Collisionless electrode sheath in an rf discharge

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Interest in low-pressure rf discharges has increased considerably on account of their use in preparing epitaxial films. For these purposes it is important to know the dc voltage drop between the plasma and the electrode, the sheath thickness, and other of its characteristics. Estimates of these quantities have been published in Ref. 1. Numerical calculations for the sheath, by a solution of the complete system of equations2 is quite difficult. An attempt has been made at an analytical calculation3 based on an incorrect expression for the electron density. In the present investigation we obtain simple analytic expressions for the parameters of the sheath using the method of averaging over the fast electron motion.

Let us assume that the sheath thickness L is small compared to the mean free path of the particles and the interelectrode gap and large compared to the Debye length r_D and that the frequency of the field satisfies $\omega = \omega_D, \omega (Maxw)$, where $\omega_D$ is the Debye frequency and $\omega (Maxw)$ is the Maxwellian frequency.

In this case the field in the plasma is much less than that in the sheath and $\sim j\varepsilon_0$. The displacement of the ions during a period of the field is small compared to L, since their motion is determined by the average field. The boundary of the electron profile (thickness on the order of $r_D$) can thus be considered sharp, and the potential drop in the sheath large compared to $\omega$. Ionization in the sheath and the initial velocities of the ions can be neglected. Then the ion flux in the sheath is conserved and $j_i = n_i, j (T/M)$, where $n_i$ is the ion density ahead of the sheath, $T$ is the electron temperature, and $\gamma$ is a number of the order of unity and depends on the nature of the distribution function. The density of ions in the sheath is

\[ n(x) = \frac{1}{\sqrt{2 \pi}} e^{-x^2/2}, \]

where $x = \gamma_0, (T/M)$ and $\phi(x)$ is the dc potential in the sheath.

Writing the current density as $j = j_i \sin \omega t$ and introducing the new variable $z(x) = \omega t, (T/M)$, the phase at which the boundary of the plasma sheath reaches the point $x$, we find a closed system of equations4 for $\phi(z)$:

\[ dp/dz = 2L_0/(\omega z^2), \]

\[ \sin \omega t dz = e^{j0}/(L_0^2), \]

\[ \phi(z) = 2L_0/\omega, \]

where $z = 0$ corresponds to the plasma and $z = \infty$ corresponds to the electrode.

The solution is found in parametric form

\[ n(x) = \frac{1}{\sqrt{2 \pi}} e^{-x^2/2}, \]

\[ \phi(z) = 2L_0/\omega, \]

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It is convenient to write the following relation between the sheet thickness and the ac potential drop, $\theta$, in the sheet.

$$Z = \frac{\gamma}{\sqrt{2}} \left( \frac{r}{2} \right)^{1/2} \left( \frac{\theta}{\sqrt{2}} \right)^{1/2}.$$  \hspace{1cm} (4)

We note that as the discharge current increases, the sheet thickness increases in proportion to $\theta$, and if the rf voltage is dropped mainly in the sheets, then $L = U$.

Figure 1 (curve 1) shows the profile of the ac potential in the sheet (3) as compared with the results of numerical calculation \(^7\) (curve 2).

Although the calculation in Ref. 5 was carried out for the case of a given sinusoidal voltage on a single sheet, the calculated $\theta(x)$ and the thickness of the sheet for various values of $a$ and $\theta_0$ differ little from (3) and (4). Formula (3) is accurate up to terms of order $(\theta_0/\gamma)^3/2$. It can be seen from Fig. 1 that the function $\theta(x)/\theta_0 = (x/L)^{3/2}$ (curve 3), which is an accurate solution for small $x$, gives a good approximation to the profile over the whole sheet.

With the potential profile $\theta(x)$ that has been found it is easy to find the ac field in the sheet

$$E(x,t) = \frac{\theta_0}{\gamma} \left( \frac{x}{L} \right)^{1/2} \frac{\partial \theta}{\partial x}$$  \hspace{1cm} (5)

or with the use of (2)

$$E(x,t) = \frac{\theta_0}{\gamma} \left( \frac{x}{L} \right)^{1/2} \left( \cos \omega t - \cos \omega z(t) \right),$$  \hspace{1cm} (6)

where $z$ varies with $t$ to $x$.

Integrating (6) we find that in a discharge between identical plane-parallel electrodes (symmetric case) the total rf voltage drop in the sheets is

$$U'(w) = \frac{\theta_0}{\gamma} \left[ \frac{\cos \omega t - \cos \omega z(t)}{\cos \omega t} \right] \left( \frac{x}{L} \right)^{1/2} \frac{\partial \theta}{\partial x} dx.$$  \hspace{1cm} (7)

A curve of $U'(\omega)$ is plotted in Fig. 2. It can be seen that $U(\omega)$ is an anharmonic function. Since $U(\omega)$ is an even function and $U(\omega - \omega_0) = -U(\omega + \omega_0)$, we have expanded $U(\omega)$ in the functions $\cos(n\omega_0 - \omega)t$. For example, the amplitude of the third harmonic ($n = 3$) is 2.3% of that of the first harmonic. The voltage in the discharge is 3.3 times higher than the $\omega$ drop in the sheet. This is very close to the voltage predicted and verified experimentally in Ref. 1.

Result (7) is not difficult to generalize to the case of an arbitrary nonsinusoidal periodic variation of the current density. If we assume that $z = \psi(x)$, where $f(x) = H(x)$, we retain instead of (7) the result

$$U'(\omega) = \frac{\theta_0}{\gamma} \left[ \frac{\cos \omega t - \cos \omega \psi(x)}{\cos \omega t} \right] \left( \frac{x}{L} \right)^{1/2} \frac{\partial \theta}{\partial x} dx,$$

FIG. 1. ac potential and density in the sheet.

\hspace{1cm} FIG. 2. rf potential drop in the sheet of a symmetric discharge. The dashed line corresponds to a nonsinusoidal dependence.
It can be seen from the numerical calculations of Ref. 2 that near the plane \( y = 0 \), where all \( T \) and the electrons approach a distance \(-r_T\) from the electrode, features appear in the displacement current behavior. On this scale, the ion density can be considered constant \( n_i = n_i(x = L) \). By integrating Poisson's equation we find the electric field at the electrode

\[
\mathbf{E}(t) = \mathbf{E}_0 \left( \frac{\varepsilon_0 \varepsilon_n}{\varepsilon_n + 1} \right) \left( -\frac{1}{\varepsilon_n + 1} \right) \mathbf{F}(x) \right) ^{\frac{1}{2}},
\]

(10)

where \( \mathbf{F}(x) = \mathbf{F}(x + \varepsilon_0 \varepsilon_n \mathbf{F}/2) \) is the potential difference between the electrode and the plasma, and \( \mathbf{U}(t) \) is given by formula (10).

The displacement current at the electrode is

\[
J(t) = \left( \frac{\varepsilon_0 \varepsilon_n}{\varepsilon_n + 1} \right) \left( -\frac{1}{\varepsilon_n + 1} \right) \mathbf{F}(x) \right) ^{\frac{1}{2}} \frac{d\mathbf{F}(x)}{dt},
\]

(11)

The function \( G(t) \) is plotted in Fig. 3. Far from \( \varepsilon_0 = 0 \) the function \( J(t) \) depends only weakly on \( \varepsilon_0 \), near the time that \( \varepsilon_0 = 0 \), for \( \varepsilon_0 < 0 \), for \( \varepsilon_0 > 0 \), for \( \varepsilon_0 = 0 \), the displacement current increases sharply in a time of the order of

\[
\omega_n \left( \frac{\varepsilon_0}{\varepsilon_n + 1} \right) ^{\frac{1}{2}}, \quad \text{where} \quad \omega_n = \begin{cases} 0 \varepsilon_0 = 0, & \varepsilon_0 \varepsilon_n \mathbf{F} > \frac{1}{2} \mathbf{F}(x) \right) \left( -\frac{1}{\varepsilon_n + 1} \right) \mathbf{F}(x) \right) ^{\frac{1}{2}} \frac{d\mathbf{F}(x)}{dt},
\]

(12)

in agreement with the results of Ref. 2 (see Fig. 3). In the opposite case expression (12) also gives a sharp increase in the displacement current for \( \varepsilon_0 = 0 \). However, at this instant the electrons are conduct-