Predictive Stability Analysis

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The challenge of predictive stability-1

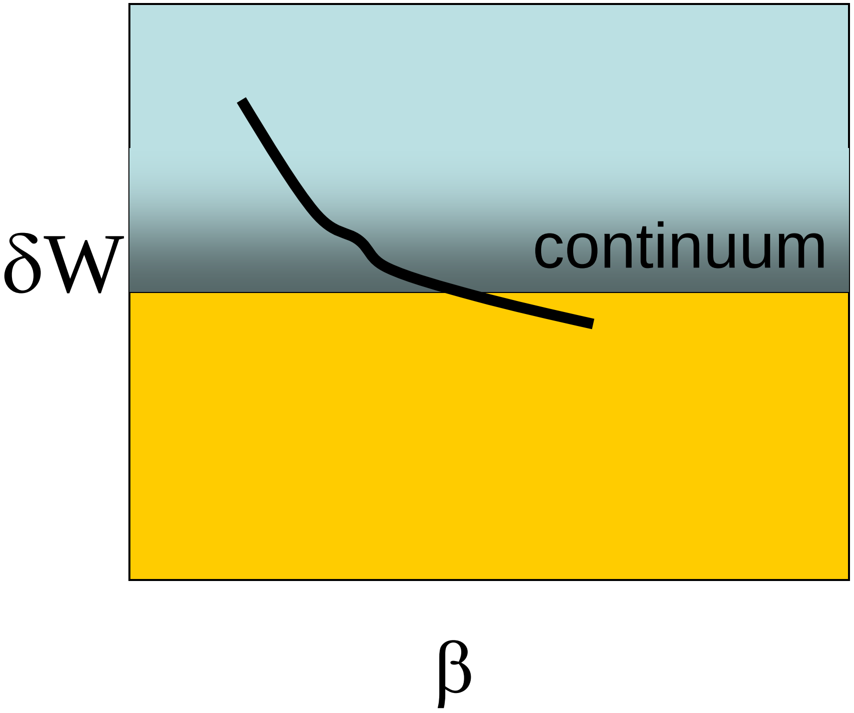
- The goal is to provide near real-time information on the MHD stability of the discharge, as it evolves
- In particular to identify proximity to the instability threshold
- The greatest difficulty lies in determination of the equilibrium
- Magnetic data quality has improved but kinetic pressure profiles require iteration

The challenge of predictive stability-2

- Even if you have perfect equilibria, stability analysis requires addressing multiple modes
  - sawteeth, ELMs, NTMs, RWMs ….
- The next difficulty is the time required for stability analysis, e. g.,
  - A linear ideal MHD stability code may take between 10 and 200 secs., depending on the complexity of the geometry and profiles
- Need to reduce this to ~msecs.
- Reminder: the results are sensitive to the accuracy of the equilibrium
- This study addresses large β-collapses

Ideal MHD stability

- Ideal MHD stability sets a hard upper limit
- If we can monitor this, we can predict instability
- The energy principle offers an approach
- But the ideal MHD spectrum makes it difficult to use the energy principle



A model for determining stability

- If δW is a weak function of plasma parameters then an approximate value is easily obtained using an approximate eigenvector
- However as the plasma evolves, so does δW
- It should reach its minimum when the real plasma becomes unstable
- Using model vectors it often peaks before the true beta limit

Alternate approach

1. Test function method
   - Create a set of test functions $\{\xi_i\}$
   - Use equilibria from similar shots
   - Determine δW_i for $\{\xi_i\}$
   - Track the minimum

Approach - Predicting δW

- We usually solve

$$\int d\tau \xi^* \delta W \xi = \omega^2 \int d\tau \rho \xi^* \xi$$

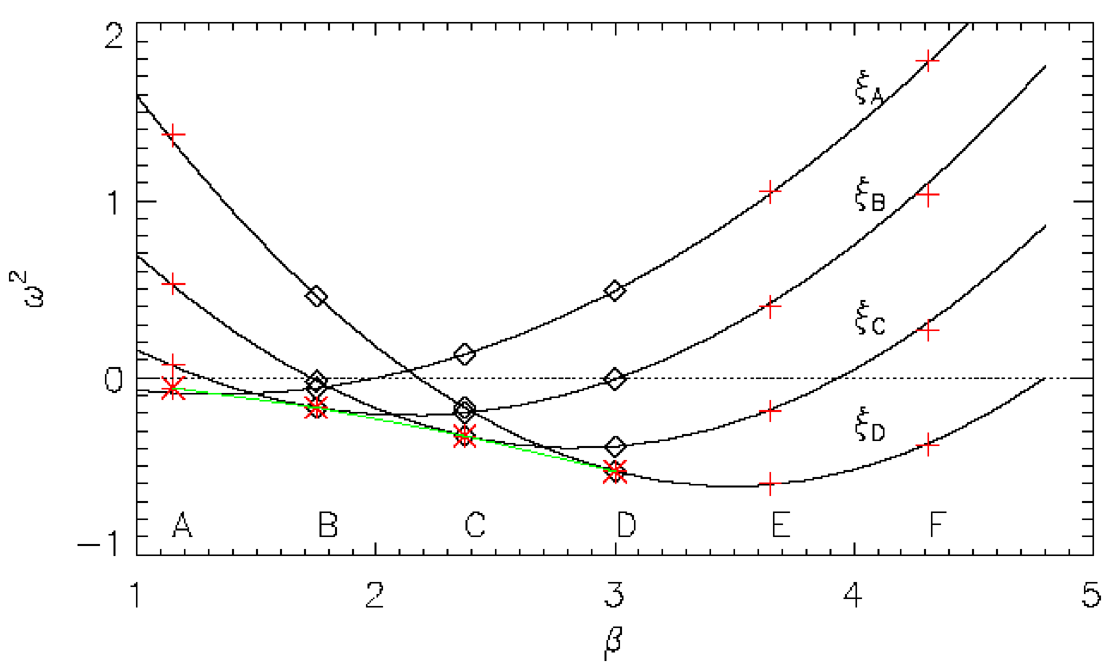
- We now evaluate

$$\int d\tau \xi_i^* \delta W \xi_i$$

- Using $\xi \in \{\xi_1, \xi_2, \xi_3, \cdots\}$

- Where the ξ are obtained from analysis of similar equilibria

ω^2 has a quadratic dependence away from the self-consistent equilibrium and eigenvector

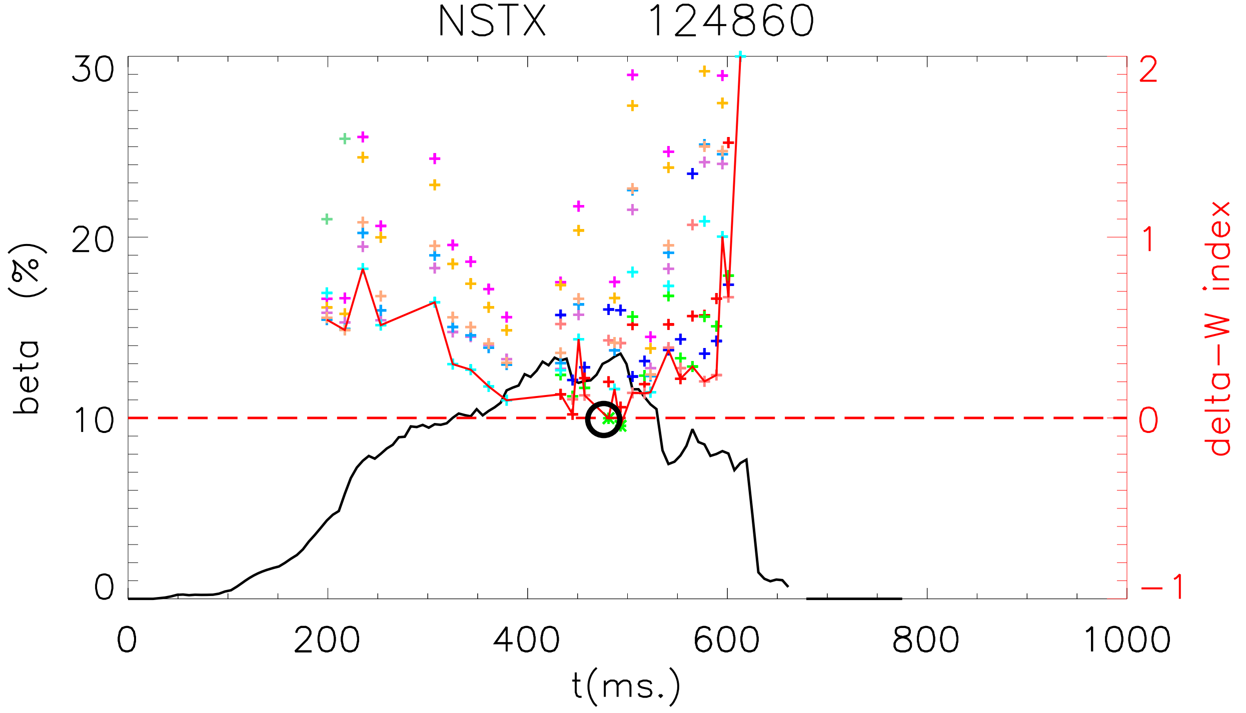


$$\omega^2 = \frac{\int d\tau \xi_i^* \delta W(j) \xi_i}{\int d\tau \rho \xi_i^* \xi_i}$$

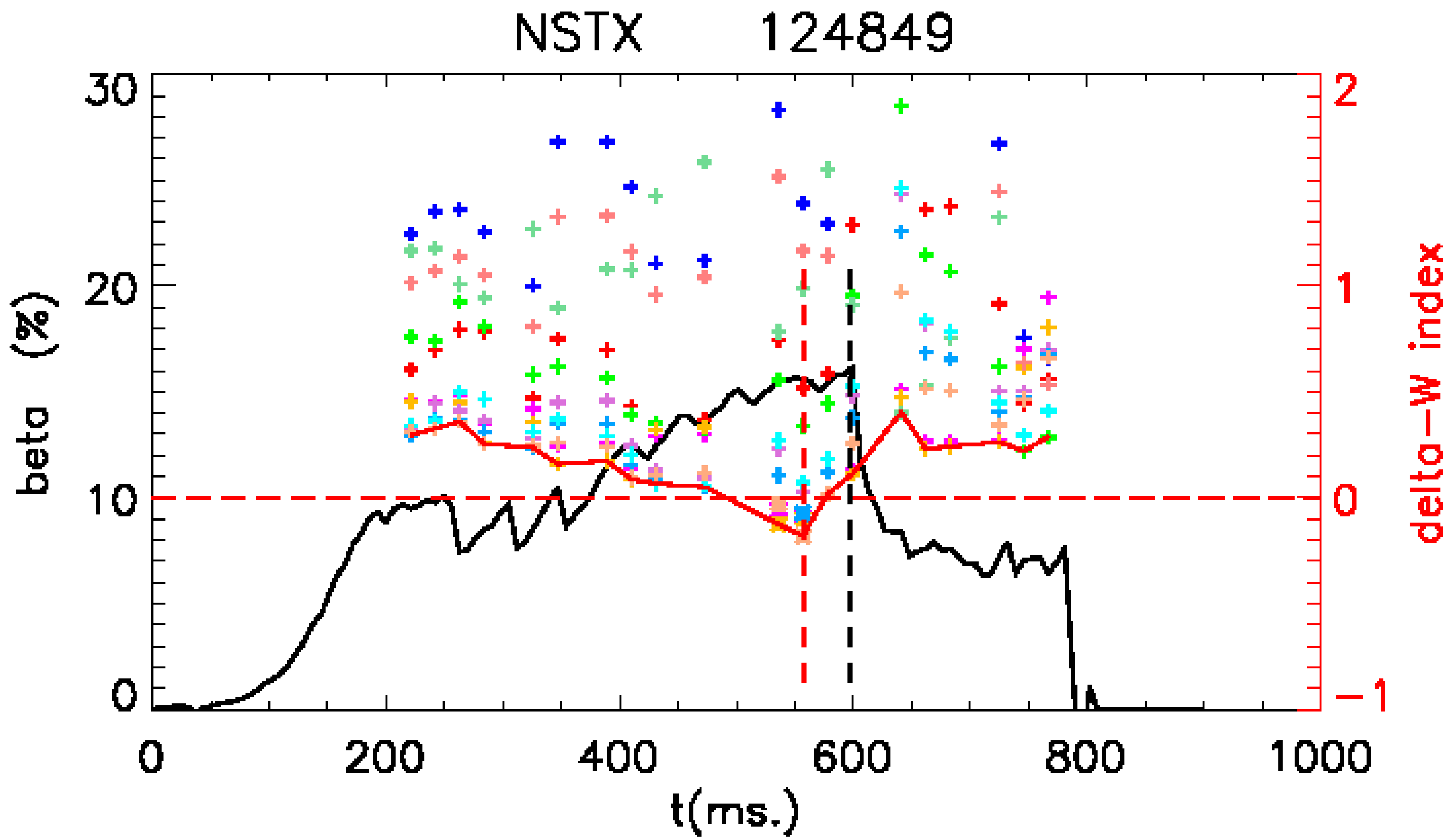
$$i, j = A, B\ldots$$

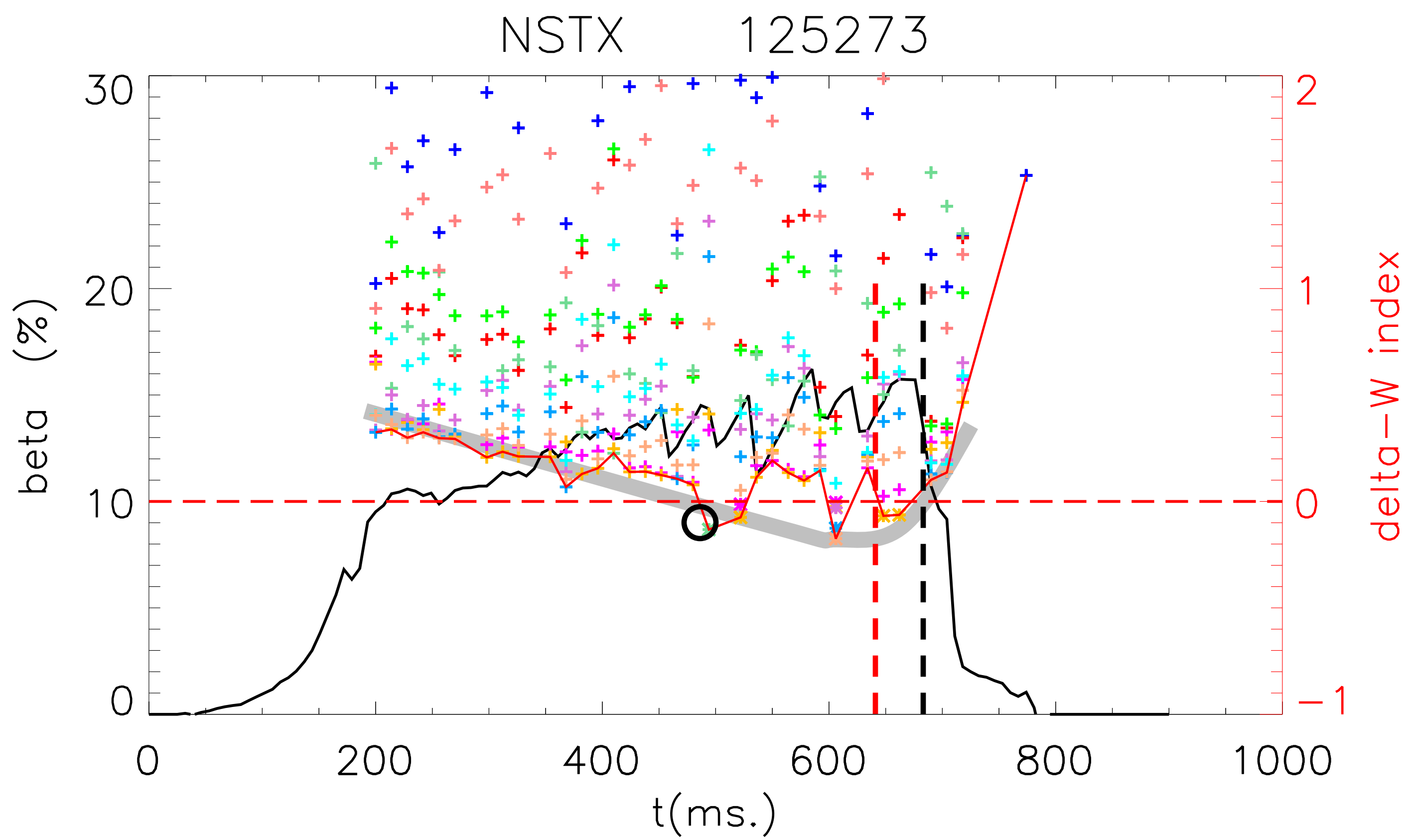
A test function obtained from NSTX 125271 at 497 ms.

A test function obtained from NSTX 124860 at 487 ms.



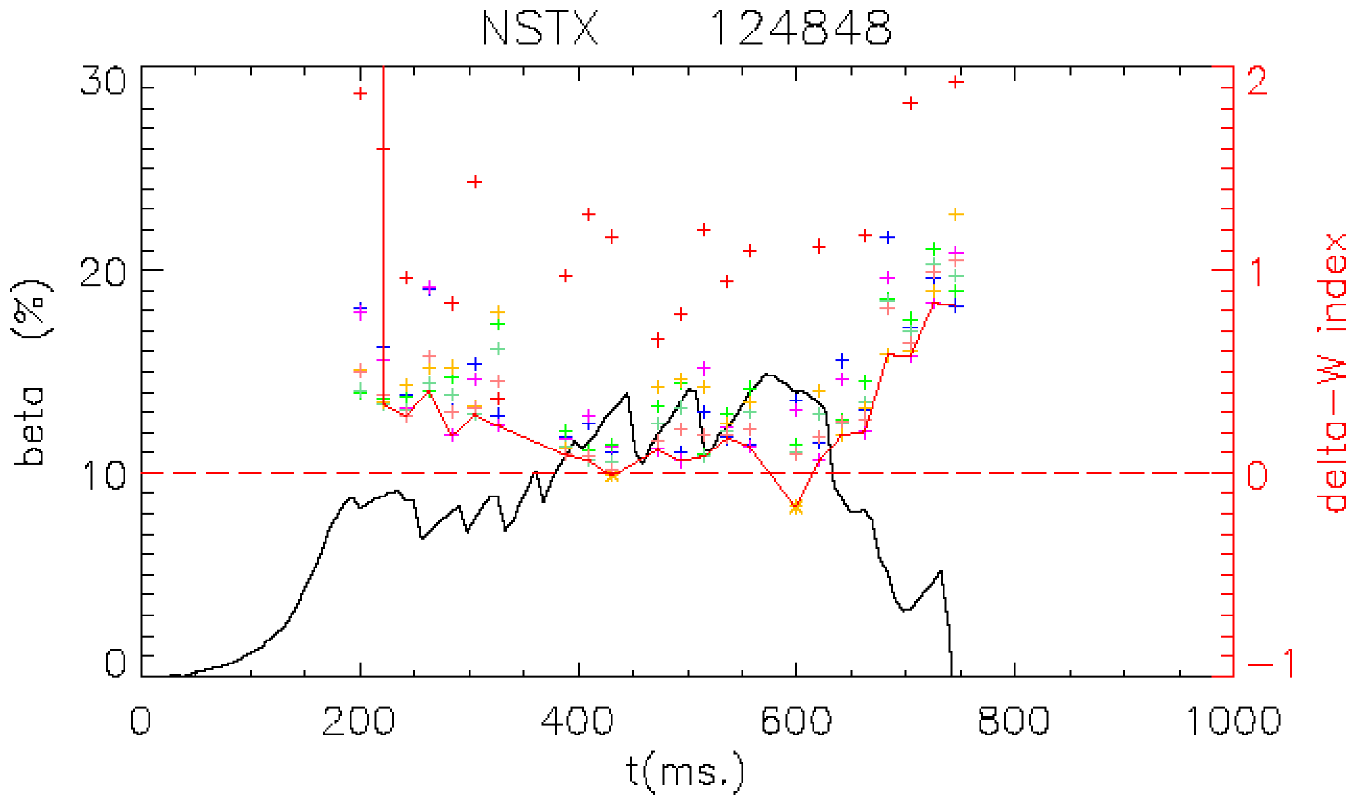
The δW(ξ) approach predicts the β-collapse 50 msecs ahead

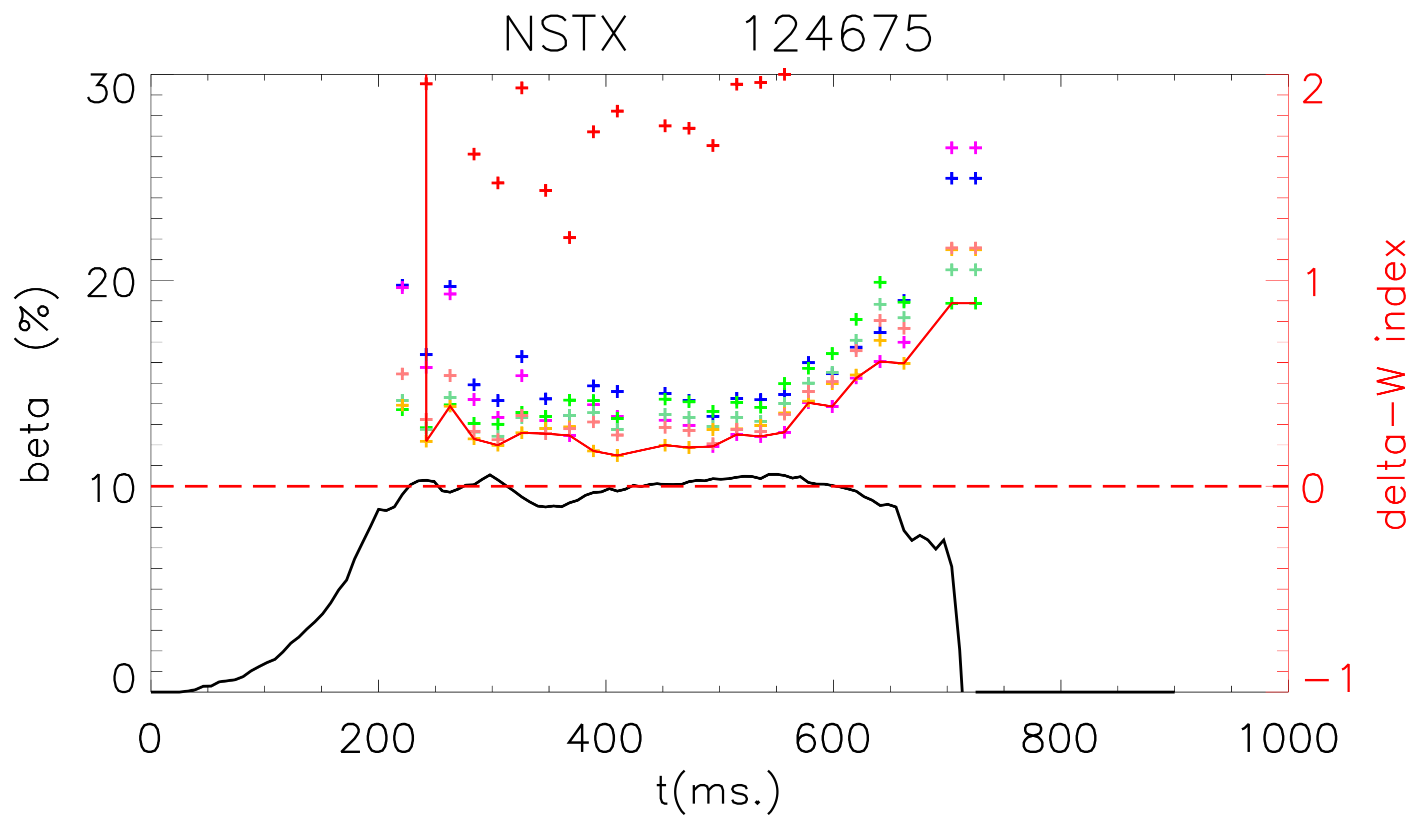




NSTX 125273
beta (%)
delta−W index
t(ms.)
0
200
400
600
800
1000
0
10
20
30
−1
0
1
2

124848


NSTX 124848
30
20
10
0
0
200
400
600
800
1000
t(ms.)
beta (%)
2
1
0
−1
delta−W index


NSTX 124675
beta (%)
delta−W index
t(ms.)
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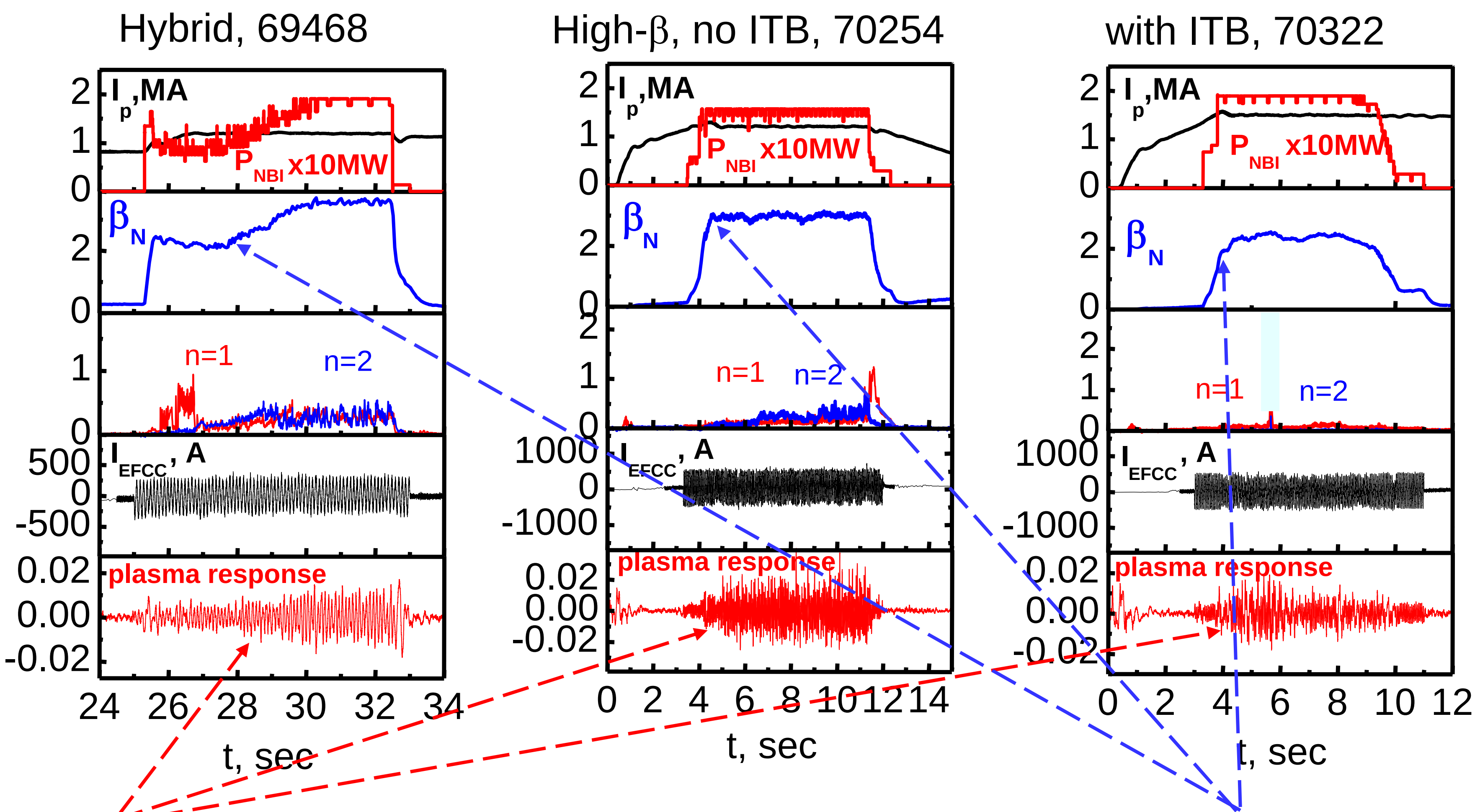
400
600
800
1000
0
10
20
30
−1
0
1
2

RFA probed in three advanced regimes



- ***RFA* clearly increases when β_N exceeds *a critical value***

To simulate the RFA response Identify the contributions due to different harmonics

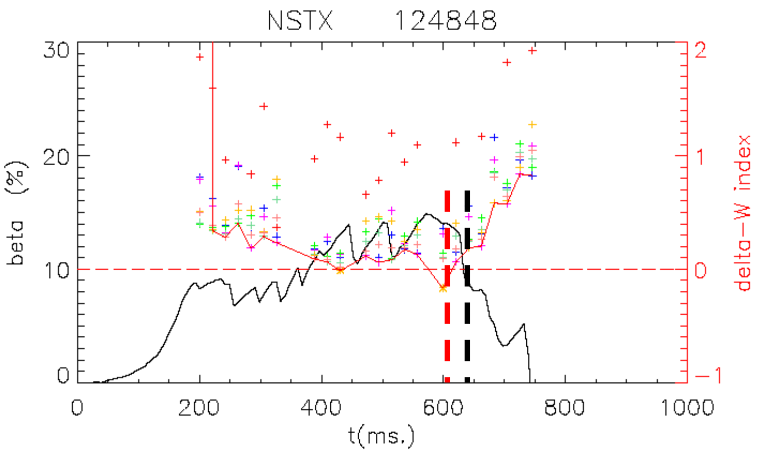
$$W = \int d\tau \xi_i^* \delta W \xi_i$$

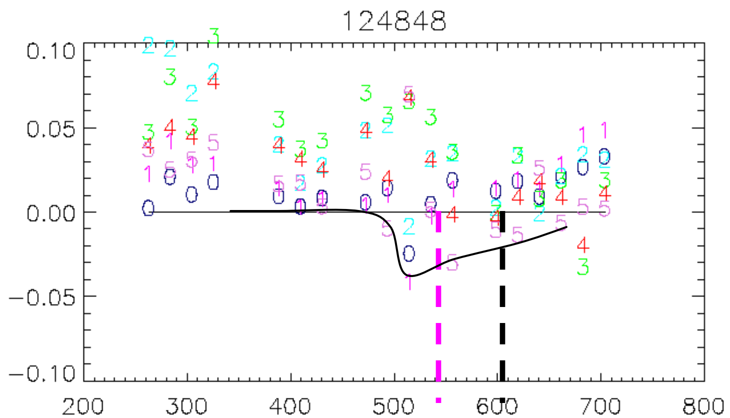
$$\xi = \sum_{l,m} a_m^l(\psi) e^{i(l\theta - n\varphi)}$$

$$\xi = \sum_l \xi_l$$

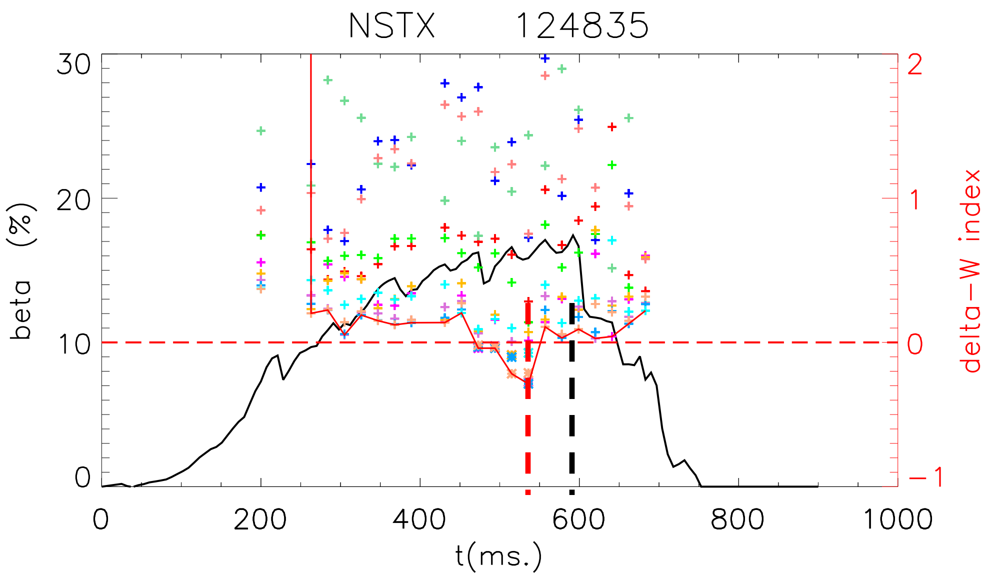
$$W_l = \int d\tau \xi_l^* \delta W \xi_l$$

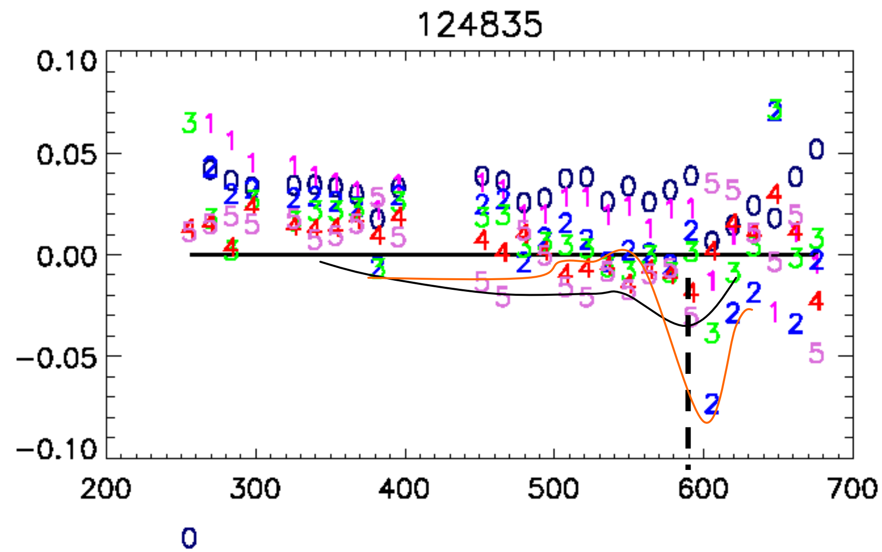
Some modes peak before the β-collapse





Some modes peak near the β-collapse





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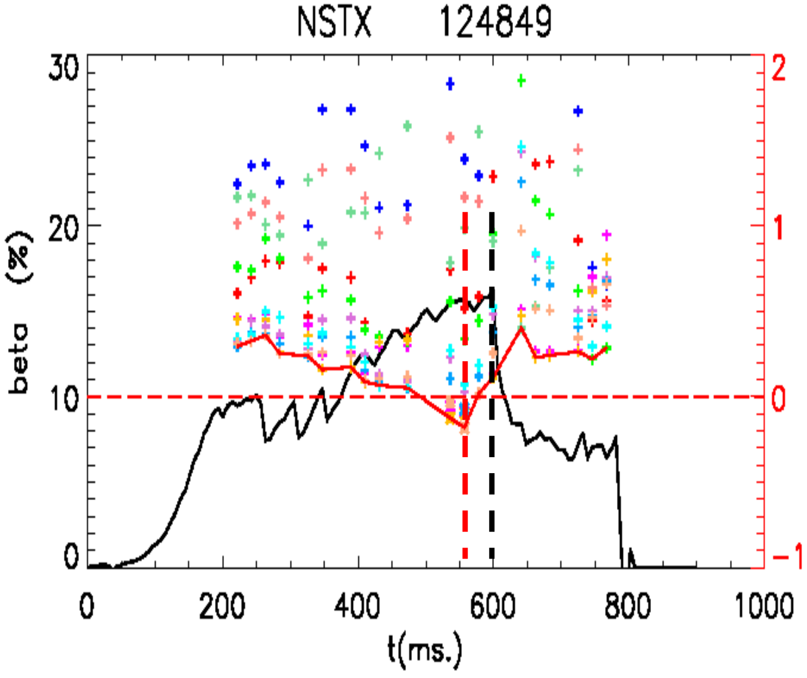
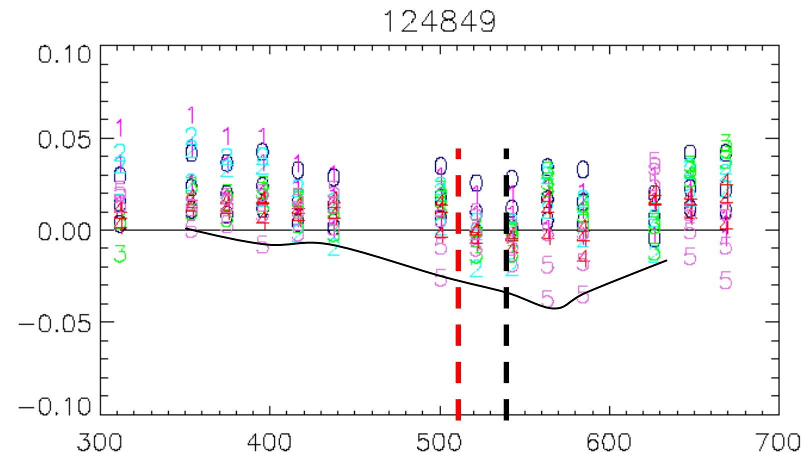
Some harmonics peak after the β-collapse





Simulating the resonant field

$$\xi_{ls} = (1 + s(\psi))\xi_l$$

$$W_l^s = \int d\tau \xi_{ls}^* \delta W \xi_{ls}$$

$$W_l^s = \int d\tau \xi_l^* \delta W \xi_l + \underbrace{\int d\tau \xi_s^* \delta W \xi_l + \int d\tau \xi_l^* \delta W \xi_s + \int d\tau \xi_s^* \delta W \xi_s}_{W_s}$$

$$W_s = \int d\tau \xi_s^* \delta W \xi_l + \int d\tau \xi_l^* \delta W \xi_s + \int d\tau \xi_s^* \delta W \xi_s$$

Plasma response ~ W_s

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Simulating the resonant field-2

$$B = B_0 + \delta B$$

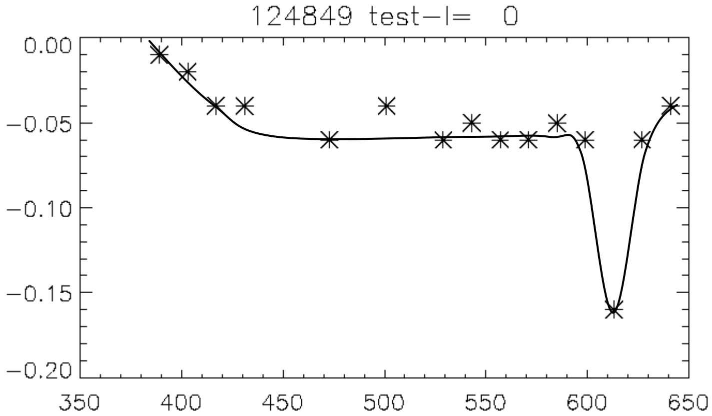
$$W_l^s = \int d\tau \xi_l^* \delta W(B) \xi_l$$

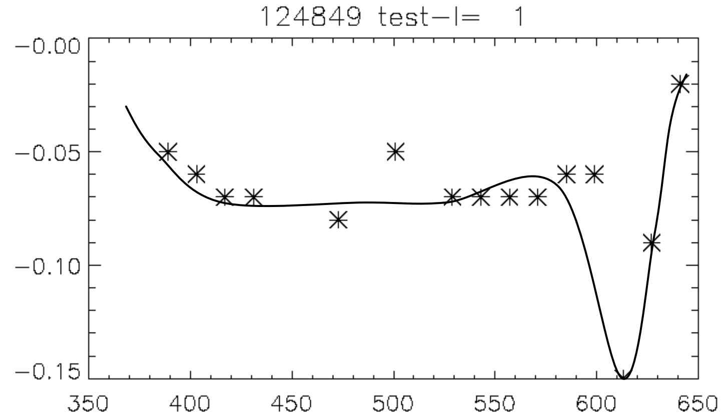
$$W_l^s = \int d\tau \xi_l^* \delta W(B_0) \xi_l + \int d\tau \xi_l^* \delta W(\delta B_0) \xi_l$$

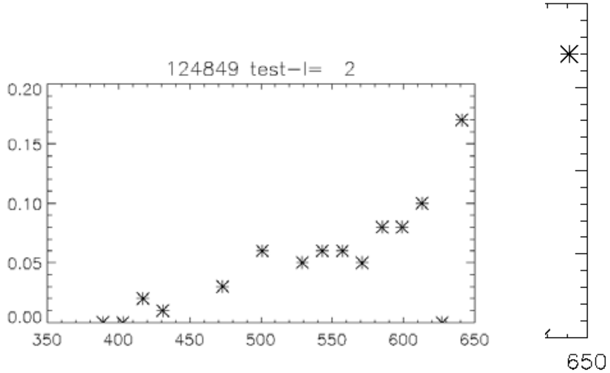
$$W_s = \int d\tau \xi_l^* \delta W(\delta B_0) \xi_l$$

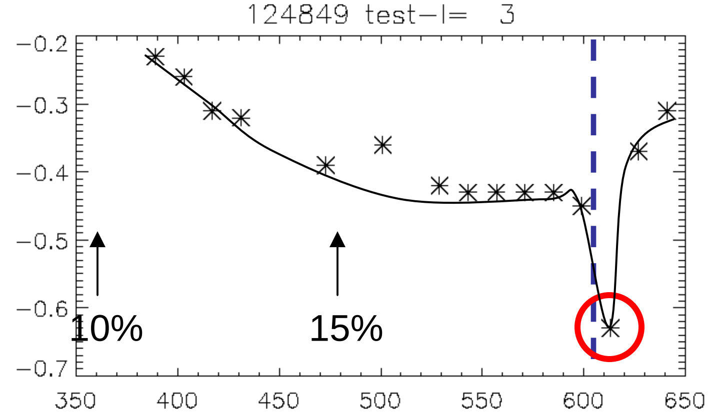
Not reported here

Response functions for 124849

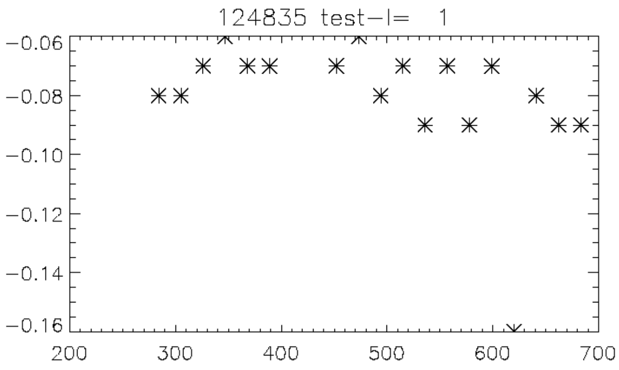


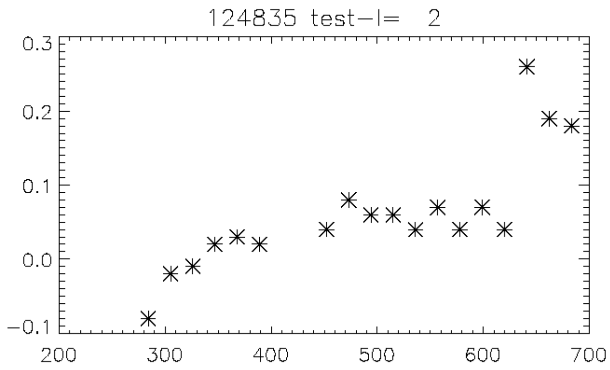


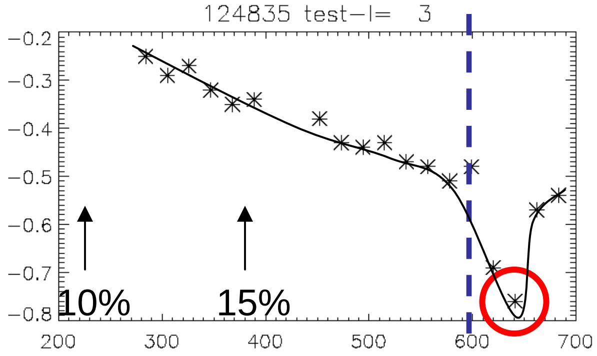


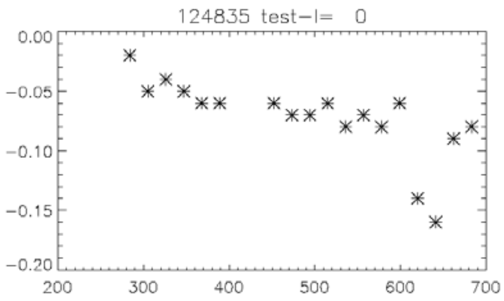


Response functions for 124835

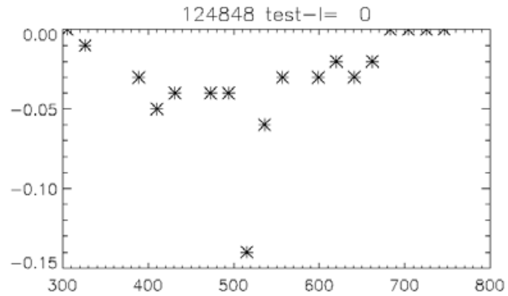


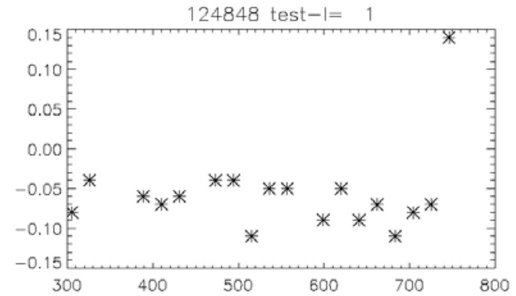


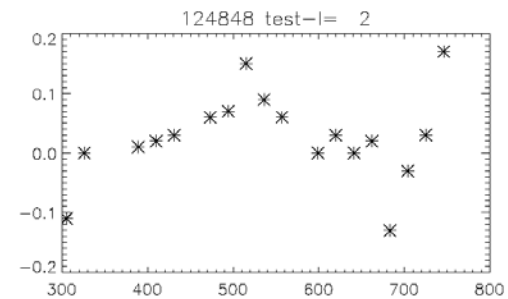


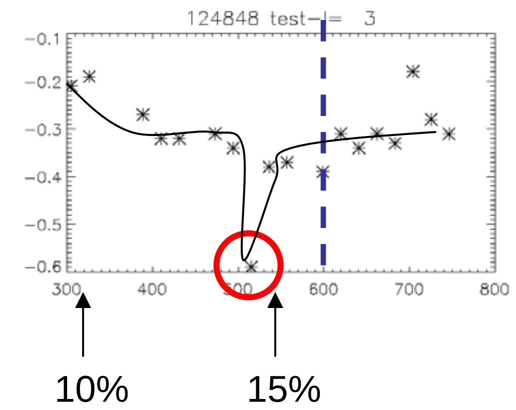


Response functions for 124848









SUMMARY

- We have examined approaches to predicting ideal stability with the potential for real-time application
- Test vector approach – faster than full calculation by a factor of 40
- Another approach is to monitor the response to an applied field
- For NSTX the m=3 is a good candidate