About the toroidal magnetic field of a tokamak burning plasma experiment with superconducting coils

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ABSTRACT

In tokamaks, the strong dependence on the toroidal magnetic field of both plasma pressure and energy confinement is what makes possible the construction of small and relatively inexpensive burning plasma experiments using high-field resistive coils. On the other hand, the toroidal magnetic field of tokamaks using superconducting coils is limited by the critical field of superconductivity. In this article, we examine the relative merit of raising the magnetic field of a tokamak plasma by increasing its aspect ratio at a constant value of the peak field in the toroidal magnet. Taking ITER-FEAT as an example, we find that it is possible to reach thermonuclear ignition using an aspect ratio of ~4.5 and a toroidal magnetic field of 7.3 T. Under these conditions, fusion power density and neutron wall loading are the same as in ITER, but the normalized plasma beta is substantially smaller. Furthermore, such a tokamak would be able to reach an energy gain of ~15 even with the deterioration in plasma confinement that is known to occur near the density limit where ITER is forced to operate.

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I. INTRODUCTION

It is widely recognized that the next step in the development of a tokamak fusion reactor is a DT burning plasma experiment for the exploration of the physics of α-dominated plasmas, i.e., plasmas where the kinetic energy of charged fusion products is the dominant source of plasma heating.

Presently, there are three burning plasma proposals under development: IGNITOR,[1] FIRE,[2] and ITER-FEAT.[3] All three proposals share a common interest in the study of the physics of burning plasmas, and the goal of achieving an energy gain of approximately 10 (here the energy gain Q is defined as the ratio of the total fusion power to the auxiliary heating power). While both IGNITOR and FIRE are based on the assumption that the study of the physics of burning plasmas must take precedence over technological issues, ITER-FEAT (in the following referred to as ITER) is designed to address the physics and the engineering of a fusion reactor in an integrated fashion. This makes ITER large and expensive, as is obvious from the list of parameters in Table 1.

The size of a tokamak burning plasma experiment is essentially determined by the value of the toroidal magnetic field. Because of its mission, ITER will employ reactor-relevant superconducting coils, capable of producing a maximum field of 5.3 T at the center of the plasma torus. This is much smaller than the magnetic field of IGNITOR and FIRE (13 and 10 T, respectively), whose designs are based instead on copper alloy magnets.

The operational mode foreseen for ITER is the ELMy H-mode, for which a number of empirical scaling laws have been published. In general, these scaling laws are cast in the form[4]

$$\tau_E \omega_c = \rho^{*\alpha} F(\beta, \nu^*, \{p_i\}) , \qquad (1)$$

where τ_E is the plasma energy confinement time, ω_c is the ion cyclotron frequency, $\rho^* = \rho / a$ is the normalized ion Larmor radius (with a the minor radius), and F is a function of the toroidal plasma beta β, the effective collision frequency $\nu*$, and a set $\{p_i\}$ of dimensionless parameters including the safety factor q_{95}, the aspect ratio $A = R/a$ (with R the major radius), the elongation k, the triangularity δ and the average isotopic number M. The scaling used to predict the performance of ITER is[4]

$$\tau_E \omega_c \propto \rho*^{-2.70} \ \beta^{-0.90} \nu*^{-0.01} \ M^{0.96} q_{95}^{-3.0} A^{-0.73} k^{2.3} . \tag{2}$$

For a constant value of q_{95} and plasma elongation, Eq. (2) predicts that the fusion figure of merit $F \equiv nT\tau_E$ (where n and T are plasma density and temperature) should scale like

$$F \propto n^{0.1} T^{-1.25} B^{3.5} a^{2.7} A^{-0.73} , \tag{3}$$

which demonstrates the crucial importance of the toroidal magnetic field for the operation of a burning plasma experiment. In the present ITER design,[3] the toroidal magnetic field on axis corresponds to a maximum field of 11.8 T on the TF coil conductor. As suggested in Ref. 5, the adoption of different engineering solutions could increase the magnetic field on axis to 6.4 T. Since this would certainly reduce the machine flexibility, in the following we discuss a less demanding technical solution to the problem of raising the magnetic field in tokamaks with superconducting magnets.

II. ASPECT RATIO VS. MAGNETIC FIELD

The toroidal magnetic field at the center of a tokamak plasma is

$$B = \frac{B_{\max}[A - (1+\gamma)]}{A} , \tag{4}$$

where $B_{\max}$ is the maximum field in the TF coil conductor and $\Delta = \gamma a$ is the radial gap between the inner circumference of the plasma column and the point in the conductor where the magnetic field is maximum. In the following, we will examine the relative merit of raising B by increasing the plasma major radius at constant values of a, γ and $B_{\max}$. Since B is a decreasing function of γ [Eq. (4)], here we are interested in tokamaks where the radial distance Δ is comparable to the plasma minor radius itself. Inevitably, this will be the case in tokamak fusion reactors because of a variety of cooling, shielding and breeding blanket components.

Figure 1 displays B as a function of A for the ITER parameters of a=2 m, γ=0.71 and B_{max}=11.8 T. In this figure, the toroidal field varies from 5.3 T for A=3.1, to 8.4 T for A=6. The corresponding plasma current ($I_p \propto B/A$) is shown in Fig. 2 for the same safety factor and plasma elongation as in ITER.

To assess the relative merit of the aspect ratio and the magnetic field on the performance of a burning plasma experiment, we have used a simple global power balance analysis[6] with the leading energy loss represented by Eq. (2). The latter can be written in terms of physical quantities as[4]

$$\tau_E = 0.144 I_p^{0.93} B^{0.15} P^{-0.69} \bar{n}^{0.41} M^{0.19} R^{1.97} A^{-0.58} k^{0.78} , \qquad (5)$$

where $\bar{n}$ is the line average density, P is the total heating power, and the units are s, MA, T, MW, 10^{20} m^{-3}, amu and m. In performing the global energy balance, all averages of plasma parameters were calculated using realistic magnetic configurations. Two examples are shown in Fig. 3 with aspect ratios of 3.1 and 5.0.

In the operation of tokamaks, two parameters of critical importance are the normalized plasma beta $\beta_N = 10^2 \beta /(I_p / Ba)$ and the plasma density $n_G = \bar{n} / n_{GR}$, where $n_{GR} = I_p / \pi a^2$ is the Greenwald density limit.[7] In the reference scenario of ITER,[3] both of these parameters (β_N=1.8 and n_G=0.85) are close to the operational limits of tokamaks. Since any burning plasma experiment will operate in a narrow range of temperatures (~10 keV), the beta limit can be considered a density limit as well ($\propto B^2 / A$). Consequently, since any increase in B makes the beta limit less restrictive than the Greenwald limit ($\propto B/R$), the power balance analysis was performed keeping constant the value of n_G (=0.85 as in ITER).

Figures 4 and 5 display the total fusion power and the average fusion power density for seven plasma configurations with Z_{eff}=1.65 (mostly Beryllium), 5% of α-particles, and two values of plasma temperature (defined as the volume average $T_n \equiv \langle Tn \rangle / \langle n \rangle$). In these figures, the solid lines represent the scaling for a constant value of n_G (B^2/A and B^2/A^2, respectively). For A=3.1 and T_n=10.5 keV, the total fusion power and the power density are the same as in the ITER reference scenario.[3] In Fig. 5, the power density peaks at an aspect ratio of ~3.5, and for A=4.5 it is only 10% smaller than in ITER. Keeping β_N constant (instead of n_G) would have given larger values for both fusion power and power density, with the latter peaking at A~5 (Fig. 6). However, as mentioned above, we have not considered this scenario since it makes the normalized plasma density scale linearly with B, and therefore becomes quickly larger than one as the aspect ratio is raised above that of ITER. For instance, for A=4.5 we get n_G=1.15. As we shall see in the next section, this has deleterious consequences for plasma confinement in the ELMy H-mode.

The average neutron flux (Fig. 7) at the plasma boundary (95% flux surface) is very similar to the average fusion power density (since both quantities vary like B^2/A^2 and a=2 m). Hence, by keeping the value of γa constant we can achieve an equal or greater level of radiation shielding than in ITER.

The normalized beta is a decreasing function of aspect ratio, as shown in Fig. 8 where β_N is displayed together with the poloidal beta (β_p). The latter increases only slightly with A, which makes the ratio of the bootstrap current to the total plasma current very insensitive to the aspect ratio, with changes of less than 10% over the entire range of A.

The calculated energy gains are displayed in Fig. 9 as a function of aspect ratio. For T_n=10.5 keV, Q starts from a value of 10 at A=3.1 (as in ITER) and reaches quickly large values (i.e., ignition) when $A \geq 4.5$. For comparison, at the same temperature we obtain Q=15 for IGNITOR and Q=4 for FIRE. Finally, for a higher temperature (T_n=12 keV) we get lower values of Q (in agreement with Eq. (2)), which however remain quite large

(30-60) for aspect ratios larger than four.

III. DISCUSSION

The results of the previous section must be considered a clear demonstration of the crucial importance of the toroidal magnetic field for the operation of a burning plasma experiment. This is a consequence of the ELMy H-mode scaling of Eq. (2). For plasma operation in this regime, the total heating power (P) must exceed a threshold (P_{LH}) given by[8] (as in ITER)

$$P_{LH} = 2.84 M^{-1} B^{0.82} \bar{n}^{0.58} R a^{0.81} , \qquad (6)$$

where units are the same as in Eq. (5). Since P_{LH} increases with both magnetic field and major radius, in this article it is an increasing function of the aspect ratio. On the other hand, plasma heating from α-particles increases with aspect ratio because of an increase in fusion power (Fig. 4), while the auxiliary heating power decreases because of a strong increase of Q with A (Fig. 9). The H-mode power threshold and the total heating power are displayed in Fig. 10, showing that $P > P_{LH}$ for T_n=10.5 keV and A<5, and on the entire range of aspect ratios when T_n=12 keV. However, experimental evidence from tokamak experiments indicates that the heating power must be 20-30% larger than P_{LH} to obtain a good H-mode plasma confinement. Consequently, from Fig. 10 we conclude that a safe range of operation is A<4.5 for T_n=10.5 keV, and A<5.5 for T_n=12 keV. Finally, we add that plasma operation at 12 keV requires ~80 MW of auxiliary heating power for ITER (Q=7) and ~16 MW for A=4.5 (Q=45).

Existing experiments[9] indicate that the plasma confinement degrades very quickly as the value of n_G approaches unity. Indeed, the database that was used for deriving the empirical scaling of Eq. (5) contains a small number of cases with $n_G \geq 0.85$. More recently, this gap has been filled with a number of high-density discharges that were obtained with new gas fueling and power controlled techniques.[10-13] From this new database, an extra factor for the ELMy H-mode scaling was derived having the form[14]

$$H = 0.71 + 0.33\delta - 1.58(n_G - 0.63)^2 + 0.58(\bar{n}/n_{ped} - 1) \ , \qquad (7)$$

where n_{ped} is the edge density. For the ITER reference discharge (with δ=0.5 and $\bar{n}/n_{ped}$=0), we obtain H=0.8, while for the previous ITER-EDA[15] design (with δ=0.3, n_G=1.15 and $\bar{n}/n_{ped}$=0) we get H=0.4. A peaking factor of $\bar{n}/n_{ped}$=1.3 would give an improved H factor of 0.96 for ITER-FEAT. However, since the fueling techniques employed in present experiments are not necessarily applicable to very large tokamaks, we have repeated the above calculations with H=0.85 for assessing the effect of a small deterioration in energy confinement. The results are displayed in Fig. 11, which shows that a mere drop of 15% in the H factor has the effect of lowering the energy gain of ITER to ~4, and to 15-20 for A≥4.5.

Another matter of concern is the low value of Z_{eff} (=1.65) in the reference scenario of ITER compared to existing tokamak experiments. Although the possibility of reaching a Z_{eff} of 1.65 cannot be ruled out completely, it is obvious that an ITER-like device must be designed to reach Q=10 even at larger values of Z_{eff}. Indeed this is not possible, as demonstrated by Fig. 11 where it is shown that a mere increase of 15% in Z_{eff} (to 1.9 as in ITER-EDA) lowers the Q of ITER to ~5. On the contrary, for A=4.5 we get a Q of ~15.

An additional benefit of using large aspect ratios is a high magnetic flux in the OH transformer. In the present ITER design,[3] of the available 277 Vs (largely produced by the Central Solenoid (CS)), only 37 Vs are used for sustaining a current flat top of 400 s. As an example, then, when the aspect ratio is increased from 3.1 to 4.5, the plasma major radius increases by 2.8 m. If only half of this is used for enlarging the radius of the CS coil (presently with inner/outer radius of ~1.3/2.0 m), with the rest used for strengthening the TF and CS coils, the total flux in the OH transformer increases by almost a factor of four. This would increase the available magnetic flux for sustaining the plasma current by almost an order of magnitude – making possible flat tops longer than one hour.

Obviously, the result of enlarging the aspect ratio of ITER is an increase in the cost of its core. However, a failure of ITER to reach Q=10 would be much costlier to the credibility of controlled fusion energy. Furthermore, since fusion power increases with aspect ratio (Fig. 4), any cost assessment must be performed at constant fusion power. Figure 12 shows two tokamak configurations, one with a=2 m and R=9.0 m (A=4.5), the second with a=2.72 m and R=8.5 m (A=3.1). Note that the latter dimensions are very similar to those of ITER-EDA.[15] In both cases, the fusion power is 800 MW and Q=45 for identical values of γ=0.71, $B_{\max}$=11.8 T, n_G=0.85, T_n=12 keV, Z_{eff}=1.65 and H=1. A volume ratio of 1.70 (in favor of the configuration with A=3.1) clearly demonstrates the advantages of using large aspect ratios.

IV. SUMMARY

The optimization of the toroidal magnetic field is of crucial importance for the feasibility of a tokamak burning plasma experiments, regardless of whether it employs resistive or superconductive coils. A case in point is that of ITER-FEAT, where the need to operate near the Greenwald density limit with a high degree of plasma purity casts serious doubts on the feasibility of its main objective – an energy gain of at least 10. In this article, we have discussed the relative merit of increasing the toroidal magnetic field of a tokamak plasma by increasing its aspect ratio at constant values of the maximum field in the TF magnet, the plasma minor radius and the radial distance between the high-field plasma edge and the point in the conductor where the field is maximum. Taking ITER-FEAT as an example and making similar assumptions on plasma confinement, normalized density and Z_{eff}., we have found that ignition can be reached using an aspect ratio of ~4.5 and a toroidal magnetic field of 7.3 T. Under these conditions, the fusion power density and the neutron wall loading are the same as in ITER, but the value of normalized beta is substantially smaller. Furthermore, such a tokamak would be able to reach an energy gain of ~15 even with the deterioration in plasma confinement that is known to occur near the density limit where ITER is forced to operate, or with of an increase of 15% in the level of impurities. On the contrary, the same conditions would lower the Q of the present ITER configuration to ~5.

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Table 1. Parameters of Burning Plasma Experiments

	IGNITOR	FIRE	ITER-FEAT
minor radius *a* [m]	0.47	0.595	2.0
aspect ratio *A*	2.8	3.6	3.1
elongation *k*	1.83	1.81	1.70
toroidal field *B* [T]	13.0	10.0	5.3
plasma current I_p [MA]	11.0	7.7	15.0
flat top [s]	10	20	400

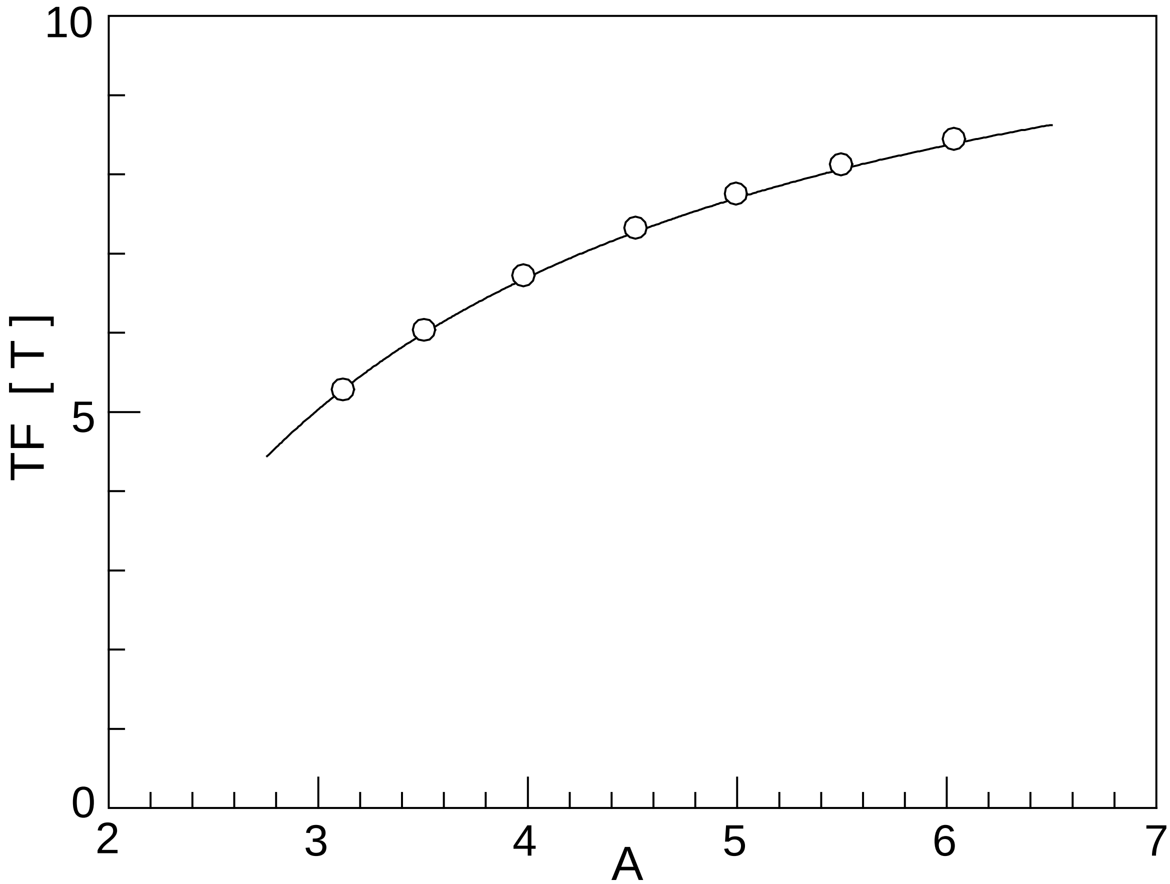

Figure 1. Toroidal magnetic field as a function of aspect ratio at constant values of a, γ and B_{max}. Circles are for tokamaks used in the global power balance; solid line is from Eq. (4).

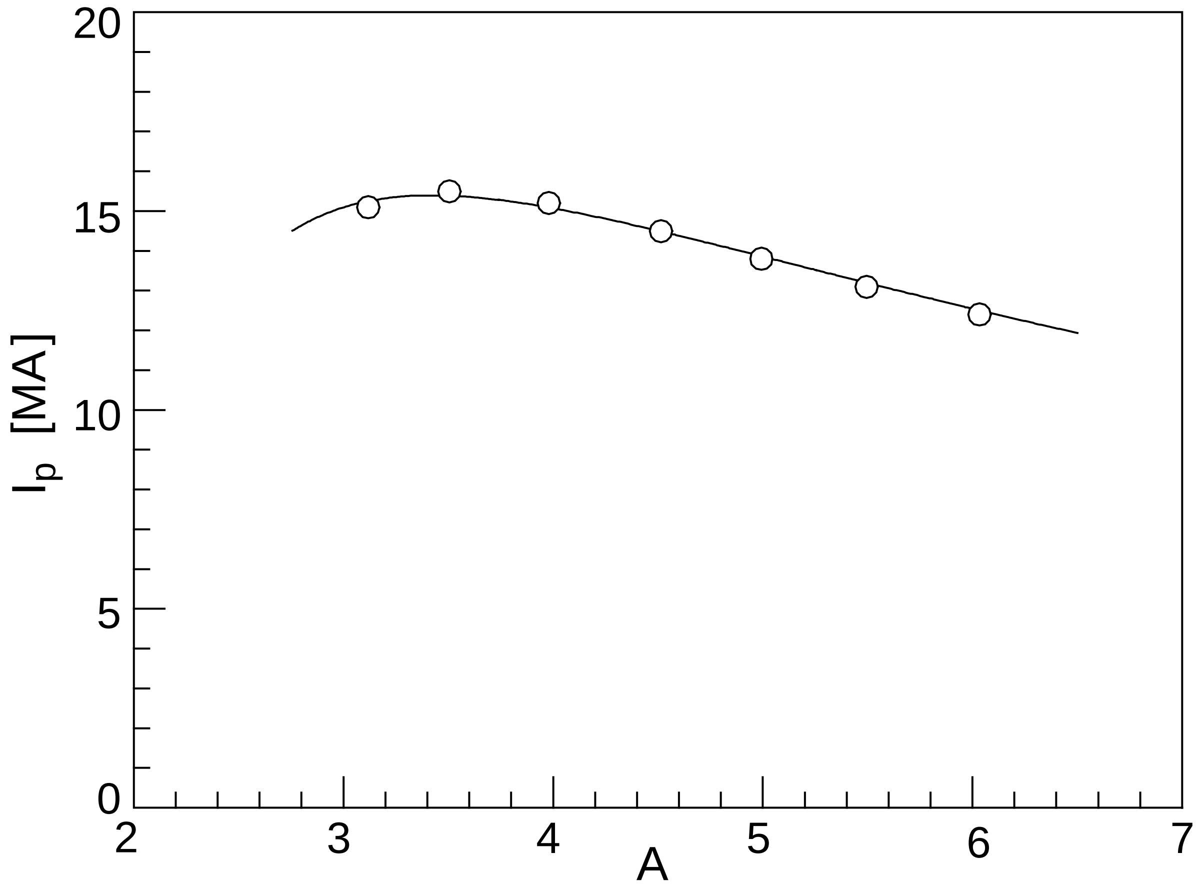

Figure 2. Plasma current as a function of aspect ratio. Circles are for tokamaks used in the global power balance; solid line is $\propto B/A$.

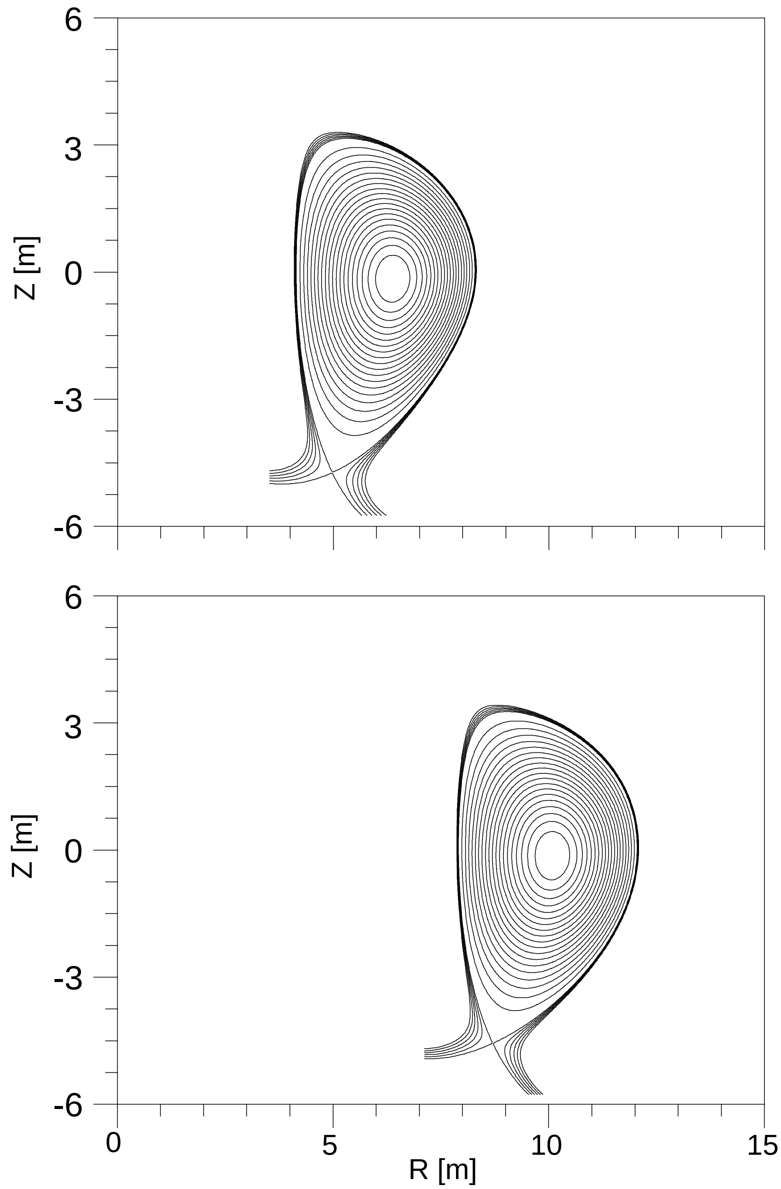

Figure 3. ITER-like magnetic configurations with an aspect ratio of 3.1 (top) and 5.0 (bottom).

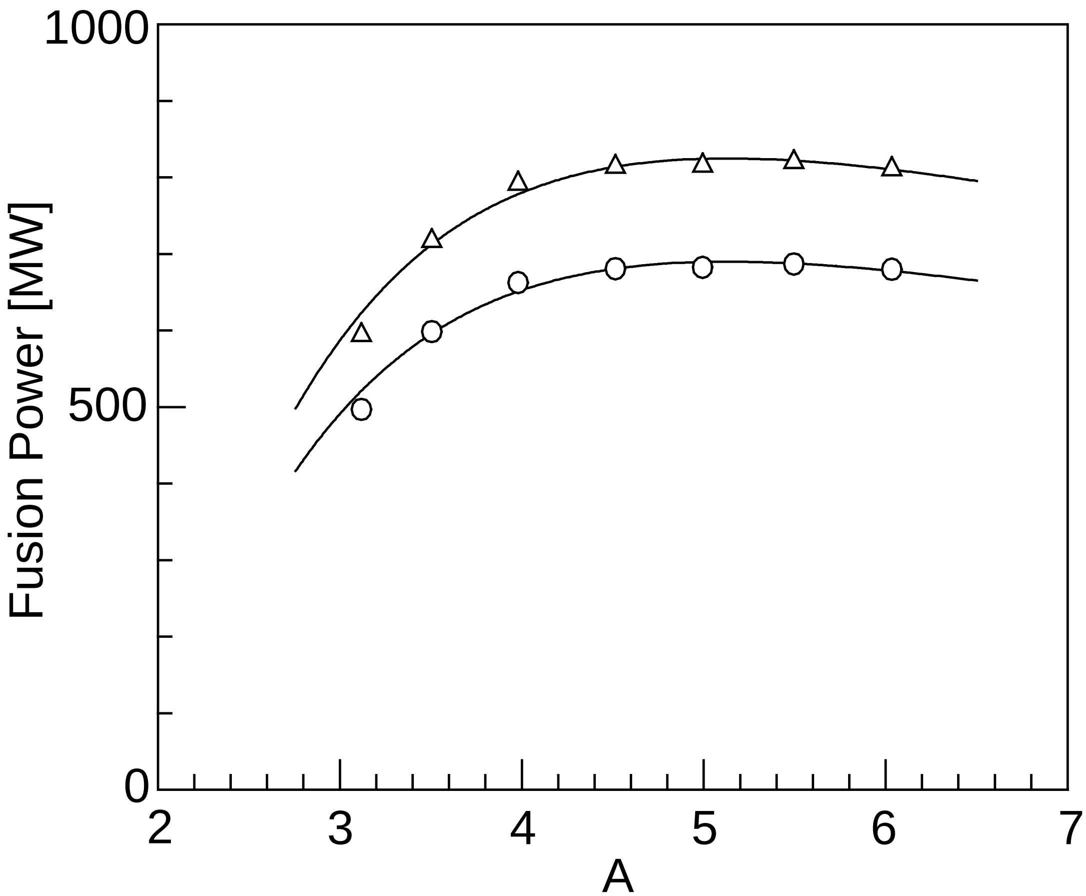

Figure 4. Total fusion power as a function of aspect ratio. Circles: T_n=10.5 keV; triangles: T_n=12 keV. Solid lines are $\propto B^2 / A$.

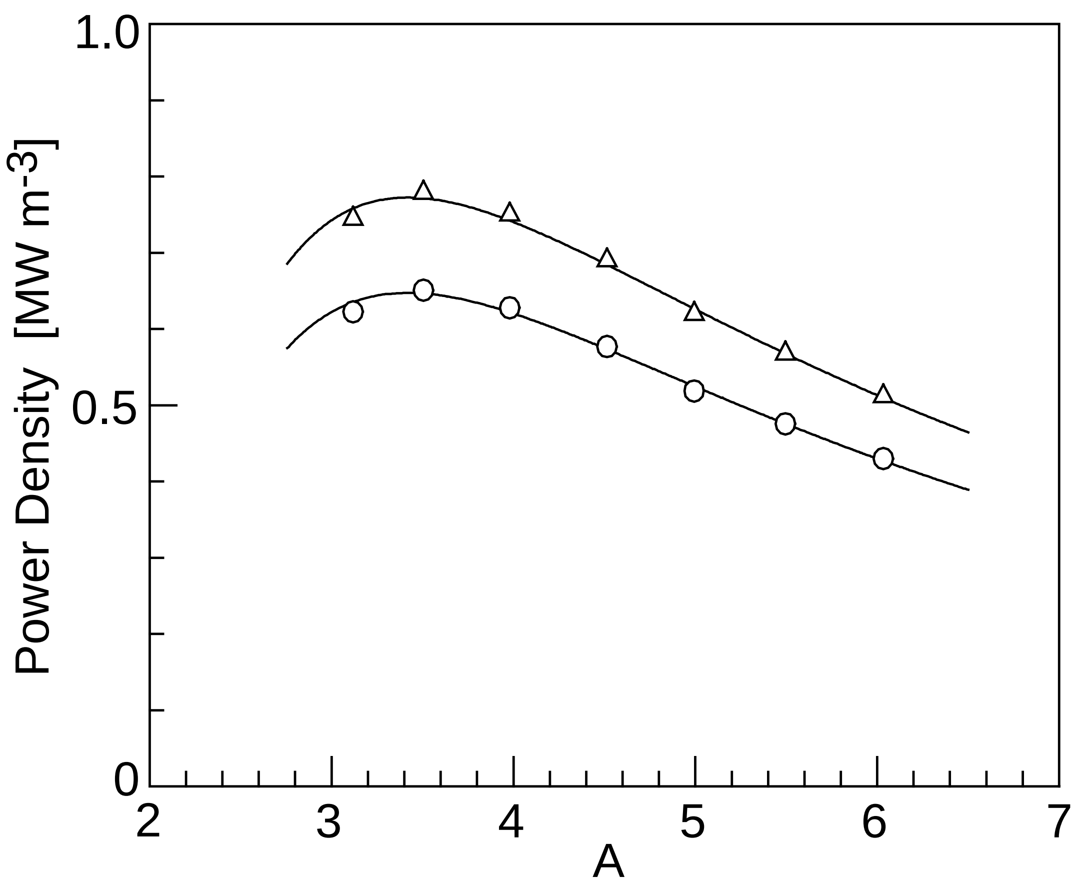

Figure 5. Average fusion power density as a function of aspect ratio. Circles: T_n=10.5 keV; triangles: T_n=12 keV. Solid lines are $\propto B^2 / A^2$.

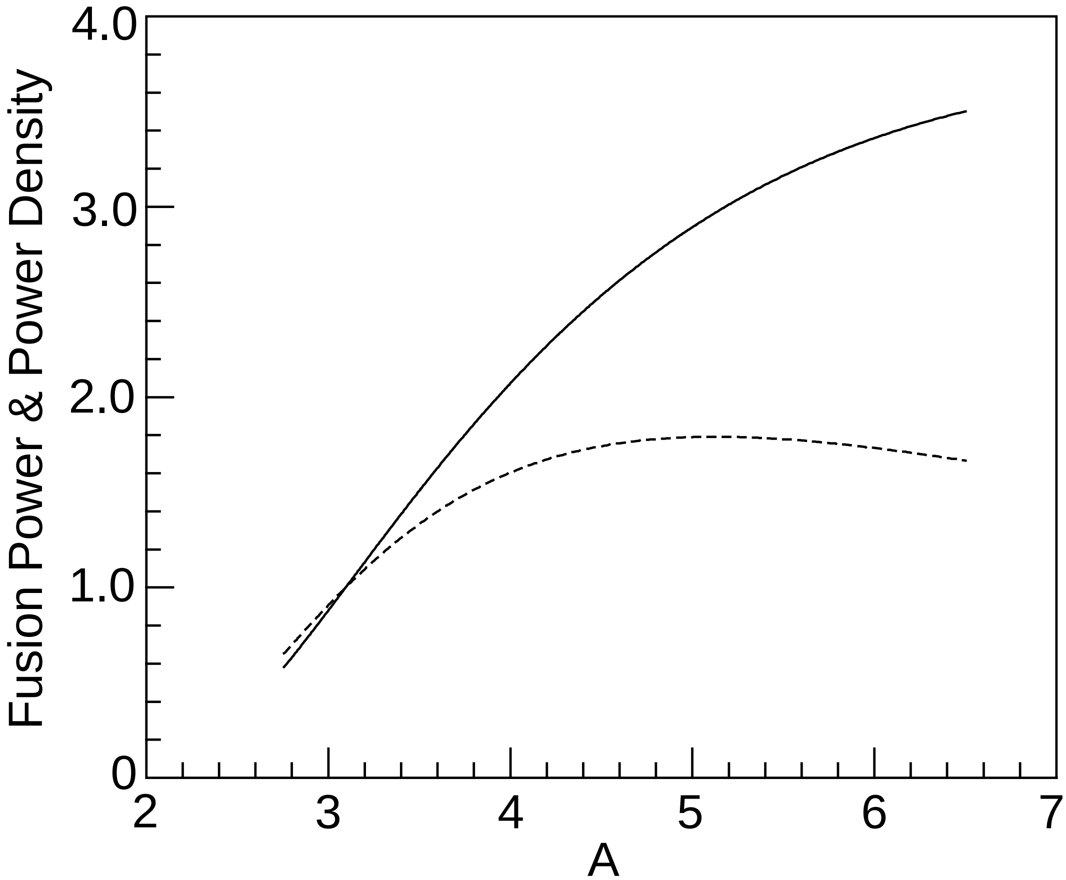

Figure 6. Total fusion power (solid line) and average fusion power density (dashed line) for a constant β_N. Values are normalized to one for *A*=3.1 (ITER).

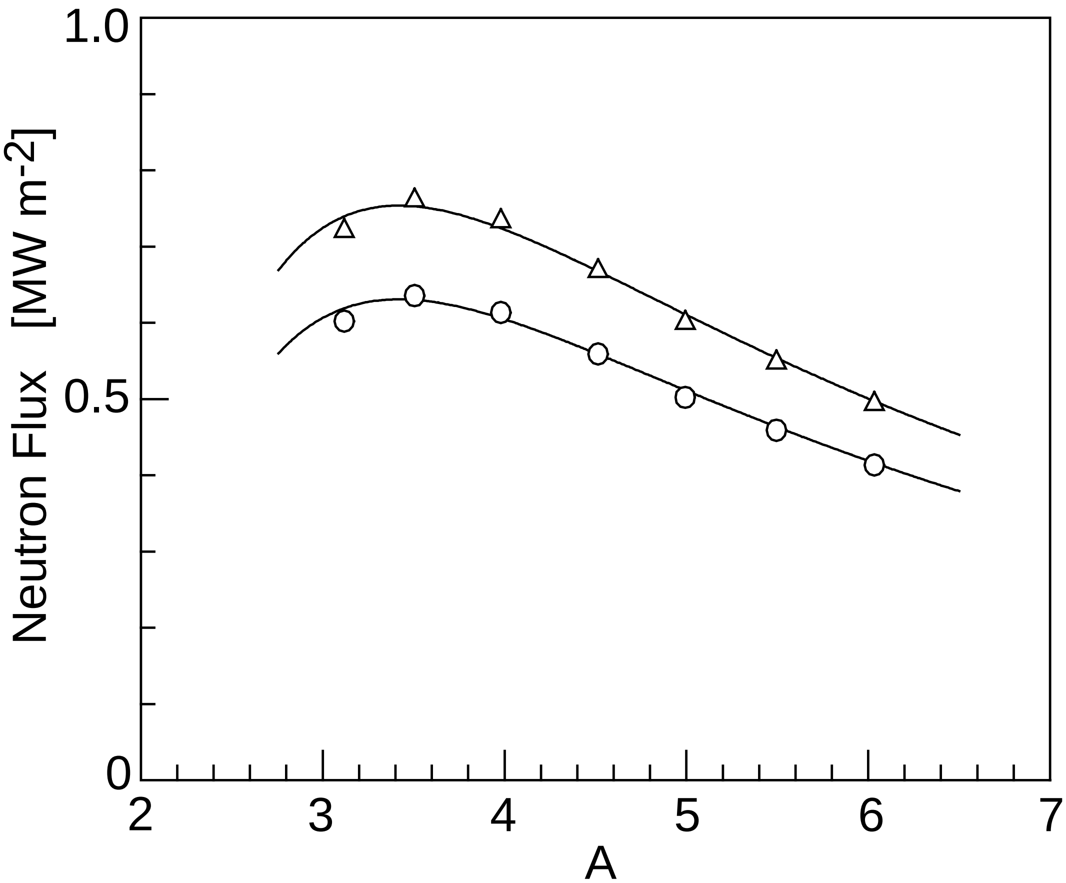

Figure 7. Average neutron flux at the plasma boundary (95% flux surface). Circles: T_n=10.5 keV; triangles: T_n=12 keV. Solid lines are $\propto B^2 / A^2$.

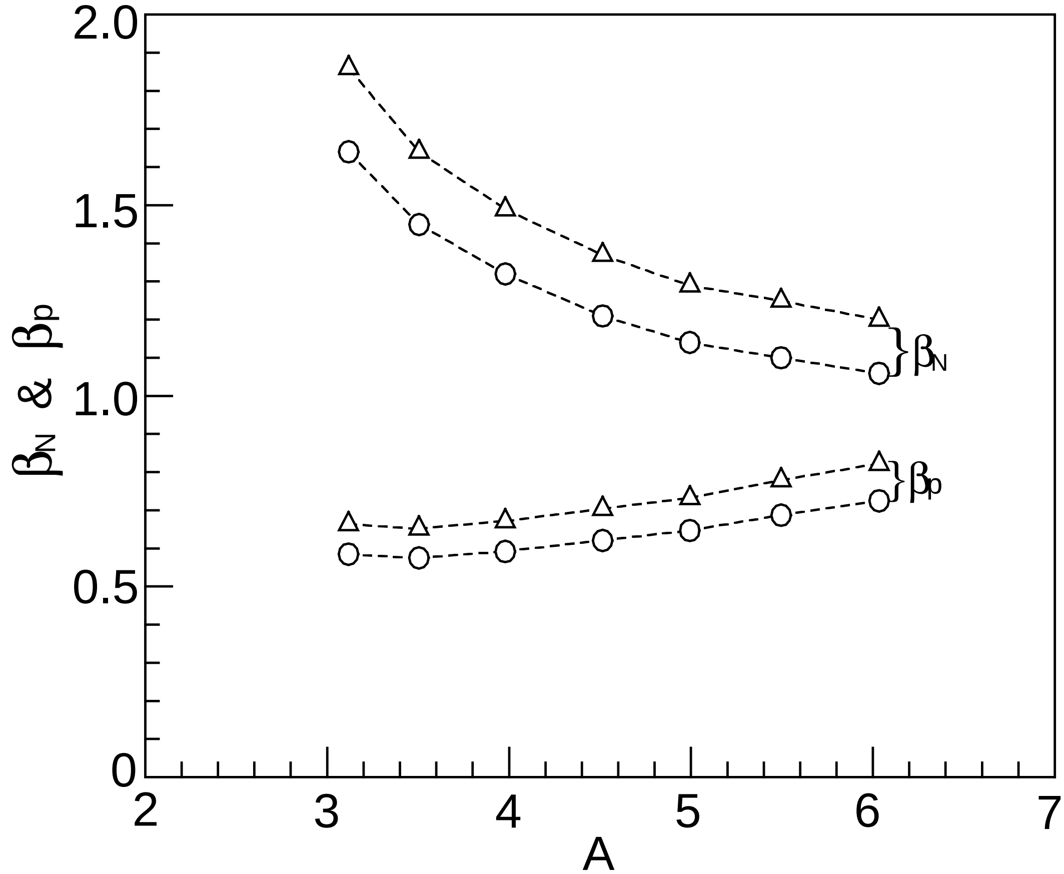

Figure 8. Normalized beta (β_N) and poloidal beta (β_p). Circles: T_n=10.5 keV; triangles: T_n=12 keV.

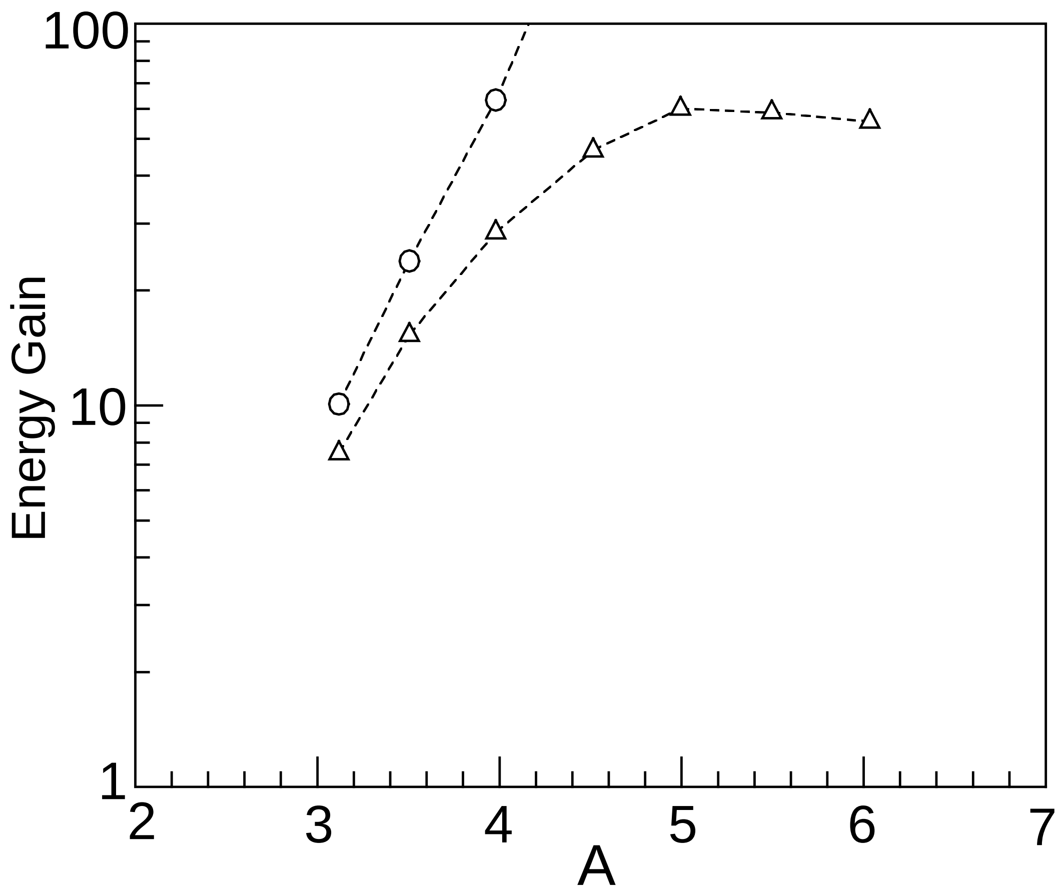

Figure 9. Energy gain as a function of aspect ratio. Circles: T_n=10.5 keV; triangles: T_n=12 keV.

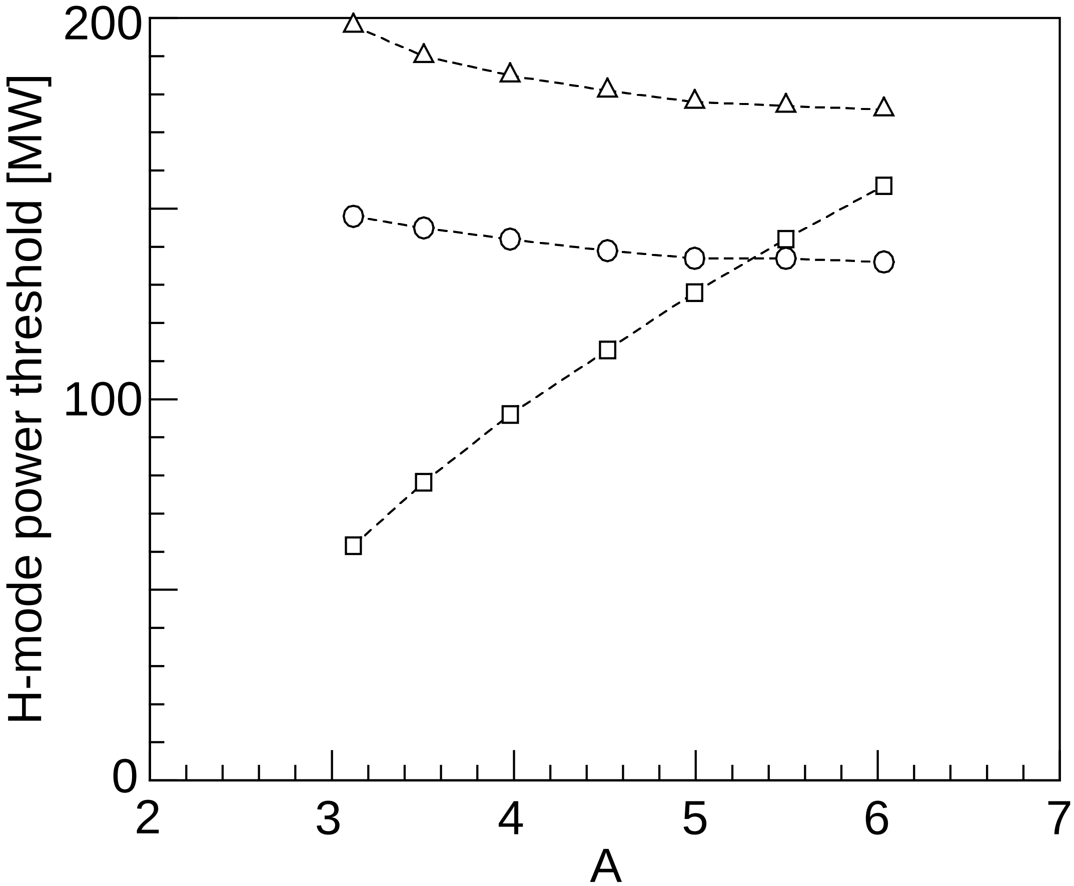

Figure 10. H-mode power threshold (squares) and total heating power (circles and triangles). Circles: T_n=10.5 keV; triangles: T_n=12 keV.

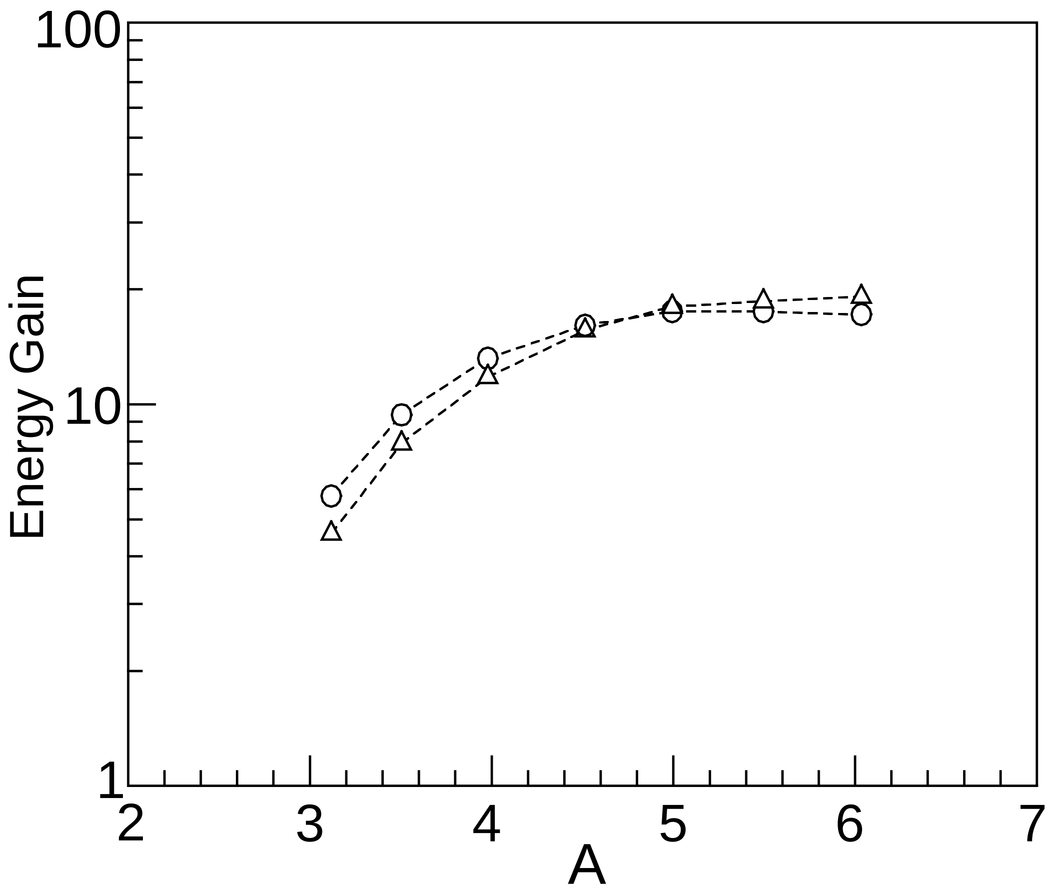

Figure 11. Energy gain for T_n=10.5 keV. Triangles: H=0.85 and Z_{eff}=1.65; circles: H=1.0 and Z_{eff}=1.9.

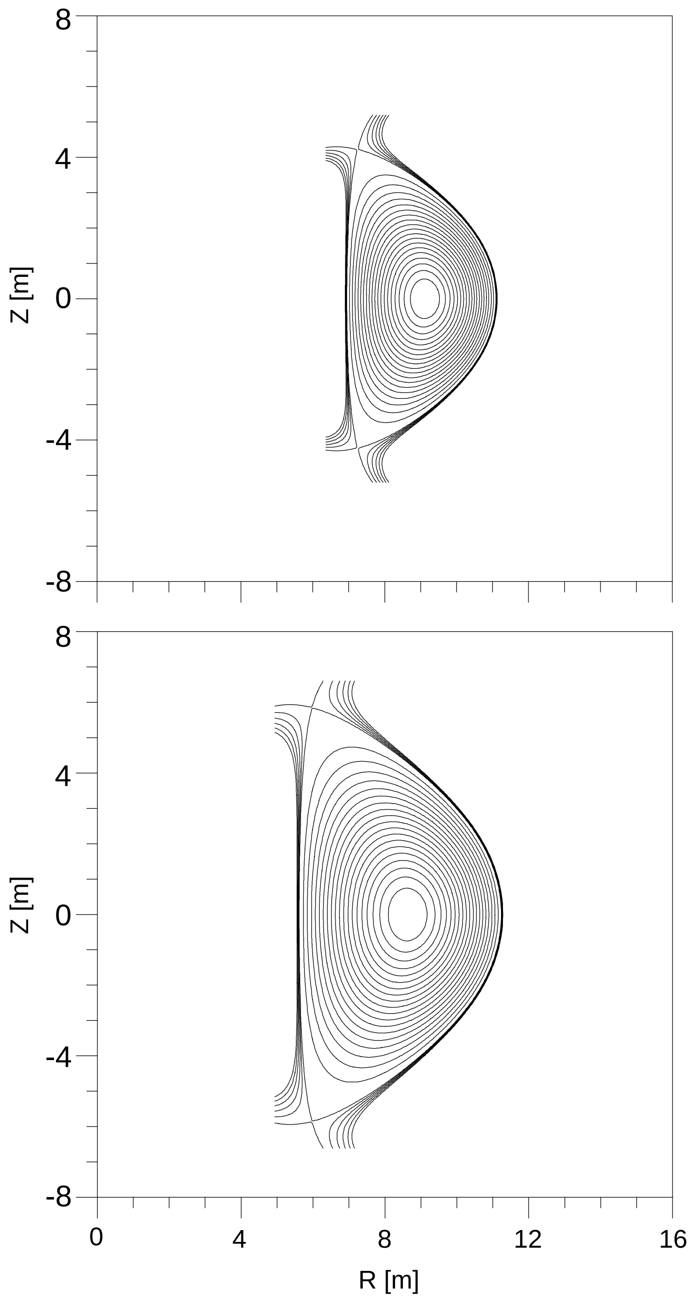

Figure 12. Tokamak configurations with γ=0.71, $B_{\max}$=11.8 T and k=1.75. Top: a=2 m, A=4.5, B=7.3 T, I_p=14.5 MA. Bottom: a=2.72 m, A=3.1, B=5.3 T, I_p=20 MA. In both cases, the total fusion power is 800 MW (with n_G=0.85, T_n=12 keV, Z_{eff}=1.65 and 5% of α particles) and Q=45 (H=1.0)