FREQUENCY SWEEPING: A NEW TECHNIQUE FOR ENERGY-SELECTIVE TRANSPORT

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ABSTRACT. A new method is described for inducing energy-selective transport by 'sweeping' the frequency of applied low n magnetic perturbations. The mechanism, formally analogous to the 'rising buckets' concept in accelerator physics, can move particles with a selected velocity in a non-diffusive fashion from one specified radius to another. The technique is considered principally as a means for removal of helium ash. Other likely applications are as a method for burn control, profile control, as a diagnostic and perhaps as a non-stochastic means of effecting the direct coupling of alpha power recently discussed by Fisch and Rax.

1. INTRODUCTION

In this work, we introduce a new technique for inducing energy-selective transport in a tokamak, achieved by 'sweeping' the frequency of applied low n magnetic perturbations. The mechanism, formally analogous to the 'rising buckets' concept [1] for accelerating particles, can non-diffusively move particles with a selected velocity from one specified radius to another, a distance comparable with the tokamak minor radius $r = a$, in the order of a hundred toroidal transit times $\tau_\zeta \simeq 2\pi R/v_\parallel$. Here we focus on the use of the technique as a means of ash removal, but the same method could be employed for burn control, profile control, as a diagnostic, and perhaps as a non-stochastic means of effecting the direct coupling of alpha power recently proposed by Fisch and Rax [2].

Earlier, we have proposed [3, 4] another possible method for ash removal, based on the energy-selective properties of a *stochastic* transport mechanism. The technique considered here shares with the earlier stochastic method the energy-selective property, and the virtue of being energetically inexpensive, and may be optimized by making use of the energy scalings underlying the earlier mechanism. It has the additional strength of requiring substantially smaller applied perturbation amplitudes, and in addition probably requires less careful tailoring than the stochastic mechanism of the spatial structure of the needed perturbation.

The basic idea of the mechanism is simple. For fixed frequency ω, a single-harmonic perturbation

$$\tilde{b} \equiv \delta B_r/B_0 = b_{mn}(r)\sin(n\zeta - m\theta - \omega t)$$

induces an island in the particle drift surfaces having the same (m, n) symmetry (plus sideband islands [3], subdominant for current considerations), whose resonant surface $r_{\rm res}$ depends upon ω, as given by the resonance condition $\omega = n\Omega_\zeta - m\Omega_{\rm b}$. (Here, $\Omega_{\rm b}$ is the poloidal transit/bounce frequency and $\Omega_\zeta \equiv 2\pi/\tau_\zeta$ is the toroidal transit/drift frequency.) For passing particles, one has $\Omega_\zeta \simeq \bar{v}_\parallel/R$, $\Omega_{\rm b} \simeq \Omega_\zeta/q(r)$ ($\bar{v}_\parallel$ is the transit averaged parallel velocity), and thus

$$q_{\rm res}(\omega) \equiv q[r_{\rm res}(\omega)] \simeq q_{mn}/[1 - (\omega/n\Omega_\zeta)] \quad (1)$$

where $q_{mn} \equiv m/n$. This is illustrated in Fig. 1, showing three Poincaré plots of the drift surfaces for particles at one tenth the alpha birth energy $E_0 = 3.5$ MeV and with pitch $\lambda \equiv v_\parallel/v = 1$, using TFTR parameters ($R = 2.62$ m, $a = 0.96$ m, $B = 5$ T, and taking $q_0 \equiv q(r = 0) = 1$, $q_a \equiv q(a) = 4$). For each, the same $(m, n, b_{mn}^{\rm max}) = (2, 1, 0.8 \times 10^{-3})$ perturbation is used, but with the frequency assuming three values. One notes that the primary (2, 1) island moves outward as the frequency changes, from near $r = 0$ (Fig. 1(a)) to near the machine edge (Fig. 1(c)).

If ω is varied slowly enough, a particle starting inside an island (or 'bucket') at $r_{\rm res}(\omega)$ will move adiabatically with that island, at radial velocity $\dot{r}_{\rm res} = \dot{q}_{\rm res}/q'_{\rm res}$, with $\dot{q}_{\rm res} = q_{\rm res}\dot{\omega}/(n\Omega_\zeta - \omega)$, where $\dot{\omega} \equiv {\rm d}\omega/{\rm d}t$ is the rate at which the frequency is swept. In this way, particles at a specified phase point $(r, v)_1$ can be picked up in a bucket at frequency ω_1, and then walked non-stochastically to another radius r_2, the bucket radius for frequency ω_2. Particles initially outside the island remain outside the island, receiving only a small radial kick as the island sweeps over them.

As will be shown, the limit on the rapidity $\dot{\omega}$ of the frequency sweep is that the non-linear trapping frequency $\omega_{\rm tr}$ of a particle in the island be greater than the characteristic frequency for the island moving radially an island halfwidth $\delta r_{\rm d}$. Including the numerical factors found later,

$$\omega_{\rm tr} \gtrsim 2\dot{r}_{\rm res}/\delta r_{\rm d} \quad (2)$$

The characteristics just summarized can be used to achieve the energy selectivity needed for the various applications of the technique mentioned earlier. For ash removal, one wants to enhance greatly the transport of the alphas at an energy E_1 small compared with E_0, while not greatly enhancing the transport of both energetic alphas, with $E \sim E_0$, or of particles in the thermal bulk, with $E \sim T$. Since one has $E_0/T \sim 10^2$, we choose the target ash energy E_1 equal to $E_0/10$. Thus, particles at E_1 have already given most of their original energy to the plasma, but may still be readily distinguished in their

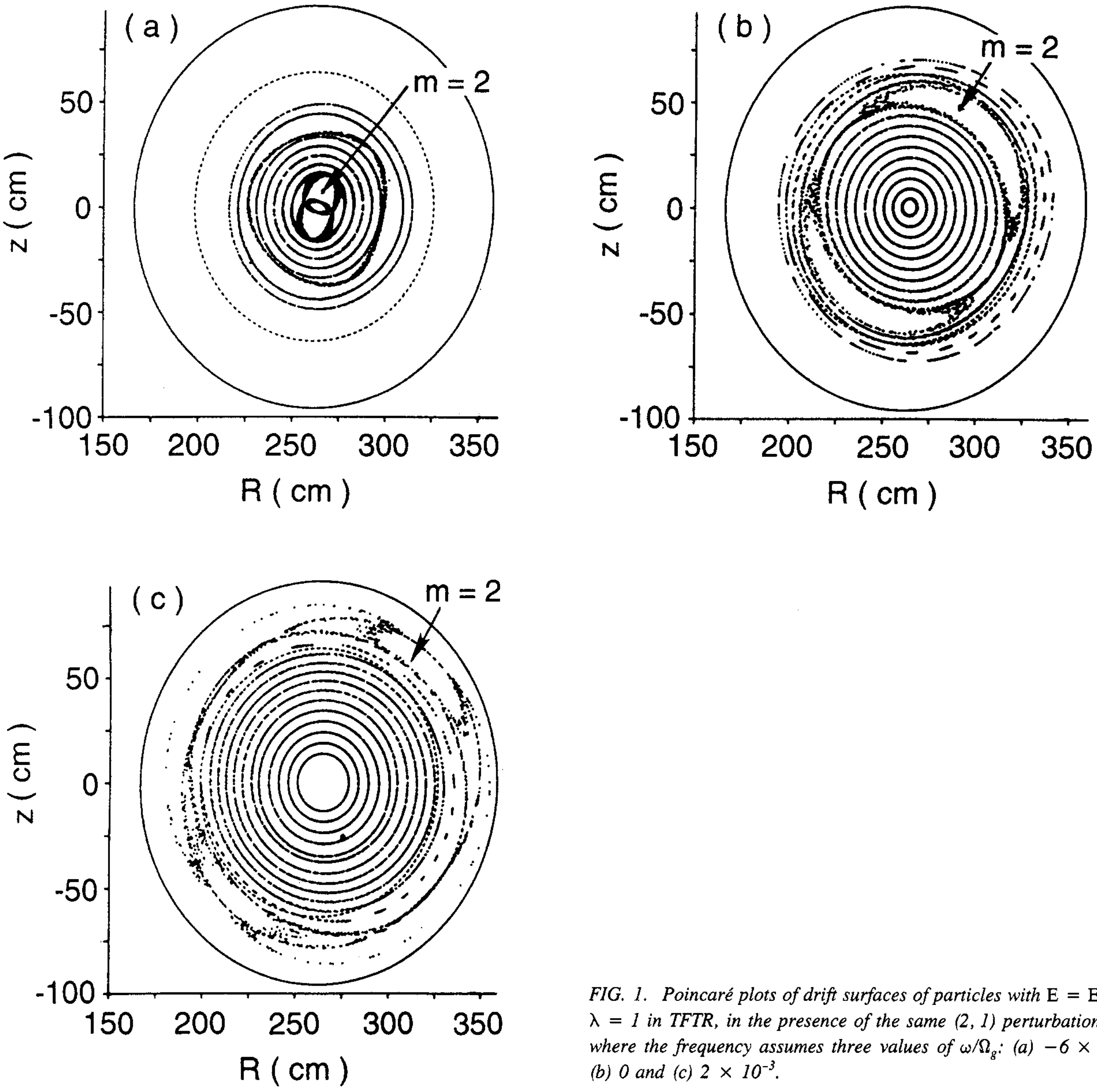

FIG. 1. Poincaré plots of drift surfaces of particles with $E = E_0/10$, $\lambda = 1$ *in TFTR, in the presence of the same (2, 1) perturbation, but where the frequency assumes three values of* ω/Ω_g*: (a)* -6×10^{-3}*, (b) 0 and (c)* 2×10^{-3}.

transport characteristics from thermal ions. In the earlier stochastic mechanism, energy selectivity was provided by orbit averaging effects. Frequency sweeping provides three additional parameters to achieve this, namely ω_1, ω_2, and $\dot{\omega}$. As indicated above, for a typical particle of energy E_1, with $\Omega_\zeta = \Omega_{\zeta 1}$, one can set ω_1 so that the induced bucket lies deep inside the plasma ($r_1(\Omega_{\zeta 1}) \simeq 0$) and ω_2 so that the bucket has moved the target particle nearly or fully outside the machine ($r_2(\Omega_{\zeta 1}) \simeq a$). From Eq. (1), for the (2, 1) perturbation, particles with $E \sim E_0$, having a much larger typical Ω_ζ than those with $E \sim E_1$, will have an r_1 much further out (near the $q = 2$ surface) and an r_2 much further in. Thus, many fewer birth alphas will be swept, both because the bucket picks up particles where not many birth alphas exist, and because the distance $r_2 - r_1$ swept is much smaller. Similarly, thermal particles, having Ω_ζ small compared with those with $E = E_1$, will have a $q_1 \equiv q(r_1) < q_0$, i.e. the bucket picks up no thermal particles, because the initial bucket position does not exist within the plasma. Moreover, one can choose $\dot{\omega}$ so that criterion (2) holds for particles with $E \gtrsim E_1$, but is violated for those with $E \sim T$, further diminishing the effect of the swept buckets. Further energy selectivity can be provided, if

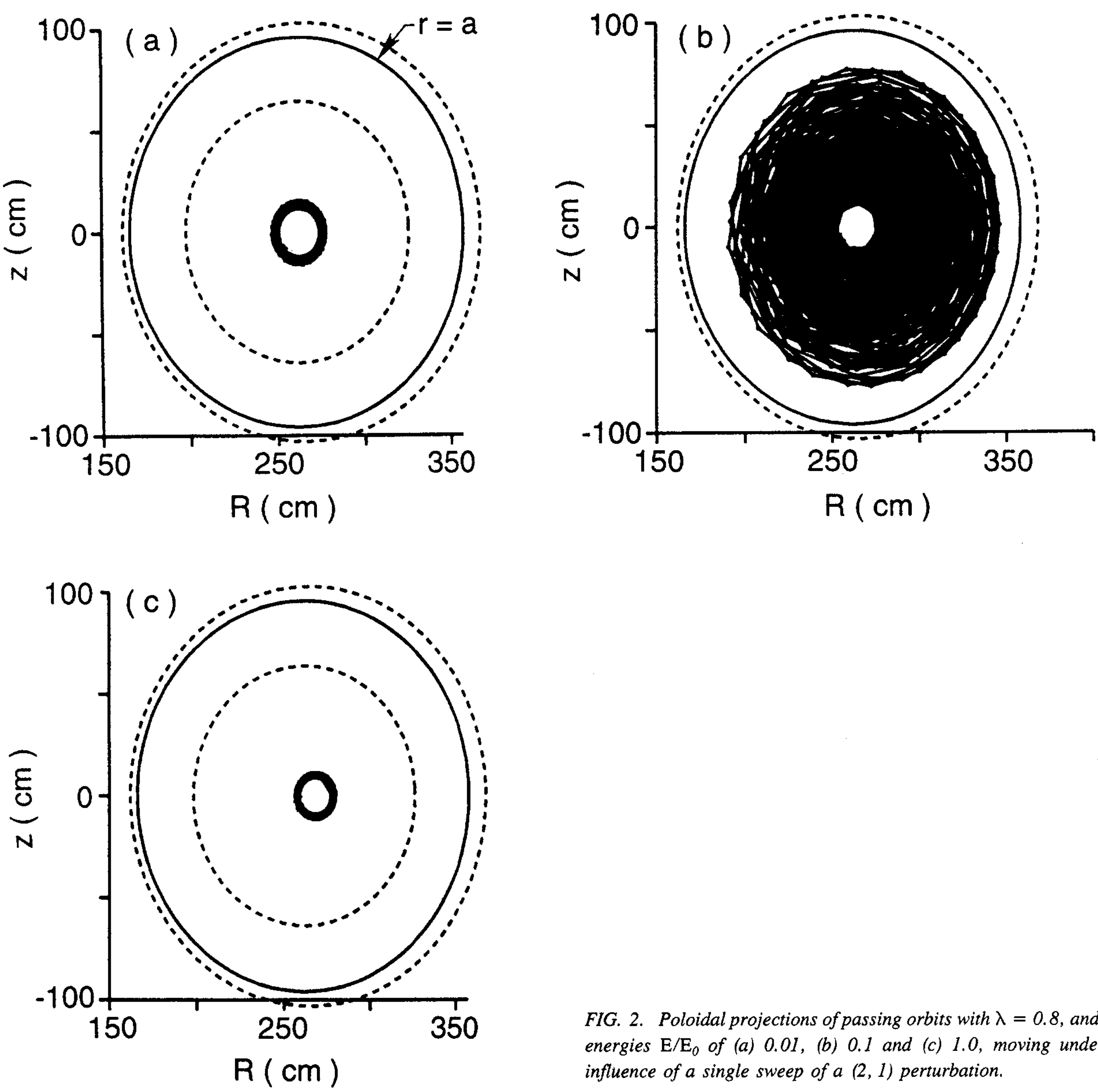

FIG. 2. Poloidal projections of passing orbits with $\lambda = 0.8$, and with energies E/E_0 of (a) 0.01, (b) 0.1 and (c) 1.0, moving under the influence of a single sweep of a (2, 1) perturbation.

needed, by making use of the orbit averaging mechanism on which the stochastic scheme is based. We shall also see that, for electrons, collisionality provides a further means of discriminating between species with large velocities.

The energy selectivity is illustrated by the orbits shown in Fig. 2, for $E/E_0 = 0.01$, 0.1 and 1.0 ((a), (b) and (c), respectively) and for each particle having pitch $\lambda = 0.8$. The same (2, 1) perturbation is used as that in Fig. 1, but with frequency swept from $\omega/\Omega_g = -5 \times 10^{-3}$ to 2.6×10^{-3} in a time $\tau_{sw} = 200\tau_{\zeta 1}$, where $\tau_{\zeta 1} \equiv \tau_\zeta(E = E_1, \lambda = 1)$. ($\Omega_g$ is the alpha gyrofrequency, equal to 2.38×10^8 s^{-1} for $B = 5$ T.)

2. THEORY

We analyse the action of frequency sweeping analytically. The radial motion of the particle due to the perturbation is given by considering $\bar{r}$, the transit/bounce averaged minor radius, which evolves according to [3, 4]

$$\dot{\bar{r}} = \dot{\bar{r}}_A + \dot{\bar{r}}_B = \sum_{l_b, n} v_{l_b} \sin \eta_{l_b} \tag{3}$$

where $\eta_{l_b} \equiv n\bar{\zeta} + l_b\theta_b + \phi_b$, $\bar{\zeta}$ is the transit/bounce averaged toroidal azimuth ζ, θ_b is the bounce phase and $l_b = l - \sigma m$ is the bounce harmonic, with $l = 0, \pm 1, \pm 2, \ldots$, and $l = 0$ corresponding to the primary island,

and with trapping-state index σ, equal to 1(0) for passing (trapped) particles. ϕ_b is the perturbation phase, with $\dot{\phi}_b = -\omega$. In the absence of a perturbation, one has $\dot{\theta}_b = \Omega_b$ and $\dot{\bar{\zeta}} = \Omega_\zeta$, so that

$$\dot{\eta}_{l_b} \simeq n\Omega_\zeta + l_b\Omega_b - \omega \qquad (4)$$

The location of the l_bth resonant surface is thus given by $\dot{\eta}_{l_b} = 0$. As discussed in detail in Refs [3] and [4], $\dot{\bar{r}}_A \sim \tilde{b}$ is the portion of $\dot{\bar{r}}$ due to the perturbing radial field δB_r, which induces 'magnetic braiding', while $\dot{\bar{r}}_B \sim \delta B/B_0$ is the contribution due to 'grad B' drifts from ripple perturbations δB in the magnitude B of the field. For each l_b and n, the amplitudes v_{l_b} contain both the braiding and the ripple contributions, and a sum over the contributions from all magnetic harmonics m having that n. For simplicity, here we drop the ripple contribution, take a single n, and consider a single island l_b. Transforming $\bar{r}$ to the frame $\bar{r}' \equiv \bar{r} - r_{res}$ moving with the swept island and expanding $\dot{\eta}_{l_b}$ in r about the resonant surface, one has

$$\dot{\bar{r}}' \simeq v_{l_b} \sin \eta_{l_b} - \dot{r}_{res}, \qquad \dot{\eta}_{l_b} \simeq \bar{r}'/M_{l_b} \qquad (5)$$

formally the equations for one dimensional motion of a particle with mass (for passing particles) $M_{l_b} \equiv -q/(l_b\Omega_b q')$, with canonically conjugate coordinate η_{l_b} and momentum $\bar{r}'$, moving according to Hamilton's equations $\dot{\eta}_{l_b} = \partial_{\bar{r}'}H$ and $\dot{\bar{r}}' = -\partial_{\eta_{l_b}}H$, with Hamiltonian

$$H(\eta_{l_b}, \bar{r}') = \bar{r}'^2/2M_{l_b} + V(\eta_{l_b})$$

and with potential

$$V(\eta_{l_b}) \equiv v_{l_b} \cos \eta_{l_b} + \eta_{l_b} \dot{r}_{res}$$

For non-zero $\dot{r}_{res} \propto \dot{\omega}$, the sinusoidal wells in V lie on a slope. Thus, as is well known from the formally similar problem of particle motion in stellarators or rippled tokamaks [5], the separatrices (buckets) in phase space have single X points, with orbits outside the separatrices passing through the $r' = 0$ resonant surface, and a reduced well depth

$$v_{ef} \equiv [V(\eta_{l_b}^{max}) - V(\eta_{l_b}^{min})]/2 \simeq v_{l_b}\{[1 - (\dot{r}_{res}/v_{l_b})^2]^{1/2} - (\dot{r}_{res}/v_{l_b})[\pi/2 - \sin^{-1}(\dot{r}_{res}/v_{l_b})]\}$$

The island halfwidth and trapping frequency are given by $\delta r_d = (4v_{ef}M_{-m})^{1/2}$ and $\omega_{tr} \simeq (v_{ef}/M_{-m})^{1/2}$, respectively. In the limit $\dot{r}_{res}/v_{l_b} \to 1$, $v_{ef} \to 0$, and the separatrix width vanishes, yielding criterion (2) limiting the size of $\dot{\omega}$.

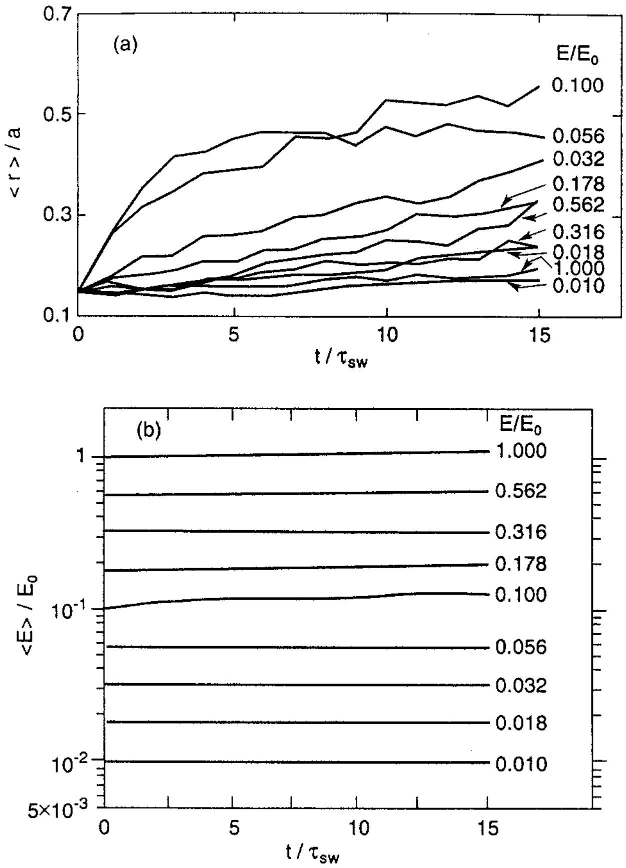

FIG. 3. (a) Plot of ensemble averaged radius of particles versus time, in units of the number of sweeps applied, for an ensemble of co-going particles, with energies $E/E_0 = \{10^{-2}, 10^{-1.75}, \ldots, 1\}$*. (b) Ensemble averaged energy versus time, for the same simulation as in (a).*

3. MULTIPLE PARTICLES, MULTIPLE SWEEPS

A single island sweep will not capture all the particles in the target distribution, both because the initial particle phase will place particles outside the bucket, and because the ash distribution exists over a range of r and λ. Since a typical sweep time $\tau_{sw} \simeq 100\tau_{\zeta 1} \simeq 400$ μs is very small compared with a typical slowing down time $\tau_{s\ell} \sim 200$ ms, many sweeps may be used, with frequencies $\omega_{1,2}$ tuned to access $E \sim E_1$ particles at different values of (r, λ). In Fig. 3(a) we show the effect of a series of identical sweeps, each of duration $200\tau_{\zeta 1}$ for nine monoenergetic ensembles of co-going particles, at $E = E_0 \times \{10^{-2}, 10^{-1.75}, \ldots, 1\}$, with initial radii in the range $r/R = 0.01$ to 0.10, and pitch in the range $\lambda = 0.6$ to 0.9. The sweeps are from $\omega_1/\Omega_g = -4 \times 10^{-3}$ to $\omega_2 = -\omega_1/2$. We plot the ensemble averaged radius $\langle r \rangle$ of the particles versus time, in units of the number of sweeps. One notes, as in Fig. 2, that the $E \simeq E_1$ particles are selectively swept. One also notes a diminution with sweep number of the rate of change $\langle \dot{r} \rangle$ of $\langle r \rangle$, as the region of phase space swept by that particular (ω_1, ω_2) bucket is cleared out, indicating time to move to a new (ω_1, ω_2) setting. Multiplying $\langle \dot{r} \rangle$ by the number $(\propto v^2 f_0(v))$ of particles at each energy yields the flux $\Gamma_{sw}(E)$ of particles swept at each energy. Assuming a slowing down distribution [6]

$$f_0(v) = N_0 H(v_0 - v)/(v_c^3 + v^3)$$

with $H(x)$ the unit step function and $v_c \simeq 0.18 v_0$ the speed at which the drags from ions and electrons are equal, results in Fig. 4, which shows a peak in flux in the vicinity of $v = v_1$ or $E = E_1$. (The normalization here is arbitrary: we choose N_0 so that $v^2 f_0$ has a maximum of unity.)

The earlier stochastic mechanism [3] required that the perturbation strength $\tilde{b}$ exceed an overlap threshold in order to be operative. For the (2, 1) perturbation, a threshold of $b_{mn} \simeq 4 \times 10^{-3}$ was found, a very large perturbation. In contrast, the present mechanism has no such lower bound on b_{mn}, thus avoiding potential deleterious effects on other aspects of the machine operation. The figures shown in this work have all used $b_{mn}^{max} = 0.8 \times 10^{-3}$, only a fifth of the stochastic threshold. This is still a rather large perturbation, used to provide easy visibility of the sweeping. In a real implementation $\tilde{b}$ could readily be taken well below this. Smaller $\tilde{b}$ and so bucket width would require an increased number of sweeps to access the full target distribution, and a correspondingly increased τ_{sw} to satisfy condition (2) for the ash. Another lower limit on $\tilde{b}$ is that the swept perturbation not be overwhelmed by the presence of naturally occurring low n MHD, with radial maxima typically in the range $\tilde{b} \sim 10^{-5}$–10^{-4}.

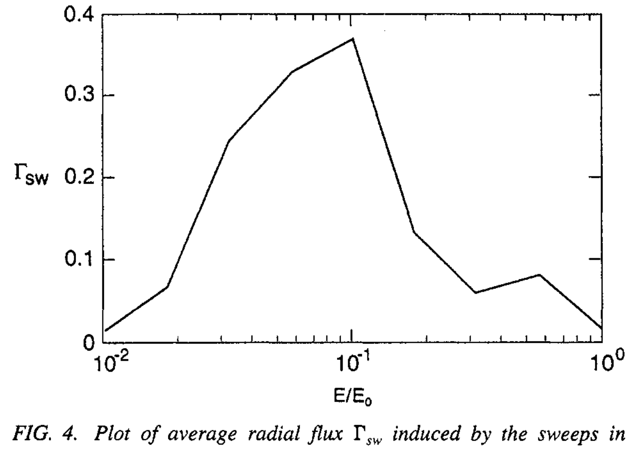

FIG. 4. Plot of average radial flux Γ_{sw} induced by the sweeps in Fig. 3 versus energy.

4. OTHER CONSIDERATIONS

Thermal electrons have a velocity v_e comparable with that of newborn alphas

$$(v_0 \equiv (2E_0/M_\alpha)^{1/2} \simeq 1.3 \times 10^7 \text{ m/s})$$

so the potential for the mechanism to also induce appreciable transport in electrons might be of concern. However, for typical fusion parameters and those in current large machines, this effect should be small. For $T_e = 10$ keV, one has $v_e \simeq 4.1 \times 10^9$ m/s, about 10 times the ash velocity $v_1 \equiv (2E_1/M_\alpha)^{1/2}$. As seen in Figs 2, 3 and 4, the energy selective mechanisms described above are adequate to make the transport of alphas at $v_0 = \sqrt{10 v_1}$ far smaller than for those at v_1, so the effect on the still faster electrons will be smaller still. Even were the electrons collisionless, one finds from Eq. (1) that the buckets for particles at this velocity would move only from $r \simeq 47$ to 55 cm as the bucket for the ash moves from $r = 0$ to a. Moreover, as opposed to the alphas, collisionality may *not* be neglected for the action of this mechanism on electrons. At $T_e = 10$ keV, $n_e = 10^{14}$ cm^{-3}, the scattering frequency is $\nu_{\perp e} \simeq 6 \times 10^3$ s^{-1}. A collisional change $\delta_\nu \lambda$ in λ causes a corresponding change $\delta_\nu r_{res}$ in r_{res}, through $\Omega_\zeta \propto \lambda$ in Eq. (1). Taking $\delta_\nu r_{res}$ equal to δr_d, the time $1/\nu_{ef}$ for an electron to scatter out of the bucket is given by the effective frequency

$$\nu_{ef} = \nu_{\perp e}/(\delta_\nu \lambda)^2 \simeq \nu_{\perp e}(\partial_\lambda q_{res}/q' \delta r_d)^2$$

with $\partial_\lambda q_{res} = -(q_{res}^2/q_{mn})(\omega/n\Omega_\zeta \lambda)$. Using $\tau_{sw} = 200 \tau_{\zeta 1}$, one estimates $\nu_{ef} \tau_{sw} \simeq 4/(b_{mn}/10^{-3})$. Thus, for $b_{mn} = 1 \times 10^{-4}$, an electron initially in the bucket will scatter out after only $\tau_{sw}/40$, i.e. after moving only from 47 to 47.2 cm. Electrons not initially in the bucket will be even less affected. Thus, the mechanism should not add appreciably to the level of electron transport already present in the plasma due to intrinsic effects.

A virtue of the earlier stochastic mechanism was that, because it can operate at zero or low frequencies, it does little work in moving the ash to the outside. The frequency $\omega/\Omega_g = -6 \times 10^{-3}$ used in Fig. 1(a) to place the bucket near $r = 0$ translates to $\omega/2\pi = -230$ kHz, no longer a low frequency. As a result, there can be significant energy exchange $\dot{E}$ between the particles and the perturbation. However, as can be understood by averaging the Hamilton's equation $\dot{E} = -\partial_t h$ over a trapping period $\tau_{tr} \equiv 2\pi/\omega_{tr}$, the sign of $\langle \dot{E} \rangle_{\tau_{tr}}$ reverses when the sign of ω does, positive for $\omega < 0$, and negative for $\omega > 0$ for co-going particles. Because a sweep with a (2, 1) perturbation requires both $\omega < 0$ and $\omega > 0$, the net energy change over a sweep is again small. One sees this in Fig. 3(b), which shows the ensemble average of the energy at each sweep versus time, for the same ensemble as in Fig. 3(a). It is thus possible to make the ash removal mechanism energetically nearly neutral, by such a choice of q_{mn}. However, we note that it is also possible to cause *negative* net work to be done on the particles by choosing q_{mn} near q_0. Tuned to resonate with alphas with $E \sim E_0$ rather than E_1, such a perturbation could draw net energy from the newborn alphas, perhaps permitting a non-stochastic implementation of the scheme of Fisch and Rax [2] or an approach for direct conversion.

However, q_{mn} is also constrained from below. One notes from Eq. (1) that reversing Ω_ζ or $\bar{v}_\parallel$ requires reversing $\omega_{1,2}$ to sweep particles across the same $r_1 \rightarrow r_2$ interval. If $|\omega_1/\omega_2| \leq 1$, therefore, sweeps tuned to sweep co-going particles from $r = 0$ outward will also sweep counter-going particles from inside $r = a$ inward, tending to cancel the effect of sweeps tuned to move counter-going particles out. From Eq. (1), one finds

$$\omega_1/\omega_2 = (q_a/q_0)(q_0 - q_{mn})/(q_a - q_{mn})$$

negative and with magnitude greater than unity only for $q_{mn} > 2q_0q_a/(q_0 + q_a)$. For the q_0, q_a used here, this requires $q_{mn} > 8/5$, satisfied by the (2, 1) perturbation, for which $\omega_1/\omega_2 = -2$.

The energy dependence in Eq. (1) arises through $\bar{v}_\parallel$ or Ω_ζ. While this equation is only approximate, neglecting the additional E dependence in the particle drifts, it raises the difficulty that particles with $E \sim E_0$, but with small λ (namely, trapped or nearly trapped particles), can have the same values of Ω_ζ as those typical of the $E \sim E_1$ ash, and so might be swept as well. To suppress the transport of such energetic particles, as noted earlier, one can employ the further energy selectivity arising from orbit averaging [3, 4]. For example, for a single-m braiding perturbation, one has $v_{l_b} \simeq \bar{v}_\parallel b_{mn} J_l(\eta_1)$, with perturbing amplitude b_{mn}, orbit averaging parameter $\eta_1 \simeq k_\perp \rho_b$ and $\rho_b \propto E^{1/2}$, the particle drift-orbit excursion. As shown in Refs [3, 4], taking $n \simeq 3$ is large enough to put J_0 at its first zero for particles with $(E, \lambda) = (E_0, 1)$, thereby causing the primary bucket width to vanish. Energetic trapped particles have still larger η_1, and are therefore even more susceptible to orbit averaging effects.

5. CONCLUSIONS

Summarizing, we have proposed a new technique for selectively transporting particles from one minor radius to another, and from a limited region of velocity space. A number of potential applications of this technique needed for a tokamak reactor have been noted, perhaps the most demanding of which is ash removal, the principal focus of this work. A theory of the frequency sweep mechanism has been developed, most fully for passing particles, numerical evidence has been presented supporting the efficacy of the proposed ash removal scheme, and some of the constraints and potential difficulties of the technique have been indicated. The emphasis in this Letter has been on perturbations induced by applied fields (see Ref. [7] for concurrent related work with naturally occurring perturbations). The constraints discussed serve to narrow the possible types of perturbation needed to implement the scheme. Low n magnetic perturbations are required, with frequencies up to a few hundred kHz, so that they can resonate with energetic alphas. An antenna system with a tunable frequency, similar to the saddle-coil array proposed [8] to excite TAE and EAE modes in JET, seems appropriate to test the present scheme, particularly as an acceptable test could be done with a perturbation strength as small ($\tilde{b} \sim 10^{-5}$) as that for which that system is designed.

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