Poloidal trapping of the high-frequency Alfvén continuum and eigenmodes in stellarators

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Outline

Introduction

- Spectra of Alfvén waves are of interest, first of all, because the waves can be destabilized by fast ions
- 1D plasmas: Continuous spectrum $\omega = k_{\|} v_A$; Damping of oscillations at continuum frequencies dur to phase mixing; Discrete spectrum (GAE modes) outside the Alfvén continuum (AC)
- 2D (toroidal) plasmas: Gaps in AC due to poloidal asymmetry; New eigenmodes (TAE etc.) inside the gaps
- 3D plasmas (stellarators): New gaps in AC with new types of AEs (helicity- and mirror-induced AEs, HAEs and MAEs) [Nakajima et al., Phys. Fluids B (1992); Kolesnichenko et al., Report IPP III/261 (2000); Nührenberg, ISSP-19 (2000)]
- AC becomes a Cantor set; possibility of continuum equation solutions localized on a single field line [Salat, Plasma Phys. Control. Fusion (1992)]
- This paper: the waves may be trapped in "waveguides" due to interference of Fourier harmonics of the equilibrium

The Alfvén Continuum Equation

The AC equation is a generalization of the Alfvén dispersion to non-uniform plasmas [Kolesnichenko et al., Phys. Plasmas **8** (2001) 491; Salat & Tataronis, Ibid. 1200]:

$$\frac{d}{d\phi}\left(h_g^{\psi\psi}\frac{d\Phi}{d\phi}\right) + \Omega^2 h_c^{\psi\psi}\Phi = 0,$$

$d/d\phi$ is the derivative along field lines,

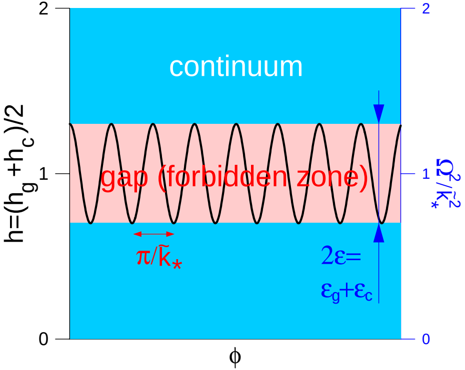
$$\Omega = \omega\frac{R}{\langle v_A\rangle}, \quad h_g^{\psi\psi} = \frac{g^{\psi\psi}}{\langle g^{\psi\psi}\rangle}, \quad h_c^{\psi\psi} = \frac{h_g^{\psi\psi}}{h_B^4},$$

$$h_{g,c}^{\psi\psi} = 1 + \frac{1}{2}\sum_{\mu,\nu=-\infty}^{\infty}\epsilon_{g,c(\mu,\nu)}^{\psi\psi}(\psi)\exp(i\mu\theta - i\nu N\phi);$$

$\epsilon_{g(\mu,\nu)}^{\psi\psi}$ and $\epsilon_{c(\mu,\nu)}^{\psi\psi}$ are "coupling parameters"; (μ,ν), "coupling numbers".

Gaps in the Alfvén Continuum

Isolated gap in the continuum

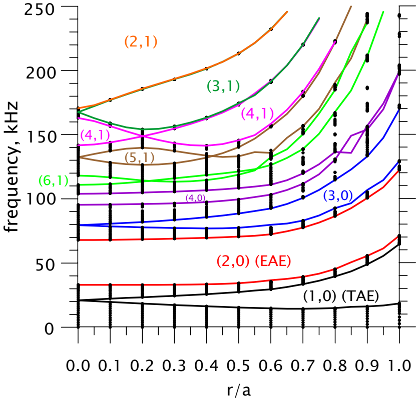


cf. electron wave in a crystal

- A single (μ, ν) harmonic forms a gap near $\Omega(r) = |\tilde{k}_{*\mu,\nu}(r)|$, where $\tilde{k}_{*\mu,\nu} = [\mu\iota(r) - \nu N]/2$.
- How the AC looks in a 3D case, when different helicities (μ, ν) are present in the configuration?

Example of the AC in W7-AS, Shot #56936

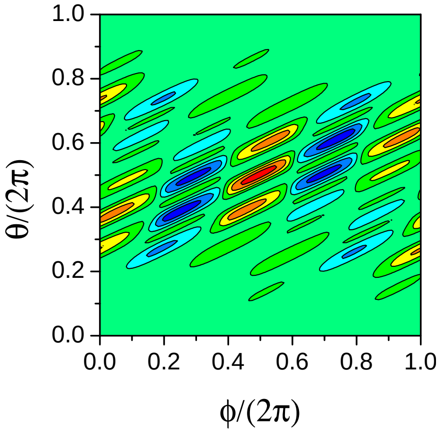
Black dots show the AC at several radii



The gaps are labelled by the corresponding coupling numbers (μ, ν)

- The high-frequency part of the continuum is reduced to extremely thin walls; e.g., $\Delta\omega/\omega \leq 6 \times 10^{-4}$ for the wall between the (2,1) and (3,1) gaps at $r/a = 0.3$.
- Gaps are wide and close ($\epsilon^{\psi\psi}_{(2,1)} = 0.59$, $\epsilon^{\psi\psi}_{(3,1)} = 0.35$, $|(\tilde{k}_{*2,1} - \tilde{k}_{*3,1})/\tilde{k}_{*2,1}| = 0.12$).
  $\Rightarrow$ The compression of the wall seems natural.
- Nevertheless, how can 6×10^{-4} be obtained from these parameters?

Example of an AC Wave Function, Shot #56936



$r/a = 0.3$, $f = \omega/(2\pi) = 193\,\mathrm{kHz}$, the continuum branch $(m, n) = (0, 2)$

- The frequency is on the the wall between the gaps $(2, 1)$ and $(3, 1)$ $\Rightarrow$ strong influence of the equilibrium harmonics $(2, 1)$ and $(3, 1)$ can be expected.
- The Fourier structure is complicated, with noticeable $(m, n) = (0, 2)$ and $(2, N)$ contributions.
- Very strong anti-ballooning: the wave is trapped at the inner circumference

The Case of Close Gaps

- Let us study the case when the longitudinal periods of two eqauilibrium harmonics are close,

$$||\tilde{k}_{*1}| - |\tilde{k}_{*2}||/|\tilde{k}_{*1,2}| < |\epsilon_{c,g}|,$$

with $\tilde{k}_{*i} \equiv \tilde{k}_{*(\mu_i,\nu_i)}$.

- Along each field line ($\phi = \alpha + \iota\theta$)

$$\begin{aligned} h_{g,c}^{\psi\psi} = 1 \quad &+ \quad \epsilon_{g,c1}^{\psi\psi} \cos\left[(2\tilde{k}_X - d)\phi + \mu_1\alpha\right] \\ &+ \quad \epsilon_{g,c2}^{\psi\psi} \cos\left[(2\tilde{k}_X + d)\phi + \mu_2\alpha\right] \end{aligned}$$

with $\tilde{k}_X = (\tilde{k}_{*2} + \tilde{k}_{*1})/2$, $d = \tilde{k}_{*2} - \tilde{k}_{*1}$.

- When $d \ll \tilde{k}_X$, the two harmonics can be considered as one harmonic with slowly varying amplitude (beatings).

Annihilation of Gaps

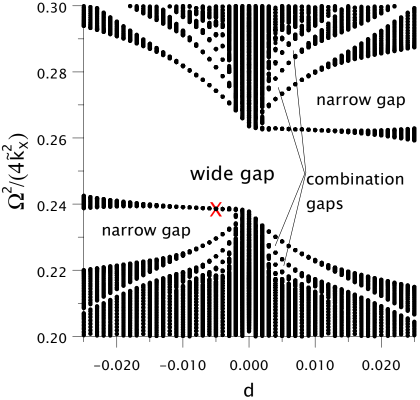
Exact crossing:

$$\tilde{k}_{*1} \equiv \frac{\mu_1 \iota - \nu_1 N}{2} = \pm \tilde{k}_{*2} = \tilde{k}_X \qquad \Rightarrow \qquad \iota = \iota_X = \frac{(\nu_1 \mp \nu_2) N}{\mu_1 \mp \mu_2}$$

- Both equilibrium harmonics, $\cos(\mu_1\theta - \nu_1 N\phi)$ and $\cos(\mu_2\theta - \nu_2 N\phi)$, have the same period on each field line.
- Their relative phases depend on the field line.
- The gap width is determined by the field line where the phases are opposite
  $\Rightarrow$ equals the difference of the widths that the two gaps would have if they were alone (the gaps "annihilate").
- The continuum wave functions are localized on field lines
  $\Rightarrow$ consist of infinite number of Fourier harmonics.

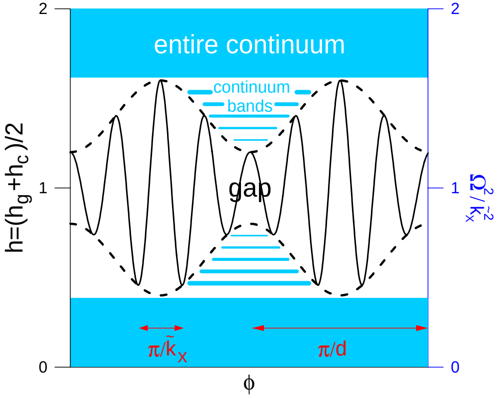
Numerically Calculated Continuum near Two Close Gaps

One gap is twice wider than the other



- Partial annihilation at the crossing point
- Multiple combination gaps with the coupling numbers $(\mu, \nu) = 2(\mu_1, \nu_1) - (\mu_2, \nu_2)$, $3(\mu_1, \nu_1) - 2(\mu_2, \nu_2)$, etc., appear.
- The continuum walls are extremely narrow (e.g., $\Delta\Omega^2/\Omega^2 = 3 \times 10^{-8}$ (!) at **X**).
- The thread width depends exponentially on $|d|^{-1}$.

Trapped Continuum Waves, Qualitative Consideration



- Two equilibrium harmonics form a joint gap with beating width
- Intermediate zones of "trapped" continuum bands
- The band width $\propto \exp(Cd^{-1})$ is due to tunnelling through the evanescence regions (in agreement with numerical calculations).

Trapped Continuum Waves, Amalytical Solution

- Averaging over fast scale, $\Delta\phi \sim \pi/\tilde{k}_X$, we reduce the AC equation to a Schrödinger equation
- Solutions agree with numerical calculations
- The wave is trapped (the tunnelling is weak) when

$$2\frac{||\tilde{k}_{*1}| - |\tilde{k}_{*2}||}{|\tilde{k}_{*1}| + |\tilde{k}_{*2}|} = 2\frac{|\Omega_{*1} - \Omega_{*2}|}{\Omega_{*1} + \Omega_{*2}} \ll \frac{\pi^2}{4}|\epsilon_1^{\psi\psi}\epsilon_2^{\psi\psi}|^{1/2}$$

(the case that we intended to consider).

- The waveguides are

$$|(\sigma\mu_2 - \mu_1)\theta - (\sigma\nu_2 - \nu_1)\phi - \arccos(-s) + 2\pi M| \\ \leq C|d/\tilde{k}_X|^{1/2}/|\epsilon_1^{\psi\psi}\epsilon_2^{\psi\psi}|^{1/4}$$

with $s = \mathrm{sgn}(\epsilon_1^{\psi\psi}\epsilon_2^{\psi\psi})$.

- For helicity-induced gaps $(2, 1)$ and $(3, 1)$, the "waveguides" are on either outer or inner circumference of the torus.

Eigenmodes in the HF Part of the Alfvén Spectrum

- The ballooning equation (BE) for pressureless plasma is (cf. [Dewar & Glasser, Phys. Fluids **36** (1983) 3038]):

$$\frac{d}{d\phi}\left(\Delta\frac{d\Phi}{d\phi}\right)+\Omega^2\frac{\Delta}{h_B^4}\Phi=0,$$

$$\Delta=h_g^{\theta\theta}+2\iota\hat{s}(\phi-\phi_k)h_g^{\psi\theta}+\iota^2\hat{s}^2(\phi-\phi_k)^2h_g^{\psi\psi},$$

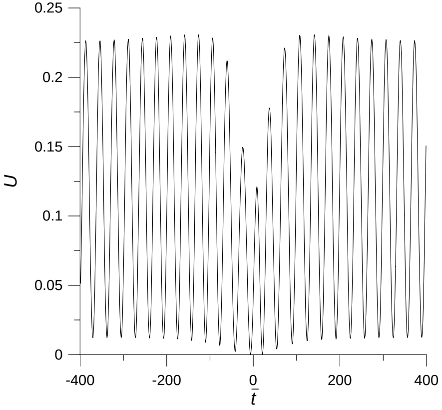
  $\hat{s}$ is magnetic shear, h^{ij} are normalized metric tensor components.

- When $\phi\to\pm\infty$, the BE is reduced to the AC equation.
- Three scales: $\Delta\phi\sim\pi/\tilde{k}_X$ (fast), $\Delta\phi\sim\pi/d$ (beatings), $\Delta\phi\sim(\iota\hat{s})^{-1}$ (usually appears in ballooning formalism)
- Averaging over fast scale reduces BE to a Schrödinger equation

$$\frac{d^2\Phi}{d\bar{t}^2}+[E-U(\bar{t})]\Phi=0,$$

  with $E=(\Omega^2/\tilde{k}_X^2-1)^2/4$, $\bar{t}=\tilde{k}_X(\phi-\phi_k)$.

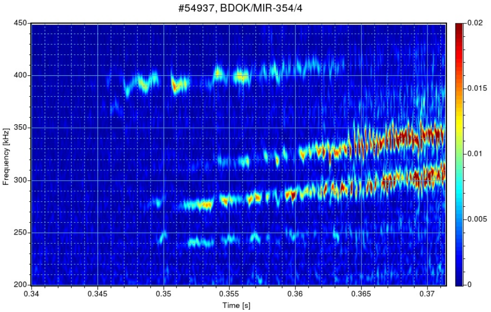
Potential and Eigenmodes



- Potential for the gaps $(\mu_1, \nu_1) = (2, 1)$ and $(\mu_2, \nu_2) = (3, 1)$ in the W7-AS shot No. 54937 at $r/a = 0.45$
- There are bounded states (eigenmodes) in the wells.
- The eigenmodes are trapped at the same places as the continuum waves.
- For the harmonics $(2, 1)$ and $(3, 1)$, the eigenmodes are localized at either inner or outer circumference.

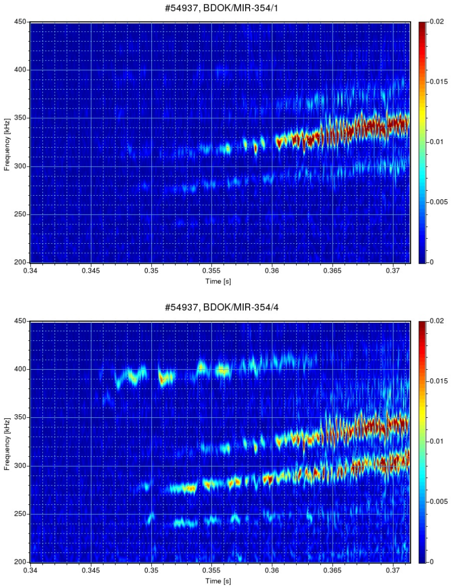
High-frequency Alfvén Activity in W7-AS Shot # 54937

Frequency spectrum of signals at a Mirnov coil at the late stage of shot # 54937.



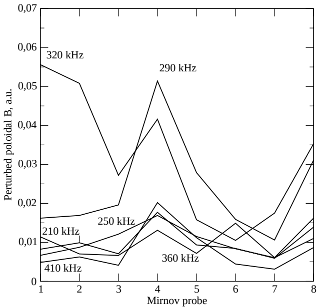
- Rather high frequencies (TAEs and EAEs have $f < 100$ kHz in the core)
- Several instabilities with different dependence of amplitudes on time.
- Alfvénic character of $f(t)$ (growth with density decrese)
- Bursting behavior but no or almost no chirping: $\Delta f \Delta t \sim 2$.

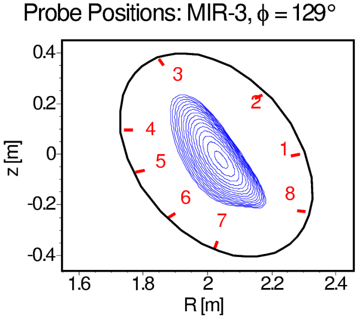
Inner Circumference vs Outer Circumference



- Signals at the Mirnov coils 1 and 4 are shown.
- Some frequency bands are much stronger at the high-field side (coil 1) than at the low-field side (coil 4).

Amplitudes of Spectral Lines on Different Mirnov Coils

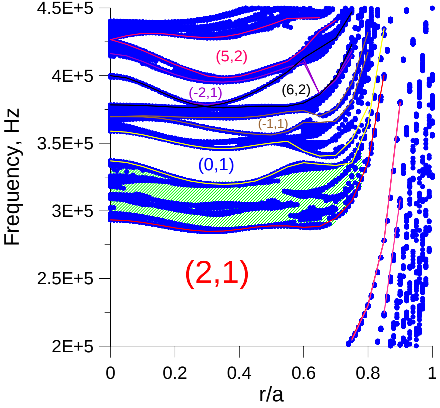




- Amplitudes on one of the Mirnov arrays are shown
- Correlating variations of intensities of all spectral bands (see coils 4, 5, 6, which are in similar positions).
- May be caused by field corrugations (typical for high-ι discharges).
- Nevertheless, clear signs of anti-ballooning behaviour are observed (for 210-, 250- and 410-kHz spectral bands)

Alfvén Continuum in W7-AS Shot # 54937

The AC calculated for $t = 0.36\,\mathrm{s}$:



The calculations did not converge in the green zone.

- The frequencies observed are in the range of HAEs and MAEs.
- The harmonics $(2, 1)$ and $(3, 1)$ (elongation and triangularity) are dominant in the shaping
  $\Rightarrow$ waveguides at the high-field side (where these harmonics tend to cancel).
- Crossing of the gaps $(7, 0)$ and $(1, 1)$ in the range of 290–340 kHz
  $\Rightarrow$ waveguides of the structure $(\mu, \nu) = (8, 1)$
  $\Rightarrow$ poloidal inhomogeneity should not be observed.

Summary

- The interaction of two sufficiently large equilibrium harmonics with sufficiently close periods along the field lines can result in trapping of the AC wave functions in certain "waveguides".
- When the periods of the two harmonics exactly coincide (i.e., the continuum gaps cross), the wave functions are localized at single field lines, which leads to "annihilation" of the gaps.
- The trapping is typical in the high-frequency part of the AC (at least, for high N). In the typical case when this part is dominated by the harmonics $(2, 1)$ and $(3, 1)$, the waves are trapped at the inner circumference of the flux surface.
- Wave trapping takes place also near crossings of continuum gaps.
- Trapping may affect the energy absorption of Alfvén waves.
- HF eigenmodes are also trapped at the same places as the continuum waves.
- There are indications that trapping of HF Alfvén instabilities at the inner circumference of the torus was indeed observed in W7-AS.