# Electron transport in closed E×B drift devices

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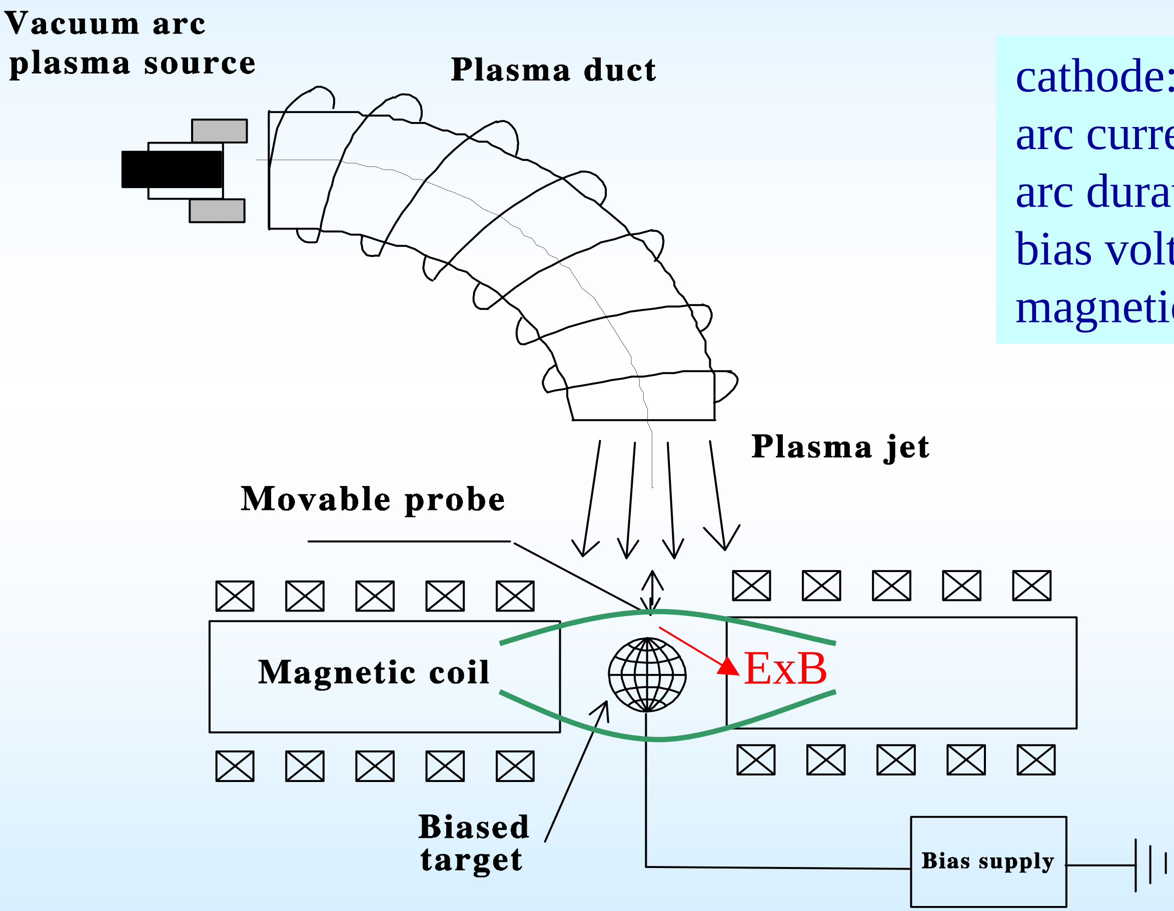
# Outline

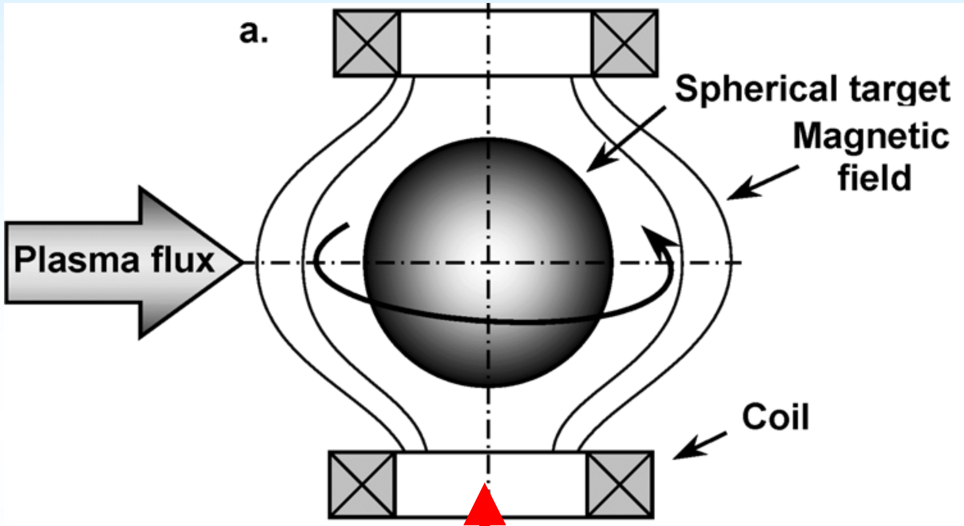
- Plasma devices with E×B drift
  - PIII with magnetic control
  - Magnetrons
  - Hall thruster
- Electron transport issues overview
- Electron transport model
  - near wall conductivity
- Summary

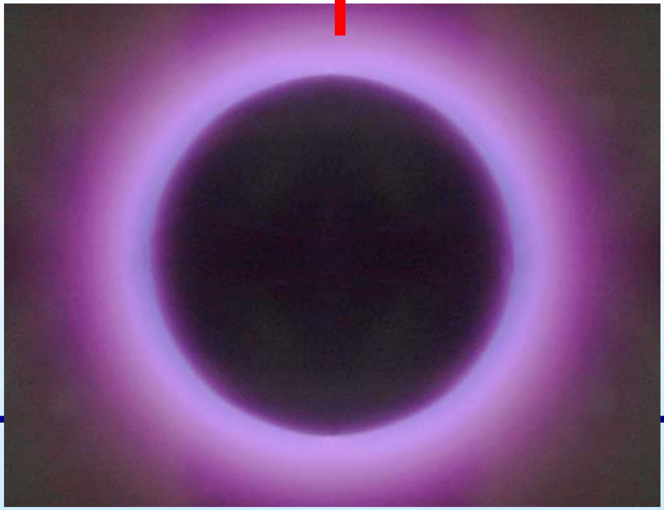
# Plasma immersion ion implantation



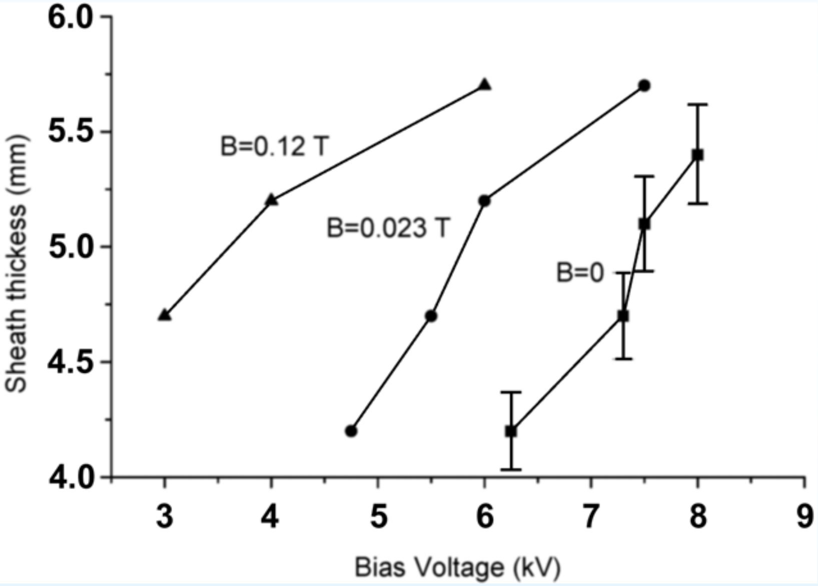
| | |
|---|---|
| cathode: | Titanium |
| arc current: | 500 A |
| arc duration: | 250 ms |
| bias voltage: | 2-8 kV |
| magnetic field: | 0-0.12 T |

# Electron drift



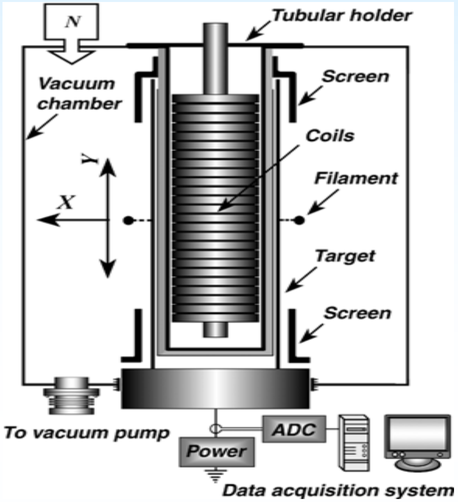


To control sheath
(breakdown vs expansion)



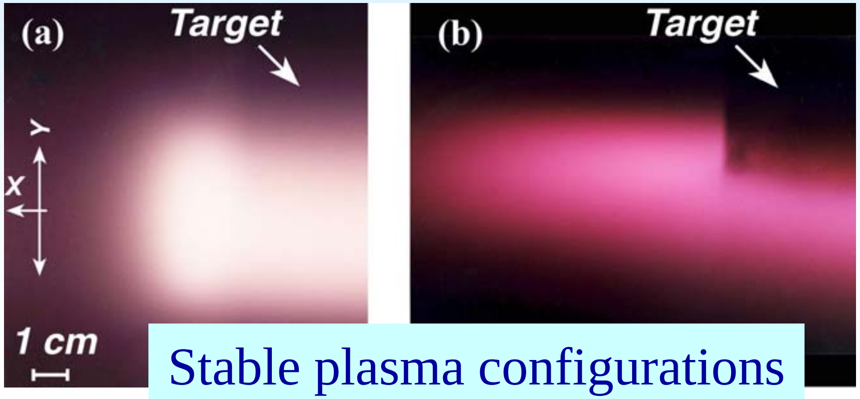
Keidar *et al.* Appl. Phys. Lett, 2002

# Magnetrons



Wang & Cohen, 1999

Abrupt transition



Stable plasma configurations

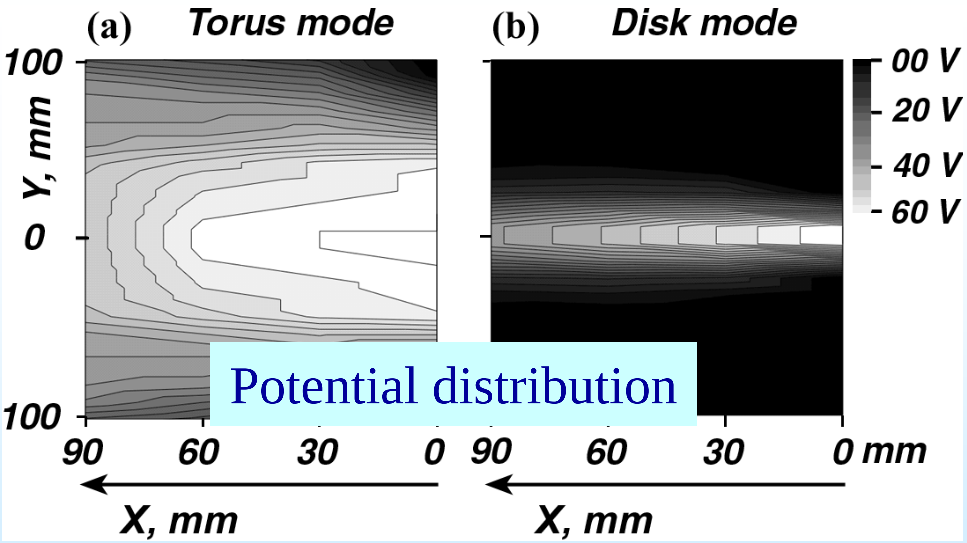
transition
~400 V

$$U_b\gamma_i = \varepsilon_c + \frac{E_{loss}}{\tau_e \nu_{iz}}$$

Energy balance

$$\frac{1}{\nu_{iz}} \cong \frac{L}{\mu_B E}$$

Particle balance



Potential distribution

Appl. Phys. Lett, 2004

# Magnetrons

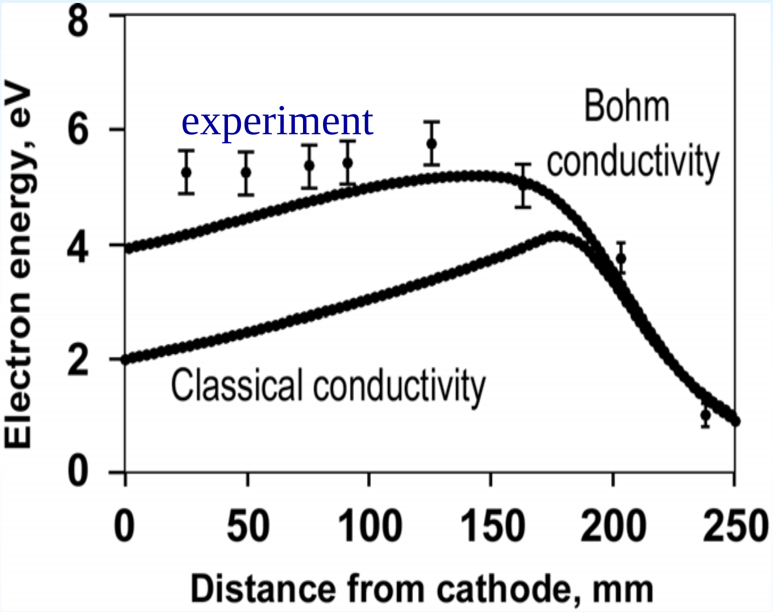
$$\frac{\partial \varepsilon_e}{\partial r} = E - \psi_e \varepsilon_i$$

$$\mu_{et.B} = \frac{1}{\alpha B}$$

$$\frac{\partial f_i}{\partial r} + \frac{eE}{m_i V_i}\frac{\partial f_i}{\partial V_i} = \frac{n_a}{V_0 V_i}\int_0^\infty \sigma_i V_e f_e dV_e$$

$$j_e(r) = e \cdot n_e \cdot \mu_{et.B} E = \frac{e \cdot n_e \cdot E}{\alpha B}$$

$E$ from experiment



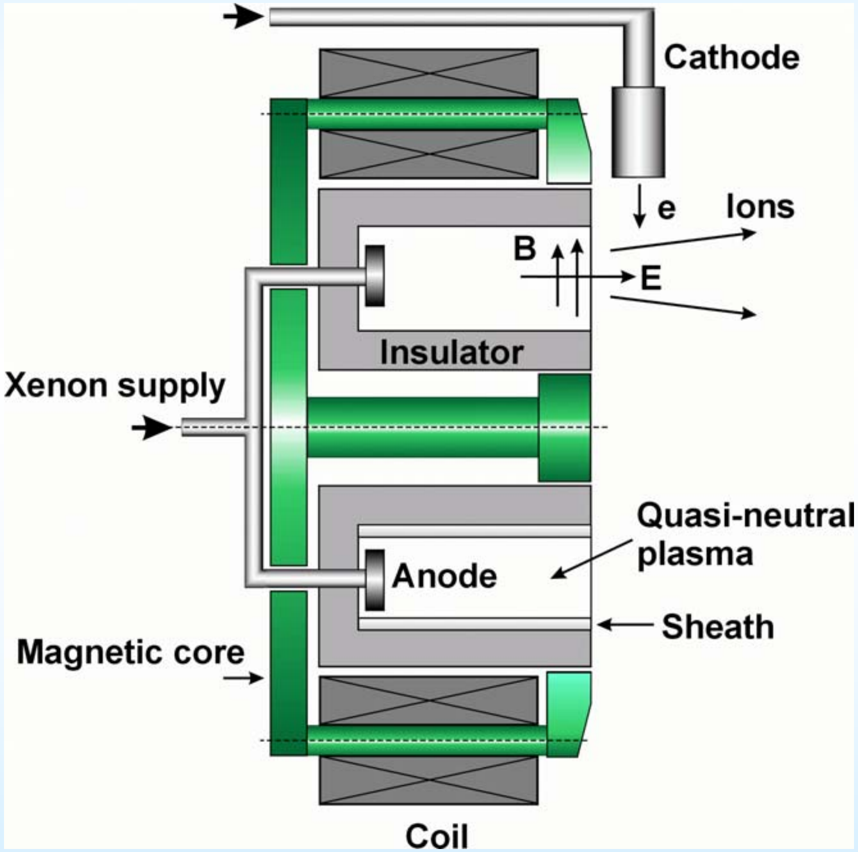
Rossnagel and Kaufman, 1987

I (classical)<I $_{\text{exp}}$

Plasma oscillations to support anomalous transport

Levchenko *et al*. submitted Phys. Plasmas

# Hall thruster



B~0.01 T
E~$10^4$ V/m
$n_e$~$10^{17}$ $m^{-3}$
h~1 cm

# Plasma flow modeling approach in Hall thrusters

$$\left.\begin{array}{l}
\bullet\ \partial(nV_z)/\partial z + \partial(nV_r)/\partial r + nV_r/r = \beta n_i n_a \\[2ex]
\bullet\ V_z\partial V_z\partial z = - V_r\,\partial V_z\partial r + e/mE_z - \beta V_a n_a \\[2ex]
\bullet\ V_z\partial V_r\partial z = - V_r\partial V_r\partial r + E_r
\end{array}\right\}\ \text{2D part}$$

$$\left.\begin{array}{l}
\bullet\ j_{er}=0; \qquad \boxed{\varphi - T_e \mathit{ln} n = \mathrm{const}} \\
\bullet\ j_{ez} = en\mu/(1+(\omega\nu)^2)\ (E_z + \partial T_e/\partial z + T_e\partial \mathit{ln} n/\partial z) \\
\bullet\ 3/2\partial(j_e T_e)/\partial z = Q_j - Q_w - Q_{ion}
\end{array}\right\}\ \text{1D part}$$

# Electron collisions

**Neutrals** $\nu_{en} = n_a \sigma_{ea} V^e_{th}$

**Bohm** $\nu_B = \alpha \omega_e$ $(\alpha \sim 1/16)$

**wall** $\nu_{ew} \sim V^e_{th}/h$

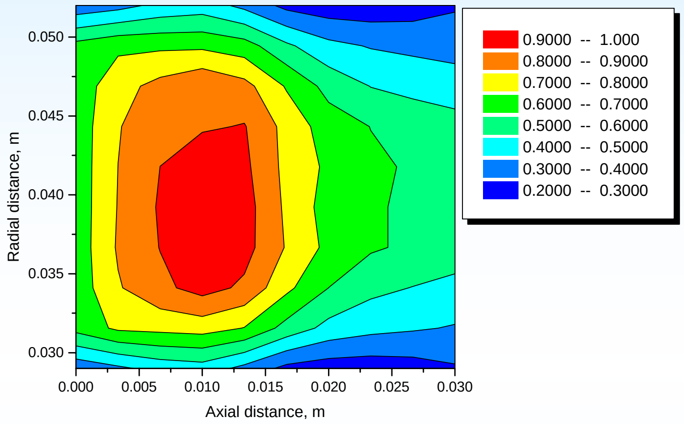
- Bohm-type:
  - Ahedo: 1/80
  - Fife: 1/100
  - Keidar: 1/40

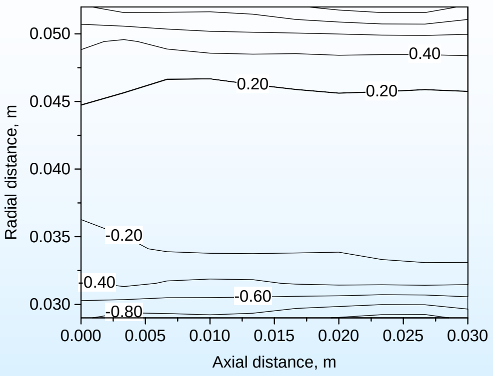
- NWC:
  - Garrigues: 0.2e7 $s^{-1}$
  - Koo: (0.2-0.3)e7 $s^{-1}$

Various models were developed PIC, hybrid, hydrodynamic
All relay on some anomalous coefficient

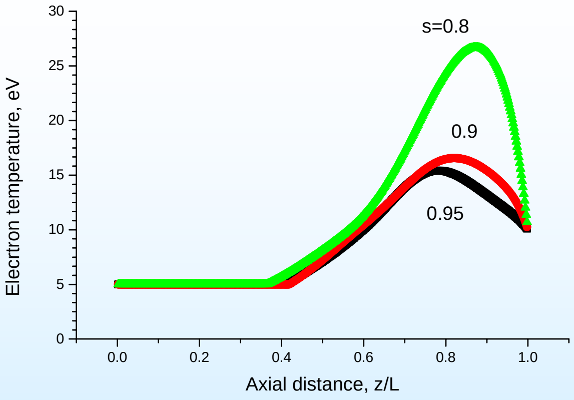
# Density & velocity distribution

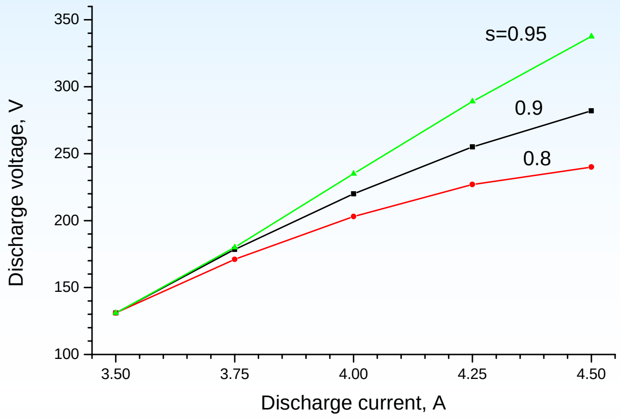
density



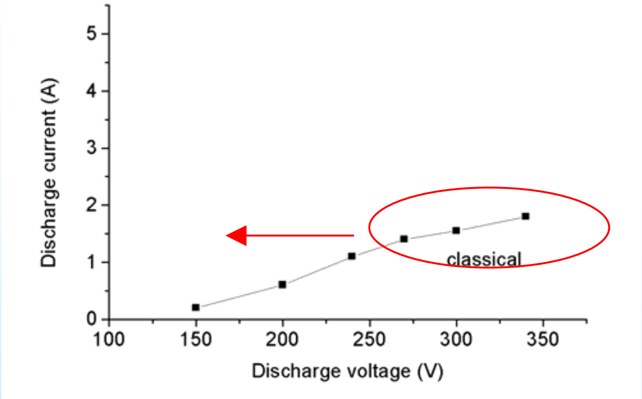


Electron energy



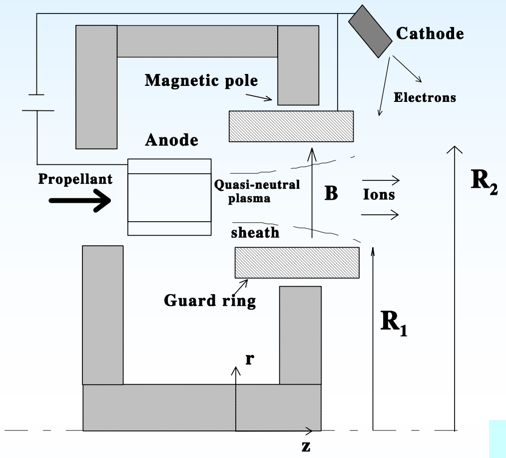


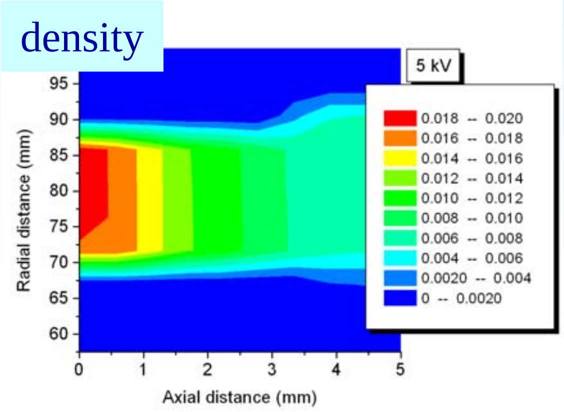
Experiment:I~4.5 A, $U_d$=300 V



Keidar *et al.* Phys. Plasmas, 2001

# Thruster with Anode Layer





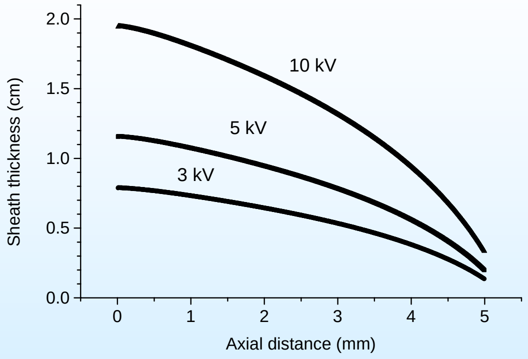
$e\mu_\perp n_e U/d \cong I_e/A(s)$

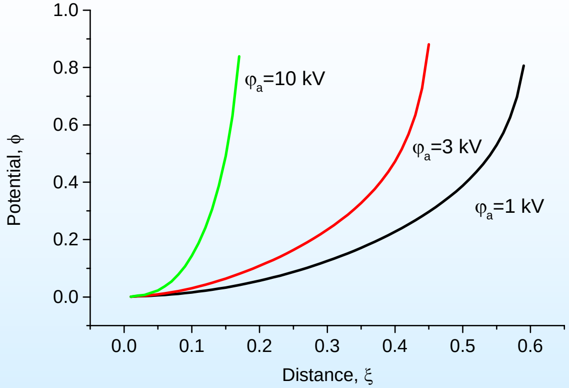
A(s)-channel cross section

$\mu_\perp = 1/(\alpha B)$ [Bohm]

Plasma detaches from the wall

$d \sim r_{Ce} A(s)$





Sheath expansion leads to shorter acceleration region

Keidar *et al.* Phys. Plasmas, 2004

# Modeling assumptions
# State-of-the- art

- Electron conductivity mechanism
  - Bohm (plasma turbulence)
  - Near wall conductivity
- Current (potential) distribution
  - “thermalized” potential
  - Uniform current

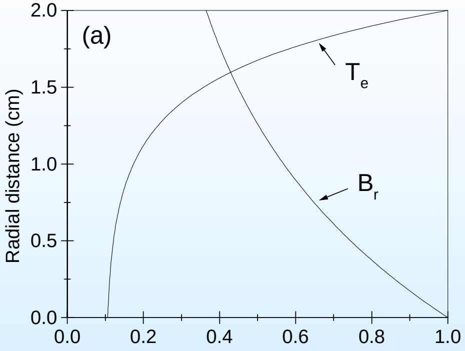
# Non-uniform electron transport in HT

current conservation

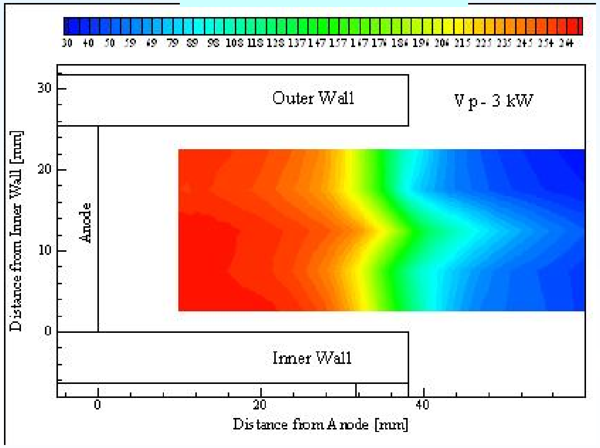
$$j_r = \sigma\left(-\frac{\partial\varphi}{\partial r} + \frac{\partial T_e}{\partial r} + T_e\frac{\partial \ln n}{dr}\right)$$

$$j_z = \frac{\sigma}{(1+\beta^2)}\left(-\frac{\partial\varphi}{\partial z} + \frac{\partial T_e}{\partial z} + T_e\frac{\partial \ln n}{\partial z}\right)$$

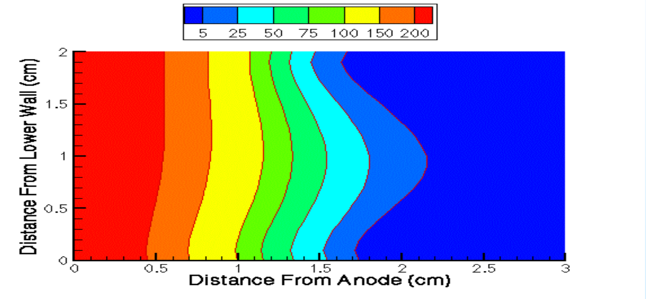
$$\frac{\partial j_r}{\partial r} + \frac{\partial j_z}{\partial z} + \frac{j_r}{r} = 0$$



experiment



simulations

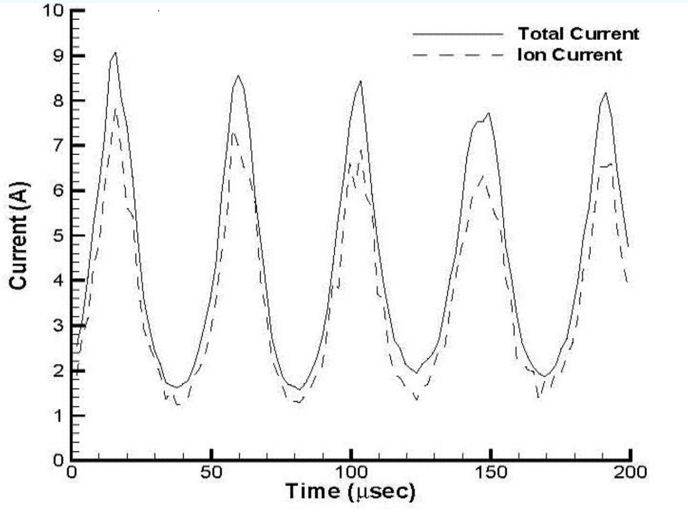


Keidar *et al.* Appl. Phys. Lett, 2004

# Plasma oscillations

- May be in support of Bohm anomalous mobility
- Experimental evidence in Hall thruster
  - Dependent on mass flow rate, discharge voltage, facility, magnetic field, cathode

2D PIC-MCC



# Oscillations

B~0.01 T
E~$10^4$ V/m
$n_e$~$10^{17}$ $m^{-3}$
h~1 cm

| **Band** | **Nature** | **conditions** | **Experiment** | **Simulations** |
|---|---|---|---|---|
| 1-20 kHz | Discharge oscillations (contour)<br>Ionization “breathing mode” | PPU | yes | yes |
| 5-25 kHz | Rotating spoke<br>azimuthal | Ionization process | yes | |
| 20-60 Hz | Gradient induced | dB/dz<0 | yes | Theory (Morozov) |
| 20-100 kHz | Azimuthal waves | Gradient-driven or ionization | yes | |
| 70-500 kHz | Transient-time<br>Axial electrostatic wave | $U_i/L$ | yes | Hybrid code |
| 0.5-5 MHz | Azimuthal, 5-8 MHz | Rayleigh | yes | Theory (Litvak et al)<br>Gradient magnetic field, density, electron drift velocity |

Choueiri, Phys. Plasmas, 2001

# Oscillations

- Drift-dissipative

$$\omega_d = k\left(\frac{T_e}{B}\right)\frac{\langle n \rangle}{n}$$

- Maximum increment

$$\gamma \sim \omega_d \sim \left(\frac{T_e}{B}\right)\frac{\langle n \rangle}{n}$$

- Diffusion

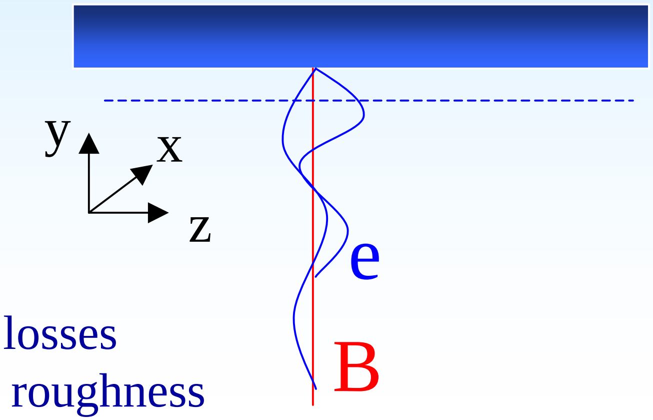
$$D \sim \lambda_\perp^2 \gamma$$

- $D \sim T_e/B$

$$D = \frac{1}{16} \cdot \frac{T_e}{B}$$ Bohm

Alcock & Keen, 1970 Afterglow plasmas
Conditions similar to Hall thrusters

# Near wall conductivity



Energy losses
Surface roughness
SEE

*Original model*
Morozov, 1968-2000
Bugrova, 1985 (experiment)

*Recent*
Ivanov & Bacal (neutralization)
Barral (sheath effect)

| | |
|---|---|
| Mean free path: | ~1 m |
| Distance between wall: | ~1 cm |

# Near wall current

Collisionless kinetic equation

$$\frac{\partial f}{\partial t}+V\frac{\partial f}{\partial r}-\frac{e}{m}(E+V\times B)\frac{\partial f}{\partial v}=0$$

Emitted electrons

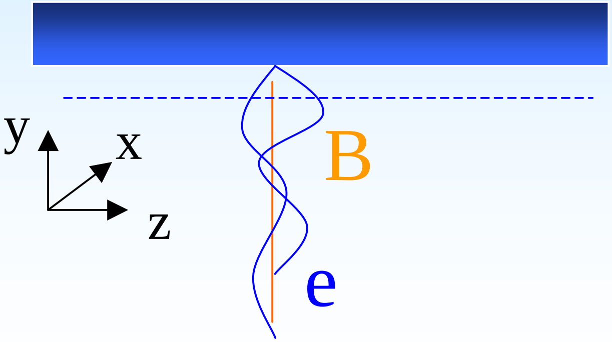
$$f(v)=n_0\left(\frac{m}{2\pi kT_w}\right)^{3/2}\exp\left(-\frac{mV^2}{2kT_w}\right)$$

Electron dynamics

$$V_{ez}=(V_{ex}{}^{o}-V_E)\sin(\omega t)+V_{ez}^{o}\cos(\omega t)$$

$$V_{ex}=V_E+(V_{ex}{}^{o}-V_E)\cos(\omega t)-V_{ez}^{o}\sin(\omega t)$$

$$V_{ey}=\sqrt{V_{ey}^{w}+\frac{2e\varphi_w}{m}}$$



Characteristics

$$\frac{dV_x}{dt}=-\omega V_z$$

$$\frac{dV_y}{dt}=0$$

$$\frac{dV_z}{dt}=-\frac{eE}{m}+\omega V_x$$

Electron current

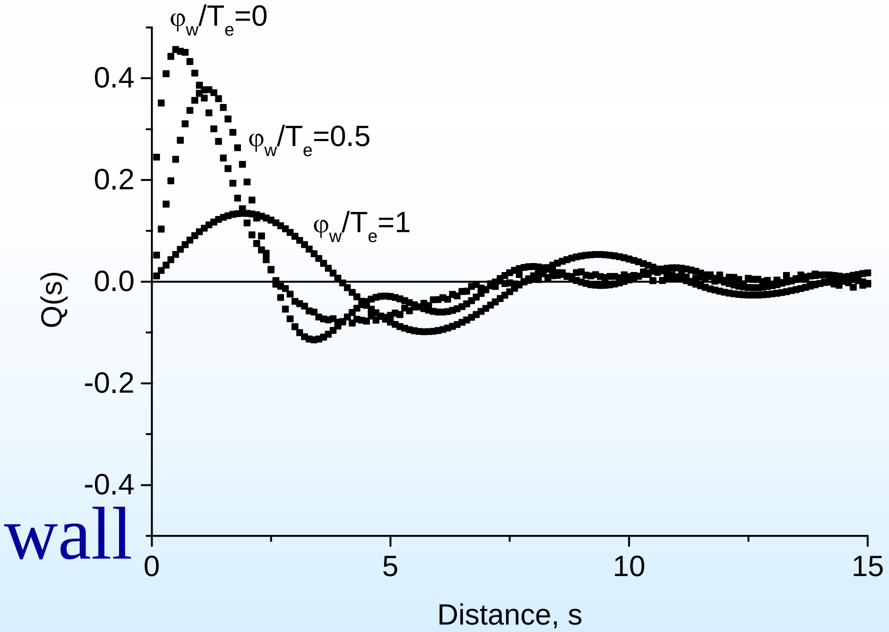
$$j_{ez}=\int_{-\infty}^{\infty}\int_{\alpha_w}^{\infty}\int_{\alpha_z}^{\infty}f(v)V_z\,dV_x\,dV_z\,dV_y$$

# Near wall current

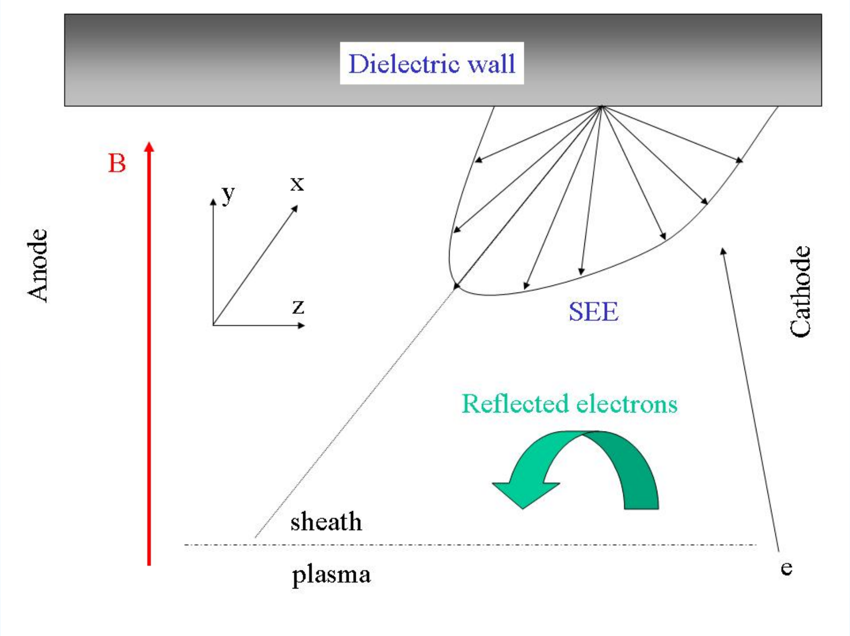
$$j_{ew} = n_0 \frac{E}{B}\left(\frac{m}{2\pi kT_w}\right)^{1/2} \exp\left(\frac{e\varphi_z}{kT_w}\right)\exp\left(\frac{e\varphi_w}{kT_w}\right)\int_{\sqrt{\frac{e\varphi_w}{m}}}^{\infty}\exp\left(\frac{mV_y^2}{2\pi kT_w}\right)\sin\left(\omega\frac{y}{V_y}\right)dV_y$$

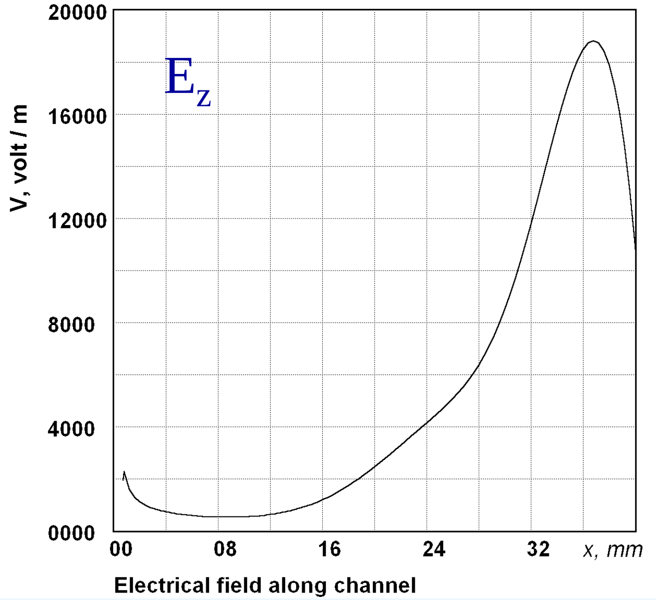
$$Q(s) = \int_{\sqrt{\frac{e\varphi_w}{m}}}^{\infty}\exp(-\alpha^2)\sin\left(\frac{s}{\alpha}\right)d\alpha$$

$$\alpha = \frac{V_y}{\sqrt{\frac{2kT_w}{m}}} \qquad s = \frac{\omega y}{\alpha\sqrt{\frac{2kT_w}{m}}}$$



# Axial electric field effect on SEE

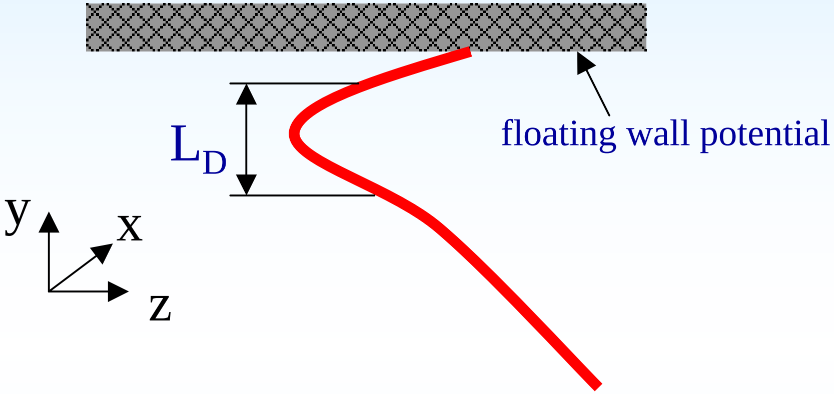




In sheath: $E_z \sim 10^4$ V/m; $E_r \sim 10^5$ V/m

# Axial E-field.
# Current enhancement

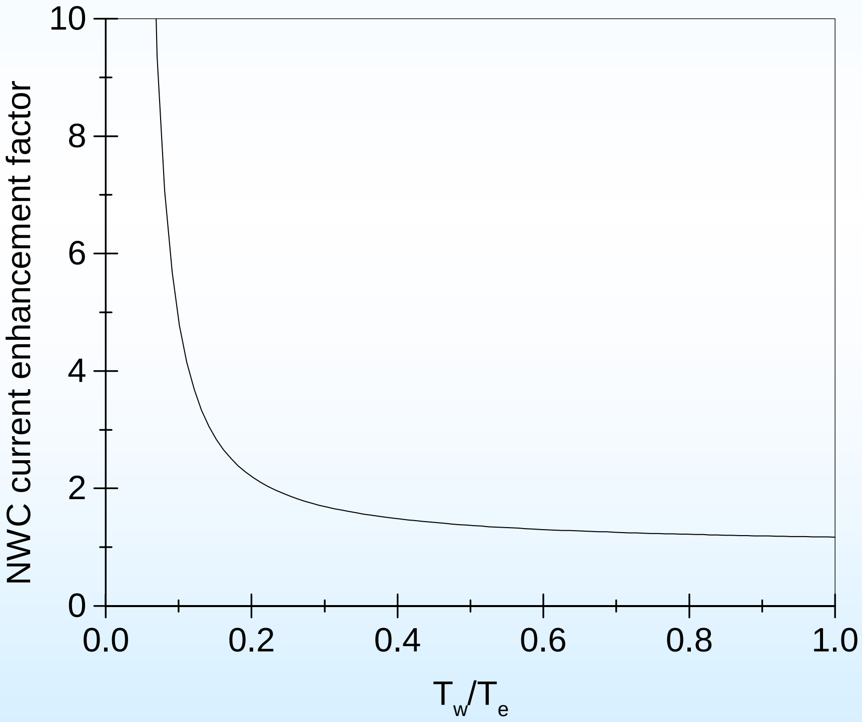
space charge saturated sheath



$$\Delta\varphi_z = E_z^2 \int_{y_{\min}}^{y_{\max}} \frac{dy}{E_y(y)} \approx E_z^2 \frac{L_D}{E_y}$$

$$\Delta\varphi_z \approx E_z L_D$$

NWC depends on wall to bulk $T_e$ ratio



# Summary

- PIII, magnetrons, Hall thrusters have similar physics, ExB drift. This leads to efficient ionization and high electric field in the quasi-neutral plasma region
- Electron transport is largely determined by properties of the drift region (walls, plasma oscillation)
- Anomalous transport mechanism is needed to explain electron current in most cases (Bohm, NWC)
- Conditions for both mechanisms are different and required further evaluation for each specific device