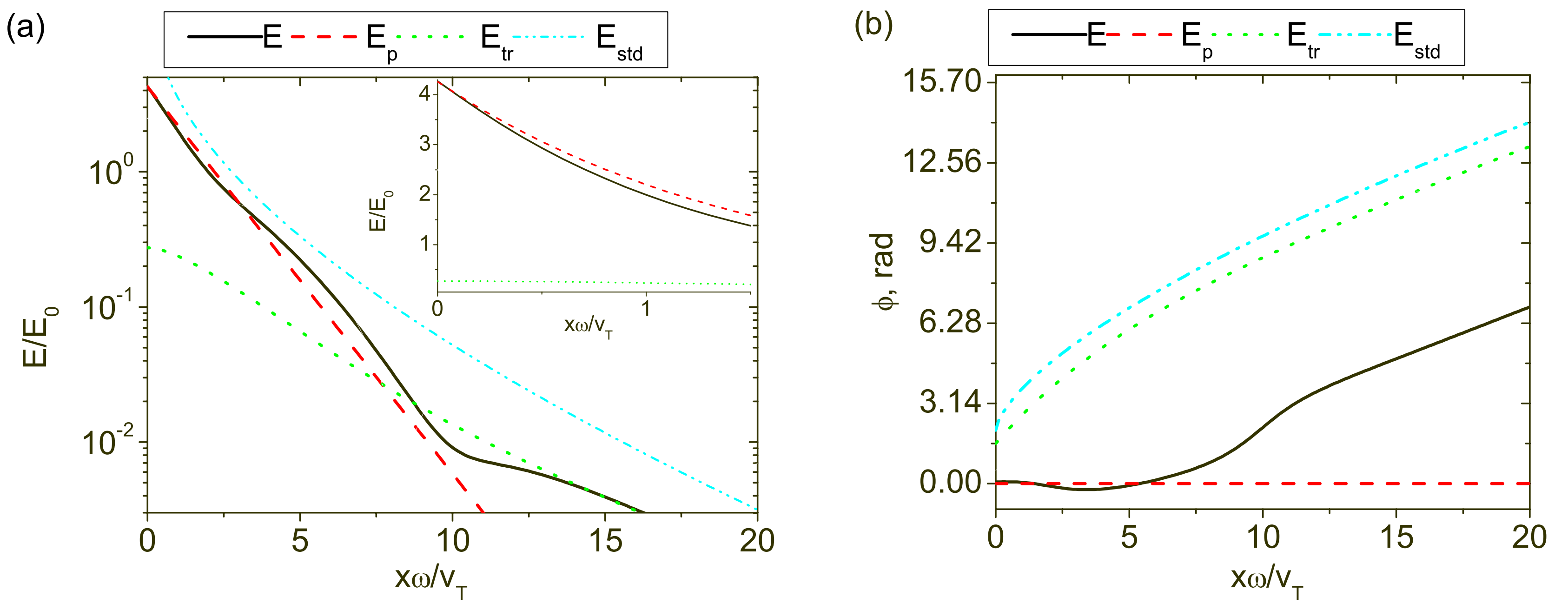
$$E_y = E_0 e^{-x/\delta_e} + E_1 e^{-(x\omega/v_T)^{2/3}}$$



For more info: I. Kaganovich, et al, submitted to IEEE Trans. on Plasma Sci., 2005.

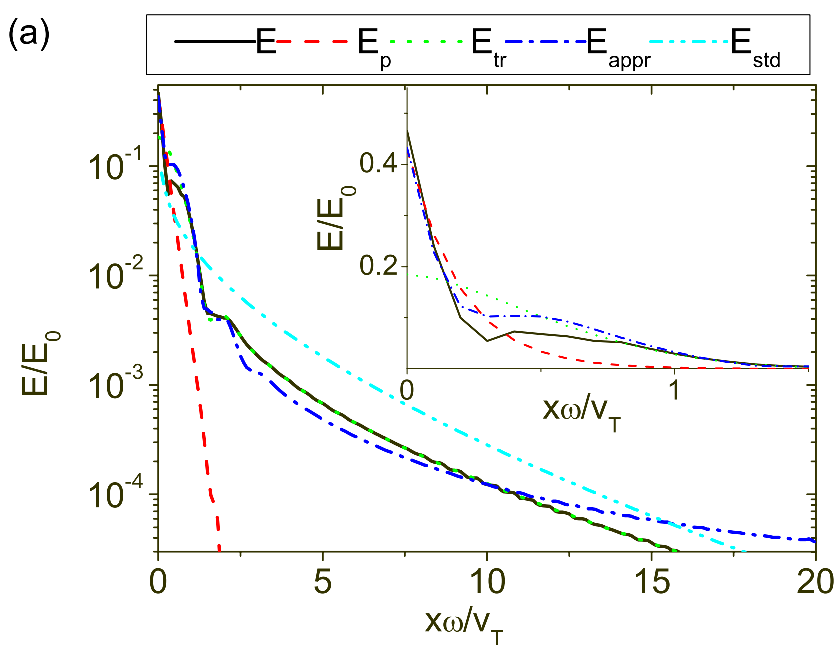
Profile of the Electric Field $\Lambda = \frac{v_T}{\omega}\frac{\omega_p}{c} \sim 1$

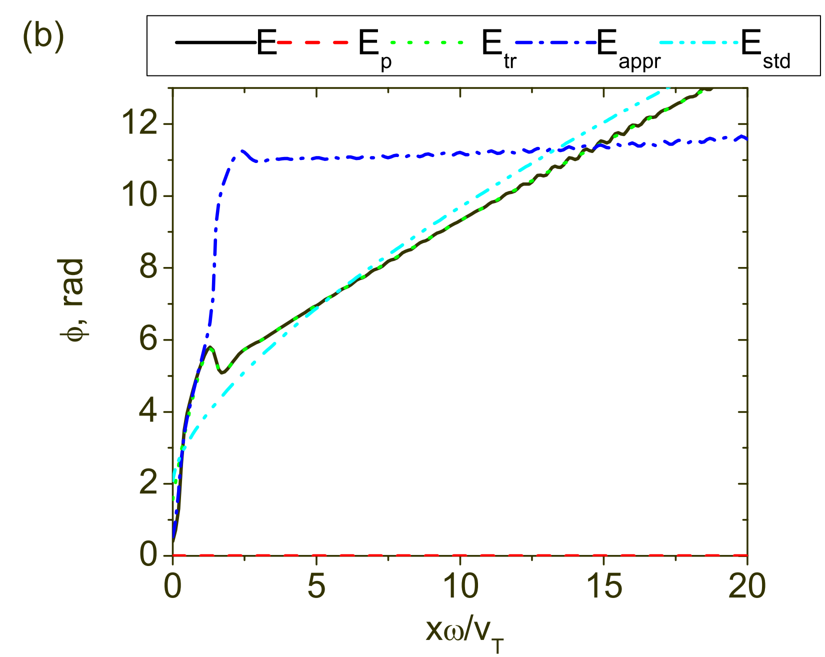
Plot of the rf electric field and *electron current* as a function of the normalized coordinate $x\omega/v_T$. Plasma parameters $n=10^{11}cm^{-3}$, T_e=3eV, f=13.56MHz. Shown are (a) amplitude, and (b) phase. Solid lines show the exact profile E(x); dashed (red) line, the exponential part of the electric field; dotted line (green), the difference of the two; and, chain (cyan) line, shows the asymptotic calculation.



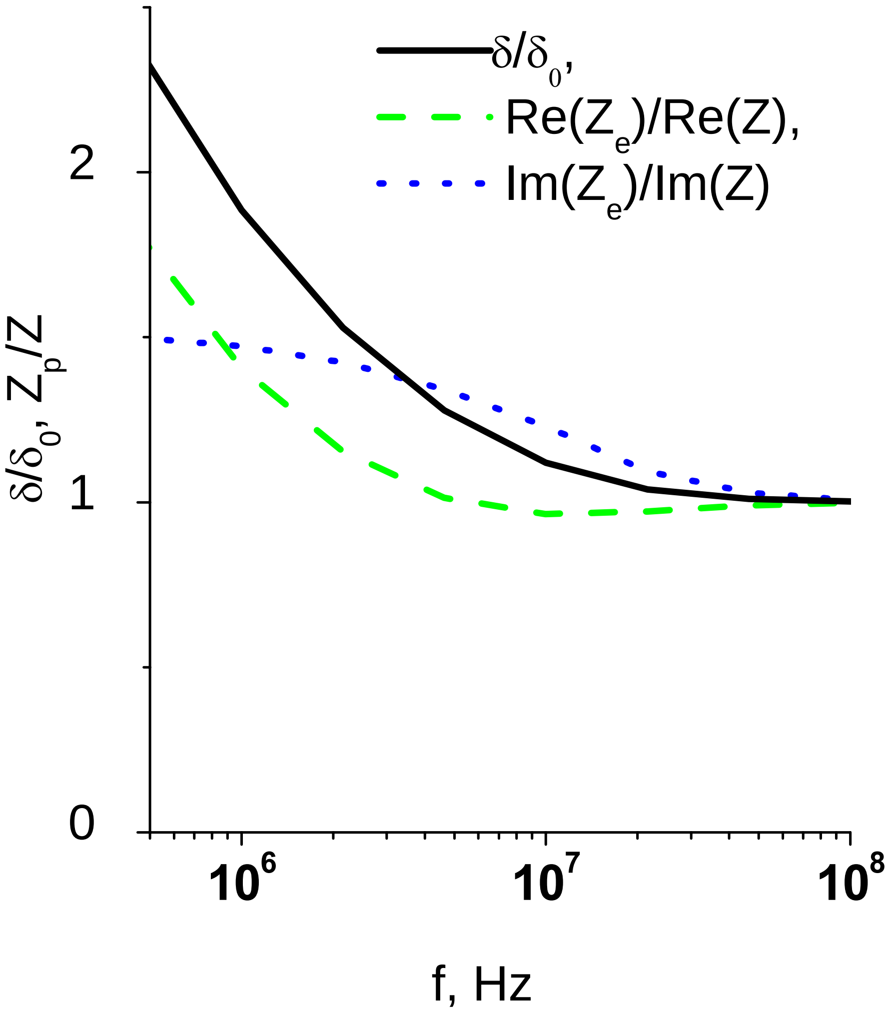
Profile of the Electric Field $\Lambda = \frac{v_T}{\omega}\frac{\omega_p}{c} \gg 1$

Plasma parameters $n=10^{11}cm^{-3}$, T_e=3eV, f=1.MHz. Shown are (a) amplitude, and (b) phase. Solid lines show the exact profile E(x); dashed (red) line, the exponential part of the electric field; dotted line (green), the difference of the two; and, chain (blue) line represents the limiting case of strong anomalous skin effect Λ→∞, and dashed and double dotted (chain) line shows the asymptotic calculation.





Surface Impedance



Plot of the real and imaginary parts of the surface impedance versus discharge frequency calculated exactly and approximately using exponential part only. Also shown is the ratio of the actual skin depth δ to the skin depth calculated in the cold plasma approximation δ_0.

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The Anomalous Skin Effect in a Plasma with a Highly Anisotropic EVDF

- An analytical solution was derived for the electric field penetrated into plasma with the EVDF described as a Maxwellian with two temperatures $T_y >> T_x$.

- The skin layer was found to consist of two distinct regions of width of order V_{Tx}/ω and V_{Ty}/ω,
  - where V_T are the thermal electron velocities and ω is the incident wave frequency.

For more info: I. D. Kaganovich et al., Phys. Plasmas **11**, 3328 (2004).

Conditions

$V_{Ty} / \omega \gg c/\omega_p$

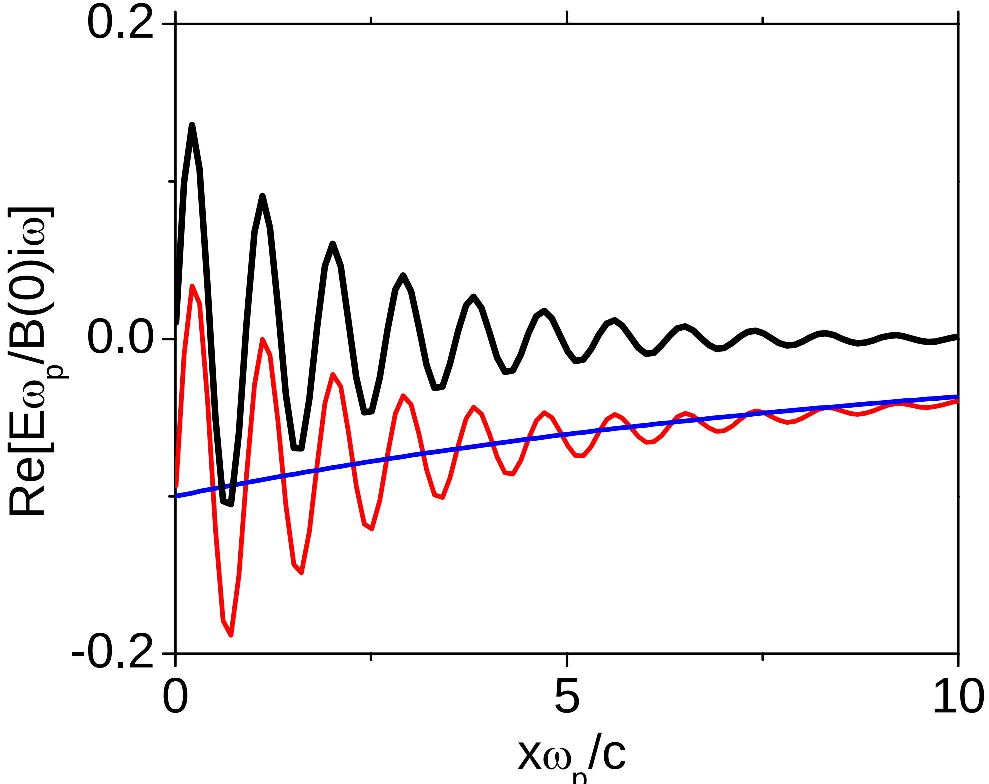
$\omega \ll \omega_p$

$T_y \gg T_x$

Then skin layer is not c/ω_p but V_{Ty} / ω [1].

Electric Field Profile

The electric field in plasma with V_{Ty} = 0.1c; ω =0.01ω_p, T_y/T_x= 50. Solid line shows the real part of the electric field profile obtained from the full solution making use of Eq.(10). Dashed line corresponds to the solution of Ref.[1] *E(z)* given by Eq.(16). Dotted line corresponds to *E(z)* given by Eq.(20).



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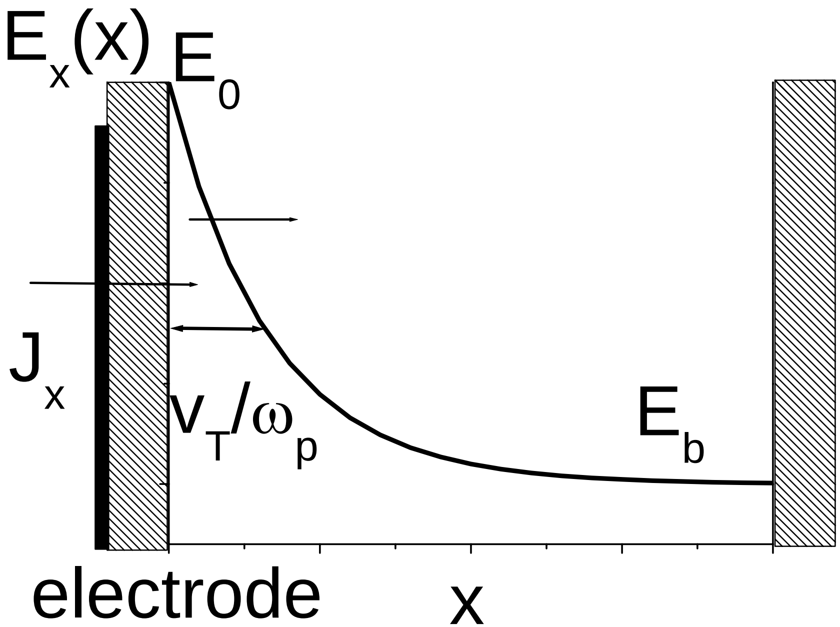
Capacitive Sheath

Conservation of total current

$$j_0 e^{-i\omega t} = \frac{1}{4\pi}\frac{\partial E_x}{\partial t} + j_x$$

$$j_x = \frac{e^2 n_e}{m}\frac{E_x}{-i\omega}$$



Small electric field penetrates plasma

$$E_b = \frac{E_0}{\varepsilon}$$

$$\varepsilon = 1 - \omega_p^2 / \omega^2$$

Landau's Linear Solution (1/2)

$$E_x(x) = \frac{1}{\varepsilon} E_0 + \frac{1}{i\pi} \int_{-\infty}^{\infty} \frac{dk e^{ikx}}{k \varepsilon_{\|}(\omega, |k|)}$$

$$\int_{-\infty}^{\infty} dk \frac{e^{ikx}}{\varepsilon_{\|}(|k|)} = \int_{-\infty}^{\infty} dk \frac{e^{ikx}}{\varepsilon_{\|}(-k)} + \int_{0}^{\infty} dk e^{ikx} \left(\frac{1}{\varepsilon_{\|}(k)} - \frac{1}{\varepsilon_{\|}(-k)} \right)$$

$\varepsilon(-k) = 0$ $\quad$ $e^{-x\omega_p / v_T}$ $\quad$ Non-exponential part

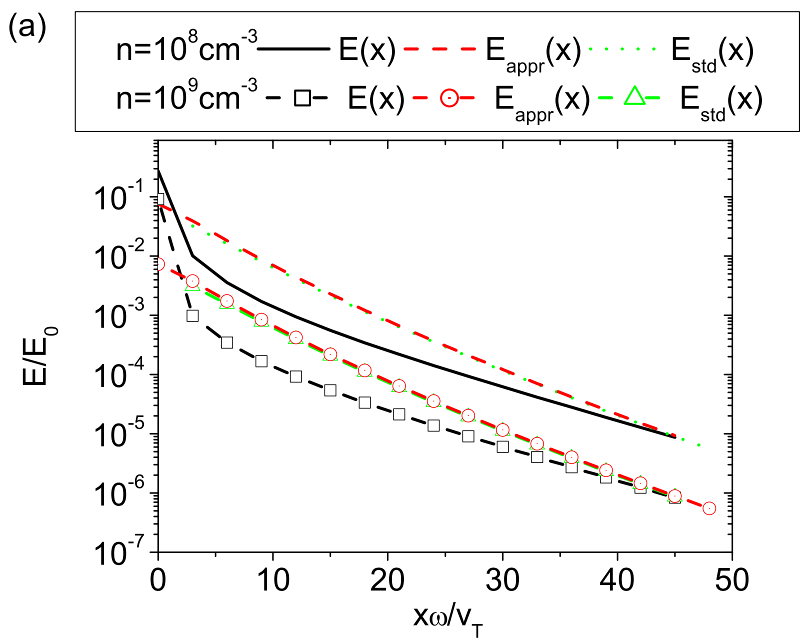
Pole gives Debye screening

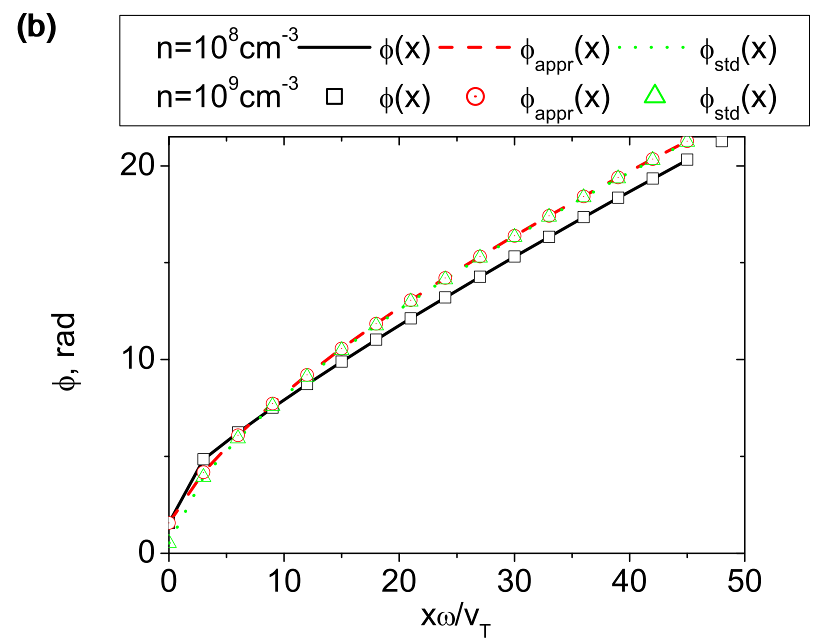
For more info: L. D. Landau., J. Phys. (USSR) **10**, 25 (1946).

Landau's Linear Solution (2/2)

$$E_x(x) = E_0/\varepsilon + E_0 e^{-x\omega_p/v_T} + E_t(x)$$

Profile of the $E_t(x)$. Solid lines show the exact solution; dashed and dotted lines correspond to the approximate solutions. f=13.56 MHz and n=10^8 cm^{-3} (lines) and 10^9 cm^{-3} (symbols).





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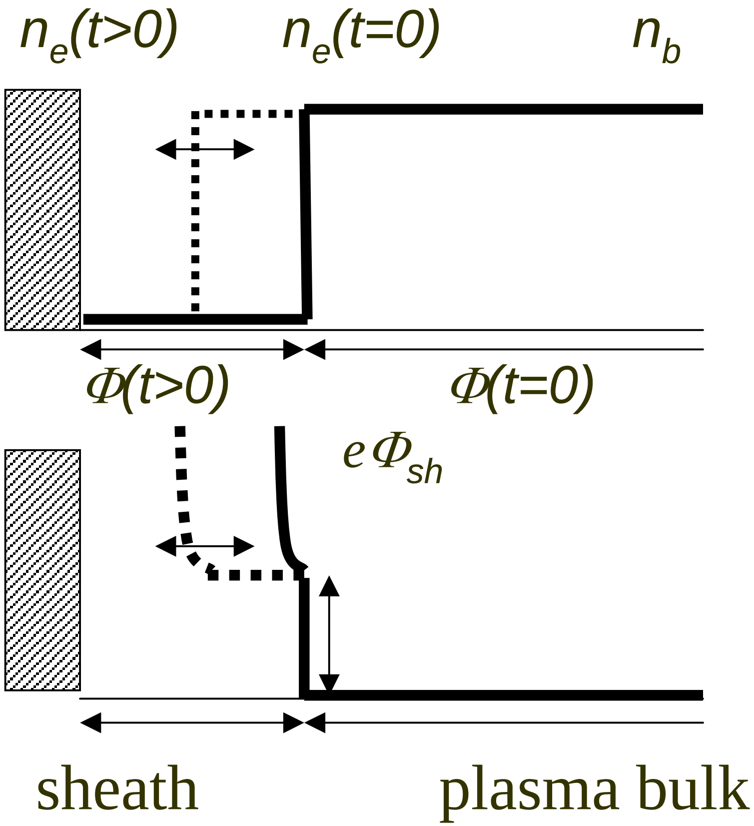
Nonlinear Solution, $V_{sh}>>T_e$

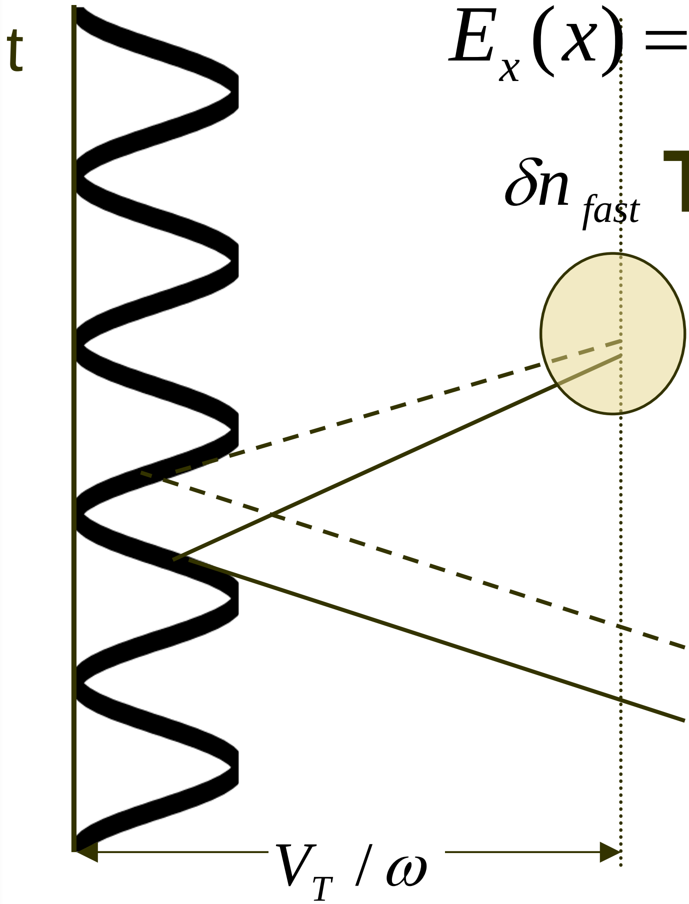


Schematic of a sheath.

The negatively charged electrode pushes electrons away by different distances depending on the strength of the electric field at the electrode. Shown are the density and potential profiles at two different times. The solid line shows the maximum sheath expansion.

Electron bunches produce electric field near sheath on distances $\sim v_T / \omega$

$$E_x(x) = E_0 / \varepsilon + E_0 e^{-x\omega_p / v_T} + E_{sh}(x) + E_t(x)$$



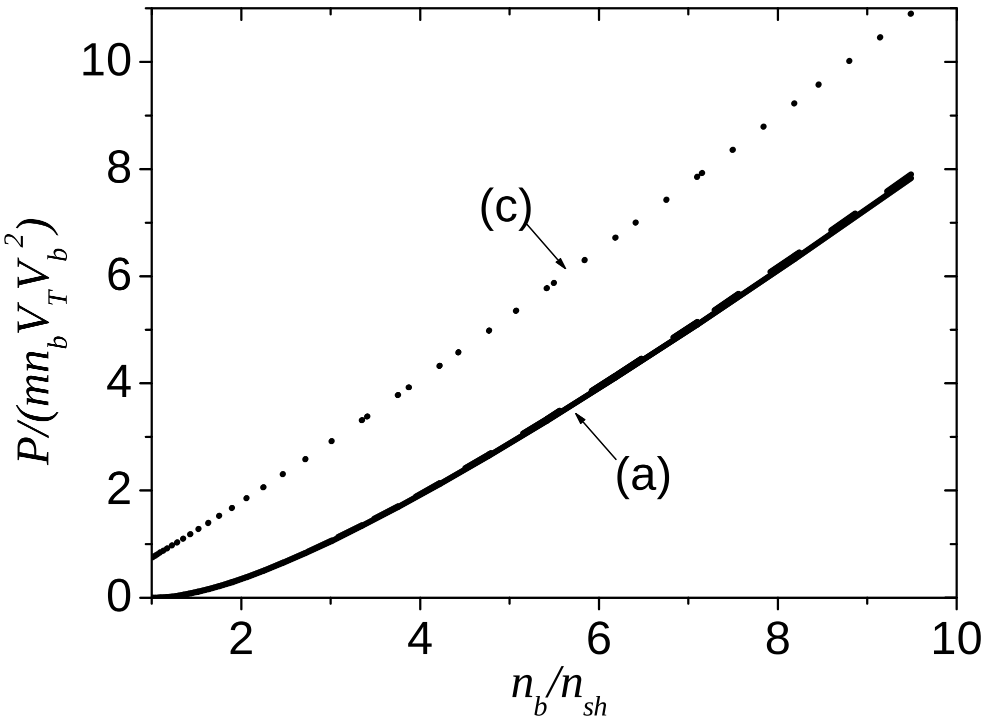
The textbook solution without $E_t(x)$ is incomplete:

E_t yields energy transfer from fast electrons to slow electrons not colliding with the sheath

For more info: I. D. Kaganovich Phys. Rev. Lett., **89**, 265006 (2002).

Effect of Self Consistency on Power Absorption

Plot of the dimensionless power density as a function of the ratio of the bulk plasma density to the sheath density, taking into account (a) self consistent treatment and (c) test particle model.



Conclusions

- **Separation of the electric field into an exponential part with width of skin depth or Debye radius and non-exponential part E_t with width $\sim V_t/\omega$.**
- **The non exponential part in capacitive sheath is missed in textbooks.**
- **E_t yields considerable change in the rate of electron heating.**