Multiplier for MC step size in NUBEAM *kick* model

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Finding the "correct" multiplier, $ntaep\_fac$

The total energy kicks are computed in the *kick* model according to

$$\Delta E = \sum_{i=1}^{n_{MC}} \sigma_i \, \Delta E_0 \, A_{mode,i} \, k_t \tag{1}$$

where σ_i is a random sign, ΔE_0 the random energy kick, A_mode the vector of mode amplitude vs. time and $k_t = \delta t_M C/\delta t_m, k$ the ratio of Montecarlo time step over the time step used to sample the probability $p(\Delta E, \Delta P_\zeta)$ (e.g. in ORBIT) for the kth mode.

The number of Montecarlo time steps, n_{MC} was originally computed as

$$n_{MC} = \frac{t_{stop} - t_{start}}{\min\{\delta t_{m,k}\}} \tag{2}$$

with t_{start}, t_{stop} the starting/ending time of the NUBEAM step. However, this definition implies that n_{MC} may be, in practice, too small and the MonteCarlo loop would not converge due to poor statistics. Moreover, simulations would not converge when a large mode amplitude is used and particles can jump from one energy bin to another (not necessarily adjacent to the original one) in only a few steps. Figure 1 shows a simple example of several MonteCarlo runs for particles starting with the same initial energy of 10 keV. Energy grid step is 4 keV and the energy kick is set to $\Delta E = 5$ keV at all steps. It can be appreciated that particle evolution over a 5 ms time interval does not converge for a small number of MonteCarlo steps, $n_{MC} = 250$, whereas all trajectories are very similar for a larger number, $n_{MC} = 5000$.

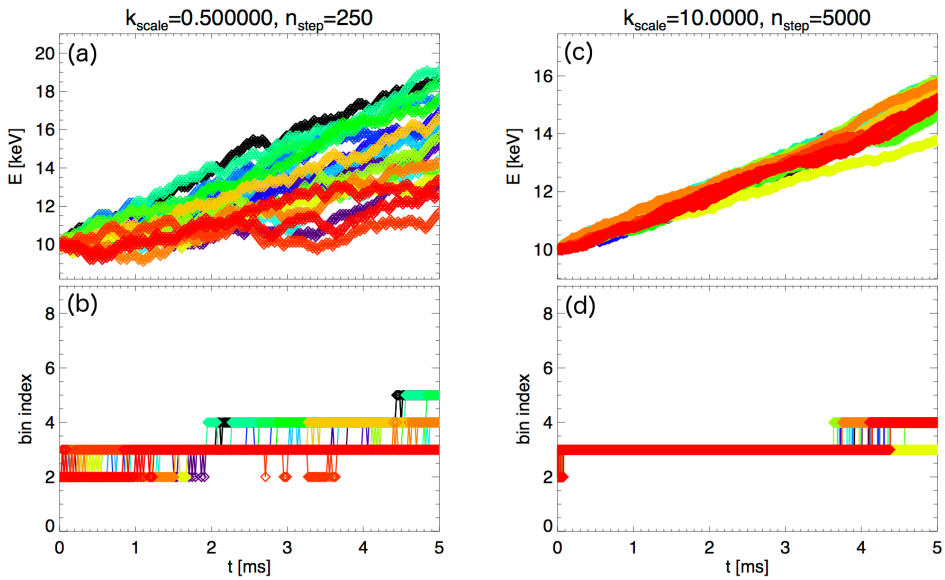

FIG. 1. Test of MonteCarlo particle (a,c) energy trajectories and (b,d) bin index vs. time. Each color represents a different MonteCarlo run for particles with the same initial energy, $E_0 = 10$ keV. The number of MonteCarlo steps to cover the 5 ms interval is increased from 250 to 5000 from (a,b) to (c,d).

To correct these issues in the NUBEAM implementation of the *kick* model, a sufficiently large number of MonteCarlo steps must be enforced. The number of steps must depend on:

- Grid steps for energy, energy steps (and, similarly, for P_ζ and P_ζ steps),
- Mode amplitude,
- Actual sampling time used to generate $p(\Delta E, \Delta P_\zeta)$.

The following procedure is proposed to compute a reasonable multiplier, $ntaep\_fac$, such that

$$n_{MC} \rightarrow n_{MC}^0 \times ntaep\_fac \tag{3}$$

where n_{MC}^0 is obtained from Eq. 2. In addition, $ntaep\_fac$ can be defined through the TRANSP nameless for users who may want to study/check the convergence of simulations as a function of the number of MonteCarlo steps.

The following procedure illustrates the reasoning behind the choice of a certain $ntaep\_fac$ based on E and ΔE grid steps. Same arguments apply based on the P_ζ and ΔP_ζ grids. The final multiplier is the maximum between the multiplier obtained from the two variables.

Consider a bin in the (E, P_ζ, μ) space over which the kick probability $p(\Delta e, \Delta P_\zeta)$ is defined. The energy and energy kick steps are E_{step} and ΔE_{step}, respectively. For a certain NUBEAM time step defined by starting/end times t_0 and t_1, define the maximum mode amplitude as A^{max}_{mode} and the minimum mode sampling time as $\delta t^{min}_m = \min\{\delta t_{m,k}\}$ (with $k = 1, N_{modes}$).

The maximum energy kick that a particle can experience at each step in the MonteCarlo evolution is ΔE_{step}. Therefore, the minimum number of steps required to push a particle outside the original (E, P_ζ, μ) bin will be such that (cf. Eq. 1):

$$n^{min}_{MC} \approx \frac{E_{step}}{\Delta E_{step}\, A^{max}_{mode}} \tag{4}$$

where it is assumed that the particle is kicked in only one direction (e.g., $\sigma_i \equiv +1$) and that $k_t \approx 1$. Ideally, we want $n^0_{MC} \gg n^{min}_{MC}$ - that is, a particle will undergo several MonteCarlo steps before eventually moving to another (E, P_ζ, μ) bin. Let's define a target number of MonteCarlo steps for each (E, P_ζ, μ) bin as n^{target}_{MC} (e.g. with $n^{target}_{MC} \sim 10^3$), and combine Eqs. 2 through 4 to estimate the integer $ntaep\_fac$:

$$ntaep\_fac = int\{\frac{\Delta E_{step}\, A^{max}_{mode}}{E_{step}} \times \frac{n^{target}_{MC}}{n^0_{MC}}\} \tag{5}$$