MULTISPECIES WEIBEL INSTABILITY FOR INTENSE ION BEAM PROPAGATION THROUGH BACKGROUND PLASMA*

R. C. Davidson, E. A. Startsev, I. Kaganovich and H. Qin
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08543

Abstract

Properties of the multi-species electromagnetic Weibel instability are investigated for an intense ion beam propagating through background plasma. Assuming that the background plasma electrons provide complete charge and current neutralization, detailed linear stability properties are calculated within the framework of a macroscopic cold-fluid model for a wide range of system parameters.

INTRODUCTION

High energy ion accelerators, transport systems and storage rings [1, 2] are used for fundamental research in high energy and nuclear physics and for applications such as heavy ion fusion. Charged particle beams are subject to various collective processes that can deteriorate the beam quality [3, 4, 5, 6, 7, 8]. In the neutralized drift compression and target chamber regions for ion-beam-driven high energy density physics applications and heavy ion fusion, the intense ion beam experiences collective interactions with the background plasma. In this paper, we investigate theoretically properties of the multi-species electromagnetic Weibel instability for an intense ion beam propagating through background plasma [5, 6, 7]. Assuming that the background plasma electrons provide complete charge and current neutralization, detailed linear stability properties are calculated within the framework of a macroscopic cold-fluid model for a wide range of system parameters.

THEORETICAL MODEL

We make use of a macroscopic fluid model [1, 5, 6] to describe the interaction of an intense ion beam $(j = b)$ with background plasma electrons and ions $(j = e, i)$. The charge and rest mass of a particle of species j $(j = b, e, i)$ are denoted by e_j and m_j, respectively. In equilibrium, the steady-state $(\partial/\partial t = 0)$ average flow velocities are taken to be in the z-direction, $\mathbf{V}_j^0(\mathbf{x}) = V_{zj}^0(r)\hat{\mathbf{e}}_z = \beta_j(r)c\hat{\mathbf{e}}_z$, and cylindrical symmetry is assumed $(\partial/\partial\theta = 0)$. Axial motions are generally allowed to be relativistic, and the directed axial kinetic energy is denoted by $(\gamma_j - 1)m_jc^2$, where $\gamma_j(r) = [1 - \beta_j^2(r)]^{-1/2}$ is the relativistic mass factor of a fluid element. Furthermore, the analysis is carried out in the paraxial approximation, treating the velocity spread of the beam particles as small in comparison with $\beta_b c$. Denoting the equilibrium density profile by $n_j^0(r)$ $(j = b, e, i)$, the corresponding equilibrium self-electric field, $\mathbf{E}^0(\mathbf{x}) = E_r^0(r)\hat{\mathbf{e}}_r$, and azimuthal self-magnetic field, $\mathbf{B}^0(\mathbf{x}) = B_\theta^0(r)\hat{\mathbf{e}}_\theta$, are determined self-consistently from the steady-state Maxwell equations, where $r = (x^2 + y^2)^{1/2}$ is the radial distance from the axis of symmetry.

In the macroscopic stability analysis of the Weibel instability, we specialize to the case of axisymmetric, electromagnetic perturbations with $\partial/\partial\theta = 0$ and $\partial/\partial z = 0$, and perturbed quantities are expressed as $\delta\psi(r, t) = \delta\psi(r)\exp(-i\omega t)$ where $Im\omega > 0$ corresponds to instability (temporal growth). For the perturbations, the perturbed field components are $\boldsymbol{\delta E}(\mathbf{x}, t) = \delta E_r(r, t)\hat{\mathbf{e}}_r + \delta E_z(r, t)\hat{\mathbf{e}}_z$ and $\boldsymbol{\delta B}(\mathbf{x}, t) = \delta B_\theta(r, t)\hat{\mathbf{e}}_\theta$. It has been shown previously that a sufficiently strong self-magnetic field $B_\theta^0(r) \neq 0$ tends to reduce the growth rate of the Weibel instability in intense beam-plasma systems [8]. For present purposes, we specialize to the case of a charge-neutralized and current-neutralized beam-plasma system with

$$\sum_{j=b,e,i} n_j^0(r)e_j = 0\,, \quad \sum_{j=b,e,i} n_j^0(r)\beta_j e_j = 0\,, \quad (1)$$

where β_j is taken to be independent of r for simplicity. It then follows from the steady-state Maxwell equations that $E_r^0 = 0 = B_\theta^0$. Making use of a macroscopic cold-fluid model based on the linearized fluid-Maxwell equations [1], some straightforward algebraical manipulation yields the eigenvalue equation [5, 6]

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\left(1 + \sum_{j=b,e,i}\frac{\beta_j^2\omega_{pj}^2(r)}{\omega^2} + \frac{(\sum_{j=b,e,i}\beta_j\omega_{pj}^2(r))^2}{\omega^2 - \sum_{j=b,e,i}\omega_{pj}^2(r)}\right)\right.$$
$$\left.\times\frac{\partial}{\partial r}\delta E_z\right] + \left(\frac{\omega^2}{c^2} - \sum_{j=b,e,i}\frac{\omega_{pj}^2(r)}{\gamma_j^2c^2}\right)\delta E_z = 0, \quad (2)$$

where $\gamma_j = (1-\beta_j^2)^{-1/2}$ is the relativistic mass factor, and $\omega_{pj}^2(r) = 4\pi n_j^0(r)e_j^2/\gamma_j m_j$.

Equation (2) is the desired eigenvalue equation for axisymmetric, electromagnetic perturbations with polarization $\boldsymbol{\delta E} = \delta E_r\hat{\mathbf{e}}_r + \delta E_z\hat{\mathbf{e}}_z$ and $\boldsymbol{\delta B} = \delta B_\theta\hat{\mathbf{e}}_\theta$, with the terms proportional to $\sum_{j=b,e,i}\beta_j^2\omega_{pj}^2(r)$ and $\sum_{j=b,e,i}\beta_j\omega_{pj}^2(r)$ providing the free energy to drive the Weibel instability. Equation (2) can be integrated numerically to determine the eigenvalue ω^2 and eigenfunction $\delta E_z(r)$ for a wide range of beam-plasma density profiles $n_j^0(r)$ [6]. As discussed in Sec. 3, analytical solutions are also tractable for the case of flat-top (step-function) density profiles. As a general remark, when $\sum_{j=b,e,i}\beta_j^2\omega_{pj}^2(r) \neq 0$ and $\sum_{j=b,e,i}\beta_j\omega_{pj}^2(r) \neq 0$, Eq. (2) supports both stable fast-wave solutions $(Im\omega = 0,\ |\omega/ck_\perp| > 1)$ and unstable slow-wave solutions $(Im\omega > 0,\ |\omega/ck_\perp| < 1)$. Here, $|k_\perp| \sim |\partial/\partial r|$ is the characteristic radial wavenumber of

* Research supported by the U. S. Department of Energy.

the perturbation. Moreover, Eq. (2) also supports stable plasma oscillation solutions with predominantly longitudinal polarization associated with the factor proportional to $[\omega^2 - \sum_{j=b,e,i}\omega_{pj}^2(r)]^{-1}$. Finally, for a perfectly conducting cylindrical wall located at $r = r_w$, the eigenvalue equation (2) is to be solved subject to the boundary condition

$$\delta E_z(r = r_w) = 0\ . \tag{3}$$

MULTISPECIES WEIBEL INSTABILITY

As an example that is analytically tractable, we consider the case where the density profiles are uniform both inside and outside the beam (Fig. 1) with

$$n_j^0(r) = \hat{n}_j^i = const.,\ \ j = b, e, i\ , \tag{4}$$

for $0 \le r < r_b$, and

$$n_j^0(r) = \hat{n}_j^0 = const.,\ \ j = e, i\ , \tag{5}$$

for $r_b < r \le r_w$. Here, the superscript "i" ("o") denotes inside (outside) the beam, and $\hat{n}_b^0 = 0$ is assumed. Consistent with Eq. (1), $\sum_{j=b,e,i}\hat{n}_j^i e_j = 0 = \sum_{j=b,e,i}\hat{n}_j^i\beta_j e_j$, and $\sum_{j=e,i}\hat{n}_j^0 e_j = 0 = \sum_{j=e,i}\hat{n}_j^0\beta_j e_j$ are assumed. We also take $\beta_j = 0$ $(j = e, i)$ in the region outside the beam $(r_b < r \le r_w)$. The subsequent analysis of the eigenvalue equation (2) is able to treat the three cases: (a) beam-plasma-filled waveguide $(r_b = r_w)$; (b) vacuum region outside the beam ($r_b < r_w$ and $\hat{n}_j^0 = 0,\ j = e, i$); and (c) plasma outside the beam ($r_b < r_w$ and $\hat{n}_j^0 \neq 0,\ j = e, i$).

Referring to Eqs. (2), (4) and (5) it is convenient to introduce the constant coefficients

$$T_i^2(\omega) = \left[\frac{\omega^2}{c^2} - \sum_{j=b,e,i}\frac{\hat{\omega}_{pj}^{i2}}{\gamma_j^2 c^2}\right]\times \left[1 + \sum_{j=b,e,i}\frac{\beta_j^2\hat{\omega}_{pj}^{i^2}}{\omega^2} + \frac{(\sum_{j=b,e,i}\beta_j\hat{\omega}_{pj}^{i^2})^2}{\omega^2[\omega^2 - \sum_{j=b,e,i}\hat{\omega}_{pj}^{i^2}]}\right]^{-1} \tag{6}$$

for $0 \le r < r_b$, and

$$T_0^2(\omega) = -\left[\frac{\omega^2}{c^2} - \sum_{j=e,i}\frac{\hat{\omega}_{pj}^{02}}{c^2}\right] \tag{7}$$

for $r_b < r \le r_w$, where $\hat{\omega}_{pj}^{i2} = 4\pi\hat{n}_j^i e_j^2/\gamma_j m_j$, $j = b, e, i$, and $\hat{\omega}_{pj}^{02} = 4\pi\hat{n}_j^0 e_j^2/m_j$, $j = e, i$. We denote the eigenfunction inside the beam $(0 \le r < r_b)$ by $\delta E_z^I(r)$ and the eigenfunction outside the beam $(r_b < r \le r_w)$ by $\delta E_z^{II}(r)$. From Eqs. (2), (6) and (7) the solutions to Eq. (2) that are regular at $r = 0$, continuous at $r = r_b$, and vanish at the conducting wall are given by

$$\delta E_z^I(r) = AJ_0(T_i r)\ ,\ \ 0 \le r < r_b\ , \tag{8}$$

$$\delta E_z^{II}(r) = AJ_0(T_i r_b)\times \frac{K_0(T_0 r_w)I_0(T_0 r) - K_0(T_0 r)I_0(T_0 r_w)}{K_0(T_0 r_w)I_0(T_0 r_b) - K_0(T_0 r_b)I_0(T_0 r_w)}\ \ r_b < r \le r_w\ , \tag{9}$$

where A is a constant, $J_0(x)$ is the Bessel function of the first kind of order zero, and $I_0(x)$ and $K_0(x)$ are modified Bessel functions of order zero.

The remaining boundary condition is obtained by integrating the eigenvalue equation (2) across the beam surface at $r = r_b$. Making use of Eqs. (4) and (5), we operate on Eq. (2) with $\int_{r_b(1-\epsilon)}^{r_b(1+\epsilon)} drr\cdots$ for $\epsilon \to 0_+$. This readily gives the condition [5, 6]

$$\left(1 + \sum_{j=b,e,i}\frac{\beta_j^2\hat{\omega}_{pj}^{i2}}{\omega^2} + \frac{(\sum_{j=b,e,i}\beta_j\hat{\omega}_{pj}^{i^2})^2}{\omega^2[\omega^2 - \sum_{j=b,e,i}\hat{\omega}_{pj}^{i^2}]}\right)T_i r_b\times \frac{J_0'(T_i r_b)}{J_0(T_i r_b)} = T_0 r_b\frac{K_0(T_0 r_w)I_0'(T_0 r_b) - K_0'(T_0 r_b)I_0(T_0 r_w)}{K_0(T_0 r_w)I_0(T_0 r_b) - K_0(T_0 r_b)I_0(T_0 r_w)}\ , \tag{10}$$

where $T_i(\omega)$ and $T_0(\omega)$ are defined in Eqs. (6) and (7), and $I_0'(x) = (d/dx)I_0(x)$, $J_0'(x) = -(d/dx)J_0(x)$, etc.

Equation (10) constitutes a closed transcendental dispersion relation that determines the complex oscillation frequency ω for electromagnetic perturbations about the step-function profiles in Fig. 1. As noted earlier, the dispersion relation has both fast-wave and slow-wave (Weibel-type) solutions, as well as a predominantly longitudinal (modified plasma oscillation) solution. Moreover, Eq. (10) can be applied to the case of a beam-plasma-filled waveguide $(r_b = r_w)$, or to the case where the region outside the beam $(r_b < r \le r_w)$ corresponds to vacuum $(\hat{n}_j^0 = 0, j = e, i)$ or background plasma $(\hat{n}_j^0 \neq 0, j = e, i)$ [6]. For present purposes, we consider the case where there is a stationary background plasma $(\hat{n}_j^0 \neq 0, \beta_j = 0, j = e, i)$ in the region outside the beam $(r_b < r \le r_w)$.

It is convenient to introduce the dimensionless quantities $\langle\beta^2\rangle$ and $\langle\beta\rangle$ defined by

$$\langle\beta^2\rangle = \frac{\sum_{j=b,e,i}\beta_j^2\hat{\omega}_{pj}^{i^2}}{\sum_{j=b,e,i}\hat{\omega}_{pj}^{i^2}}\ ,\quad \langle\beta\rangle = \frac{\sum_{j=b,e,i}\beta_j\hat{\omega}_{pj}^{i^2}}{\sum_{j=b,e,i}\hat{\omega}_{pj}^{i^2}}\ , \tag{11}$$

and the dimensional quantity Γ_w defined by

$$\Gamma_w \equiv \frac{[\langle\beta^2\rangle - \langle\beta\rangle^2]^{1/2}}{(1 - \langle\beta^2\rangle)^{1/2}}(\sum_{j=b,e,i}\hat{\omega}_{pj}^{i^2})^{1/2}\ . \tag{12}$$

The quantity Γ_w provides a convenient unit in which to measure the growth rate $Im\omega$ of the Weibel instability in the subsequent numerical analysis of the general dispersion relation (10). For present purposes, we consider a positively charged ion beam $(j = b)$ propagating through background plasma electrons and ions $(j = e, i)$. The charge states are denoted by $e_b = +Z_b e$, $e_e = -e$, and $e_i = +Z_i e$, and the plasma electrons are assumed to carry the neutralizing current $(\beta_e \neq 0)$, whereas the plasma ions are taken to be stationary $(\beta_i = 0)$. The conditions for charge neutralization, $\sum_{j=b,e,i}\hat{n}_j^i e_j = 0$, and current neutralization, $\sum_{j=b,e,i}\hat{n}_j^i e_j\beta_j = 0$, then give

$$\hat{n}_e^i = Z_b\hat{n}_b^i + Z_i\hat{n}_i^i\ ,\ \ \beta_e = \frac{\beta_b Z_b\hat{n}_b^i}{Z_b\hat{n}_b^i + Z_i\hat{n}_i^i}\ . \tag{13}$$

0-7803-8859-3/05/$20.00

Except for the case of a very tenuous beam ($Z_b\hat{n}_b^i \ll Z_i\hat{n}_i^i$), note from Eq. (13) that β_e can be a substantial fraction of β_b. Careful examination of the expression for Γ_w in Eq. (12) for $\beta_i = 0$ shows that

$$\Gamma_w^2 = \frac{1}{(1-\langle\beta^2\rangle)} \times \left[\frac{(\beta_e^2\hat{\omega}_{pe}^{i^2} + \beta_b^2\hat{\omega}_{pb}^{i^2})\hat{\omega}_{pi}^{i2} + (\beta_b-\beta_e)^2\hat{\omega}_{pe}^{i^2}\hat{\omega}_{pb}^{i^2}}{\sum_{j=b,e,i}\hat{\omega}_{pj}^{i2}}\right]. \quad (14)$$

For $\hat{\omega}_{pb}^{i^2}, \hat{\omega}_{pi}^{i^2} \ll \hat{\omega}_{pe}^{i^2}$, it follows that Eq. (14) is given to good approximation by

$$\Gamma_w^2 \simeq \frac{1}{(1-\beta_e^2)}[\beta_e^2\hat{\omega}_{pi}^{i^2} + (\beta_b-\beta_e)^2\hat{\omega}_{pb}^{i^2}]\,. \quad (15)$$

Note from Eq. (15) that Γ_w involves the (slow) plasma frequencies of both the beam ions and the plasma ions.

In the remainder of Sec. 3 we consider the case of a cesium ion beam with $Z_b = 1$ and $\beta_b = 0.2$ propagating through a neutralizing background argon plasma with $Z_i = 1$, $\hat{n}_i^i = (1/2)\hat{n}_e^i = \hat{n}_b^i$, and $\beta_e = 0.1$ [see Eq. (13)]. Typical numerical solutions to Eq. (10) for the unstable branch are illustrated in Fig. 2 for the choice of system parameters $r_w = 3r_b$, $\beta_b = 0.2$, $\beta_e = 0.1$, $\hat{n}_i^i = \hat{n}_e^i/2 = \hat{n}_b^i = \hat{n}_e^o = \hat{n}_i^o$, and $\hat{\omega}_{pe}^i r_b/c = 3$. Shown in Fig. 2 is a plot of the normalized growth rate $(Im\omega)/\Gamma_w$ versus radial mode number n, and a plot of the eigenfunction $\delta E_z(r)$ versus r/r_w for mode number $n = 5$. It is clear from Fig. 2 and the analysis in Sec. 3 that the Weibel instability has characteristic growth rate Γ_w and can be particularly virulent for an intense ion charge bunch propagating through background plasma that provides full charge and current neutralization. It is therefore important to assess the relative importance of the electrostatic two-stream and electromagnetic Weibel instabilities for similar system parameters [5, 6].

REFERENCES

[1] R. C. Davidson and H. Qin, *Physics of Intense Charged Particle Beams in High Energy Accelerators* (World Scientific, Singapore, 2001), and references therein.

[2] M. Reiser, *Theory and Design of Charged Particle Beams* (Wiley, New York, 1994).

[3] E. A. Startsev, R. C. Davidson and H. Qin, Phys. Rev.-ST Accel. and Beams**6**, 084401 (2003).

[4] E. G. Harris, Phys. Rev. Lett. **2**, 34 (1959).

[5] R. C. Davidson, I. Kaganovich, H. Qin, E. A. Startsev, D. R. Welch, D. V. Rose and H. S. Uhm, Phys. Rev.-ST Accel. and Beams **7**, 114801 (2004), and references therein.

[6] R. C. Davidson, I. Kaganovich and E. A. Startsev, Princeton Plasma Physics Report PPPL-3940 (2004).

[7] E. S. Weibel, Phys. Rev. Lett. **2**, 83 (1959).

[8] R. C. Davidson, *Physics of Nonneutral Plasmas* (World Scientific, 2001), pp 272-276.

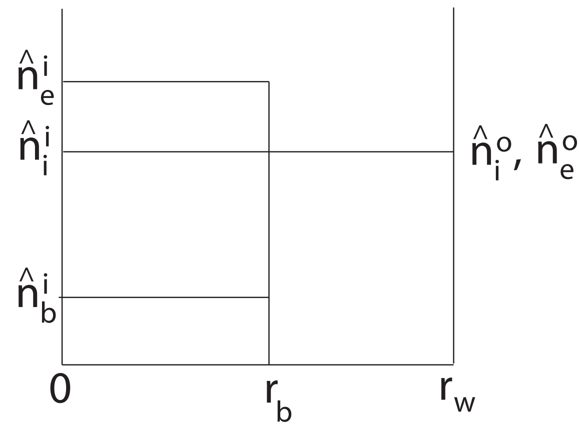

Figure 1: Schematics of the density profiles of the beam ions ($\hat{n}_b^i$) and the plasma ions and electrons inside ($\hat{n}_i^i$ and $\hat{n}_e^i$) and outside ($\hat{n}_i^o$ and $\hat{n}_e^o$) the beam.

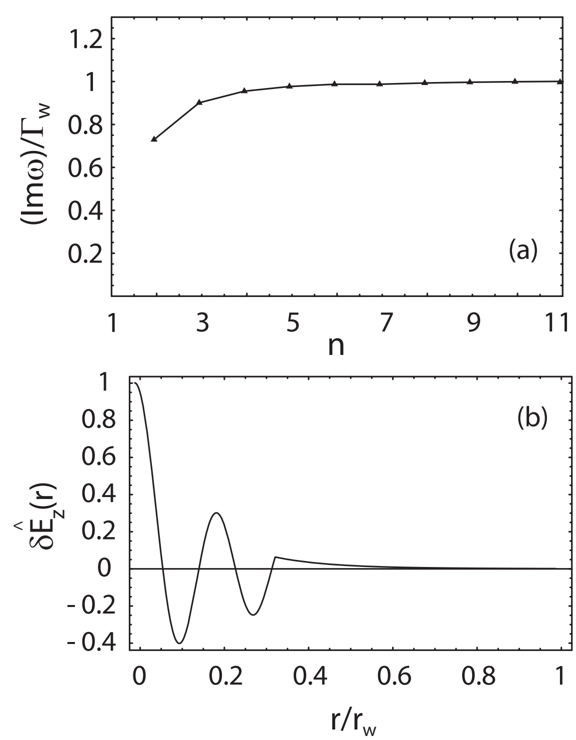

Figure 2: Plots of (a) Weibel instability growth rate $(Im\omega)/\Gamma_w$ versus radial mode number n, and (b) eigenfunction $\delta\hat{E}_z(r)$ versus r/r_w for $n = 5$ obtained from Eq. (10). System parameters are $r_b = r_w/3$, $\beta_b = 0.2$, $\beta_e = 0.1$, $\hat{n}_i^i = \hat{n}_e/2 = \hat{n}_b^i = \hat{n}_e^o = \hat{n}_i^o$, $\hat{\omega}_{pe}^i r_b/c = 3$.