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# **Electron Transport Phenomena in Plasma Devices with $E \times B$ Drift**

**(Invited Review Paper)**

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**Abstract-** A review of plasma devices involving electron drift in crossed electric and magnetic fields ( $E \times B$  drift) and electron transport phenomena is presented. There are two important peculiarities of  $E \times B$  system: possibility to maintain a large electric field in a quasi-neutral plasma which allows transport of relatively large intensity beam of charged particle and an efficient impact ionization due to closed electron drift. Several technological applications of devices based on electron drift in  $E \times B$  field are under development, including plasma immersion ion implantation, energetic deposition of materials, magnetron sputtering and plasma propulsion. Despite very different applications, the underlining physics of operation of these devices is very similar. One of the important physical phenomena is the electron transport across a magnetic field. Experimental and theoretical study reveals that electrons undergo anomalous transport and several possible mechanisms are proposed and studied previously. Anomalous electron transport mechanisms such as Bohm diffusion and near-wall conductivity are reviewed and assessed for two  $E \times B$  devices, namely magnetron and Hall thruster. A modified model of the near-wall conductivity that takes into account various sheath effects is developed. It is shown that an axial electric field in the sheath can significantly affect the near wall conductivity.

**Keywords:** Hall thruster, magnetron, anomalous electron transport, near-wall conductivity

## **I. Introduction**

Devices based on a closed electron drift are currently applied in a number of areas including plasma immersion ion implantation, magnetron discharge, and electric thrusters for spacecraft propulsion. Although having different applications, these technologies are based on the same physical phenomena of using magnetic field in order to maintain large electric field in the quasi-neutral plasma. This configuration results in the closed electron drift and electrostatic control of the ion flow. While ion dynamics in most devices can be relatively easily described, the underlying physics of electron transport is not well understood. As it will be shown below the main problem lies in description of the electron transport mechanism across a magnetic field, which was found to be largely non-classical. Several possible mechanisms of anomalous electron transport were proposed and investigated over years and some progress was achieved. In this paper we summarize recent achievements and outline outstanding problems remained. Before describing the electron transport let us first briefly review various application of the plasma discharge with closed electron drift.

## **II. Devices with closed $E \times B$ electron drift**

In this section we describe several systems based on  $E \times B$  drift. The main attention will be paid to their application, specific conditions and important issues related to understanding the underlying physical phenomena.

### **1. Plasma immersion ion implantation**

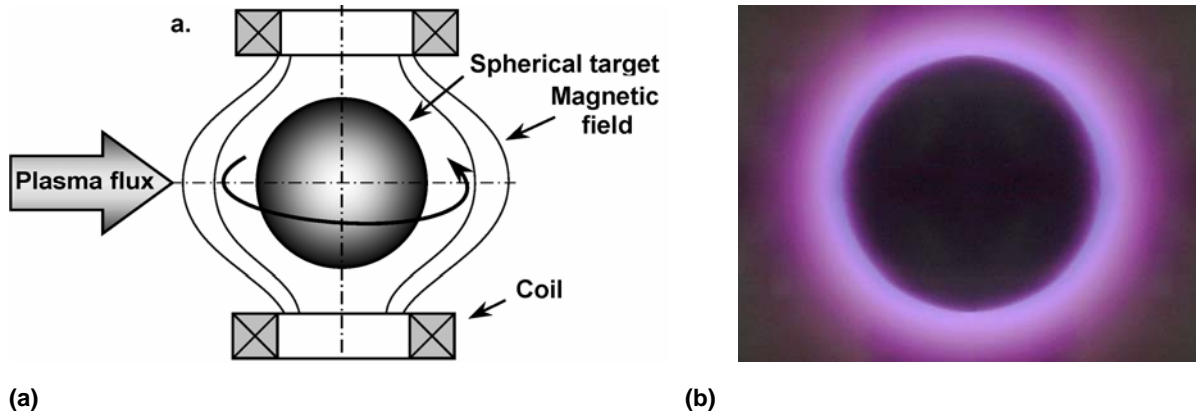
The implantation of highly energetic ions is found to be an efficient technology tool for modification of material surfaces<sup>1</sup>. Ion implantation is used to inject ions to depths of hundreds up to thousands of Angstroms below the surface that forms non-equilibrium structure that is difficult or even impossible to form in other ways. For instance, ion implantation is used as a means of doping the semiconductor elements of integrated circuits. Conventional ion implantation technology has some shortcomings such as small ion beam size, low ion current and high cost. An alternative to the usual method, so-called plasma immersion ion implantation (PIII), was proposed by Conrad and co-workers<sup>2,3</sup> and much progress has been made by a growing number of researchers around the world<sup>4,5,6,7,8</sup>. In this method, the target to be implanted is immersed in a gaseous plasma of the desired implantation species. By repetitively applying negative high-voltage bias pulses to the substrate, ions are extracted from the plasma, accelerated across the high voltage sheath and implanted into the surface. Both metallurgical and semiconductor<sup>9</sup> implantation processes have been demonstrated using PIII. PIII was shown to also be a promising method for hydrogenation of thin film transistors (TFTs) for flat panel displays<sup>10</sup>, formation

of shallow junctions, synthesis of silicon-on-insulator (SOI) structures<sup>11</sup>, and nanofabrication<sup>12</sup>. A related and very promising new kind of surface modification has been developed based on the use of vacuum arc generated metal plasmas<sup>13,14,15</sup>. Importantly, PIII with metal plasma makes it possible to establish a combined process due to the condensable nature of the plasma. Thus the vacuum arc plasmas extend the possibilities of PIII substantially. The metal plasma formed by a vacuum arc discharge is created on the cathode surface (at cathode spots) and expands away from the cathode in a direction normal to the surface with a speed that is typically 1 to 3 cm/ $\mu$ s.<sup>16</sup> This magnitude is of the order of the ion acoustic speed in the plasma or up to several times the acoustic speed<sup>17,18</sup>. The characteristics of vacuum arc plasmas have been investigated and described by a large number of authors<sup>19,20</sup>. An interesting feature of the vacuum arc in contrast to the gaseous plasma is that the vacuum arc plasma is fully ionized and has much higher plasma density (and therefore technological process efficiency). Unfortunately, the vacuum arc plasma jet contains liquid droplets or solid particles, commonly called macroparticles<sup>21</sup>. The macroparticle contamination is a major disadvantage of this source. Macroparticles generally move along straight trajectories due to their inertia, except for those that approach the duct wall at relatively small angle that can be reflected in the sheath<sup>22,23</sup>. Taking this into account, guiding of the plasma jet from the cathode to the substrate by a curved magnetic field is a technique often used for the production of a macroparticle free plasma<sup>24</sup>. The properties of the vacuum arc plasma motion through the curved magnetized duct were studied experimentally<sup>25,26,27</sup> and theoretically<sup>28,29,30</sup>. Very recently, considerable progress in advanced curved magnetized filters has been reported<sup>31</sup>.

Application of negative bias to the substrate immersed in the plasma leads to sheath formation near the substrate. Firstly, an ion matrix sheath (electron-depleted ion layer) is established on a time-scale of about the inverse electron plasma frequency. Then the ions respond on a time-scale of about the inverse ion plasma frequency. During this time, the high voltage plasma sheath propagates into the plasma at about the ion acoustic speed<sup>32,33</sup>. A growing sheath may deplete the plasma and may stop the process when the sheath expands all the way to the chamber wall, however, this will only occur at relatively low plasma densities and high bias voltages. Using a vacuum arc plasma source for PIII leads to several effects. The ions in the vacuum arc plasma jet have supersonic velocity (gained in the cathode spot region), which can significantly affect sheath expansion<sup>34</sup>. The plasma density along the plasma jet is non-uniform, which may lead to a plasma density increase at the sheath edge during sheath expansion<sup>35</sup>. In addition, ion current increases with increasing bias voltage, which was also explained in terms of plasma non-uniformity<sup>36</sup>. If the sheath is too thin there is a problem associated with electrical breakdown due to very high electric field strength at the target surface<sup>37</sup>. Sheath optimization issues were considered recently<sup>38</sup>. It is clear that keeping the sheath thickness in the optimal range is an important issue.

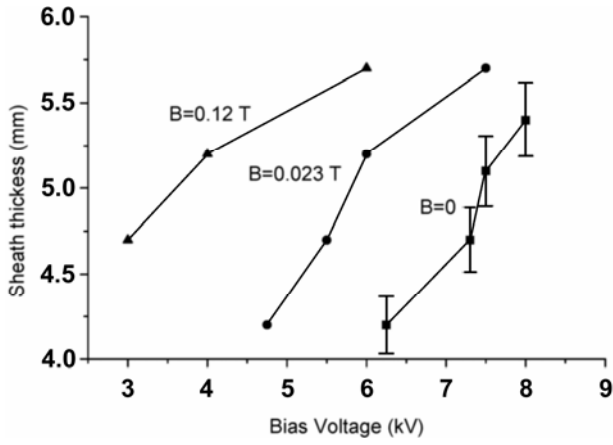
It is important to increase the sheath thickness in a dense plasma in order to avoid electrical breakdown.<sup>37</sup> One of the ways to control sheath thickness may be the use of a magnetic field. It is well known that a sheath is significantly affected by a magnetic field, especially in the case when the magnetic field is parallel to the surface or intersects the surface at small angles<sup>39</sup>. Therefore it is expected that a magnetic field will affect the sheath

dynamics under PIII conditions as well. Previously, magnetic fields have been considered in PIII to suppress secondary electrons and the associated generation of X-rays.<sup>8,40</sup>



**Fig. 1 (Color online). a: scheme of PIII process in cross-field setup with spherical target and external magnetic coils; b: Photo of discharge around spherical target**

One particular way of control of the high-voltage sheath by a magnetic field was recently studied.<sup>41</sup> A vacuum arc plasma gun with a silver cathode was operated with an arc current of 500 A in pulses of 250  $\mu$ s duration. The voltage (up to  $-8$  kV) was applied to the target. The magnetic field has its main component parallel to the surface area facing the plasma flow as shown in Fig.1. The measured steady state sheath thickness dependence on the bias voltage is shown in Fig. 2 with magnetic field as a parameter. One can see that steady-state sheath thickness increases with magnetic field. Generally these results suggest that the magnetic field has a significant effect on the high voltage sheath development and requires further detailed study. It is interesting to note that magnetic field leads to electron drift in  $E \times B$  direction; thus creating spherical electron drift cloud around the target shown in Fig. 1b. Sheath formation closely related to the electron transport across the magnetic field region as it was suggested in Ref. 41. In order to explain above observation a following rather simple arguments can be invoked. The magnetic field range considered in the above experiments is well below the magnitude required for ion magnetization. Therefore, in the steady state, the ions are accelerated toward the substrate by the electric field of the sheath without an effect of the magnetic field. The current density in the sheath can be calculated according to Child-Langmuir law.<sup>42,43</sup> The sheath thickness is determined by the applied voltage and ion density and velocity at the sheath edge. When a negative voltage is applied to a substrate immersed in a plasma, electrons are repelled from the substrate, leading to sheath formation. Electrons drift away from the target across the magnetic field due to the presence of the high electric field and rare collisions with ions and neutrals. In the steady state, the ions are then accelerated toward the substrate by the electric field of the sheath.



**Figure 2. Sheath thickness as a function of the applied bias, with the magnetic field as a parameter (Reprinted with permission from M. Keidar *et al.*, *Applied Physics Letters*, 81, 1183 (2002). Copyright 2002, American Institute of Physics.).**

While, in general, vacuum arc plasma jets are well understood<sup>44,45</sup>, the plasma density and velocity at the sheath edge in the specific configuration of considered experiment are determined by the plasma jet dynamics across the magnetic field layer in the target vicinity. The problem of the plasma propagation across a magnetic field is a long standing one having numerous applications.<sup>46,47,48</sup> In a simple quasi-one-dimensional approximation the electric field across the magnetic layer can be calculated as<sup>41,48</sup>:

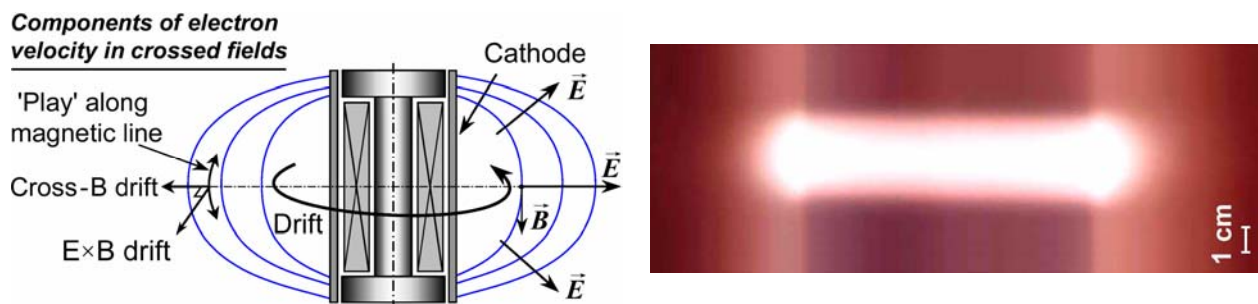
$$E = -\frac{kT_e}{e} \frac{d \ln N}{dx} - \beta BV + \frac{j}{\sigma} \quad (1)$$

where  $x$  is the direction along the plasma jet normal to the target,  $E$  is the electric field,  $\beta$  is the Hall parameter,  $B$  is the magnetic field,  $j$  is the current density,  $T_e$  is the electron temperature,  $V$  is the electron velocity along the electric field and  $\sigma$  is the conductivity. When a partially magnetized plasma (i.e., only electrons are magnetized) flows across a magnetic field, the plasma potential increases in order to provide quasi-neutrality across the layer. As a result, the ion velocity decreases in this electric field as the plasma propagates across the layer. When the plasma drift velocity decreases (as a result of the magnetic field effect) it leads to plasma spreading along magnetic field lines followed by plasma density decreases. Thus, plasma jet propagation across a magnetic field layer leads to decreases in the plasma density and ion velocity. According to this simple quasi-1d approximation argument the steady state sheath thickness would increase, which is in line with experimental observations. However it is clear that a detailed 2D model of plasma propagation across the magnetic field and electron transport in this configuration is required. Of particular interest for MPIII is the time- dependent response of the electrons and ions during the sheath formation in the magnetic field.

## 2. Magnetron discharges

Another example of plasma device based on  $E \times B$  drift, magnetron, is electrical discharge device used for sputter deposition of thin films<sup>49, 50</sup>. Modern magnetrons use crossed  $E \times B$  fields that cause a Hall current (Hall drift), directed across both field vectors. It is clear that uncontrolled Hall current will lead to electrons escape from the discharge. Electron losses can be limited by closing the drift current, i.e. by the use of such field configuration that will provide a circular pass of the drift current. In this case the use of closed-drift configurations ensured a significant increase in the plasma density and discharge efficiency.

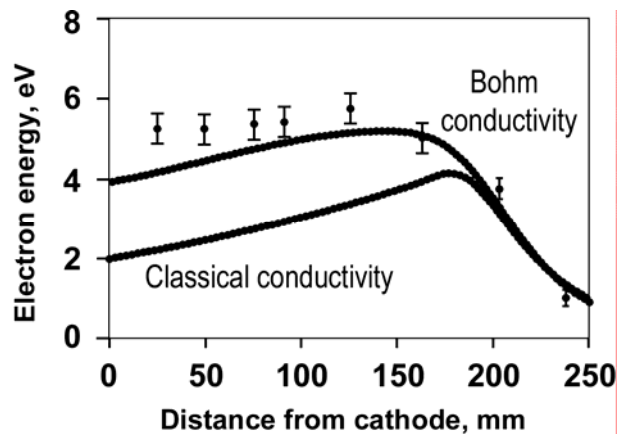
Currently, two main layouts are used, namely the planar one and cylindrical one. In planar devices, the closed drift current flows between the anode and cathode, while in the cylindrical, or axial magnetrons the drift current circulates around the electrode (usually cathode). Let us examine the physics of a cylindrical magnetron using the scheme shown in Fig. 3. The shape of a magnetic field in this setup resembles a semi-torus without an internal part. With the cathode biased and anode placed outside of the magnetic field (not shown in the scheme), a system of crossed electrical and magnetic fields will be created. A divergent electrical field intersects the torus-like magnetic field, and thus the main components of the  $E$  and  $B$  vectors are approximately orthogonal, creating a system of crossed fields. Electrons are trapped in the region of a high magnetic field and rotate around cylindrical cathode due to Hall effect, thus creating a Hall current. In addition electrons diffuse across the magnetic field due to collisions thus sustaining the main discharge current.



**Fig. 3 (Color online). Scheme of cylindrical magnetron and photo of cylindrical magnetron discharge**

The complex motion of the electrons results in discharge ignition around the cathode. As a rule, plasma bunch represents a torus located mainly in the central zone of magnetic field where an electrical field is mainly orthogonal to the cathode surface. Photos of typical magnetron plasmas are shown in fig. 3b, for classical magnetic field configuration shown in Fig. 3a.

Both reciprocating (along magnetic field) and Hall currents are rather important for sustaining plasma since they ensure an intense gas ionization, thus providing an influx of ions which are the main outcome of the process. Recently plasma configurations and electron transport in cylindrical magnetron discharges was studied.<sup>50,51</sup> It was found that two stable plasma configurations are possible around the negatively biased cylindrical target, namely torus and thin disk. Diffuse plasma torus changes the shape with magnetic field to form a thin disk when the target voltage is less than 400 V. Experiments with low-current magnetrons show that the measured electron mobility across the magnetic field scales as  $1/B$  and not according to classical scaling as  $1/B^2$  [52]. As a result the concept of anomalous or Bohm diffusion was introduced to describe the electron transport in magnetrons. Recently there was an attempt to analyze the mechanisms of electron cross-field mobility and its impact on the discharge characteristics.<sup>53</sup> A hybrid quasi-neutral model was based on phenomenological description of the electron transport. While it was not a self-consistent description i.e. experimentally measured electric field distribution was used, it reproduced very well major plasma properties. From the analysis of the electron energy distribution (shown in Fig. 4) it was concluded that the Bohm anomalous diffusion can properly describe the electron transport in cylindrical magnetron. It is interesting to note that in related study of the miniature Hall thruster (that will be discussed largely in the next Sections) it was concluded that the Bohm-type diffusion can properly describe the electron cross-field transport.<sup>54</sup> Thus existing experimental evidence and simulations support the idea of the anomalous transport, however it is not clear what physical phenomena may lead to anomalous electron mobility. It was shown by Sheridan and Goree<sup>55</sup> that low-frequency plasma turbulence is not responsible for electron transport across the magnetic field. Thus further study of electron transport and plasma turbulence spectra in magnetron plasmas is required in order to understand electron transport mechanism.



**Fig. 4.** Dependence of electron energy on distance from cathode for cylindrical magnetron discharge. From I. G. Levchenko, 2005 [53].

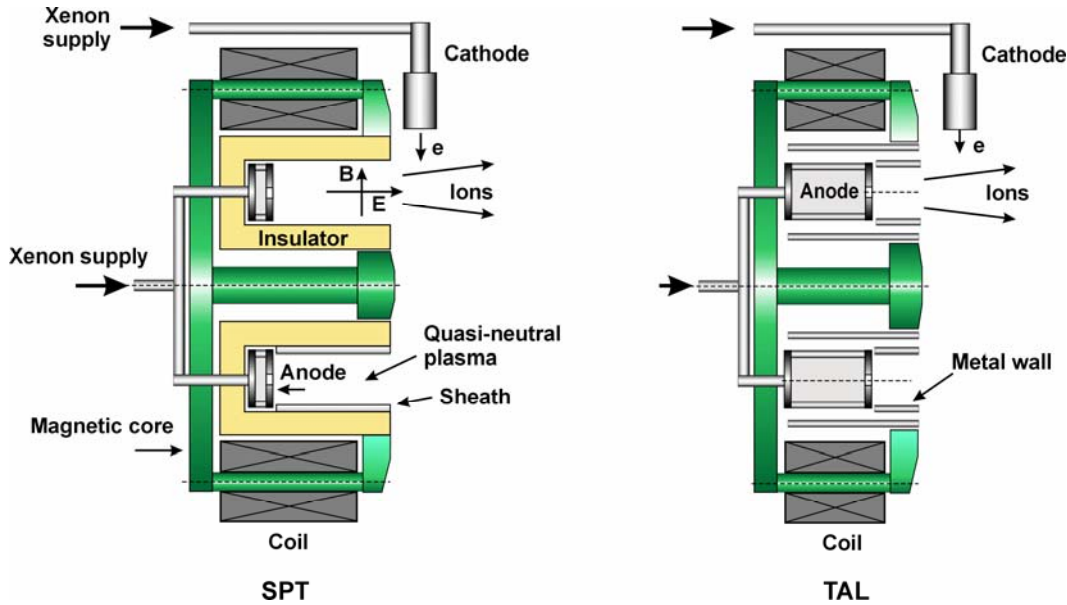
### 3. Hall thruster

Another device that involves an electrostatic acceleration of ions and a closed electron drift is so-called Hall thruster. A Hall thruster is currently one of the most advanced and efficient types of electrostatic propulsion devices for spacecraft with an efficiency of about 50%.<sup>56</sup> In particular this configuration is beneficial because the acceleration takes place in a quasi-neutral plasma and thus is not limited by space charge effects. The electrical discharge in the Hall thruster has a  $E \times B$  configuration where the external magnetic field is radial and perpendicular to the axial electric field, which accelerates the ions as shown schematically in Fig. 5. Passing the electron current across a magnetic field leads to an electron closed drift or Hall drift. The original idea of ion acceleration in crossed fields by using magnetron-type configuration was first introduced by Zharinov<sup>57,58</sup> Since initial development of this idea in 60's (see Refs. 59,60,61,62,63,64,65,66) numerous experimental and theoretical investigations have been conducted. Two different types of Hall thruster were developed: a thruster with closed electron drift and extended acceleration zone, or Stationary Plasma Thruster (SPT), and a thruster with short acceleration channel or Thruster with Anode Layer (TAL). In a SPT, the interaction of the plasma with the dielectric wall plays an important role. Due to the collisions of the electrons with the wall and secondary electron emission, the electron temperature remains relatively low in comparison to the TAL.<sup>67</sup> In TAL ion acceleration occurs in a very thin layer near the anode with thickness of about electron Larmor radius that gave the name to this thruster variant. Recently new mathematical solutions of the anode layer problem were found, i.e. so called B- and E-layers<sup>68,69</sup>. These solutions are different by the width of ion acceleration zone and by parameters of the cathode plasma. Similarly in magnetically insulated ion diodes ion acceleration occurs in a thin magnetic layer near the anode<sup>70</sup>.

Mainly the physics of the electron transport, propagation and neutralization of the ion beam, plasma interaction with a dielectric wall and the transition between the quasi-neutral plasma and the sheath, have not been understood completely. In fact, it was shown experimentally that the channel wall material has substantial effect on the discharge behavior in the Hall thruster<sup>71</sup>. Additionally it was found that use of sectioned electrodes inside the Hall thruster channel has a considerable effect on plasma properties, the discharge characteristics as well as thruster performance<sup>72,73</sup>. Two approaches for modeling plasma flows in Hall thrusters were undertaken in the past: particle simulation and hydrodynamic approach. A variation of the first approach is hybrid models in which ions and neutrals are treated as particles whereas electrons are treated as a fluid<sup>74,75,76</sup>. In this numerically expensive approach, however, very simplified boundary conditions are applied at the walls without considering the plasma-wall transition in details. In the second approach, the 1D hydrodynamic description for all species is employed<sup>77,78,79,80</sup>. However, due to restrictions of 1D analyses, the real boundary conditions at the wall were not considered. 2D hydrodynamic model of plasma flow in a Hall thruster suggests that plasma-wall interactions are rather complicated.<sup>81,82,83,84,85</sup> Any state-of-the-art Hall thruster model employ some anomalous cross-field



electron mobility in order to reproduce experimental features. Generally there is no clear convincing evidence regarding which one of the possible anomalous transport mechanisms prevails in Hall thruster thus leaving this question unresolved. We just want to point out that very recent experimental and theoretical study of the Hall thruster with variable channel width shed some light on this revealing that near wall conductivity mechanism may be responsible for electron transport in Hall thruster.<sup>86</sup>



**Fig. 5. Schematic of Hall thrusters**

Thus the problem associated with closed electron drift configurations (MPIII, magnetrons and Hall thruster) can be formulated in a more general manner: what electron transport mechanism sustains an electric field in the partially magnetized quasi-neutral plasma? Earlier it was shown that dependent on the electron transport across the magnetic field, an electric field is established, resulting in ion acceleration or deceleration.<sup>87</sup> From this point of view magnetic PIII system and Hall thruster represent two limiting cases with ion deceleration or acceleration conditions across the region with a closed electron drift. Similarly, ion detachment from the magnetized region in the Hall thruster (near the thruster exit plane) and ion flux entrance into the magnetized region in a MPIII are governed by very similar physics. Therefore despite the fact that the two aforementioned devices (magnetic PIII and Hall thruster) have very different applications, the underlying physics, i.e. closed electron drift, is very similar.

### **III. Anomalous electron transport mechanisms**

Electron conductivity across magnetic field is one of the long-standing problems related to Hall thrusters. The electron conductivity has various implications on fundamental issues of the Hall thrusters, such as current continuity, energy balance and ultimately on thruster efficiency. It was known for a long time that classical

mechanism of conductivity based on electron collisions could not explain the electron current experimentally observed in Hall thrusters. In this section we review two possible non-classical mechanisms of electron transport across a magnetic field.

## 1. Plasma oscillations

The non-classical or anomalous electron diffusion across a magnetic field has a long history since it was proposed by Bohm to explain high electron transport in magnetic confinement devices.<sup>88</sup> Presence of the turbulent electric field  $\delta E$  results in a random drift across the magnetic field. This leads to so-called anomalous diffusion coefficient  $D_{\perp}$  to be proportional to  $\langle \delta E^2 \rangle$  [55], which is typically larger than the classical diffusion coefficient.

In order to assess a possibility of Bohm-type anomalous transport in specific device such as Hall thruster we start from review experimental evidence on plasma oscillations. Hall thrusters have complex wave structure that goes across wide frequency spectrum.<sup>89</sup> It was established that amplitude and frequency of the oscillations in the Hall thruster depend on mass flow rate, discharge voltage, geometry, facility, magnetic field profile and cathode operation mode. In addition it was found that the oscillation spectrum depends on the location inside the Hall thruster channel.<sup>89,90</sup> The way to characterize the oscillations is to look at the oscillation spectrum as a function of a magnetic field or as a function of discharge voltage<sup>91</sup>. Generally several typical oscillations band were identified in Hall thruster, such as 10-20 kHz discharge oscillations, 5-25 kHz rotating spokes (attributed to ionization process), 20-60 kHz azimuthal modes (drift type instability associated with gradient of density and magnetic field), 70-500 kHz transient time (ion residence time in the channel), 0.5-5 MHz azimuthal wave. Last oscillations were recently detected by Litvak *et al.*<sup>92</sup> Parallel study of the Rayleigh instability<sup>93</sup> suggest that axial density, magnetic field and electron velocity gradients can drive this type azimuthal instability. High-frequency instabilities (1-10 MHz) were studied in the Hall-effect thruster.<sup>94</sup> It was found that these instabilities have the highest level near the thruster exit plane. Particle-in-cell (PIC) simulation suggests that high-frequency oscillations with very short wavelength can be developed in the Hall thruster.<sup>95</sup> It was suggested that these oscillations can be responsible for anomalous electron transport. On the other hand it was argued that axial oscillation (beam-plasma parametric instability type) can promote anomalous electron transport.<sup>96</sup> This type of instability was observed in the hybrid particle-fluid simulations of the Hall thruster<sup>96</sup>. A relationship between the Bunemann instability and low-frequency (1-20 KHz) oscillations was shown theoretically.<sup>97</sup> Thus one can see that Hall thruster has very wide oscillation spectrum. Oscillations in Hall thruster determine the efficiency of the system, may affect the divergence of the ion beam and electron transport across the magnetic field.

From the plasma theory point of view there are several instabilities that can lead to anomalous electron transport of the Bohm-type. In the low temperature plasma with parameters typical for the Hall thruster, it was found that the drift-dissipative instability can develop<sup>98</sup>. This instability has the following maximum increment<sup>98</sup>:

$$\gamma \sim \left( \frac{T_e}{B} \right) \cdot \frac{1}{n} \cdot \frac{dn}{dr} \quad (2)$$

where  $T_e$  is the electron temperature,  $B$  is the magnetic field,  $n$  is the plasma density and  $dn/dr$  is the plasma density gradient. One can see that the maximum increment inversely proportional to the magnetic field. In this case the diffusion coefficient can be estimated as follows:

$$D \sim \lambda_{\perp}^2 \gamma \quad (3)$$

where  $\lambda_{\perp}^2$  is the characteristic size of the plasma turbulence pulsing across the magnetic field, which can be approximated as a wavelength. If this is the case, one can see that the maximum increment has the same dependence as empirical coefficient proposed by Bohm<sup>88</sup>:

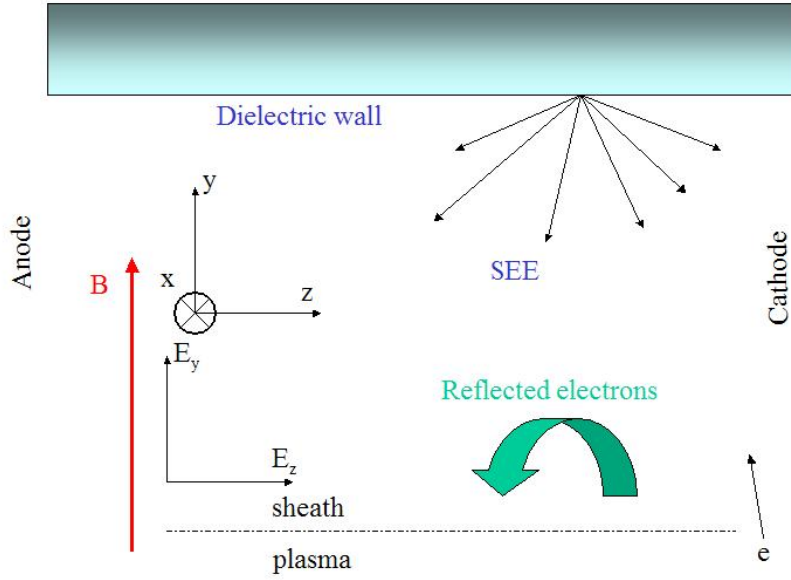
$$D = \frac{1}{16} \cdot \frac{T_e}{B} \quad (4)$$

It should be pointed out that while this instability was experimentally detected in the afterglow plasma having plasma parameter range somewhat similar to Hall thrusters [98], there is no clear experimental evidence that this type of instability is present in Hall thrusters.

In the following sections we will describe another mechanism that can lead to enhanced electron transport in Hall thrusters, so-called near wall conductivity.

## 2. Near-wall conductivity

The idea of near wall conductivity (NWC) stems from the fact that, typically in the Hall thruster channel, the mean free path for electron neutral collisions is about 1 m, while the distance between walls is about 1 cm. Therefore electron collisions with the wall happen much more often than collisions with neutral particles (the same conclusion is true for electron-ion collisions, electron-electron collisions since ionization degree is about 0.01 or less). Without the presence of the axial electric field, electron reflection in the sheath is mirror-type and therefore cannot contribute to the conductivity. This makes the axial electric field one of the most important factors in determining the electron transport. In the next section (Section IV) we will examine the effect of the axial electric field on the electron transport across the magnetic field.



**Figure 6. Schematic of electron interactions in the sheath**

Possible electron trajectories in the sheath near the dielectric are shown schematically in Fig.6 dependent on the initial velocity at the sheath edge. Two electron populations exist dependent on the electron energy distribution function and sheath potential drop. Reflected electrons are the low energy population of the energy distribution having energy smaller than the potential drop in the sheath. On the other hand, energetic electrons transit through the sheath and collide with the wall thus leading to secondary electron emission (SEE) as shown schematically in Fig.6. For typical Hall thruster parameters (electron temperature 20-30 eV, wall material is boron nitride) the SEE coefficient is about 1. Therefore, sheath reaches the space charge saturated regime associated with a non-monotonic potential profile. In this case, the sheath voltage drop  $U_w$  is relatively small and is about  $T_e$ . [99] Under these conditions the fraction of electron current colliding with the wall is large. Therefore one should consider the SEE effect on the transport across the magnetic field. SEE electrons have an angular distribution that depends on the energy of the primary electrons and angle of incidence. In the presence of the axial electric field in the sheath, the SEE angular distribution could change so that electrons would have some preferable injection in the direction of the electric field. This effect should also contribute to electron transport across the magnetic field. The frequency of electron collisions with walls can be estimated as:

$$v_{ew} \approx \langle V_e \rangle / h \quad (5)$$

where  $\langle V_e \rangle$  is the average electron velocity and  $h$  is the distance between walls. Previously, Baranov *et al.*<sup>100</sup> proposed to take into account that only a fraction of electrons will collide with walls due to reflection in the sheath, i.e

$$v_{ew} = \frac{\langle V_e \rangle}{h} \exp\left(-\frac{\Delta\phi_y}{T_e}\right) \quad (6)$$

However, taking into account that electron reflection from the sheath boundary can also contribute to the near-wall conductivity (as will be shown below), one can conclude that the collision frequency should be close to  $\langle V_e \rangle / h$ . In the next section, we describe the model of the near-wall conductivity taking into account details of electron interactions in the sheath.

## IV. Analysis of the near-wall conductivity in a Hall thruster channel

If the mean free path in the Hall thruster channel is much larger than the channel width, as happens typically in Hall thrusters, the electron collisions with the channel wall start to play a significant role. Morozov developed a model that calculates the near-wall current.<sup>64</sup> The method consists in solving equations for integrals of motion for electrons in a collisionless approach. The electron distribution function at the wall was chosen to be Maxwellian. The model was further developed over the last decades.<sup>101-102</sup> Recently, a new formulation of the NWC problem based on Morozov's approach was proposed<sup>103</sup> taking into account ion neutralization near the dielectric wall. It was shown that the resulting NWC current contains a correction factor and permits better quantitative agreement with experiment. Indeed, recent two-dimensional simulations of the ion dynamics in the Hall thruster demonstrated a significant effect of the ion neutralization near the walls.<sup>104</sup>

Phenomenologically, the NWC is the result of electron collisions with walls and consequent cycloidal motion along the magnetic field. Spatially oscillating currents is the essence of the NWC. Electrons reflecting from the wall are not monoenergetic and therefore the resulting current oscillation will rapidly decay with distance from the wall with most electron current concentration in the near wall region giving the appropriate name for this effect, near wall conductivity [64,66,101,102].

However, we want to point out an additional important effect that was not considered. Secondary electrons interact with electric fields (both axial and radial) in the sheath and thus their energy distribution function (EDF) is modified. In this section, the NWC problem taking into account this effect is formulated. It should be pointed out that EDF modification due to the radial electric field was very recently considered using a similar formalism. [102,105] We adopt the mathematical description proposed originally by Morozov<sup>64</sup>. The present model is based on the following assumptions:

- a) plasma properties are spatially uniform
- b) The EDF of the secondary electrons is Maxwellian with temperature  $T_w$  which is different from the bulk electron temperature  $T_e$ .
- c) Electrons are accelerated in the sheath by potential drops  $\Delta\varphi_z$  and  $\Delta\varphi_r$  in both axial and radial directions.

Let us describe the NWC mechanism taking into account SEE. The main idea is that the distribution function of the emitted electrons is shifted by the electric field in the sheath. Since both axial and radial electric fields are present, the distribution function is shifted in both directions and the EDF is centered along the direction of the

electric field. The electron dynamics can be fully described by the collisionless kinetic equation for distribution function  $f(t, \mathbf{r}, \mathbf{V})$ :

$$\frac{\partial f}{\partial t} + \mathbf{V} \cdot \frac{\partial f}{\partial \mathbf{r}} - \frac{e}{m} (\mathbf{E} + \mathbf{V} \times \mathbf{B}) \cdot \frac{\partial f}{\partial \mathbf{v}} = 0 \quad (7)$$

where  $\mathbf{E}$  is the electric field,  $\mathbf{B}$  is the magnetic field. The following distribution function of the emitted electrons from the wall is assumed [64,102,103,105]:

$$f(v) = 2n_0 \left( \frac{m}{2\pi k T_w} \right)^{3/2} \exp\left(-\frac{mV^2}{2kT_w}\right) \quad (8)$$

where  $n_0$  is the electron density,  $T_w$  is the temperature of emitted electrons, which is unknown and remains a free parameter of the problem. Further consideration is based on the fact that the distribution function is constant along the characteristics, which are determined by the equations of motion for electrons [64]. Assuming constant electric and magnetic fields, the solution of the equation of motion is the electron drift with constant velocity  $E/B$  along the  $x$ -axis and cyclotron rotation.

We take into account that typically, in Hall thrusters, the electron Larmor radius is much larger than the Debye length. Therefore, we neglect effect of the magnetic field in the sheath. The equations of motion for the electron (characteristics) have the following form:

$$\frac{dV_x}{dt} = -\omega V_z \quad (9)$$

$$\frac{dV_y}{dt} = 0 \quad (10)$$

$$\frac{dV_z}{dt} = -\frac{eE_z}{m} + \omega V_x \quad (11)$$

where  $\omega$  is the electron cyclotron frequency. Using the above assumptions and conditions one can integrate the equation for characteristics:

$$V_{ez} = (V_{ex}^o - V_E) \sin(\omega t) + \sqrt{(V_{ez}^o)^2 + \frac{2e\Delta\phi_z}{m}} \cos(\omega t) \quad (12)$$

$$V_{ex} = V_E + (V_{ex}^o - V_E) \cos(\omega t) - \sqrt{(V_{ez}^o)^2 + \frac{2e\Delta\phi_z}{m}} \sin(\omega t) \quad (13)$$

$$V_{ey} = \sqrt{(V_{ey}^0)^2 + \frac{2e\Delta\phi_y}{m}} \quad (14)$$

where  $V_E = \frac{E_z}{B}$ .

where  $V_{ex}^0$ ,  $V_{ez}^0$ ,  $V_{ey}^0$  are the velocity at the wall. It was taken into account that electrons are accelerated across the sheath having a potential drop of  $\Delta\phi_y$ . It should be noted that equations (9-14) describe electron motion in the crossed field outside the sheath, while in the sheath finite jumps of  $V_{ez}$  and  $V_{ey}$  are considered without considering effect of the magnetic field. The current density (z component) can be calculated as follows:

$$j_{ez} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(v) V_z dV_x dV_z dV_y \quad (15)$$

where  $\alpha_y = \sqrt{\frac{2e\Delta\phi_y}{m}}$ . Now we substitute velocity components according to equation of characteristics (Eq.12-14). In this case, one can arrive at the following expression for the electron current density:

$$j_{ew} = 2n_0 \frac{E}{B} \left( \frac{m}{2\pi kT_w} \right)^{1/2} \exp\left(\frac{e\Delta\phi_z}{kT_w}\right) \exp\left(\frac{e\Delta\phi_y}{kT_w}\right) \int_{\sqrt{\frac{2e\Delta\phi_y}{m}}}^{\infty} \exp\left(-\frac{mV_y^2}{2\pi kT_w}\right) \sin\left(\omega \frac{y}{V_y}\right) dV_y \quad (16)$$

Let us introduce the following new variables:

$$\theta = \frac{V_y}{\sqrt{\frac{2kT_w}{m}}} \quad \text{and} \quad s = \frac{\omega y}{\sqrt{\frac{2kT_w}{m}}} = \frac{y}{\rho_{Le}}, \quad \text{where } \rho_{Le} \text{ is the secondary electrons Larmor radius. In this case, a new}$$

function can be introduced which is the integral in Eq. 16

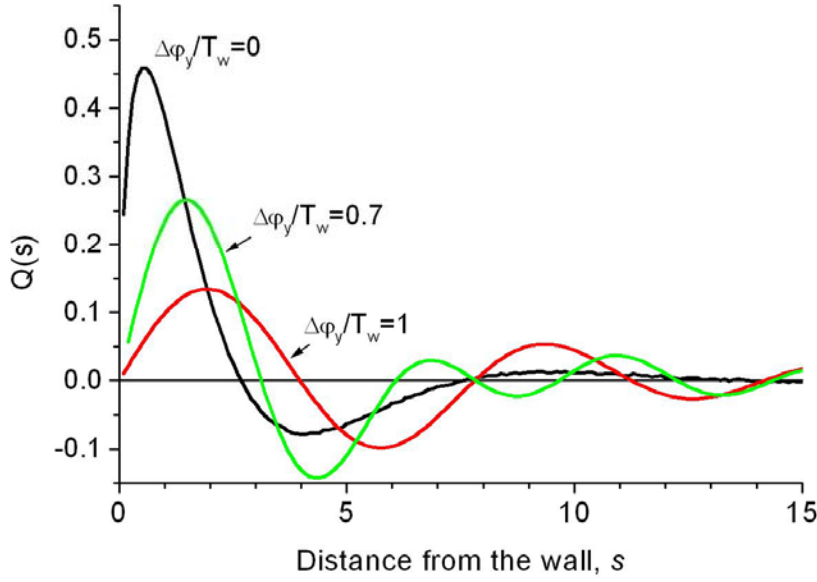
$$Q(s) = \int_{\sqrt{\frac{e\Delta\phi_y}{kT_w}}}^{\infty} \exp(-\theta^2) \sin\left(\frac{s}{\theta}\right) d\theta \quad (17)$$

where  $s$  is the non-dimensional distance from the wall ( $y$  direction) and the function  $Q(s)$  determines the current distribution as a function of that distance. In essence, this function is similar to one introduced originally by Morozov<sup>64</sup> with one exception, i.e. the potential drop in the sheath is taken into account. In that sense, the present approach is similar to recent work of Barral *et al*<sup>105</sup>. The difference is that, in addition, we take into account electron acceleration in the sheath along the axial electric field component. The current density due to NWC can be expressed as follows:

$$j_{ew} = \frac{2}{\sqrt{\pi}} n_0 \frac{E}{B} \exp\left(\frac{e\Delta\phi_z}{kT_w}\right) \exp\left(\frac{e\Delta\phi_y}{kT_w}\right) \times Q(s) \quad (18)$$

One can see that the function  $Q(s)$  provides the dependence of the current density on the distance from the wall.

The calculated dependence of  $Q(s)$  is shown in Fig. 7.



**Fig. 7. Calculated function  $Q(s)$  as a distance from the wall with sheath potential drop as a parameter**

One can see that current is concentrated near the wall at a distance of a few Larmor radii in the simplest case of  $\Delta\phi_y=0$ . This case corresponds to Morozov's original solution [64]. Generally, the sheath voltage leads to decrease of the current concentration near the wall and to more uniform current distribution across the channel between the two walls. This was a reason that led some authors to conclude that NWC may be a misnomer [105].

However it was indicated by some authors that in this formulation, the NWC current underpredicts the measured values [103]. Below we will consider an additional effect associated with electron interactions in the sheath that lead to enhancement of the NWC current density. The main idea is that the electric field along the wall can affect the near-wall current by producing an additional velocity shift in the axial direction. Typically in the Hall thruster channel, the axial electric field is  $E_z=2-3 \times 10^4$  V/m (Refs. 66,84), which is smaller than the typical radial electric field in the sheath. However, it will be shown that in some cases the axial electric field can be an important factor contributing to NWC.

The current density increases by a factor of  $\exp\left(\frac{\Delta\phi_z}{T_w}\right)$  as can be seen from Eq.16 in which we have to know the potential drop  $\Delta\phi_z$  in the axial direction. In order to estimate the effective potential drop in the axial direction let us consider in some detail the electron motion in the sheath. It was stated above that typically, in a Hall thruster acceleration channel, the saturated space charge sheath occurs.<sup>106</sup> In this case, the potential distribution in the sheath has a minimum [99]. In the location of the minimum, the corresponding (y) component of the electric field is zero. On the other hand, there is an axial electric field component which arises from the axial potential



distribution in the plasma bulk. The presence of the axial component was discussed previously.<sup>107</sup> It was suggested that the electric field along the dielectric is close to the electric field in the plasma bulk due to high dielectric strength of the wall material. The potential distribution in the axial direction can be obtained from the electron momentum equation as

$$d\varphi_z = dzE_z = E_z^2 \frac{dy}{E_y} \quad (19)$$

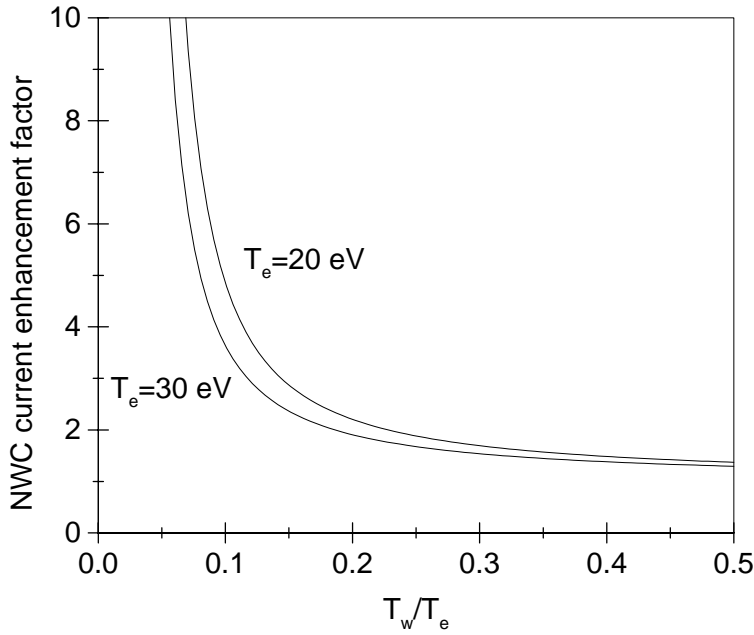
The total potential drop that electrons are experiencing while being in the sheath can be estimated as follows:

$$\Delta\varphi_z = E_z^2 \int_{y_{\min}}^{y_{\max}} \frac{dy}{E_y(y)} \approx E_z^2 \frac{L_D}{E_y} \quad (20)$$

In the last expression, it was assumed that the sheath thickness is about one Debye length. However, since the space charge limited sheath is considered, the electric field is not uniform in the  $y$  direction. The largest influence of the axial electric field is near the potential well, since typically the electric field in the  $y$  direction is much larger than in  $z$ , i.e.  $E_y/E_z \gg 1$ , except near the potential well where  $E_y/E_z \leq 1$ . The spatial extension of the potential well is about one Debye length  $L_D$  (Ref.108). Thus, the potential drop can be estimated with satisfactory accuracy as

$$\Delta\varphi_z \approx E_z L_D \quad (21)$$

It should be noted that typically the depth of the potential well is small in comparison to  $T_w$ . Thus, secondary electrons reflection back to the wall can be neglected in calculation of the distribution function outside the sheath. Taking Eq. 21 into account, one can estimate the NWC current enhancement  $\left(\exp\left(\frac{E_z L_D}{T_w}\right)\right)$ , which depends on the bulk to wall electron temperature ratio as shown in Fig. 8. In typical conditions in the Hall thruster channel, we can find that this factor may be about 10.



**Figure 8. Enhancement of the NWC by axial electric field effect . The typical conditions are considered:  $E_z=3 \times 10^4$  V/m;  $n_o=10^{17}$  m<sup>-3</sup>.**

If we take into account possible enhancement factor due to axial electric field in the sheath, the predicted NWC current will be close to that measured experimentally [103]. Thus it can be concluded that, in general, NWC can explain the high electron mobility in a Hall thruster. However, it is quite interesting to point out that higher NWC current is expected in the case of small  $T_w$  as follows from Fig. 8. On the other hand, according to this model prediction, this is the case in which NWC current is not decaying from the wall as shown in Fig. 7, thus putting a question mark on the near wall nature of the current. Bearing this in mind we can conclude that the full picture of electron transport in the Hall thruster is far from completion and further investigation is needed.

## **Concluding remarks**

In this paper we reviewed various plasma devices involving a closed electron drift. Having different applications the operation of these devices is based on the very similar physical phenomena, which is the plasma generation in a crossed ExB field region. It was identified that two major important phenomena take place in these systems: very efficient gas ionization and anomalous electron transport. Two mechanisms were found to be suitable to explain experimentally observed electron transport phenomena across the magnetic field. For instance in the case of magnetron it was found that Bohm-type mechanism can be used to describe electron transport. However, to date there is no definite experimental proof that the existing plasma oscillation spectra can support Bohm diffusion. On the other hand in Hall thrusters several indications are in favor of the near wall conductivity. In spite of the fact that Bohm-type diffusion, used properly in models, can also explain experimental evidence, thus making the question about electron transport the main intrigue of the current research on Hall thruster. A new formulation of the original near-wall conductivity model that takes into account an axial electric field in the sheath was presented. It was shown that under certain conditions i.e. small ratio of the secondary electron temperature to the bulk electron temperature, the near wall conductivity can be significantly enhanced.

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