The electron diffusion coefficient in energy in bounded collisional plasmas

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***Abstract*—**The electron energies in typical gas discharge plasmas do not exceed significantly the first ionization potential. This being the case, the momentum relaxation in collisions with neutrals is significantly faster than the energy relaxation due to collisions. It follows that the main part of the electron distribution function (EDF) is isotropic. So the interaction of an electron with an electric field is predominantly stochastic random walk process and can be described by a diffusion coefficient in energy D_ε. Both collisional and stochastic heating mechanisms can be incorporated in it.

By the proper choice of variables, the electron Boltzmann equation can be reduced to the standard diffusion one, both in space and in energy. This approach is very efficient in solution of the problems of the electron kinetics in bounded non-uniform plasmas.

Some paradoxical effects, such as the formation of a cold electron population in discharges with peripheral energy input, and non monotonic radial profiles of the excitation rates, are explained within this framework.

The expressions for D_ε in different discharges are presented.

The history of the EDF non-locality concept is discussed for stationary gas discharges.

***Index Terms*—**keywords: transport in plasmas, electron kinetics, gas discharge.

1. Introduction.

The characteristic electron energies in stationary gas discharges with weakly ionized plasmas, generated by electron impact ionization, are fixed by the plasma maintenance condition at a level of several eV. The reason lies in the fact that for the stationary discharge maintenance during the charged particle lifetime precisely one ionization event occurs. In unmagnetized plasma the lifetime is controlled by slow ion motion; so during the lifetime every electron undergoes considerable number of collisions. Since the EDF reproduces the average electron history, it follows that the EDF tail with kinetic energies w above the ionization energy ε_i contains relatively small fraction of the total electron population. In atomic gases, as a rule, it holds for the energies $w > \varepsilon_1$, too, where $\varepsilon_1 \simeq \varepsilon_i$: the excitation energy of the first atomic level. So the electrons with energies by several eV exceeding ε_1 are usually practically absent. In this energy range the elastic collisions cross-section 1-2 orders of magnitude exceeds the excitation ones. It implies that the EDF anisotropy is small: the momentum relaxation is considerably faster than the energy relaxation. In the simplest and the most familiar case, when both these processes are due to the elastic collisions, the ratio of the relaxation frequencies is extremely small and equals to $\delta = (2m/M)$. At the EDF tail this ratio is $(\nu^*/\nu) \sim 10^{-1} - 10^{-2} << 1$, is small, too. Here, ν is the (transport) collision frequency of the elastic collisions, and ν^* is the inelastic collision frequency.

It implies that the EDF in this energy range is close to isotropic, and the traditional two-term approximation (6) is valid.

This fact remains valid in the free-flight regime, $R\lambda$, too, because the majority of the electrons are trapped by the ambipolar electric field, and during its lifetime an

electron undergoes many elastic collisions.
In most practically important cases the energy input occurs by relatively small increments (in respect to the characteristic EDF energy scale), too. So the typical scenario of EDF formation can be outlined as follows. An electron starts with a low energy $w << \varepsilon_1$, and gains energy by small increments from the applied electromagnetic field. If these small energy kicks are uncorrelated, this process cal be treated as a random walk along the energy axis, i.e., as a diffusion with a diffusion coefficient D_ε. The EDF body $w < \varepsilon_1$, is formed by competition of this diffusion in energy and of energy loss in collisions. Since the elastic losses occur practically continuously, they can be described by a downward-directed convective velocity in energy

$$V_\varepsilon = -\delta\nu w. \tag{1}$$

In the molecular gases the electronic energy levels are of the order of ε_i, and the energies of the vibrational and rotational levels are more than order of magnitude lower. So the collision integral for excitation of these levels can be reduced to the Fokker-Planck form and approximated as action of an effective retarding force (1). For the EDF calculation these quasielastic collisions can be described by introducing the parameter $\delta_{eff}(w)$. It is to be noted that using for the EDF calculation the parameter $\overline{\delta_{eff}}$, defined from the total electron energy balance, can result in erroneous results.
The flux density along *w* is

$$\Gamma_\varepsilon = -D_\varepsilon \frac{df_0}{dw} + V_\varepsilon f_0.$$

The zero flux Γ_ε corresponds to the Druyveteyn-Davydov's isotropic EDF

$$f_0(w) = A\exp\left[\int^w \frac{V_\varepsilon(w')}{D_\varepsilon(w')}dw'\right].$$

The characteristic energy scale (and the average energy $\bar{\varepsilon}$) of such an EDF, which takes place at low energy input level, can be estimated as the diffusive displacement against the convective velocity (1):

$$\bar{\varepsilon} = D_\varepsilon / V_\varepsilon . \tag{2}$$

Such EDFs correspond to the situation when the energy balance is controlled by the elastic collisions. They are traditionally used as a benchmark in the electron kinetics of discharges. They were thoroughly investigated by Druyvesteyn [1], Allis [2] and Davydov [3]; a detailed review can be found in [4].
As inealstic collisions result in considerably faster energy loss, than elastic ones, the characteristic frequencies usually satisfy

$$\delta\nu << \nu^*.$$

So the EDF tail, $w > \varepsilon_1$, decreases considerably faster as energy increases, than the EDF body; its energy scale in most cases of interest is

$$T^* = \sqrt{D_\varepsilon / \nu^*} << \bar{\varepsilon}, \varepsilon_1, \varepsilon_i. \tag{3}$$

In other words, an electron undergoes an inelastic collision practically at the same moment as it gains enough energy from the field; it corresponds to the widely known black (absorbing) wall approximation of diffusion theory. In this case $\bar{\varepsilon} < \varepsilon_1, \varepsilon_i$, the main EDF part at $w < \varepsilon_1$ is determined by elastic collisions. The influence of the

inelastic collisions on the EDF "body", $w < \varepsilon_1$, is small. It is considerable on the EDF tail only in close vicinity of the threshold ε_1. The EDF "body" at $w < \varepsilon_1$, and the diffusive electron flux (along the energy axis!) Γ_ε in the zeroth in $\left(T^*/\varepsilon_1\right)$ approximation, can be found by imposing the boundary condition

$$f_0\left(w = \varepsilon_1\right) = 0. \tag{4}$$

For the existence of the Druyvesteyn-Davydov's EDFs in EDF body $w < \varepsilon_1$, it is necessary that the average energy given by the expression (2), is to be lower than the threshold of inelastic collisions ε_1. In this case it decreases exponentially in the interval $\left[\bar{\varepsilon}, \varepsilon_1\right]$. As the EDF tail decreases exponentially with a steeper slope (3), the probability of an electron to reach the ionization energy ε_i is extremely small. It is proportional to product of exponentially small probability to gain energy ε_1, multipled to the exponentially small probability to overcome the interval $[\varepsilon_1, \varepsilon_i]$. On the other hand, the lifetime of the charged particles decreases at low and medium pressure, and in order to maintain a stationary plasma a rather high energy input is necessary. It means that at low values of $\left(pR\right)$ the energy losses in the elastic collisions become negligible and the energy balance becomes controlled by inelastic collisions. The EDF in this case is analogous to a "pipe-line": electrons after an inelastic collision are continuously heated due to the diffusion in energy up to the moment when they reach the threshold energy ε_1. This boundary $\left(pR\right)$ lies for the noble gases, in the interval $\left(1-10\right)$*Torr cm.* The corresponding values for molecular gases are an order of magnitude lower than for noble gases.
The concept of electron diffusion in energy turns out to be highly efficient for the analysis of gas discharge plasmas, especially at low and medium pressures.

2. The kinetic equation.

The electron Boltzmann equation (for simplicity, without taking into account a magnetic field) is of the form:

$$\frac{\partial f}{\partial t} + \left(\vec{\mathbf{v}} \cdot \nabla\right) f - (e/m)\left(\vec{\mathbf{E}} \cdot \frac{\partial f}{\partial \vec{\mathbf{v}}}\right) - I(\vec{r}, \vec{\mathbf{v}}, t) + \\ + \sum_\beta \left[St_\beta^{el}(f, f_\beta) + St_\beta^{inel}(f, f_\beta)\right] = \frac{df}{dt} + \left(\frac{df}{dt}\right)_{\text{col}} = 0. \tag{5}$$

The two-term EDF expansion is

$$f\left(\vec{r}, \vec{\mathbf{v}}, t\right) = f_0\left(\vec{r}, v, t\right) + \sum_{k=1,2,3} f_{1k}\left(\vec{r}, v, t\right) Y_k\left(\theta, \varphi\right), \\ f_{1k}\left(\vec{r}, v, t\right) << f_0\left(\vec{r}, v, t\right), \tag{6}$$

where $\left(\theta, \varphi\right)$ are angles in the velocity space, Y_k are the first-order (vectorial) spherical harmonics. The equation for the vectorial EDF part $\overrightarrow{f_1}\left(\vec{r}, v, t\right)$ is

$$\frac{\partial \overrightarrow{f_1}}{\partial t} + \left(\vec{\mathbf{v}} \cdot \nabla\right) f_0 - \frac{e}{m}\vec{\mathbf{E}} \cdot \vec{\mathbf{v}} \frac{\partial f_0}{\partial v} + \nu \overrightarrow{f_1} = 0, \tag{7}$$

where $\nu(v)$ is the electron-neutral transport collision frequency. Here, we have used the expression

$$St\left(\overline{f_1}\right) = \nu \overline{f_1}. \tag{8}$$

The self-consistent electric field $\vec{\mathbf{E}}\left(\vec{r},t\right)$ in the vast majority of the discharge situations can be subdivided into two components. One of them is generated by the plasma inhomogeniety. Roughly speaking, this part of the electric field maintains the plasma quasineutrality. Because the plasma density profile is controlled by slow (with respect to the EDF formation characteristic times) generation-recombination processes, this part of the electric field variation is slow and it can be described by the electrostatic potential $\varphi\left(\vec{r},t\right)$. In stationary discharges this field is time-independent:

$$\vec{\mathbf{E}}\left(\vec{r},t\right) = -\nabla\varphi\left(\vec{r}\right) + \tilde{\vec{\mathbf{E}}}\left(\vec{r},t\right). \tag{9}$$

Introducing the total electron energy

$$\varepsilon = mv^2/2 - e\varphi = w - e\varphi, \tag{10}$$

equation (7) takes the form

$$\frac{\partial \overline{f_1}}{\partial t} + \left(\vec{\mathbf{v}}\cdot\nabla_\varepsilon\right) f_0 - e(\tilde{\vec{\mathbf{E}}}\cdot\vec{\mathbf{v}})\frac{\partial f_0}{\partial \varepsilon} + \nu\overline{f_1} = 0\,, \tag{11}$$

where ∇_ε is the gradient to be calculated at fixed ε. On the other hand, if the frequencies ω of RF or microwave fields $\tilde{\vec{\mathbf{E}}}\left(\vec{r},t\right)$, which maintain a discharge, exceed the energy relaxation frequency τ_ε^{-1}, the isotropic EDF part $f_0\left(\vec{r},v,t\right)$ is too inertial to follow the heating field variation and the kinetic equation can be averaged over the fast field oscillations. Substituting (11) and the standard expression for the zeroth collision integral spherical harmonic to (5) and performing the averaging over the fast oscillations, we obtain an equation for the EDF f_0:

$$\begin{aligned} &-\frac{\partial f_0}{\partial t} + \frac{1}{v}\left(\nabla_\varepsilon \cdot v\, D(w)\nabla_\varepsilon\right) f_0 + \frac{1}{v}\frac{\partial}{\partial\varepsilon}(v(D_\varepsilon(\vec{r},w,t)\frac{\partial f_0}{\partial\varepsilon} + V_\varepsilon(w)\, f_0)) = \\ &St^{inel}(f_0) = N\, v\sum_k [\sigma_k(w)\, f_0(\vec{r},\varepsilon,t) - \sqrt{1+\varepsilon_k/w}\ \sigma_k(w+\varepsilon_k)\, f_0(\vec{r},\varepsilon+\varepsilon_k,t)]. \end{aligned} \tag{12}$$

where $\sigma_k(w)$ is the excitation cross-section of the $k-$th atomic level, the diffusion coefficient in space

$$D = v^2\nu/3, \tag{13}$$

and D_ε is the diffusion coefficient in energy. For example, for a monochromatic uniform oscillatory field with amplitude $E_{0\omega}$ we have

$$D_\varepsilon = \frac{e^2E_{0\omega}^2v^2\nu}{6(\omega^2+\nu^2)}. \tag{14}$$

As a result, the Boltzmann equation is reduced to the physically transparent form of a 2D diffusion equation in space and energy. The flux density along the energy axis is

$$\Gamma_\varepsilon = -\left(D_\varepsilon(\vec{r},w,t)\frac{\partial f_0}{\partial\varepsilon} + V_\varepsilon(w)\, f_0\right), \tag{15}$$

and the differential spatial flux (i.e., the flux density of electrons with energy ε per

unit energy range)

$$\vec{\Phi}(\varepsilon,\vec{r}) = -\frac{v^3}{\nu(v)}\nabla_\varepsilon f_0(\varepsilon,\vec{r}). \tag{16}$$

In low-pressure discharges the electron energy balance is typically controlled by the inelastic collisions. So we neglect the second term in the right-hand side of (15).

The expression (14) for the diffusion coefficient in energy can be interpreted as follows. As the EDF is almost isotropic, the directed electron velocity $\overrightarrow{\Delta \mathbf{v}}$ is small. It satisfies

$$m\frac{d\overrightarrow{\Delta \mathbf{v}}}{dt} = e\vec{E}_{0\omega}\exp(-i\omega t) - m\nu\overrightarrow{\Delta \mathbf{v}}.$$

After a collision this velocity component is transformed into the chaotic velocity. So this equation can be interpreted as if every collision results in a directed velocity kick $\overrightarrow{\Delta \mathbf{v}} = \frac{e\overrightarrow{E_{0\omega}}}{m(\nu - i\omega)}$.

The energy kick is

$$\Delta\varepsilon = m\nu\Delta v\cos\theta,$$

where θ is the scattering angle. So (14) can be interpreted in a standard form of the average product of squared random walk step to the step frequency

$$D_\varepsilon = \left\langle (\Delta\varepsilon)^2\nu \right\rangle = \frac{e^2E_{0\omega}^2v^2\nu}{6\left(\omega^2+\nu^2\right)}, \tag{17}$$

the factor 1/6 resulting from averaging over RF oscillations and over $\cos^2\theta$.

3. DC positive column.

In the case of cylindrical longitudinally homogeneous positive column it is more convenient to include in the total energy (10) only the radial potential $\varphi(r)$, and to define the diffusion coefficient in energy as

$$D_\varepsilon = \frac{(\Delta\varepsilon)^2\nu}{3} = \frac{e^2E^2\lambda^2\nu}{3}, \tag{18}$$

where E is the longitudinal field, λ is an electron mean free path. The solution of the problem of the trapped electrons in a DC positive column was formulated first by Bernstein and Holstein in 1954 in [5], but remained unnoticed for two decades to be rediscovered in [6].

If the column radius R is small with respect to the energy relaxation length, the distribution function must have the form $f_0(\varepsilon)$ with small coordinate-dependent corrections responsible for the origin of spatial differential fluxes (16). Since electron displacement occurs faster than energy relaxation, the spatial fluxes are practically independent of each other at different ε and they cannot be described in terms of the conventional fluid approach; at different ε values they can even be oppositely directed [7]. The dominant spatial terms in equation (5) can be eliminated by averaging over the available region. So equations (5), (12) can be reduced to the conventional 1D form by replacing the kinetic energy w with the total energy ε. A procedure for solving (5) for this case was proposed in [5,6].

Within the traditional local approach, which is valid at large R, the simplification of the kinetic equation has been achieved by neglecting terms resulting from spatial

nonuniformity in (5). This is certainly not permitted for low pressure for which a rather simple method which relies on three main ideas exists.

(i) It is assumed that the whole electron kinetics within discharge plasma is described by a unique EDF of the total electron energy.

(ii) This EDF of the total energy can be derived from a spatially averaged kinetic equation, which is a one-dimensional ordinary differential equation, no matter how many spatial dimensions are considered!

(iii) The spatially resolved EDF of the kinetic energy and coordinate is obtained by a cutting procedure from the EDF of the total energy. This 'generalized Boltzmann relation' for a non-Maxwellian EDF was described in [8].

Collisions, heating and transport result in a small coordinate-dependent correction $\left(f_0^{(0)} >> f_0^{(1)} \right)$:

$$f_0(\varepsilon, r) = f_0^{(0)}(\varepsilon) + f_0^{(1)}(\varepsilon, r). \tag{19}$$

The main simplification within the nonlocal approach is made by the spatial averaging of the kinetic equation over the part of the discharge cross section accessible to electrons with a certain total energy (see Fig. 1). An arbitrary average quantity G is then defined by

$$\overline{G}(\varepsilon) = \frac{1}{V_0} \int_{V_{ac}} G(\varepsilon, r)\, dV\,. \tag{20}$$

Here V_0 represents the total discharge volume. The accessible volume V_{ac} is defined by

$$\varepsilon \geq -e\varphi(r)\,. \tag{21}$$

The V_{ac} boundary S_{ac} (21) is thus given by $w\left(S_{ac}\right) = 0$. The diffusion term in equation (12) can be shown to cancel by applying the Gaussian law:

$$\int_{V_{ac}} \nabla_{r'} \left(\frac{v^3}{\nu} \nabla_{r'} f_0^{(1)} \right) dV = \oint_{\text{boundary}} \left(\frac{v^3}{\nu} \nabla_{r'} f_0^{(1)} \right)_n dS_{ac} = 0. \tag{22}$$

The second integral vanishes, since the kinetic energy and the velocity are zero at the S_{ac} boundary. Thus the integrand, which represents the spatial flux of electrons, vanishes at the available region boundary. With this result, neglecting the energy loss in the quasielastic collisions and accounting for (20), the averaged kinetic equation (12) becomes

$$-\frac{d}{d\varepsilon} \overline{w^{1/2} \left(D_\varepsilon(\varepsilon) \frac{df_0^{(0)}(\varepsilon)}{d\varepsilon} + V_\varepsilon(\varepsilon) f_0^{(0)}(\varepsilon) \right)} = \overline{w^{1/2} St^{inel}(w)} \cdot f_0^{(0)}(\varepsilon). \tag{23}$$

In the right-hand side, the following terms are included:

$$\overline{w^{1/2} St^{inel}(w)} \cdot f_0^{(0)}(\varepsilon) = -\sum_k \left(\overline{w^{1/2} \nu_k^*(w)} \cdot f_0^{(0)}(\varepsilon) - \overline{w^{1/2} \nu_k^*(w + \varepsilon_k)} \cdot f_0^{(0)}(\varepsilon + \varepsilon_k) \right) \tag{24}$$

with

$$\overline{w^{1/2} \nu_k^*(w)} = \frac{1}{V_0} \int_{w(r) \geq \varepsilon_k} w^{1/2}(r) \nu_k^*[w(r)]\, dV.$$

The last averaging is performed over the region where the k-th excitation is possible. This region is marked as the 'excitation region' in Fig. 1

It should be stressed again that the averaging (23), (24) of the kinetic equation (12) is nothing other than the mathematical formulation of the physical fact that spatial

diffusion is a much faster process than diffusion in the energy space. Thus, the energy gained from the electric field is redistributed over the whole accessible cross section via rapid spatial motion. This, however, means that every point of the cross section contributes to the formation of the total energy EDF, which is equivalent to the averaging procedure employed.

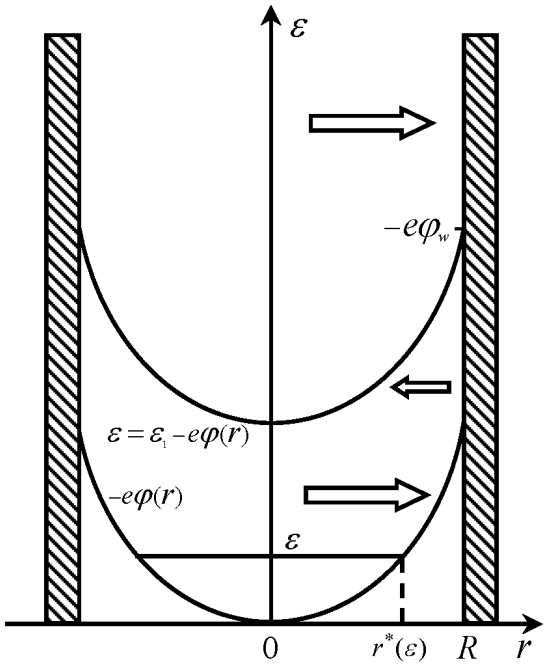

Fig. 1. Energetic conditions for trapped electrons. The accessible region for the trapped electrons in (ε, r) plane is bounded by $\varepsilon \geq -e\varphi(r)$. In the dashed part $r < r^*(\varepsilon)$ they have $w > \varepsilon_1$, and are capable to undergo the inelastic collisions with excitation of neutrals. The turning point $r^*(\varepsilon)$ is defined as $\varepsilon = -e\varphi(r^*)$. The arrows correspond to the differential fluxes.

In the low-pressure discharges contribution of the elastic collisions to the electron energy balance is, as a rule, negligible, and the second term in the left-hand side of (23) can be omitted. It corresponds to the "pipeline" EDF, which transports electrons via the diffusion in energy to the EDF tail, where the inelastic collisions occur. At $\varepsilon < \varepsilon_1$ in the absorbing wall $(\nu^*(\varepsilon) \to \infty)$ approximation the pipeline EDF is of the form

$$f_0^{(0)}(\varepsilon) = \int_{\varepsilon}^{\varepsilon_1} \frac{d\varepsilon'}{w^{1/2} D_\varepsilon(\varepsilon)}. \tag{25}$$

The boundary between the EDF tail and the body in the non-local case corresponds to the total energy $\varepsilon = \varepsilon_1$ (Fig. 1). The steep decrease of the EDF starts at $\varepsilon \geq \varepsilon_1$ instead of $w \geq \varepsilon_1$ in the local case. It seems paradoxical that on the plasma periphery (see Fig. 1) the EDF decreases due to inelastic collisions in the regions of the phase space where these collisions do not occur, namely at $w < \varepsilon_1$ with $\varepsilon > \varepsilon_1$. The reason is that at these ε values the inelastic collisions occur mainly in the central region and the small correction term $f_0^{(1)}(\varepsilon, r)$ corresponds to an inward flux of particles and energy.

Since the available area expands with ε, the differential flux at $\varepsilon < \varepsilon_1$ is outward directed. On the other hand, the electrons with $\varepsilon > e\varphi_w$ can escape to a vessel wall (φ_w is the wall potential). It means that the differential flux of these electrons at the plasma periphery is to be outward-directed, too. The divergence of all these fluxes in low-pressure discharges is considerable even though $f_0^{(1)}$ is a small correction.

It is obvious that this complex pattern cannot be described in terms of the fluid approach of a small ambipolar outward-directed particle flux. This situation cannot be improved by accounting of the thermal diffusion.

An attempt to describe the electron energy flux results in similar paradoxes. Neglecting the net particle flux of the trapped electrons, the energy flux in the fluid approximation is due only to thermal conductivity. Since there is no particle flux for trapped electrons, the energy flux in this approximation is due only to thermal conductivity. The local "electron temperature" profile $T_e(x)$ is determined by the EDF body at $\varepsilon < \varepsilon_1$. So the $T_e(x)$ gradient, which is responsible for the energy flux, depends on the $\nu(w)$ and $\varphi(r)$ functions. This may result in a physically meaningless conclusion about an outward energy flux.

It is surely one of the most interesting aspects of the nonlocal approach that the spatially resolved EDF of kinetic energy can be found from the EDF of the total energy. At first glance, it seems paradoxical that spatially resolved information should be gained from a spatially averaged kinetic equation. However, if the single EDF of the total energy has been found, the kinetic energy EDF at every position can be calculated unambiguously via the ambipolar space charge potential. With a simple back-substitution, one obtains the kinetic energy EDF

$$f_0(w,r) = f_0^{(0)}[\varepsilon = w - e\varphi(r)]. \tag{26}$$

The physical interpretation of equation ((26)) is simple (cf. Fig. 2).

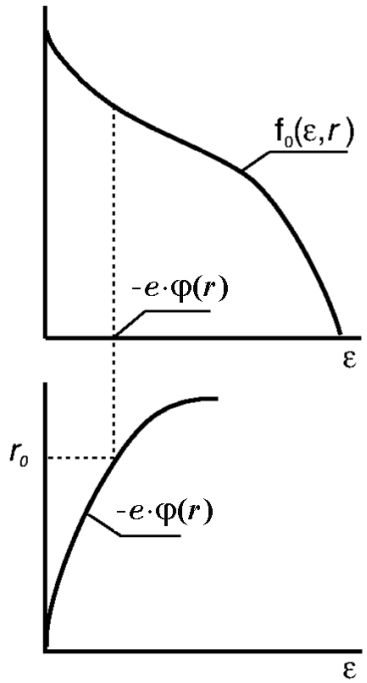

Fig. 2. Scheme for finding the EDF of kinetic energy $f_0^{(0)}(w,r)$ from the EDF of total energy $f_0^{(0)}(\varepsilon)$ and the ambipolar potential $\varphi(r)$.

At any position r, the space charge potential $\varphi(r)$ determines the minimum total energy needed for an electron to reach this position. Electrons with a lower total energy are confined to regions of lower potential energy. The total energy EDF is thus cut off at $\varepsilon < -e\varphi(r)$, and only the part with $\varepsilon \geq -e\varphi(r)$ forms the EDF of the kinetic energy at r. This relation represents, in a sense, a generalized Boltzmann relation [8] for a non-Maxwellian EDF. For the electron density we have

$$n_e(r) = \frac{4\pi\sqrt{2}}{m^{3/2}} \int_{-e\varphi(r)}^{\infty} \sqrt{\varepsilon + e\varphi(r)}\, f_0(\varepsilon)\, d\varepsilon. \tag{27}$$

In the case of a Maxwellian EDF the expression (27) corresponds to the traditional Boltzmann relation.

The simplifications of modeling the electron kinetics in the nonlocal approach are enormous. Regardless of the number of spatial dimensions which are included in an electron kinetic model, the EDF is determined by a one-dimensional ordinary differential equation. This kind of equation can be solved by various efficient and well-tested algorithms. Of course, the number of spatial dimensions is still included in the definition of the averaged coefficients of this differential equation. Nevertheless, the computation of multidimensional integrals can be performed much more effectively than the solution of a partial differential equation.

It should be mentioned that much evidence has been found for the validity of the nonlocal approach. The first experimental demonstration was given by Wiesemann in 1969 in Ref.[9], but unfortunately this finding has not been widely recognized. A convincing demonstration of the nonlocality of the EDF was reported by Godyak and Piejak [10] in a capacitive RF discharge and by Kortshagen [11] in a surface wave plasma.

The applicability of the non-local approach at the EDF tail is restricted by:

$$L \ll \lambda^* = \sqrt{\frac{D}{\nu^*}} \simeq \sqrt{\lambda\lambda^*}, \tag{28}$$

where L is the vessel size, λ, λ^* are the mean free paths with respect to the elastic and inelastic collisions. It should be emphasized that if the energy input in the non-local situation above is eccentric such that the coefficient D_ε varies with the coordinate, the equation (23) for $f_0(\varepsilon)$ contains only the averaged values of D_ε. Even if the energy input is strongly asymmetrical, the resulting plasma density distribution will remain symmetric with respect to the vessel center. The self-consistent ion (plasma) density profile and the potential profile are symmetric with a maximum at the vessel center, whereas the asymmetry in the energy input D_ε can manifest itself only as a small shift of the resulting profiles, of the order of the non-locality violation).

There are many situations, in which the oscillatory heating field has a sharp localized maximum, so the energy gain by an electron occurs in the form of occasional energy kicks during its passage through this zone. The most common is the case of stochastic electron heating in capacitively and inductively coupled RF (CCP and ICP) discharges [20], [24]. It is possible also to derive a simple expression for the diffusion coefficient in energy for the non-local EDF.

The non-local EDF $f_0(\varepsilon)$ is defined by the space-averaged diffusion coefficient in energy $\overline{w^{1/2} D_\varepsilon}(\varepsilon)$ with D_ε defined by (14), (17), (18). The averaging is performed according to (20). For a cylindrical DC positive column of radius R, homogeneous

along the $z-$axis, we have

$$\overline{w^{1/2}D_\varepsilon}(\varepsilon)=\frac{2}{a^2}\int_0^{r_1(\varepsilon)}\left(eE_z\lambda\right)^2\nu r dr/3, \tag{29}$$

where $r_1(\varepsilon)$ satisfies $-e\varphi\left(r_1(\varepsilon)\right)=\varepsilon.$ The expression (29) can be interpreted as the product of the averaged squared energy kick $\Delta\varepsilon=\left(eE\lambda\cos\vartheta\right)$ by the kick frequency ν. It follows that the contribution of this mechanism to the averaged diffusion coefficient $\overline{w^{1/2}D_\varepsilon}(\varepsilon)$ equals the product of the average squared kick in energy, which an electron gains from the field and the frequency of electron interaction with the field localization region.

One of the first experimental observations of the EDF nonlocality [9] is presented in Fig.3. The second derivatives of the probe current, which are proportional to the isotropic part of the EDF f_0, for the non-local case, coincide with each other everywhere in the discharge volume.

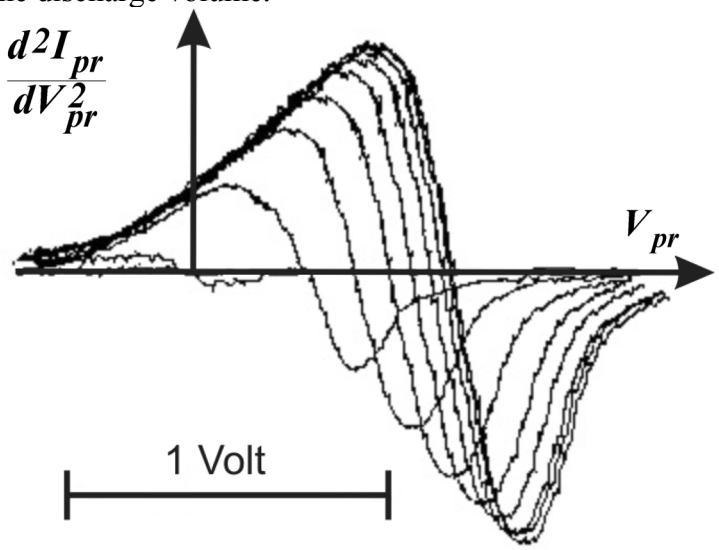

Fig. 3. The second derivatives of the probe current in different points of the discharge gap [9]. Discharge in Xe **at** $0.1Torr$, **and** $I=2\text{mA}$**. The coincidence of the left parts of the curves corresponds to the EDF dependence on the full energy** ε**.**

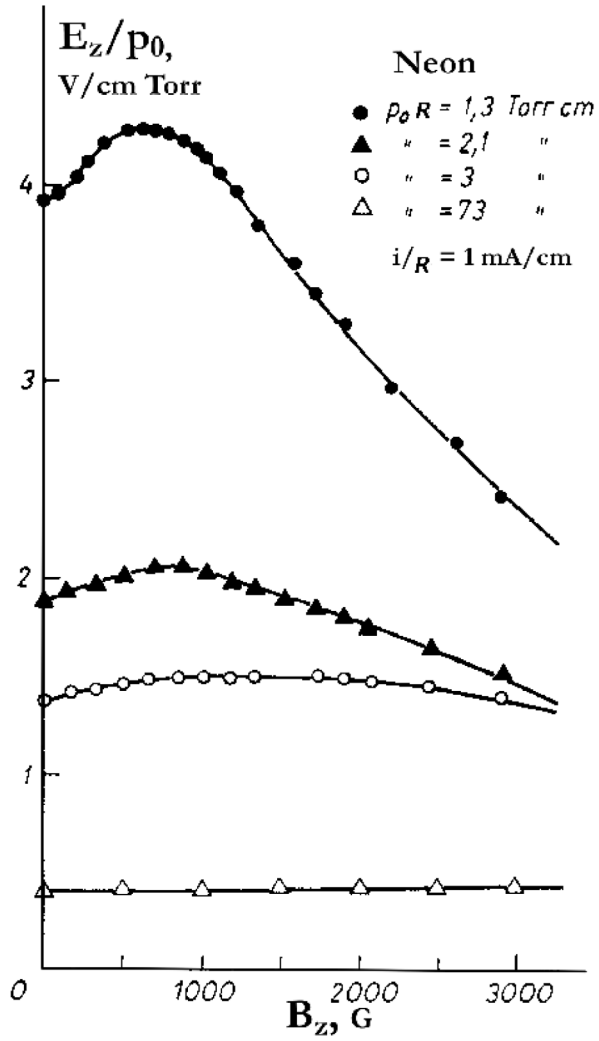

Fig. 4. The dependence $E_z(B_z)$ in Ne at $I/R = 1 mA/cm$ [12]. The numbers at the curves are the (Rp) values: 1, 1.3, 2.0, 2.1, 3, 3.0, 4, 7.3.

Convincing evidence of the EDF nonlocality in a positive column can be seen in Fig.4. Here the dependence of the self consistent longitudinal electric field E_z as a function of the externally applied magnetic field B_z is shown [12]. Since the electric field E_z is determined by the discharge maintenance condition, it rises as the charged particle lifetime decreases. As a magnetic field suppresses the transversal charged particle transport and increases the lifetime, the falling branch of the $E_z(B_z)$ dependence seems quite natural. As the ambipolar lifetime decreases with B_z, the lower ionization rate becomes necessary for the plasma stationary maintenance. It demands the lower field E_z, which creates the EDF with lower electron fraction in the EDF tail. Far more surprising was the ascending branch of the $E_z(B_z)$ dependence. It was observed at rather weak magnetic field, when the plasma lifetime, which was controlled by ions, remained practically unaffected by B_z. The estimates show that the maxima of the $E_z(B_z)$ dependences corresponded to the transition from the local to the nonlocal EDF. As in the nonlocal case the EDF depends from the total energy

(10), the steep EDF decrease starts at $\varepsilon > \varepsilon_1$. At the plasma periphery it corresponds to the kinetic energy $w > \varepsilon_1 - e\varphi(r)$, which is considerably lower than ε_1. So the excitation and ionization processes are substantionally suppressed at the plasma periphery. It follows that the plasma generation rate is more strongly peaked at the plasma center, than the plasma density profile. In other words, the average distance between the generation place and the tube wall, where the charged particles recombine, decreases as the transition from the nonlocal to the local case occurs. It means that the average lifetime decreases, too. So, the necessary for stationary discharge maintenance E_z value increases with B_z. In the self-consistent simulations [13] this concept was clearly demonstrated.

Convincing manifestations of EDF nonlocality can be found among phenomena of plasma luminosity. The transition from the local to the nonlocal EDFs can be seen in behavior of discharge luminosity with pressure reduction in discharges with peripheral energy input. At high pressure, when the EDF is local, the luminosity is maximal at the peripheral region, where the energy input is maximal. In CCP , for example, Joule heating is maximal in the peripheral region, since the electron conductivity current, which is practically equal to the total current, is uniform in a plasma, and the plasma density is low here. In ICP the skin-effect leads to a similar result. On the other hand, since the nonlocal EDF depends on the total energy ε, and the ambipolar potential $\varphi(r)$ is maximal at the vessel center, the fraction of the fast electrons, and the discharge luminosity, shift to the discharge center. This effect was observed in [14].

An interesting exception from the rule that at low pressure the luminosity is to be maximal at the plasma center, can be seen in a spherical discharge vessel for which $R > \lambda$ [15]. In this case the isotropic EDF f_0 depends on two variables: the total energy ε, and the angular momentum μ. In the vessel center only the electrons with small μ are present. On the other hand, these electrons with small μ fall to the vessel wall almost normally**.** So the electrons with $\varepsilon > e\varphi_{fl}$, where φ_{fl} is the floating potential of dielectric wall, can escape to it, and are to be practically absent. As a result, the radiation of the lines with upper level energies exceeding $e\varphi_{fl}$ from a central discharge region is to be strongly suppressed.

At higher pressure the EDF depends only from the total energy ε, and the excitation rates become maximal at the discharge center, independently from the vessel form. Since the energy relaxation time at the EDF tail, $\left(\nu^*\right)^{-1}$, is considerably shorter, than the relaxation time of the EDF body, the with the pressure rise the non-locality condition is violated first at the EDF tail, and the EDF f_0 here becomes coordinate- and ε-dependent. As a result, in the intermediate pressure range, when the EDF tail is local, and the EDF body is nonlocal, the excitation rates profiles become non-monotonic [16], see Fig.5.

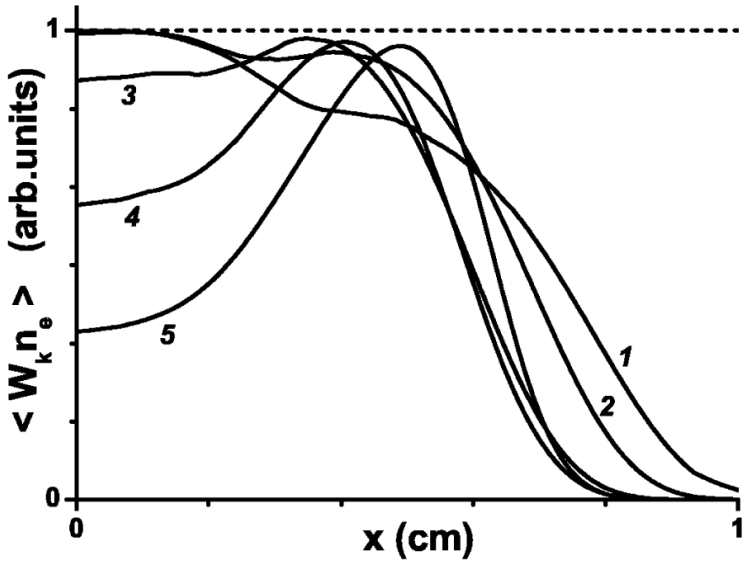

Fig. 5. Spatial profiles [16] of the excitation rate for $E_z = 5V/cm,\ p = 3\,Torr$ **and** $-e\varphi(x) = a\varepsilon_1 (x/R)^3$ **for different** a **: 0 (dotted line); 0.25(1); 1(2); 1.5(3).**

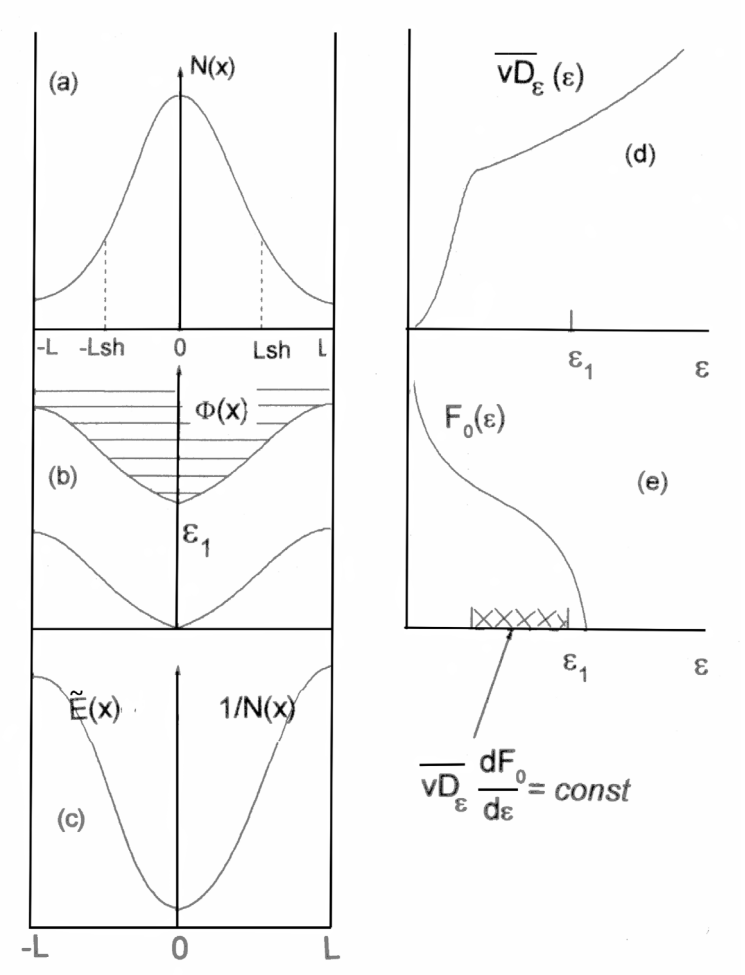

Fig. 6. Sketch of the parameters profiles in a EFC discharge [15].

4. RFC discharges.

The electric field in RFC discharges in the usual parameter range is a potential one and can be subdivided into several strikingly distinct parts. The central part of a discharge is filled by quasineutral plasma. The electric field here consists of a (quasi)stationary ambipolar part with a potential component $\varphi(r)$, responsible for trapping electrons in the discharge center, and of an oscillatory RF component $\tilde{E}(r,t)$, which maintains the RF electron current. The strong electric field in the ion space charge region consists of stationary and of RF components, too. As the thickness of the transition layer between the plasma and the space charge regions is thin - of the order of the Debye radius - it can be treated as infinitely thin (with respect to the sheath thickness L_{sh} in Fig.6) oscillating surface [17], which reflects electrons. If these reflections can be treated as statistically independent, they result in stochastic electron kicks and in electron diffusion in energy - as the supplementary electron heating mechanism, called stochastic electron heating, which replaces the traditional Joule collisional heating at low pressures [18]. We would restrict here by the simple model. In detail this problem was discussed in [19, 21].

If a discharge gap L exceeds the electron mean free path λ, an energy kick, which an electron receives in a collision with the moving reflecting potential wall of the space charge field, will obviously be transformed into the chaotic energy due to a collision with a neutral molecule. If $L << \lambda$, the subsequent kicks can be considered as statistically independent, if the Chirikov's criterion [20] is fulfilled

$$\frac{\omega L \Delta v}{v^2} > \pi, \tag{30}$$

where Δv is the velocity kick, ω is the driving frequency, or collisions are frequent enough $\nu > \frac{v(\Delta v/\omega L)^{1/2}}{L}$, see Ref. 19. The kick Δv is equal to the directed electron velocity at the plasma-space charge interface, which is small with respect to the chaotic electron velocity, if (6) is fulfilled. So if condition (30) holds, stochastic heating can be described as diffusion in energy, too. Analogously to (17), the expression for the stochastic diffusion coefficient in energy can be written as

$$D_{\varepsilon}^{(stoch)}(\varepsilon) = \left\langle (\Delta\varepsilon)^2 \Omega \right\rangle,$$

where $\Omega(\varepsilon,\theta)$ is the bounce frequency of an electron with total energy ε. The averaging is to be performed over the RF period and over the electron velocity angles. If a discharge width L is small with respect to the energy relaxation length λ_{ε} (28), and both the applied field and the electron bounce frequencies are higher than the inverse energy relaxation time, the EDF form, according to (23), depends only on averaged in space and time function of the total energy $\left\langle \overline{w^{1/2} D_{\varepsilon}} \right\rangle(\varepsilon)$. This function can be interpreted as a sum of squares of the energy kicks, which occur per second over the whole available for an electron with energy ε volume. So the EDF satisfies the averaged kinetic equation (23) with replacement of $\overline{w^{1/2} D_{\varepsilon}}(\varepsilon)$, (29) for sum of two terms: collisional

$$\left\langle \overline{w^{1/2} D_{\varepsilon}^{\mathrm{coll}}} \right\rangle(\varepsilon) = \frac{\sqrt{2/m}}{6L} \left\langle \int_0^{L(\varepsilon,t)} \frac{e^2 E_{0\omega}^2(x) v^3 \nu(v)}{\omega^2 + \nu^2(v)} dx \right\rangle,$$

and stochastic

$$\left\langle \overline{w^{1/2} D_\varepsilon^{stoch}} \right\rangle (\varepsilon) = \int_0^{\theta(\varepsilon)} \left[m v V(t) \right]^2 \frac{v}{3\pi L} d(\omega t).$$

Here $V(t)$ - the plasma-space charge interface velocity [17], $L(\varepsilon, t)$ - the boundary of the available area, $\theta(\varepsilon)$ - the phase during which an electron with energy ε can reach this interface, $v(\varepsilon, x) = \sqrt{2(\varepsilon - e\phi(x))/m}$. Here the simple model expression (17) for $D_\varepsilon^{\text{coll}}$ was used. Of course, the general definition of an energy kick remains rather delicate problem; for more detail see [20].

The scenario of the electron heating in a RFC discharge is presented in Fig.6 [22, 23]. Since the electron density profile is bell-shaped, Fig.6a, the RF field $\tilde{E}$, which heats electrons, is minimal at the gap center, and maximal in the RF sheath, Fig.6c. As the ambipolar potential $\Phi(x)$, Fig.6b, traps the electrons with low ε in the central region, where the field $\tilde{E}$ is minimal, the regions with high $\tilde{E}$ are unavailable for these electrons, and the averaged value of $\overline{w^{1/2} D_\varepsilon}(\varepsilon)$, which, according to (25), determines the EDF slope, is minimal for them, Fig.6d. This effect explains the EDF formation, which contains a considerable population of the slow electrons, Fig.6e, in discharges with a peripheral energy input [15], as it was observed in [10]. In Ref. [23] it was shown that the transition to the discharge mode with abundant slow electrons can proceed abruptly analogously to a thermal explosion. It is to be noted that both the stochastic and the collisional mechanisms, which are both maximal at the discharge periphery, contribute to this phenomenon. So to distinguish between them from experimental data remains a rather complicated problem. It needs detailed calculation and knowledge of power balance.

The collisionless stochastic electron heating in low pressure inductively coupled discharges was discussed in detail in [24].

The similar phenomena arise in the vicinity of the resonance region in a microwave field [25], [27].

5. Cathode region of a DC discharge.

The complex self-consistent nonlocal phenomena in a cathode region of a DC glow discharge are traditionally classified phenomenologically by its visual properties. So the cathode region is subdivided into the cathode dark space, negative glow, and Faraday dark space, which contacts with a homogeneous or stratified positive column. Nevertheless, the optical plasma characteristics result from a rather complicated self-consistent sequence of processes, and in order to clarify the underlying physics it seems more convenient to choose as fundamental the electrodynamic and kinetic plasma parameters [26].

First of all, the cathode region consists of quasineutral plasma domain, and of the cathode ion space charge sheath. At the cathode surface, $x = 0$, a current is transported mainly by ions; the electron (j_e) and the ion (j_i) current densities satisfy

$$j_e(0) = \gamma j_i(0),$$

where $\gamma << 1$ is the electron-ion secondary emission coefficient. The electric field in the sheath is rather strong (Fig.7a); the electrons, emitted by the cathode surface or born in the sheath, are accelerated rapidly by this strong field, and the EDF of these

fast electrons is nonlocal. It is determined not by the local (E/p) value, but by the upstream potential profile. The current of these fast electrons increases exponentially in the sheath, and is maximal at the plasma-sheath interface. The fast electrons penetrate to the plasma region and produce the nonlocal ionization there. Electrons born close to this interface, where the field decreases, have small range, and are stopped fast; the electrons, which were born close to the cathode surface, penetrate deeper. So the current of the fast electrons, which produce ionization and excitation (the curve El1 in the Fig. 7d), is maximal at the plasma-sheath interface. It means that the luminosity is maximal here, too (Fig.7c). Note, that this paradigm totally contradicts to the traditional local approach, which predicts zero ionization and excitation rates at this interface. The length of the negative glow is determined by the range of the most energetic electrons which were emitted by cathode. So the negative glow consists of two distinct regions - of the space charge part, and of the plasma part, luminosity (and the ionization rate) being maximal at the boundary between them. In other words, a considerable part of the ionization occurs in the plasma.

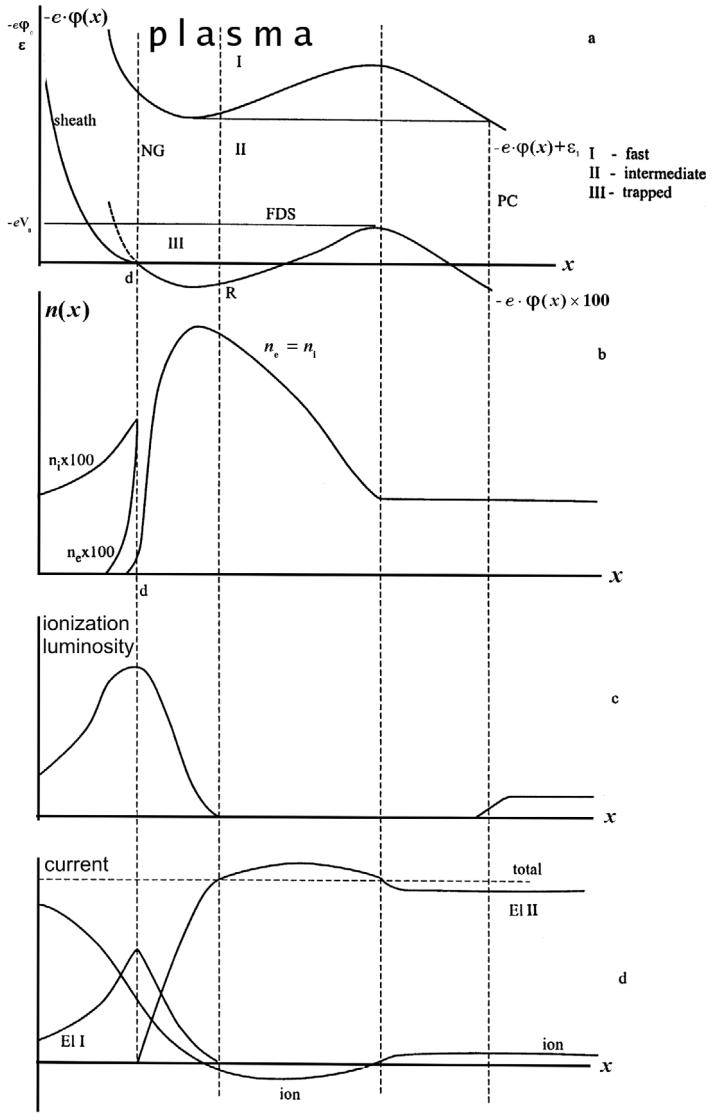

Fig. 7. Sketch of the parameters profiles in a DC cathode fall [26]; the upper curve in Fig. 7a satisfies $\varepsilon = -e\varphi(z_2(\varepsilon)) + \varepsilon_1$.

The traditional Townsend's condition of stationary discharge maintenance states that the multiplication of a cathode-emitted electron, which equals to the number of ions born by it, is to be equal to $(\gamma)^{-1}$. In other words, the ions, generated both in the space charge and in the plasma sections of the negative glow, are to be returned to the cathode. The ions from the sheath section of the negative glow are easily transported to the cathode by the strong field in the sheath. In the plasma part the only mechanism which is able to deliver the plasma-born ions to the cathode is the feeble mechanism of ambipolar diffusion. It means a large plasma density gradient and a plasma density peak arises in the plasma part of the negative glow, with density in it far (more, than order of magnitude) exceeding the density value in the positive column (Fig.7b). On the other hand, the electron diffusive current towards the anode far exceeds the total current, and a potential profile arises, which suppresses the electron diffusion towards the anode. So the potential well for electrons is formed (Fig.7a), and the electrons can be separated into three groups with clearly defined boundaries between them. The first of them consists of the fast electrons, which are emitted by the cathode, or generated in the sheath; their energies far exceed the characteristic atomic excitation and ionization energies. The second group consists of the trapped electrons. The electrons trapped in the potential well do not participate in a current transport. Since the Joule heating of these electrons is absent, they have a Maxwell-Boltzmann distribution with an electron temperature of the order of the room temperature. Their full energy lies between the potential minimum and maximum (Fig.7a). The third electron group consists of the intermediate electrons, with the energy ε, which exceeds the potential maximum (Fig.7a), and is slightly (by the potential well depth) lower, than the first excitation energy ε_1. Since in the Faraday dark space only the trapped and the intermediate electrons are present, the intermediate electrons are transporting the electron current here. Neglecting the energy loss in the elastic collisions, their EDF at $\delta \to 0$ can be written analogously to (25), as

$$f_0^{(int)}(\varepsilon) = \Phi(\varepsilon) \int_z^{z_2(\varepsilon)} \frac{dx'}{w^{1/2} D(\varepsilon, x')}$$

From the Fig.7a it is seen that in the Faraday dark space the excitation rate is totally absent, because the electrons with the full energy more than by ε_1 exceeding the potential well bottom, are absent here.

6. Conclusions.

The concept of the electron diffusion coefficient in energy turns out to be very useful and efficient in various problems of gas discharge plasma kinetics.Using it, the physical meaning of numerous important and interesting problems can be made more transparent and clear.

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