Overview of Stellarator Flow Measurements and Modeling

Stefan Gerhardt

Thanks to D. Spong for providing slides from his APS talk, feedback from J. Callen, C. Hegna, J.N. Talmadge, and D.T. Anderson.

Motivation (1)

- Cross-field transport of particles, heat, and momentum in tokamaks is generally anomalous (greater than the transport expected due to coulomb collisions).

- In some cases, the transport in some channels can be reduced to the neoclassical level.

- These cases involve significant shear in the ExB flow, leading to suppression of turbulence.

- Hence, the study of flow drive/damping in tokamaks has been very important for total transport optimization.

Motivation (2)

- Global thermal transport in stellarators is often dominated by turbulence (ISS-95 scaling fits tokamaks and stellarators equally well!).

- It is anticipated that flow generation in stellarators will improve confinement…and evidence points this way.

- Flows in stellarators are often damped by a neoclassical damping effect which is not present in the tokamak case.

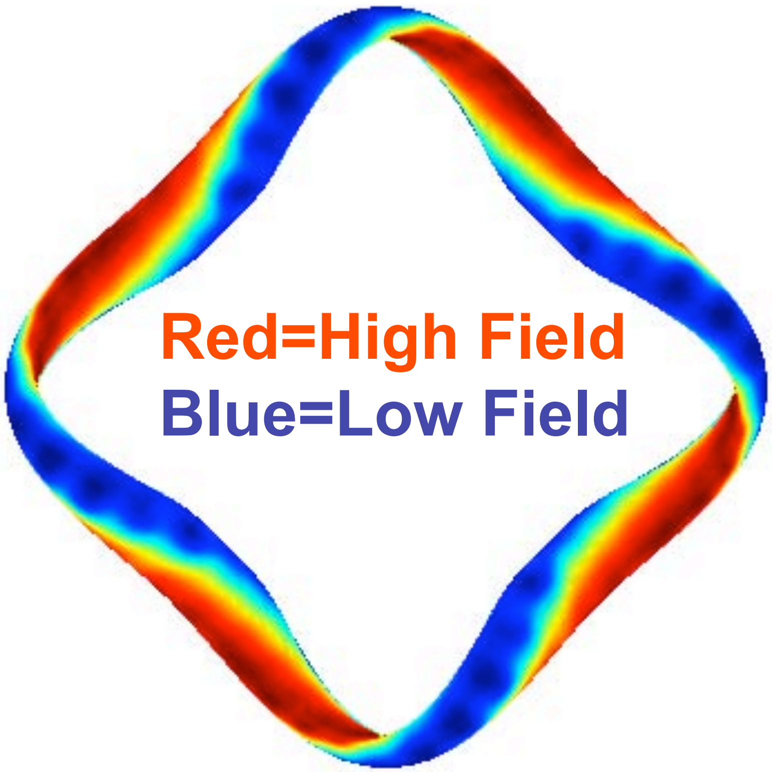
- This talk: review some results from the measurements and theory of flows in stellarators.

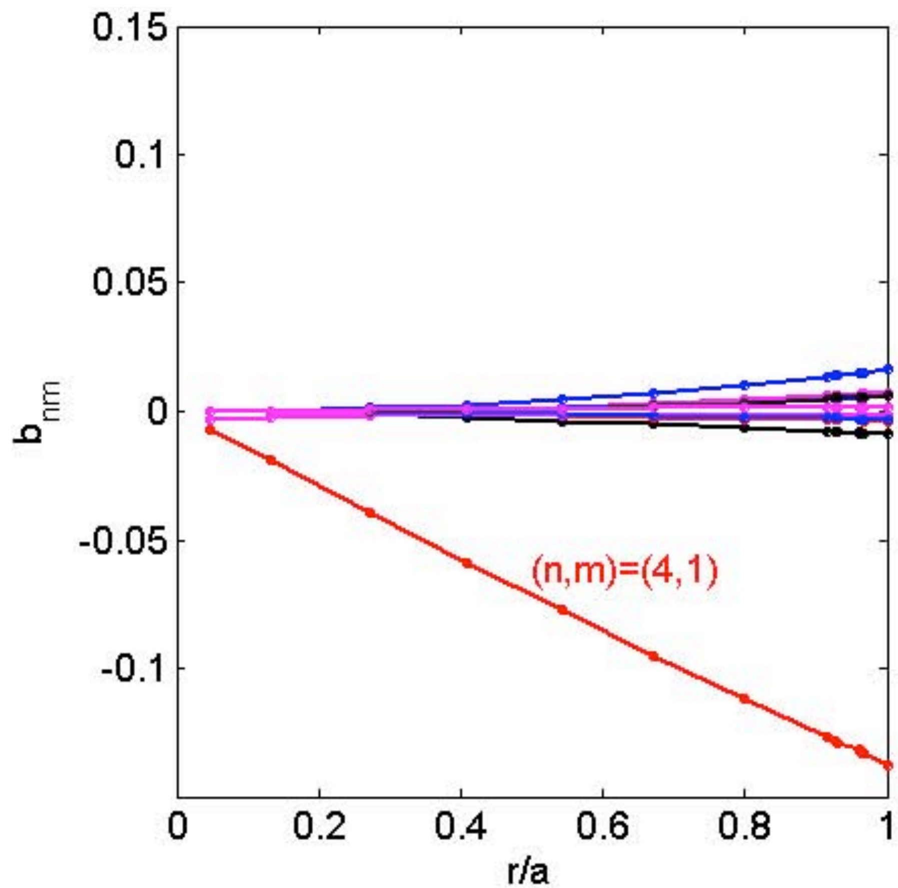
Outline

- Experimental Results: Toroidal Rotation studies in CHS and W7-AS
  - Both neoclassical and anomalous damping NBI driven flows are important.
- Simplified Theoretical Picture of Neoclassical Flows.
  - Momentum conservation in both directions on a flux surface is important
- Example from H-mode windows in W7-AS
  - Surface distortions due to islands may lead to reduced plasma flows.
- Rotation Studies with Quasisymmetry in HSX
  - Flow damping is reduced with quasisymmetry
- Recent Flow Modeling for NCSX, QPS, and HSX
  - Flow pattern in these devices reflect the underlying symmetries

Stellarators Described By Their Magnetic Field Spectrum

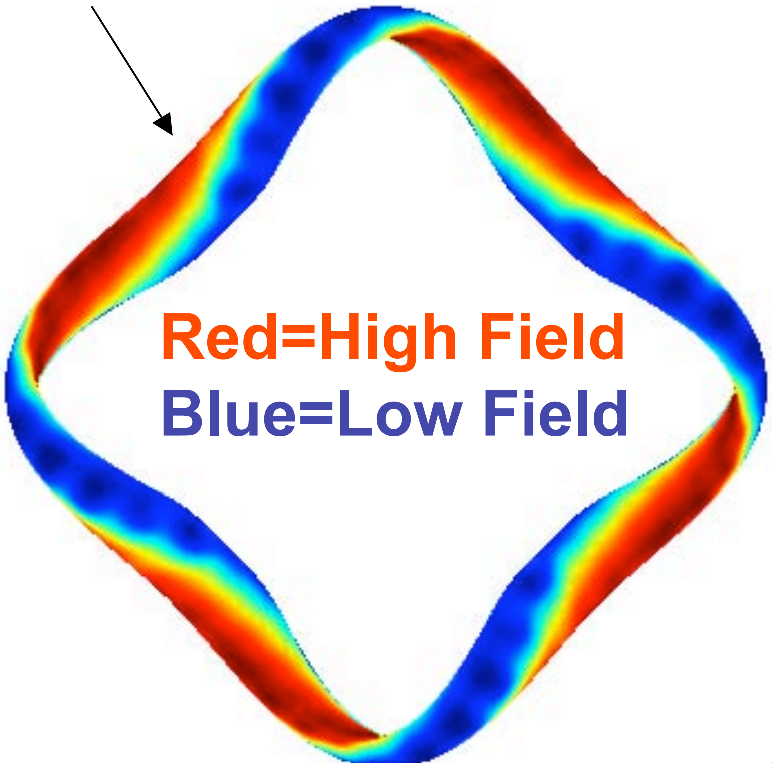
$$B(r,\varphi,\theta) = B_0(r)\left(1 + \sum_{n,m\neq 0} b_{nm}(r)\cos(n\varphi - m\theta)\right)$$

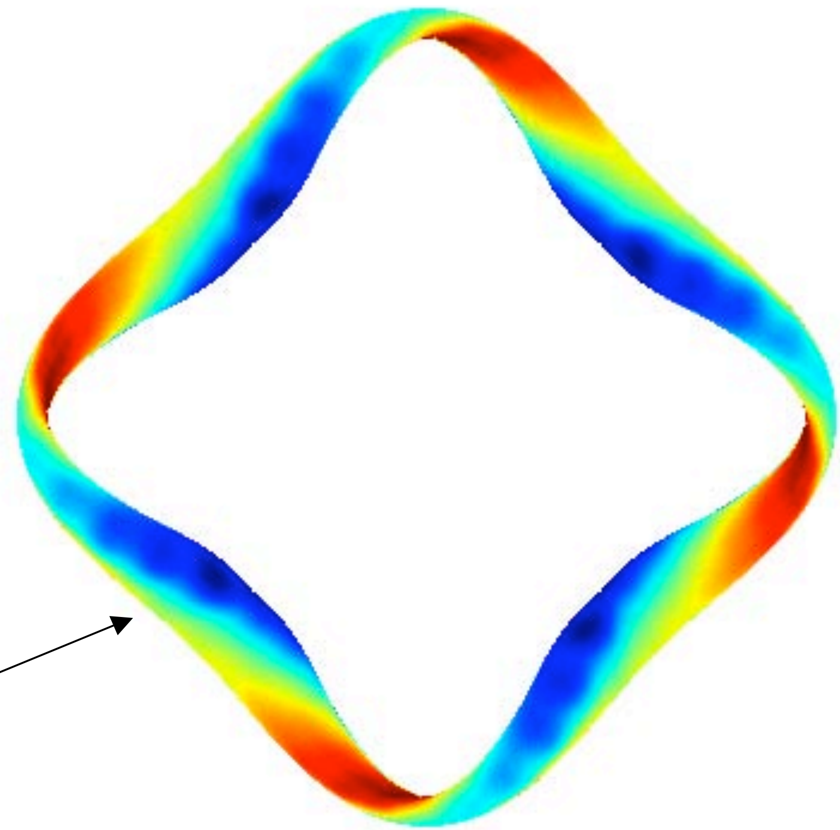




Flexible Coil Sets Often provide Experimental Flexibility

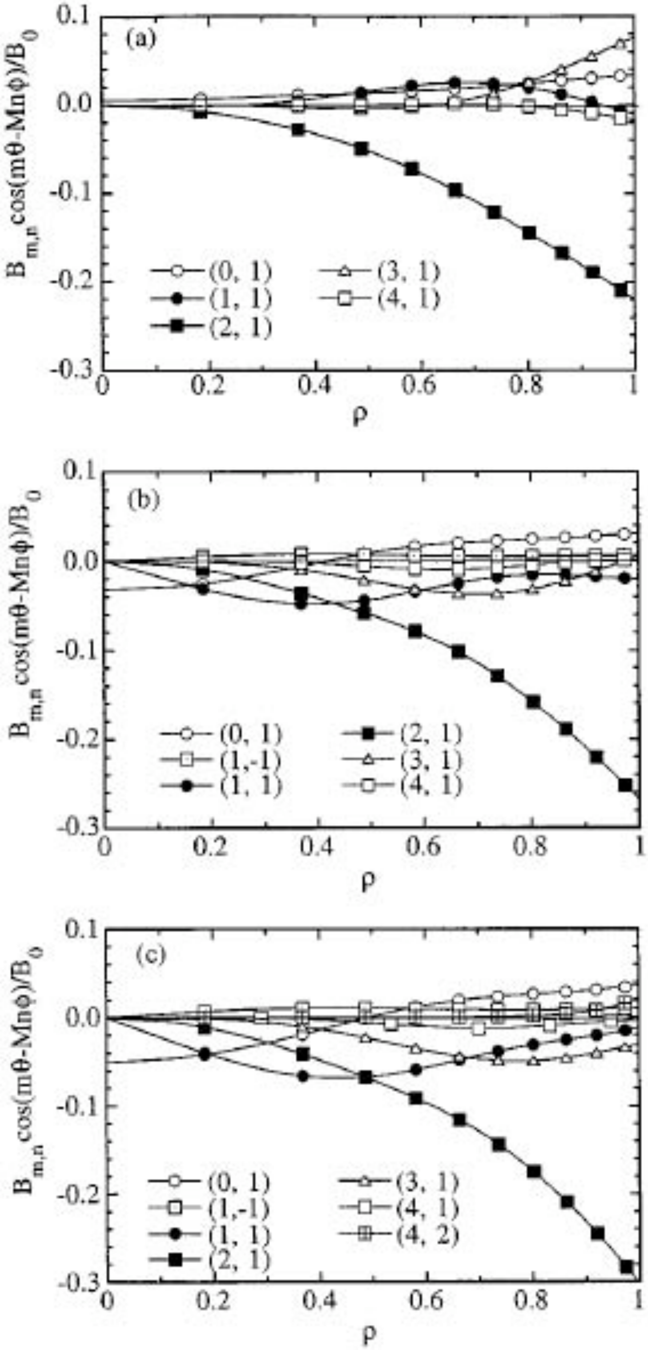
QHS: $B/B_o \approx 1 + \varepsilon_H \,\mathbf{cos}(4\varphi - \theta)$





Mirror: $B/B_o \approx 1 + \varepsilon_H \,\mathbf{cos}(4\varphi - \theta) + \varepsilon_M \,\mathbf{cos}(4\varphi)$

Shifting Axis Modifies Neoclassical Damping in CHS

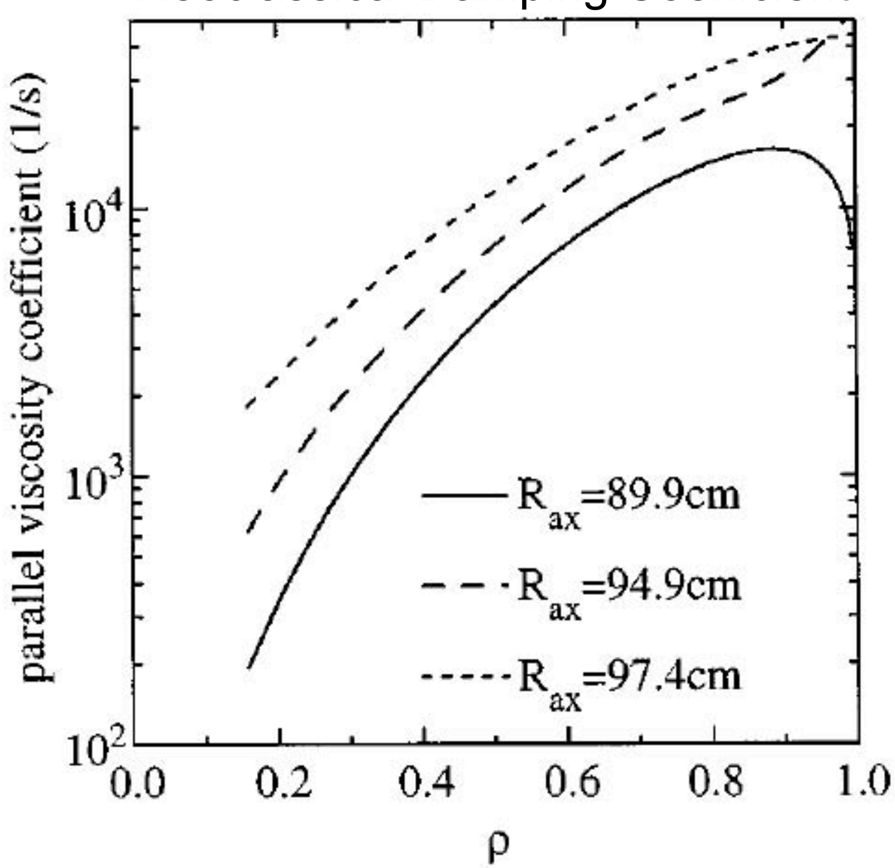


R=89.9

R=94.9

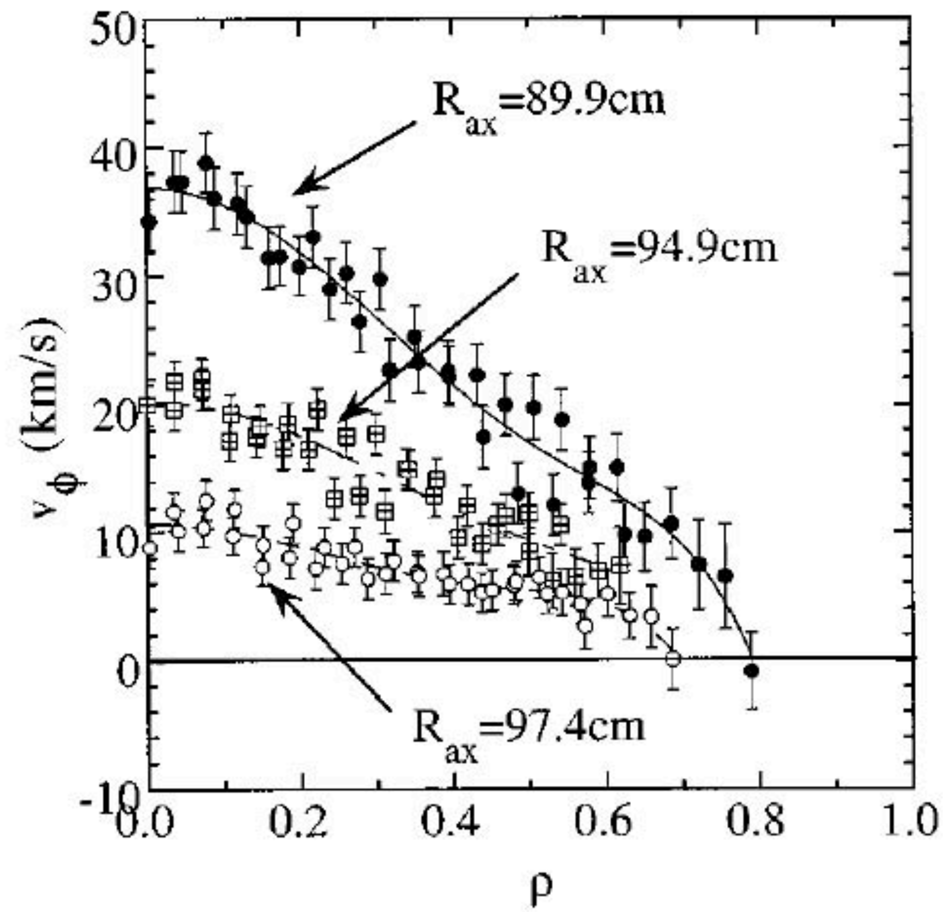
R=97.5

Neoclassical Damping Coefficient



K. Ida and N. Nakajima, Phys. Plasmas **4**, 310 (1997)

Reduced Toroidal Flows with Higher Neoclassical Viscosity



K. Ida and N. Nakajima, Phys. Plasmas **4**, 310 (1997)

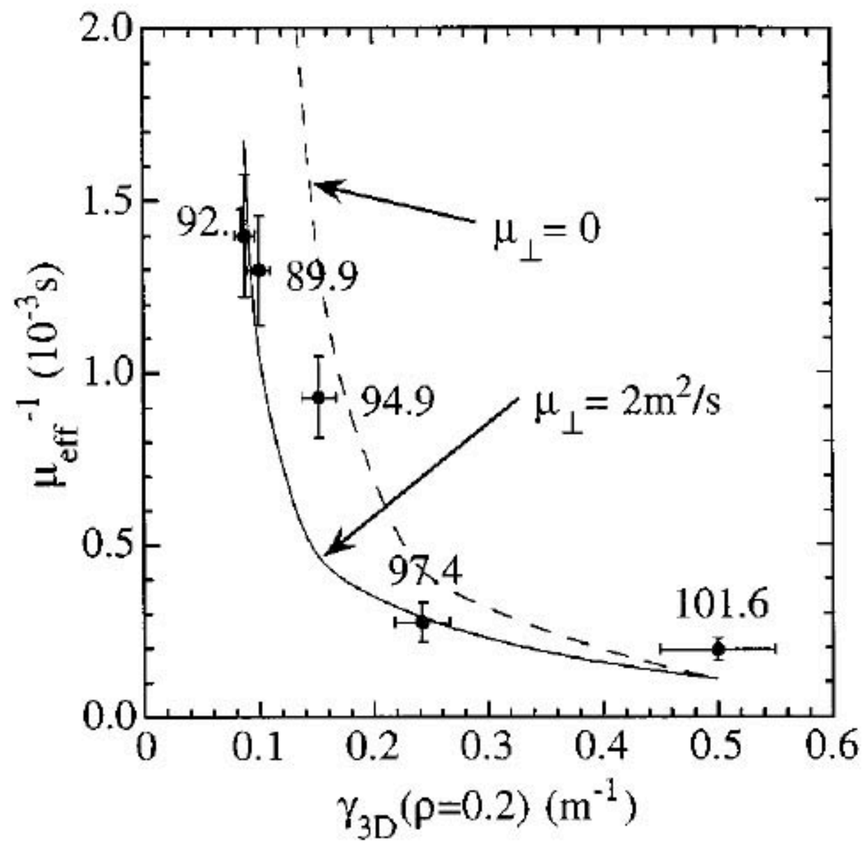
Anomalous Perpendicular Viscosity Required

NBI torque calculated including the fast ion birth distribution and shine-through

$$\frac{F_\phi^{NBI}(r)}{m_i n_i(r)} = \mu_\parallel(r)V_\phi(r) - \mu_\perp(r)\nabla^2 V_\phi(r)$$

Define a measured "effective" viscosity coefficient.

$$\mu_{eff}^{-1} = \frac{v_\phi(0) m_i n_i(0)}{F_\phi^{NBI}(0)}$$



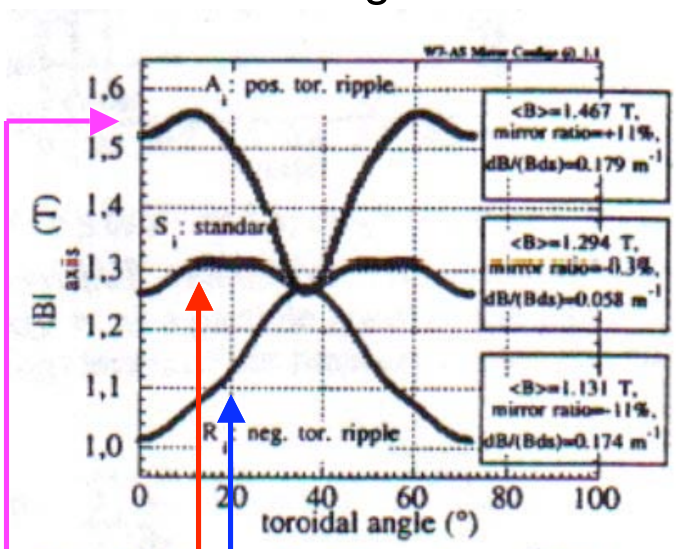
$$\gamma_{3D}^2 = \frac{\left\langle (\mathbf{n}\cdot\nabla B)^2 \right\rangle}{\left\langle B^2 \right\rangle}$$

γ_{3D} measures how "3-Dimensional" the configuration is.

K. Ida and N. Nakajima, Phys. Plasmas **4**, 310 (1997)

Reduced NBI Driven Flows in W7-AS when Large Mirrors are Present

|B| vs Toroidal Angle in 3 Configurations.



Negative Ripple

Standard

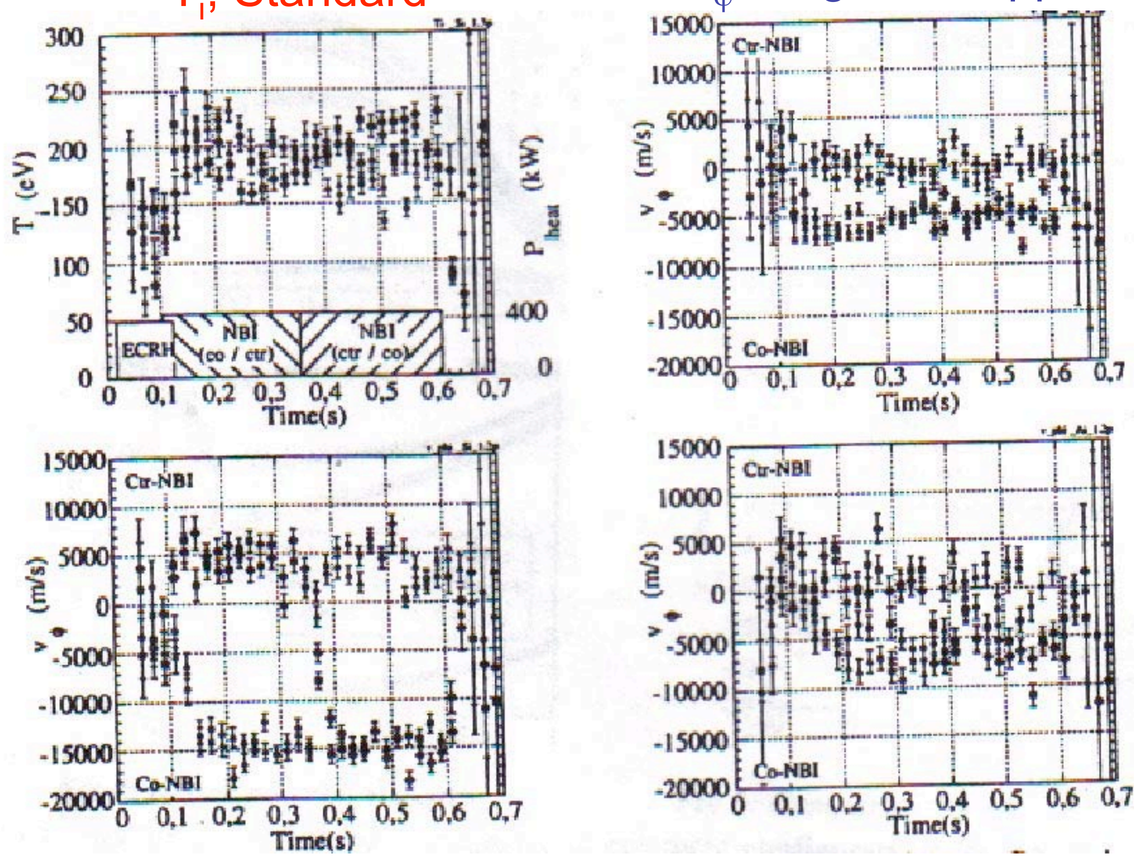
Positive Ripple

n_e=1.3x10^{19}, <30% Beam Absorption

T_i and Central Rotation Speed in the Three Configurations

T_i, Standard

V_φ, Negative Ripple



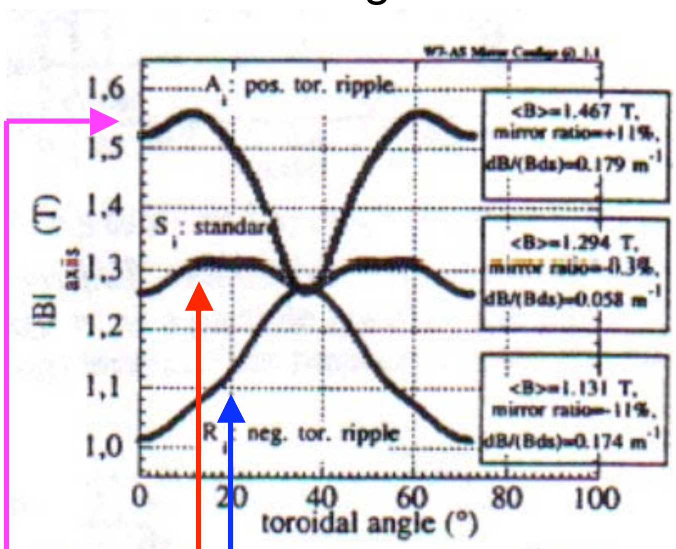
V_φ, Standard

V_φ, Positive Ripple

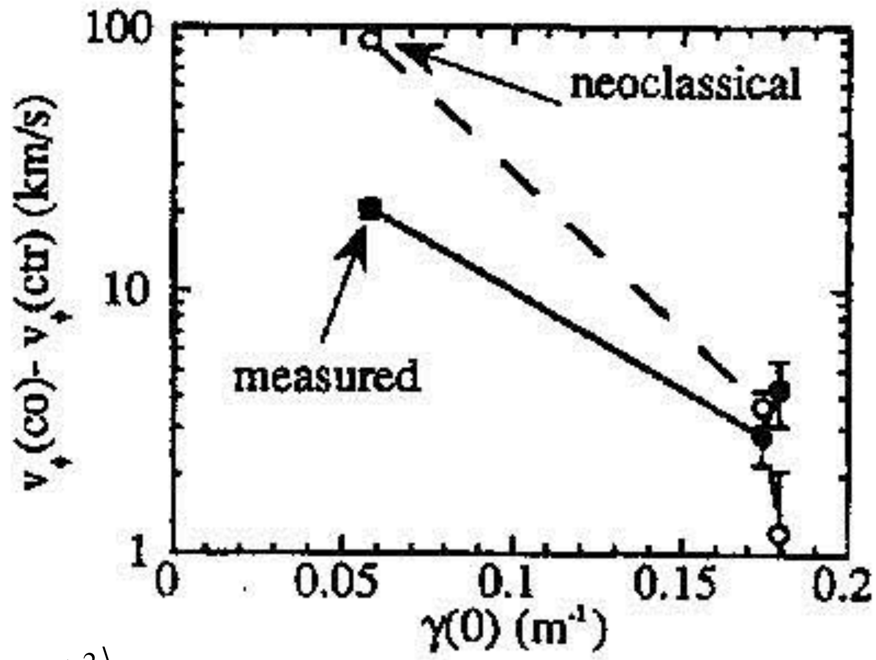
J.V. Hoffman et al., 21st EPS Conference on Controlled Fusion and Plasma Physics, Montpellier, 392 (1994),

Anomalous Flow Damping Observed in W7-AS as Well

|B| vs Toroidal Angle
in 3 Configurations.



Negative Ripple
Standard
Positive Ripple

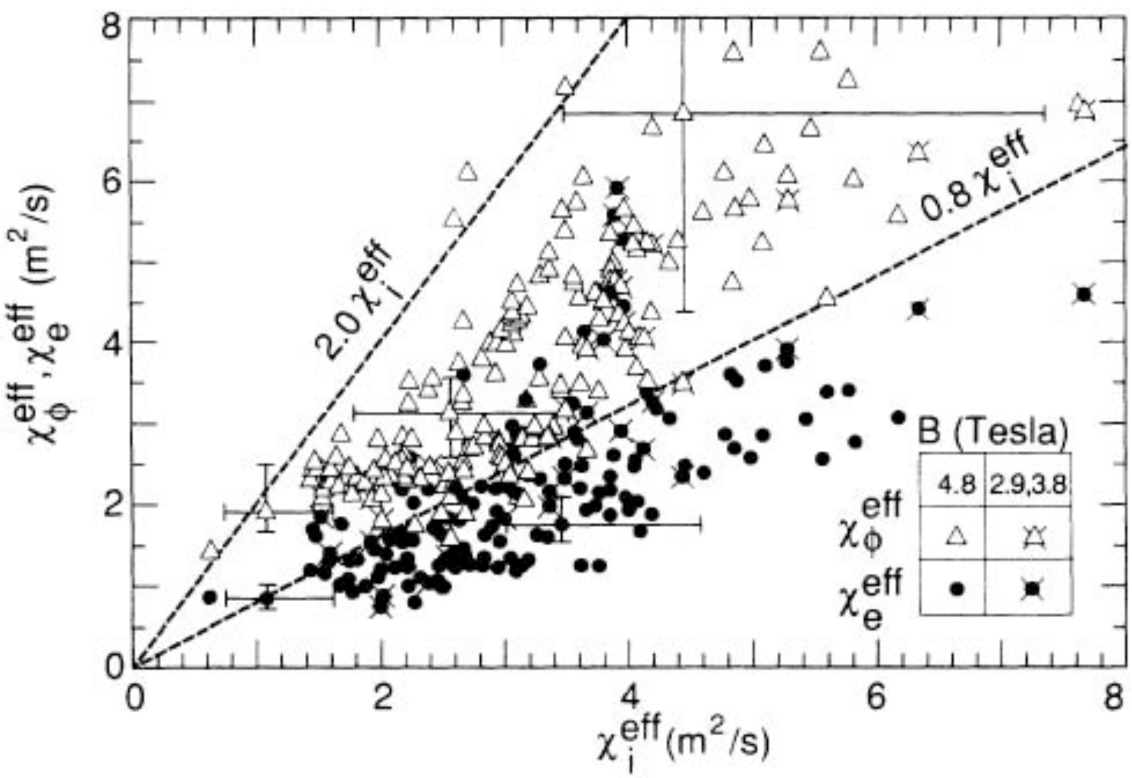


$$\gamma_{3D}^2 = \frac{\left\langle \left(\mathbf{n}\cdot\nabla B\right)^2 \right\rangle}{\left\langle B^2 \right\rangle}$$

K. Ida, et al, AIP Conference Proceedings **345**, 177 (1995)

Relationship to Tokamak Results

- Damping of toroidal flows in tokamaks is much larger than the (small) neoclassical level of damping.
- Ion thermal energy and momentum diffusivity are strongly coupled.



Scott et al, Phys. Rev. Lett. **64**, 531 (1990)

- Ion temperature gradient turbulence theory predicts $\chi_\varphi=\chi_i$.
- In CHS: "The anomalous perpendicular viscosity of 2m^2/s has a similar magnitude to that of effective thermal diffusivity (χ_{eff}=5-6m^2/s)."

Overview of Theory…

- Neoclassical theory describes the behavior of currents, flows, and cross-field transport in the inhomogeneous magnetic fields of a torus.
- Moment method for Tokamaks developed in 1970s
  - Hirshman and Sigmar, Nuclear Fusion **21**, 1079 (1981).
- Use conservation equations for particles, momentum, heat,…, to calculate plasma properties.
- Use kinetic theory to calculate viscosities, friction,…
- Moment method for non-axisymmetric torus developed since then.
  - K.C. Shaing and J.D. Callen, Phys. Fluids **26**, 3315 (1983).
  - M. Coronado and H. Wobig, Phys. Fluids **30**, 3171 (1987).
  - H. Sugama and S. Nishimura, Phys. Plasmas **9**, 4637 (2002).
  - D. Spong, Phys. Plasmas **12**,1 (2005)
- Stellarator moment method papers use magnetic coordinates, often Hamada Coordinates.

Neoclassical Parallel Flow Has Two Components

Lowest Order momentum balance…

$$e_a N_a\left(\mathbf{E}+\frac{1}{c}\mathbf{U}_a\times\mathbf{B}\right)=\nabla p_a$$

…determines $U_\perp$:

$$\mathbf{U}_{\perp a}=-\frac{c}{B^2}\left(\frac{\partial\Phi}{\partial\rho}+\frac{1}{e_a N_a}\frac{\partial p_a}{\partial\rho}\right)\mathbf{B}\times\nabla\rho$$

Incompressibility ($\nabla\cdot U = \nabla\cdot U_\perp + \nabla\cdot U_{||} = 0$) specifies the parallel flow:

$$\mathbf{U}_{\parallel a}=\underbrace{c\left(\frac{\partial\Phi_a}{\partial\rho}+\frac{1}{eN_a}\frac{\partial p_a}{\partial\rho}\right)\frac{B_\alpha}{B^\varsigma B^2\sqrt{g}}\mathbf{B}}_{\text{Pfirsch-Schlueter flow to maintain } \nabla\cdot U=0}+\underbrace{\lambda_a(\rho)\mathbf{B}}_{\lambda_a\text{: “bootstrap” like parallel flow determined through first order momentum balance}}$$

Pfirsch-Schlueter flow to maintain $\nabla\cdot U=0$

λ_a: “bootstrap” like parallel flow determined through first order momentum balance

K.C. Shaing and J.D. Callen, Phys, Fluids **26**, 3315 (1983)
M. Coronado and H. Wobig, Phys Fluids **29**, 527(1986)

Conversion to Tokamak Geometry Yields Traditional Result

- Consider a large aspect ratio tokamak:

$$\mathbf{B} = \frac{B_0}{1+\varepsilon\cos(\theta)}\left(\Theta(r)\hat{\theta} + \hat{\phi}\right), \quad \varepsilon = r/R_0, \quad \Theta(r) = \varepsilon/q(r)$$

- The Hamada coordinates for this configuration have been calculated ($\rho=\psi$):

$$B_\alpha \approx 2rqB_0\cos(\theta) \quad B^\xi = B_0/R_0 \quad \sqrt{g} = R_0/B_0$$

$$\frac{dp_a}{d\psi} = \frac{dp_a}{dr}\frac{dr}{d\psi} \sim \frac{1}{rB_0}\frac{dp_a}{dr}$$

- Traditional $\cos(\theta)$ dependence is recovered.

$$\mathbf{U}_{\|a} = c\left(\frac{\partial\Phi_a}{\partial\rho} + \frac{1}{eN_a}\frac{\partial p_a}{\partial\rho}\right)\frac{B_\alpha}{B^\varsigma B^2\sqrt{g}}\mathbf{B} + \lambda_a(\rho)\mathbf{B} \Rightarrow$$

$$\sim Const.\left(\frac{\partial\Phi_a}{\partial r} + \frac{1}{eN_a}\frac{\partial p_a}{\partial r}\right)\frac{q\cos(\theta)}{B_0^2}\mathbf{B} + \lambda_a(\rho)\mathbf{B}$$

M. Coronado and J. Galindo Trejo, Phys. Fluids B **2**, 530 (1990)

Toroidal Momentum Balance is Related to Radial Fluxes (1)

- 1st Order Momentum Balance:

K.C. Shaing and J.D. Callen, Phys. Fluids **26**, 3315 (1983).

$$m_a N_a \frac{\partial}{\partial t}\mathbf{U}_a = N_a e_a\left(\mathbf{E}+\frac{1}{c}\mathbf{U}_a\times\mathbf{B}\right)-\nabla p_a+\mathbf{F}_a-\nabla\cdot\Pi_a-m_a N_a \upsilon_{an}\mathbf{U}_a-\mathbf{R}_a$$

- $\langle B_T \cdot \ldots \rangle$ the equilibrium momentum equation, and notice:

$$\mathbf{B}_T\cdot\mathbf{U}_a\times\mathbf{B}=\mathbf{U}_a\cdot\left[\left(B^\zeta\mathbf{e}_\zeta+B^\alpha\mathbf{e}_\alpha\right)\times B^\zeta\mathbf{e}_\zeta\right]=B^\alpha B^\zeta\sqrt{g}\,\mathbf{U}_a\cdot\nabla\rho$$

- Process yields the radial flux of a given species

$$\Gamma_a=N_a\langle U_a\cdot\nabla\rho\rangle=\Gamma^a_{cl}+\Gamma^a_{PS}+\Gamma^a_{bp}+\Gamma^a_{na}+\Gamma^a_{R_a}+\Gamma^a_{an}+\frac{N_a c}{B^\alpha\sqrt{g}}\left\langle\frac{E_\parallel^{(A)}}{B}\left(\langle I\rangle\frac{B^2}{\langle B^2\rangle}-I\right)\right\rangle$$

Classical Flux, Ambipolar

$$\Gamma^a_{cl}=\frac{c}{e_a B^\zeta B^\alpha\sqrt{g}}\langle\mathbf{B}_T\cdot\mathbf{F}_{\perp a1}\rangle,\quad \Gamma^a_{na}=\frac{c}{e_a B^\zeta B^\alpha\sqrt{g}}\langle\mathbf{B}_T\cdot\nabla\cdot\Pi_a\rangle$$

Stellarator Neoclassical Flux, NonAmbipolar

Pfirsch-Schlueter Flux, Ambipolar

$$\Gamma^a_{PS}=\frac{c}{\sqrt{g}B^\alpha e_a}\left\langle\frac{F_{\parallel a1}}{B}\left(I-\langle I\rangle\frac{B^2}{\langle B^2\rangle}\right)\right\rangle,\quad \Gamma^a_{R_a}=\frac{c}{e_a B^\zeta B^\alpha\sqrt{g}}\langle\mathbf{B}_T\cdot\mathbf{R}_a\rangle$$

External Force Flux, NonAmbipolar

Bananna-Plateau Flux, Ambipolar

$$\Gamma^a_{bp}=-\frac{c}{e_a B^\alpha\sqrt{g}}\frac{\langle I\rangle\left\langle B\left(F_{\parallel a1}+n_a e_a E_\parallel^{(A)}\right)\right\rangle}{\langle B^2\rangle},\quad \Gamma^a_{an}=\frac{c}{e_a B^\zeta B^\alpha\sqrt{g}}\langle N_n\upsilon_{an}\mathbf{B}_T\cdot\mathbf{U}\rangle$$

Neutral Friction Flux, NonAmbipolar

Toroidal Momentum Balance is Related to Radial Fluxes (2)

- Sum over species to find the ambipolarity constraint:

$$\sum_a e_a N_a \langle U_a \cdot \nabla \rho \rangle = \sum_a \left(\frac{-c}{B^\xi B^\alpha \sqrt{g}} \left(\langle \mathbf{B}_T \cdot \nabla \cdot \Pi_a \rangle + \langle \mathbf{B}_T \cdot \mathbf{R}_a \rangle + m_a N_a \upsilon_{an} \langle \mathbf{B}_T \cdot \mathbf{U}_a \rangle \right) \right)$$

- If there are no neutrals, and no external forces, then

$$\sum_a e_a N_a \langle U_a \cdot \nabla \rho \rangle = \sum_a \frac{-c}{B^\xi B^\alpha \sqrt{g}} \langle \mathbf{B}_T \cdot \nabla \cdot \Pi_a \rangle$$

- For a tokamak, or any axisymmetric system (will be “shown” later.)

$$\langle \mathbf{B}_T \cdot \nabla \cdot \Pi_a \rangle = 0$$

- In this case, there is not net current (electrons and ions have equal radial fluxes).

K.C. Shaing, Phys. Fluids **29**, 2231 (1986), K.C. Shaing and J.D. Callen, Phys. Fluids **26**, 3315 (1983).

Parallel Momentum Balance Yields Second Constraint (2)

- 1st Order Momentum Balance:

$$m_a N_a \frac{\partial}{\partial t}\mathbf{U}_a = N_a e_a\left(\mathbf{E} + \frac{1}{c}\mathbf{U}_a \times \mathbf{B}\right) - \nabla p_a + \mathbf{F}_a - \nabla\cdot\Pi_a - m_a N_a \upsilon_{an}\mathbf{U}_a - \mathbf{R}_a$$

•<B·…>, and assume equilibrium :

$$m_a N_a \frac{\partial}{\partial t}\langle \mathbf{B}\cdot\mathbf{U}_a\rangle = \langle \mathbf{B}\cdot\mathbf{F}_a\rangle - \langle \mathbf{B}\cdot\nabla\cdot\Pi_a\rangle - m_a N_a \upsilon_{an}\langle \mathbf{B}\cdot\mathbf{U}_a\rangle - \langle \mathbf{B}\cdot\mathbf{R}_a\rangle$$

•Sum over species to find the parallel momentum constraints:

$$\sum_a \langle \mathbf{B}\cdot\nabla\cdot\Pi_a\rangle = -\sum_a m_a N_a \upsilon_{an}\langle \mathbf{B}\cdot\mathbf{U}_a\rangle - \sum_a \langle \mathbf{B}\cdot\mathbf{R}_a\rangle$$

•The ambipolarity constraint and the parallel momentum constraint must both be satisfied

K.C. Shaing, Phys. Fluids **29**, 2231 (1986)

The Neoclassical Viscosities are Easily Calculated in the Collisional Regimes

K.C. Shaing and J.D. Callen, Phys. Fluids **26**, 3315 (1983).

•Parallel viscosity is related to distortion of the distribution function (CGL form)

$$\overleftrightarrow{\Pi}_{\parallel a} = \left(p_{\parallel a} - p_{\perp a}\right)\left(\widehat{b}\widehat{b} - \overleftrightarrow{I}/3\right)$$

•In *collisional* (Braginski) regime, the pressure anisotropy is related to the flows as:

$$p_{\parallel a} - p_{\perp a} = -\frac{3}{2}\eta_{0a}\left(\hat{b}\cdot\overleftrightarrow{W}\cdot\hat{b}\right)$$

$$\overleftrightarrow{W} = \vec{\nabla}\vec{V} + \left(\vec{\nabla}\vec{V}\right)^T - \frac{2}{3}\overleftrightarrow{I}\left(\vec{\nabla}\cdot\vec{V}\right)$$

•Can show that the viscosity is given by

$$\langle \mathbf{F}\cdot\nabla\cdot\Pi\rangle = \frac{4.095\,p_i}{\upsilon_{ii}}\left\langle \frac{\mathbf{U}\cdot\nabla B}{B}\frac{\mathbf{F}\cdot\nabla B}{B}\right\rangle, \quad \mathbf{F} = \{\mathbf{B}_T, \mathbf{B}_P, \mathbf{B}\}$$

•Inserting the magnetic field spectrum

$$B(r,\alpha_H,\zeta_H) = B_0(r)\sum_{n,m\neq 0} b_{nm}(r)\cos(n\zeta_H - m\alpha_H)$$

$$\langle \mathbf{F}\cdot\nabla\cdot\Pi\rangle = \kappa\left[U^\zeta\left(\underline{F^\zeta\alpha_T + F^\alpha\alpha_C}\right) + U^\alpha\left(\underline{F^\zeta\alpha_C + F^\alpha\alpha_P}\right)\right],$$

$$\alpha_T = \sum\frac{n^2 b_{nm}^2}{2}, \quad \alpha_C = -\sum\frac{nm b_{nm}^2}{2}, \quad \alpha_P = \sum\frac{m^2 b_{nm}^2}{2}$$

Viscosity Coefficients contain information about magnetic geometry.

Parallel Momentum Balance Used to "Predict" Flow Pattern

- Neglect the external forces and ion neutral friction.

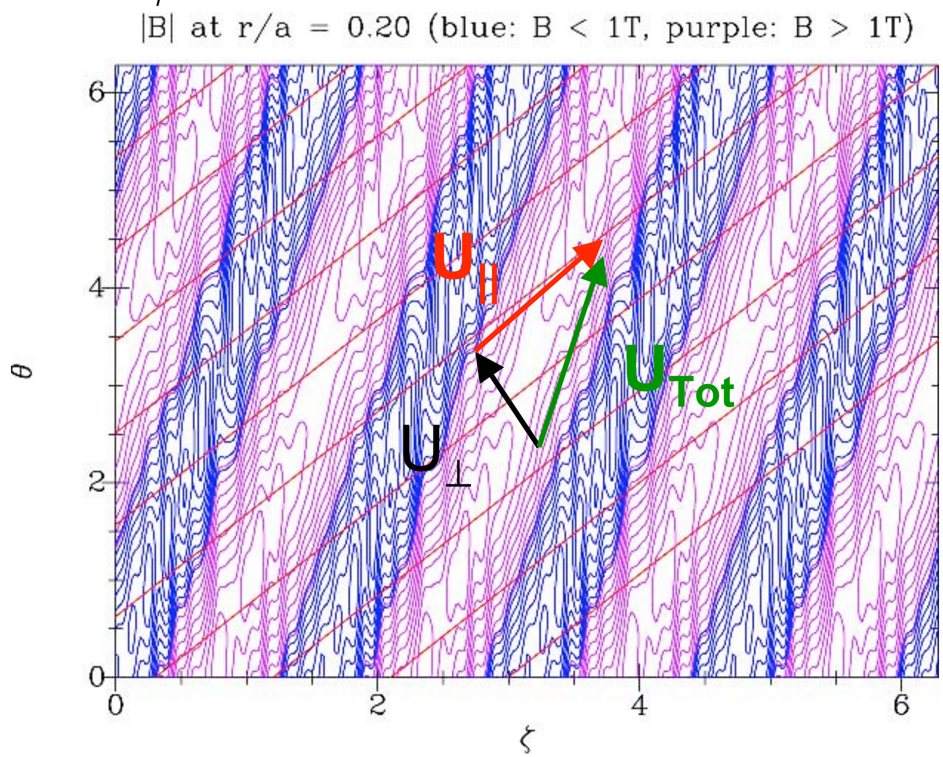
$$\sum_a \langle \mathbf{B} \cdot \nabla \cdot \Pi_a \rangle = 0$$

- Neglect electron viscosity compared to ions, and ignore temperature gradient.

$$\langle \mathbf{B} \cdot \nabla \cdot \Pi_i \rangle = 0$$

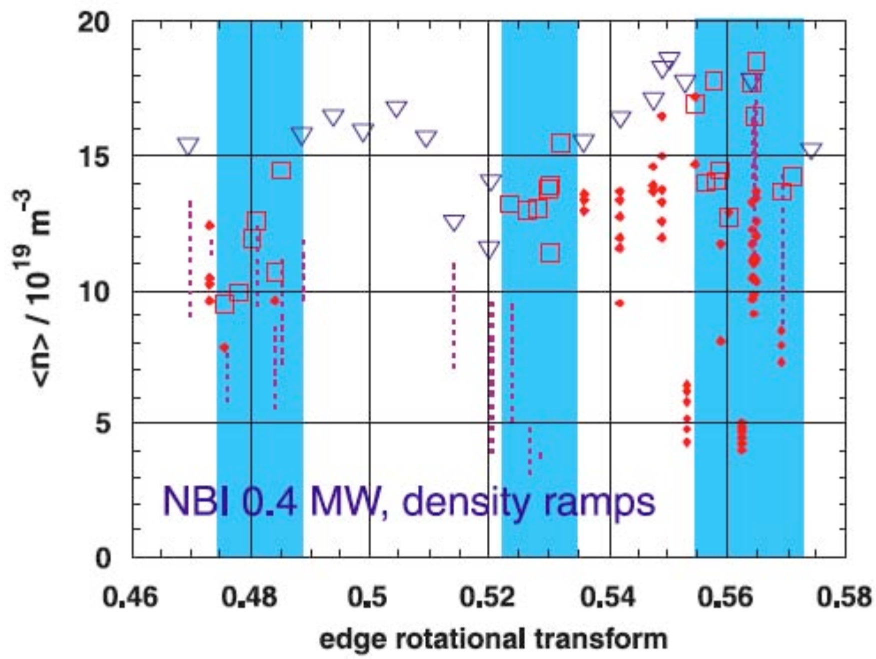
$$\langle \mathbf{B} \cdot \nabla \cdot \Pi_i \rangle = \frac{4.095 p_i}{\upsilon_{ii}} \left\langle \frac{\mathbf{U}_i \cdot \nabla B}{B} \frac{\mathbf{B} \cdot \nabla B}{B} \right\rangle = 0 \quad \Rightarrow \mathbf{U}_i \cdot \nabla B = 0$$

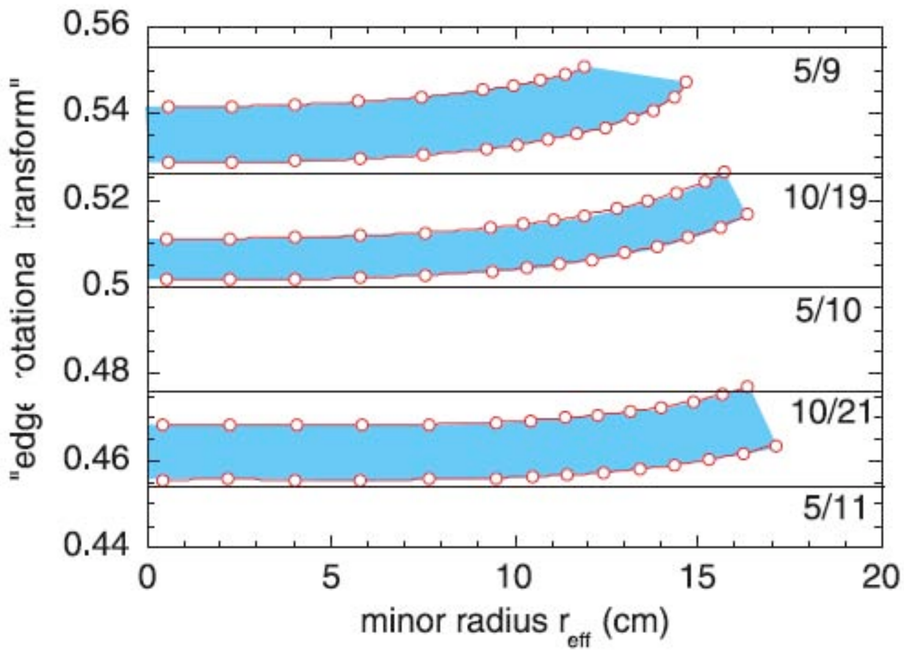
- Contours represent |B|
- Lines indicate field line trajectory.
- This flow pattern minimizes the viscous dissipation.
- Field calculated by D. Spong



K.C. Shaing and J.D. Callen, Phys. Fluids **26**, 3315 (1983)
D. Spong, Submitted to Phys. Plasmas

H-modes Observed in Only Certain Ranges of Rotational Transform in W7-AS



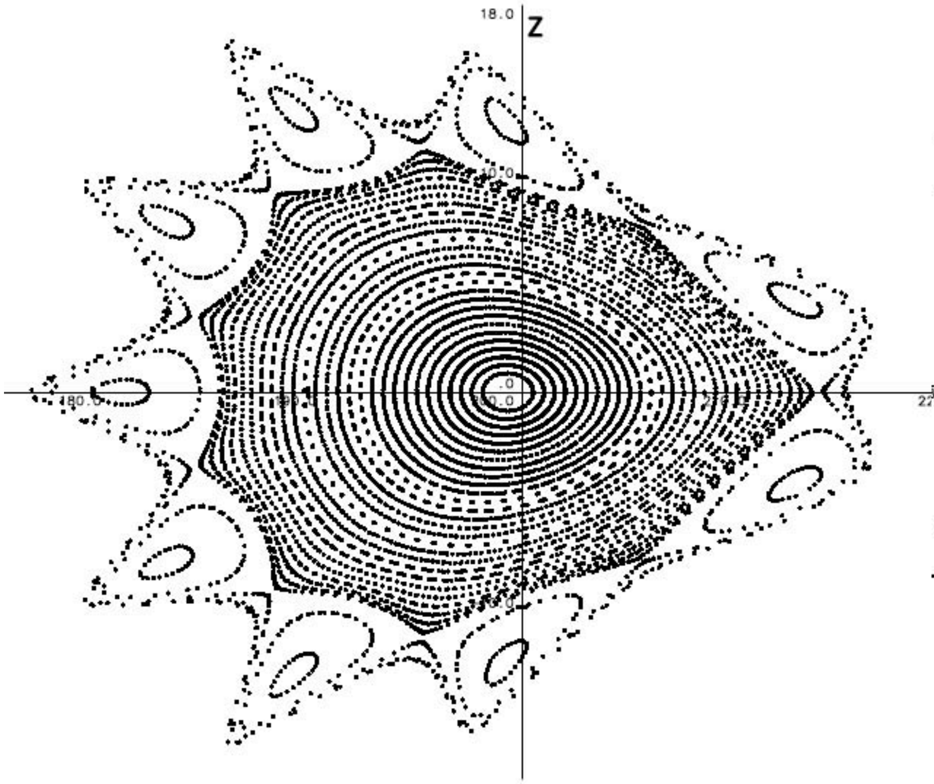


Squares: Density at transition to H-mode
Triangles: Maximum density without H-mode
Points and Broken Lines: Dithers and ELMs

M. Hirsch, et al, Plasma Physics and Controlled Fusion **42**, A231 (2000)

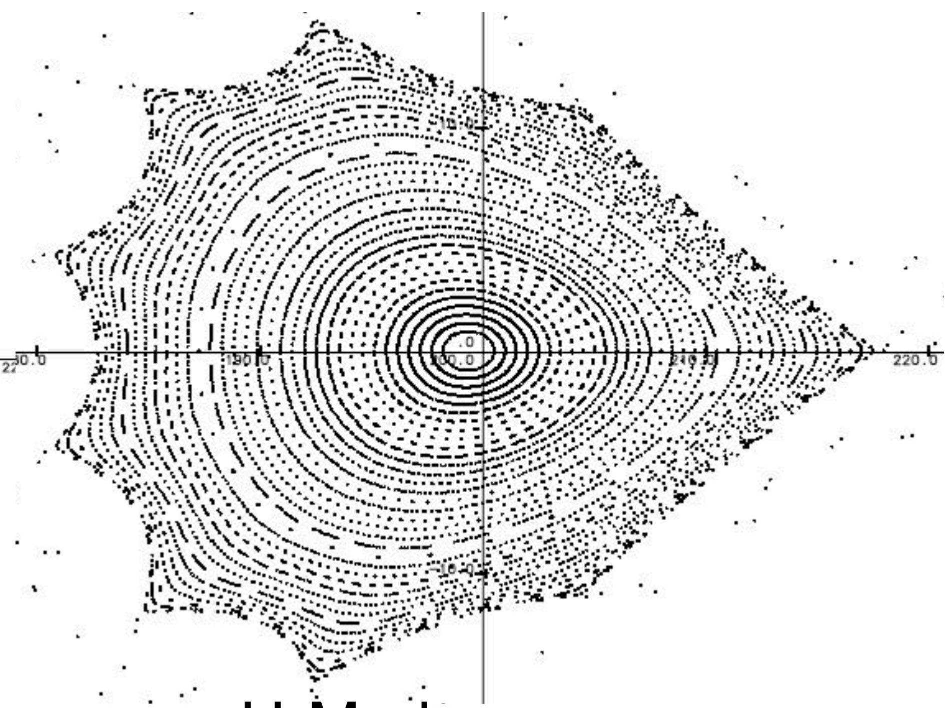
Magnetic Islands Distort Surface Shapes

$\iota/2\pi$=5/9 inside LCMS



No H-Mode

$\iota/2\pi$=5/9 Defines LCMS



H-Mode

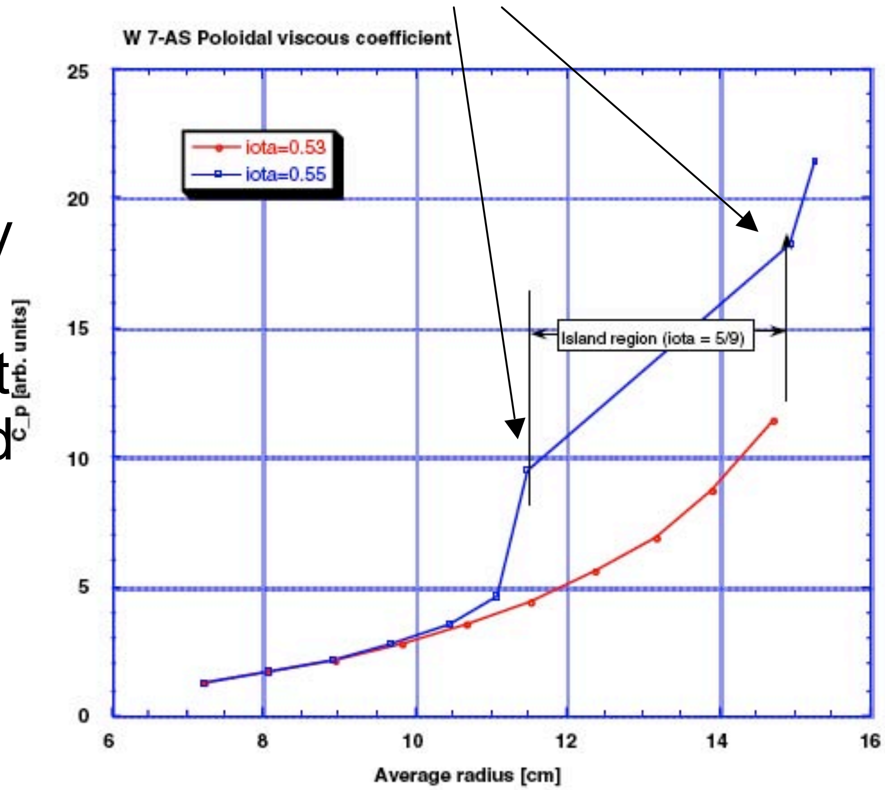
H. Wobig and J. Kisslinger, Plasma. Phys. Control. Fusion **42**, 823 (2000)

Distorted Surface Shape Increases Neoclassical Poloidal Viscous Damping

Quoting Wobig and Kisslinger:

“spin-up of poloidal flow shear and H-mode confinement is inhibited by enhanced poloidal viscosity in the presence of the following islands at ι=10/21, 5/10, 10/19, 5/9, 5/11, and lower. If these islands exist somewhere inside the plasma, H-mode is not possible.”

Enhanced Damping at Edge of Island

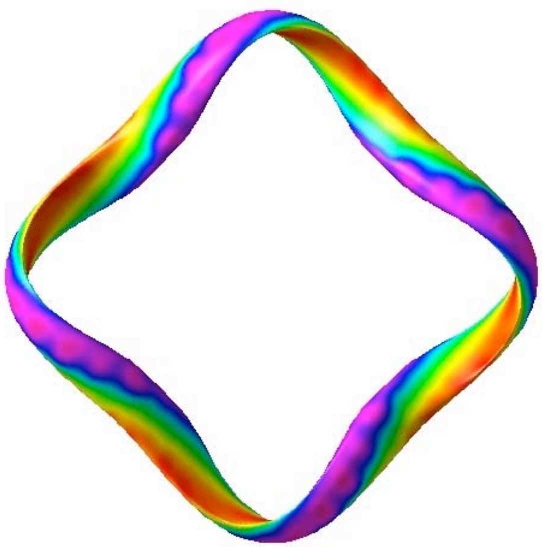


Similar Calculation Performed Configurations with Islands in HSX
S. P. Gerhardt, et al, Phys Plasmas, 2005.

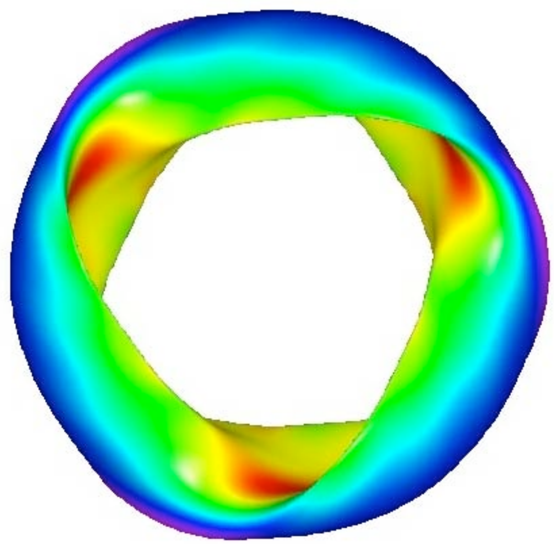
H. Wobig and J. Kisslinger, Plasma. Phys. Control. Fusion **42**, 823 (2000)

New Generation of Stellarators are Designed with Quasisymmetry

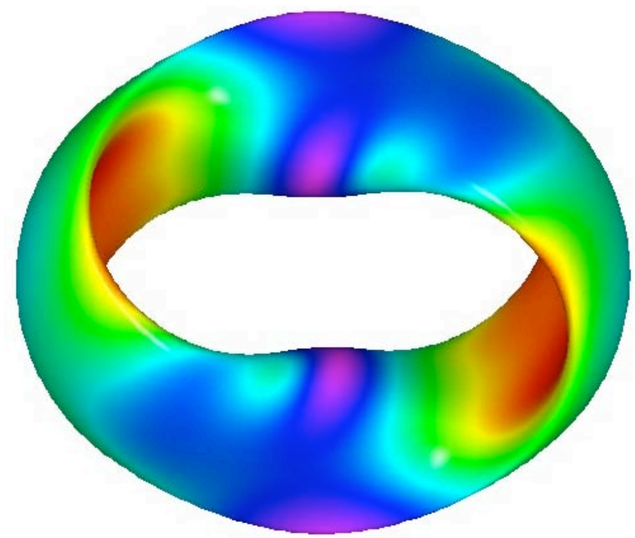
- |B| spectum dominated by a single term
- Systems possess an almost ignorable coordinate, and thus a direction of minimal neoclassical flow damping.



HSX



NCSX



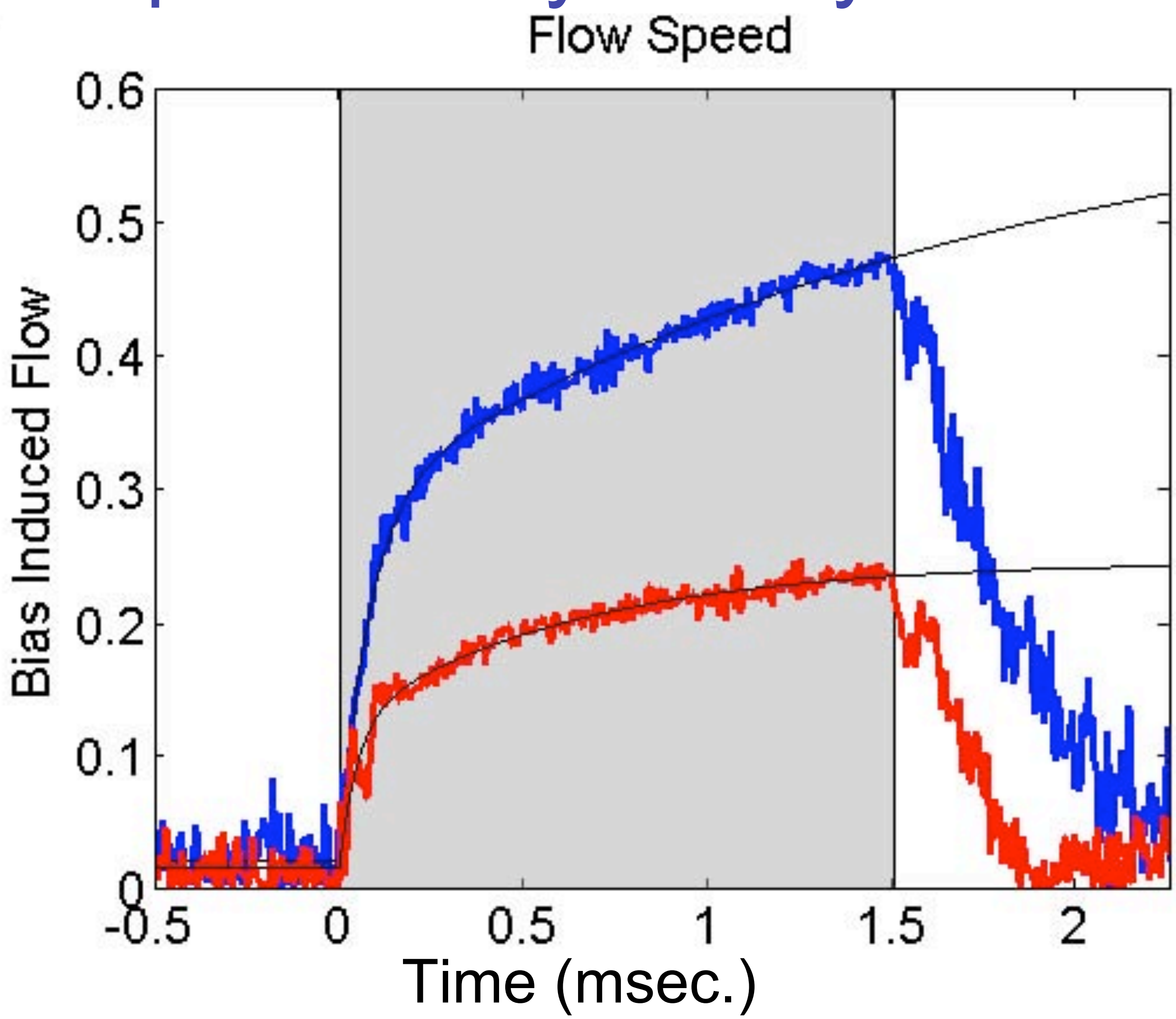
QPS

- Not the only form of transport optimization.
- Not the only step in the stellarator optimization process (surfaces, stability,…)

J. Nuhrenberg and R. Zille, Physics Letters A **114**, 129 (1986)
J. Nuhrenberg and R. Zille, Physics Letters A **119**, 113 (1988)

Images from www.ornl.gov/sci/fed/Theory/stci/stellarator_theory.html

QHS Flows Exhibit Reduced Damping Compared to Symmetry Broken Configuration



QHS: 8 A of electrode current

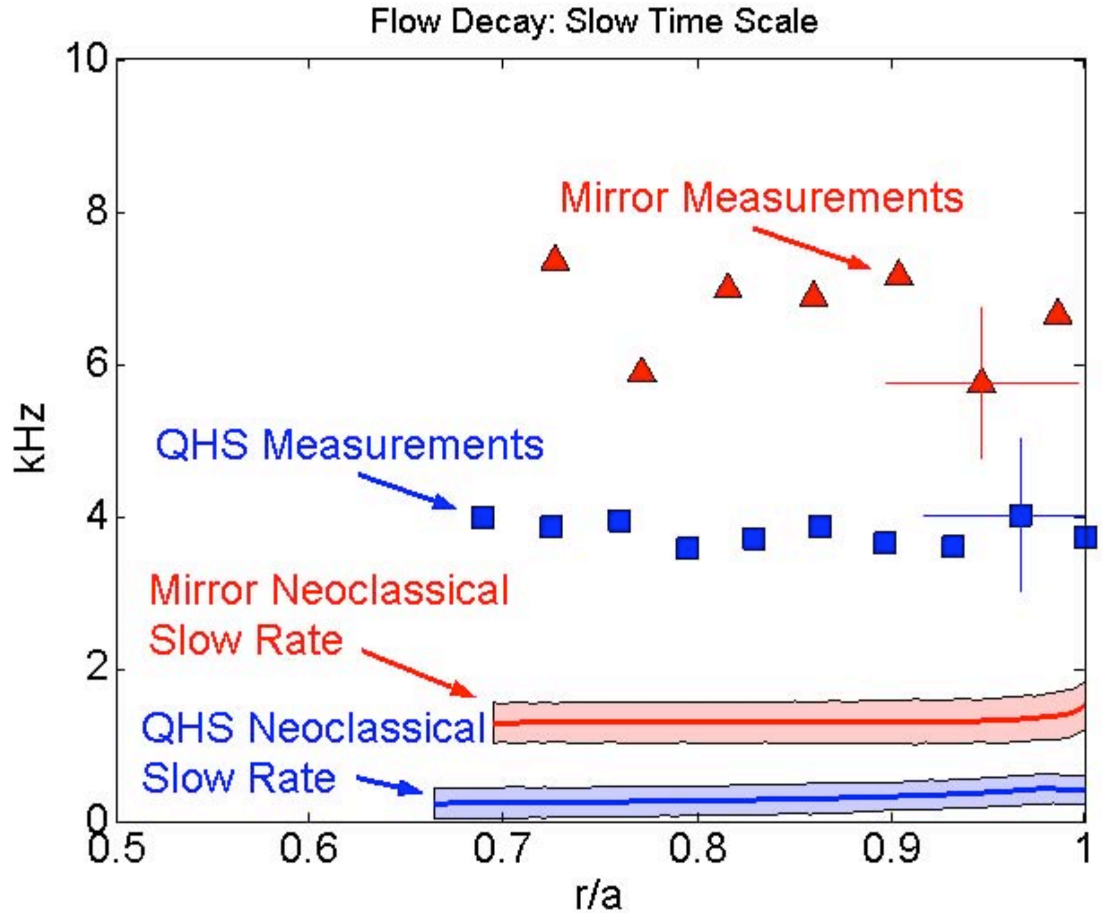
Mirror: 10 A of electrode current

S.P. Gerhardt, et. al, Phys. Rev. Lett. **94**, 015002 (2005)
S.P. Gerhardt, et al, Submitted to Phys. Plasmas.

Measurements and Modeling Show Reduced Damping in QHS Compared to Mirror

- ➢Neoclassical model predicts a much slower decay than the measurements (Factor of 10 in QHS, factor of 3-5 in Mirror).

- ➢ Difference between measurements is comparable to the difference in the model.

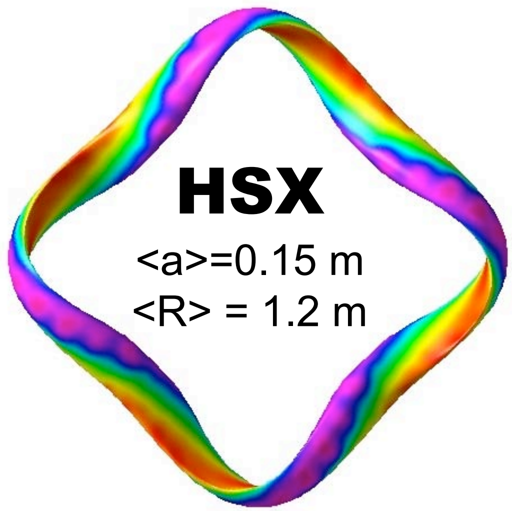


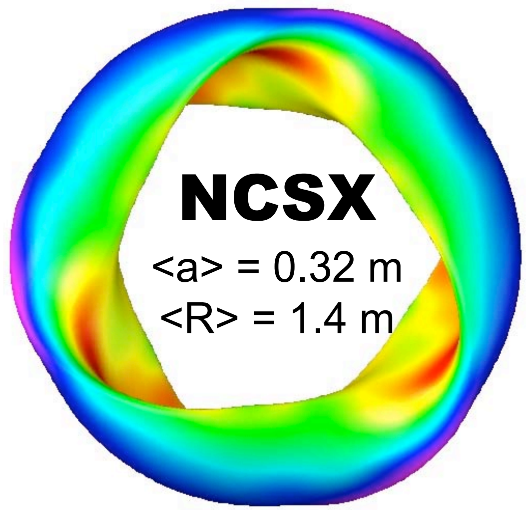
<u>Conclusion/Hypothesis</u>

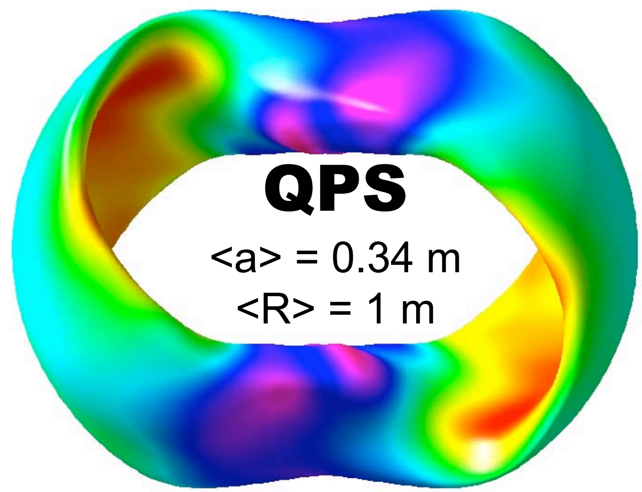
Some non-neoclassical damping mechanism obscures most, but not all, of the difference between the configurations.

S.P. Gerhardt, et. al, Phys. Rev. Lett. **94**, 015002 (2005)

Advances in stellarator optimization have allowed the design of 3D configurations with magnetic structures that approximate: straight helix/tokamak/connected mirrors:

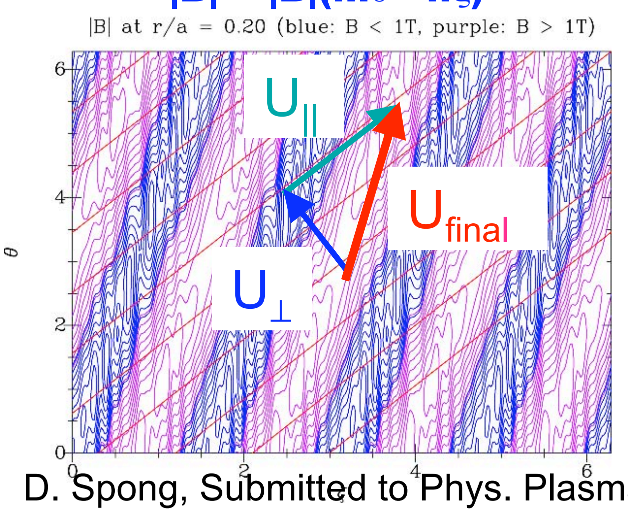






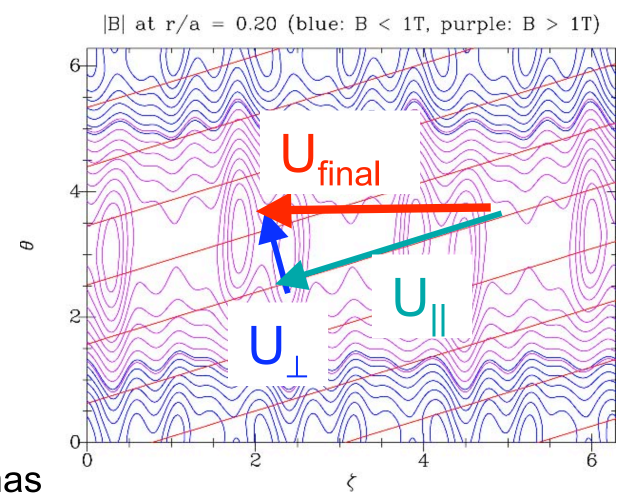
Quasi-helical symmetry

$|B| \sim |B|(m\theta - n\zeta)$



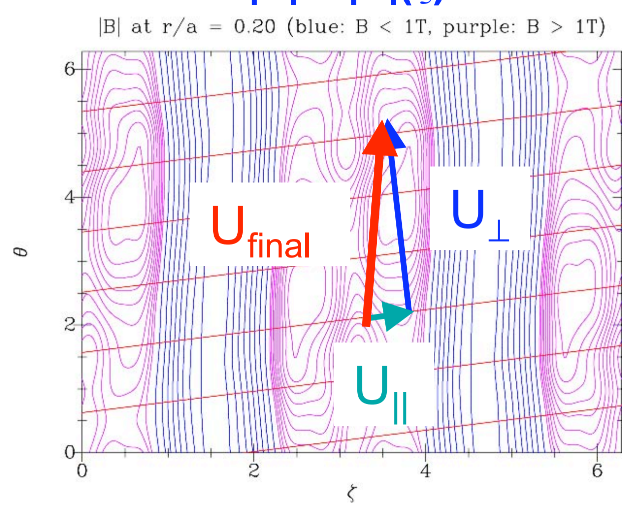
Quasi-toroidal symmetry

$|B| \sim |B|(\theta)$



Quasi-poloidal symmetry

$|B| \sim |B|(\zeta)$



D. Spong, Submitted to Phys. Plasmas

Development of stellarator moments methods

- Viscosities incorporate all needed kinetic information
  - Momentum balance can be invoked at macroscopic level rather than at kinetic level
- Multiple species can be more readily decoupled
  - Facilitates development of self-consistent impurity models
- Recent work has related viscosities to Drift Kinetic Equation Solver (DKES) transport coefficients
  - DKES: D_{11} (diffusion of n,T), D_{13} (bootstrap current), D_{33} (resistivity enhancement)

    [W. I.Van Rij and S. P. Hirshman, *Phys. Fluids B*, **1**, 563 (1989)]
  - Moments method, viscosities related to D_{11} D_{31}, D_{33}

    [H. Sugama, S. Nishimura, Phys. of Plasmas **9** (2002) 4637]

Moments Method Closures for Stellarators

The parallel viscous stresses, particle and heat flows are treated as fluxes conjugate to the forces of parallel momentum, parallel heat flow, and gradients of density, temperature and potential:

$$\begin{bmatrix} \left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Pi}_a\right)\right\rangle \\ \left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Theta}_a\right)\right\rangle \\ \Gamma_a \\ Q_a / T_a \end{bmatrix} = \begin{bmatrix} M_{a1} & M_{a2} & N_{a1} & N_{a2} \\ M_{a2} & M_{a3} & N_{a2} & N_{a3} \\ N_{a1} & N_{a2} & L_{a1} & L_{a2} \\ N_{a2} & N_{a3} & L_{a2} & L_{a3} \end{bmatrix} \begin{bmatrix} \left\langle u_{\parallel a} B\right\rangle / \left\langle B^2 \right\rangle \\ \dfrac{2}{5 p_a}\left\langle q_{\parallel a} B\right\rangle / \left\langle B^2 \right\rangle \\ -\dfrac{1}{n_a}\dfrac{\partial p_a}{\partial s} - e_a \dfrac{\partial \Phi}{\partial s} \\ -\dfrac{\partial T_a}{\partial s} \end{bmatrix}$$

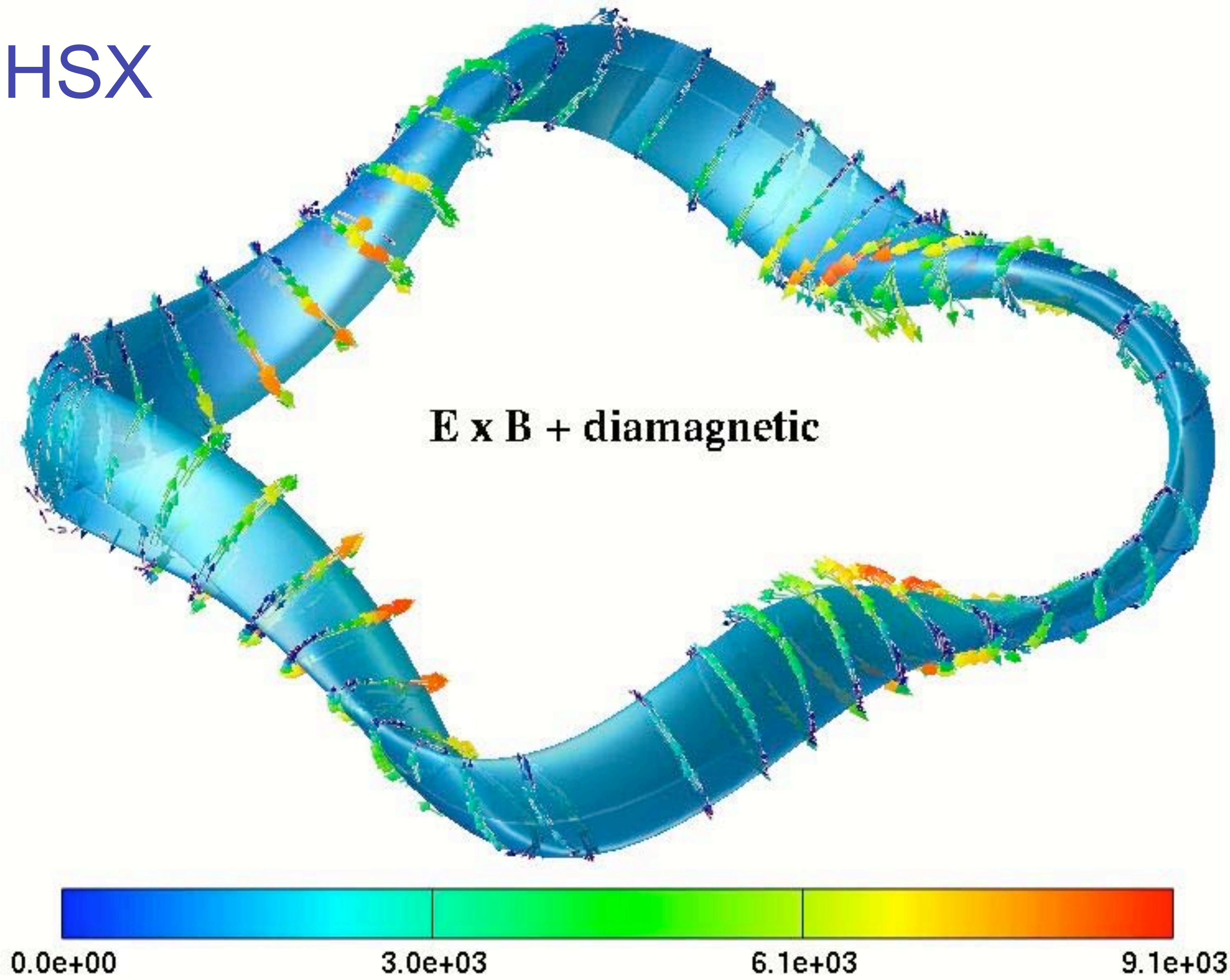
- Analysis of Sugama and Nishimura related monoenergetic forms of the M, N, L viscosity coefficients to DKES transport coefficients
- Combining the above relation with the parallel momentum balances and friction-flow relations

$$\left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Pi}_a\right)\right\rangle - n_a e_a \left\langle B E_\parallel \right\rangle = \left\langle B F_{\parallel a1} \right\rangle$$

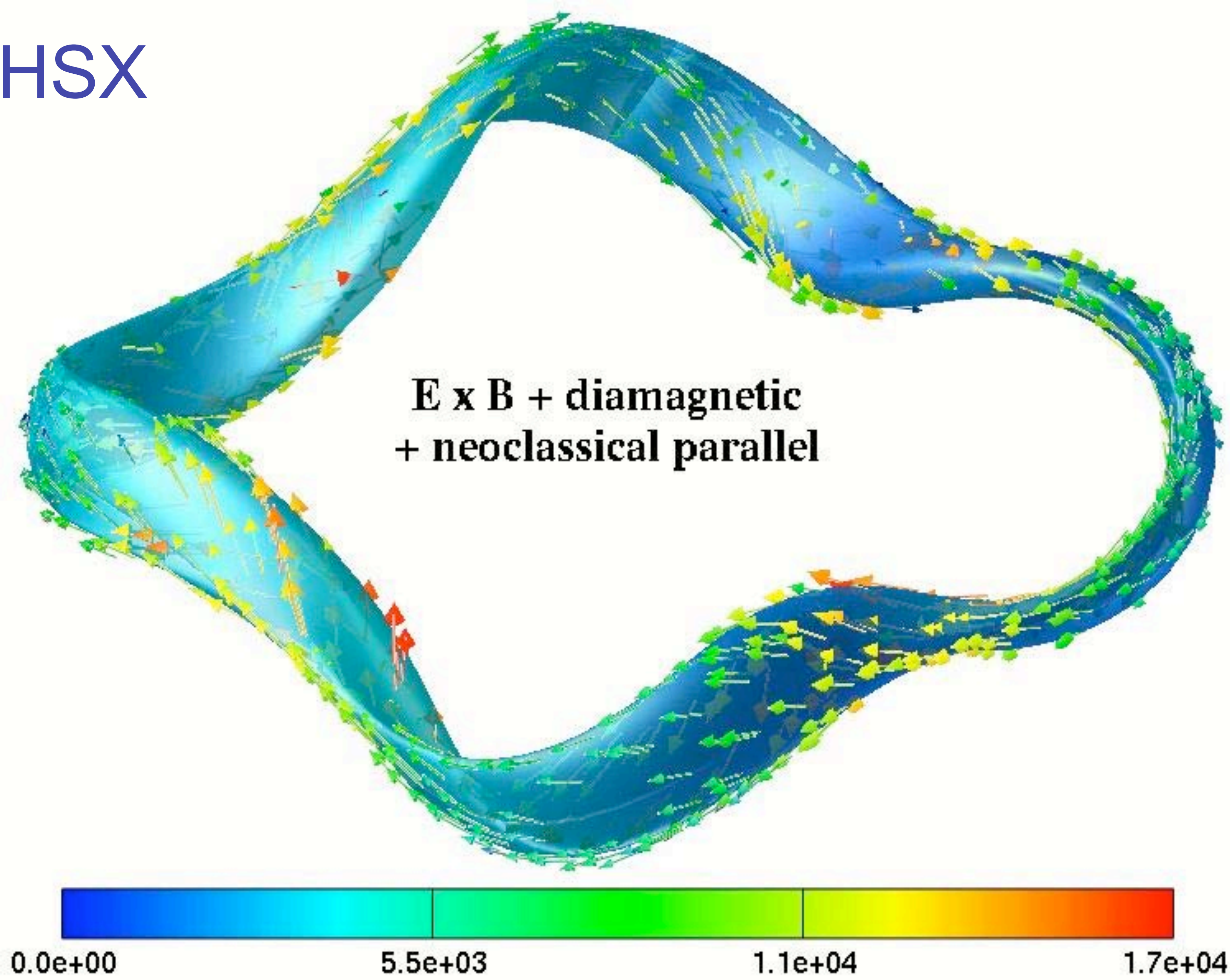
$$\left\langle \vec{B}\cdot\left(\vec{\nabla}\cdot\overleftrightarrow{\Theta}_a\right)\right\rangle = \left\langle B F_{\parallel a2} \right\rangle$$

$$\begin{bmatrix} \left\langle B F_{\parallel a1} \right\rangle \\ \left\langle B F_{\parallel a2} \right\rangle \end{bmatrix} = \sum_b \begin{bmatrix} l_{11}^{ab} & -l_{12}^{ab} \\ -l_{12}^{ab} & l_{22}^{ab} \end{bmatrix} \begin{bmatrix} \left\langle B u_{\parallel b} \right\rangle \\ \dfrac{2}{5 p_b}\left\langle B q_{\parallel b} \right\rangle \end{bmatrix}$$

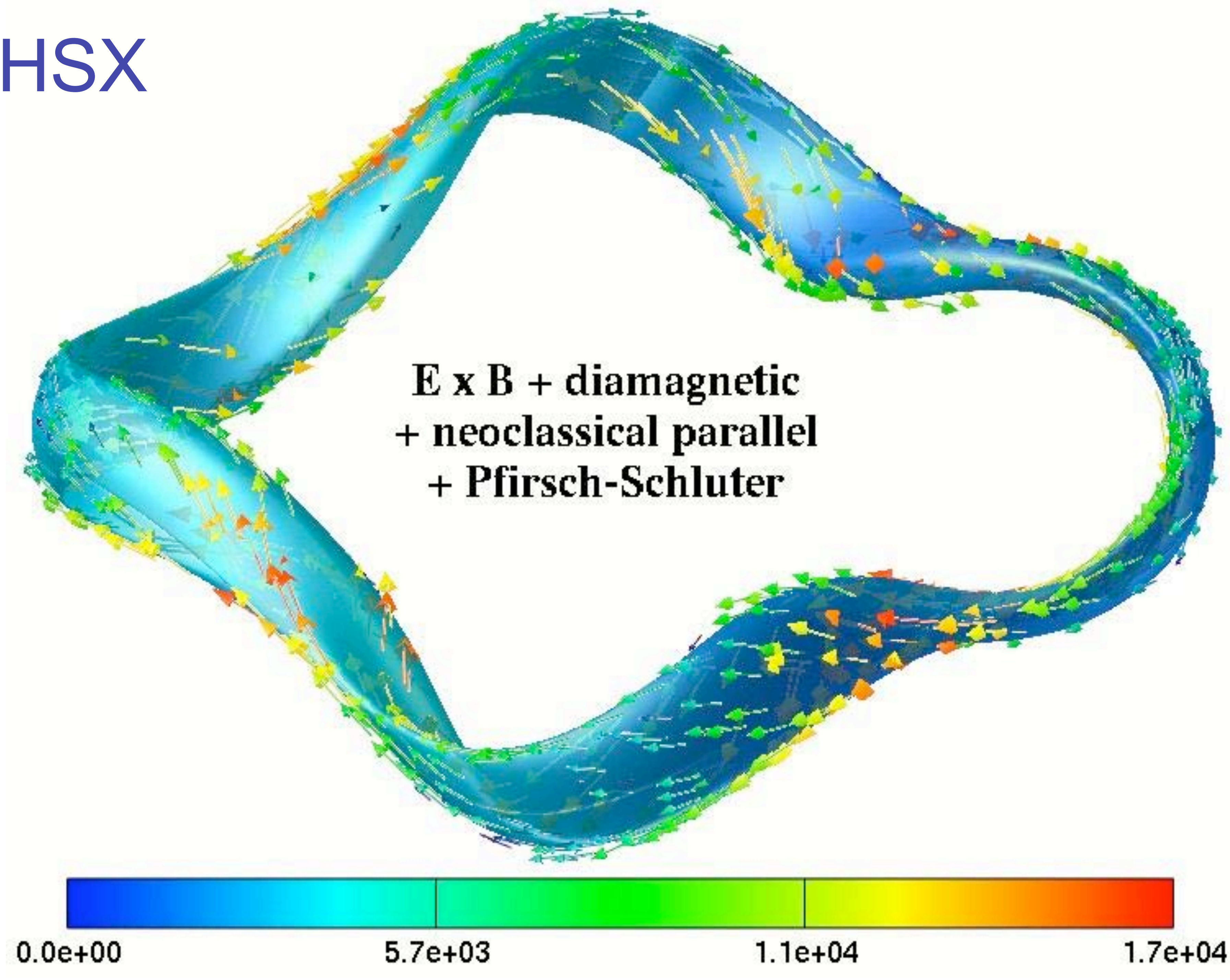
Leads to coupled equations that can be solved for $\langle u_{\parallel a} B\rangle$, $\langle q_{\parallel a} B\rangle$, Γ_a, Q_a


HSX
E x B + diamagnetic
0.0e+00
3.0e+03
6.1e+03
9.1e+03

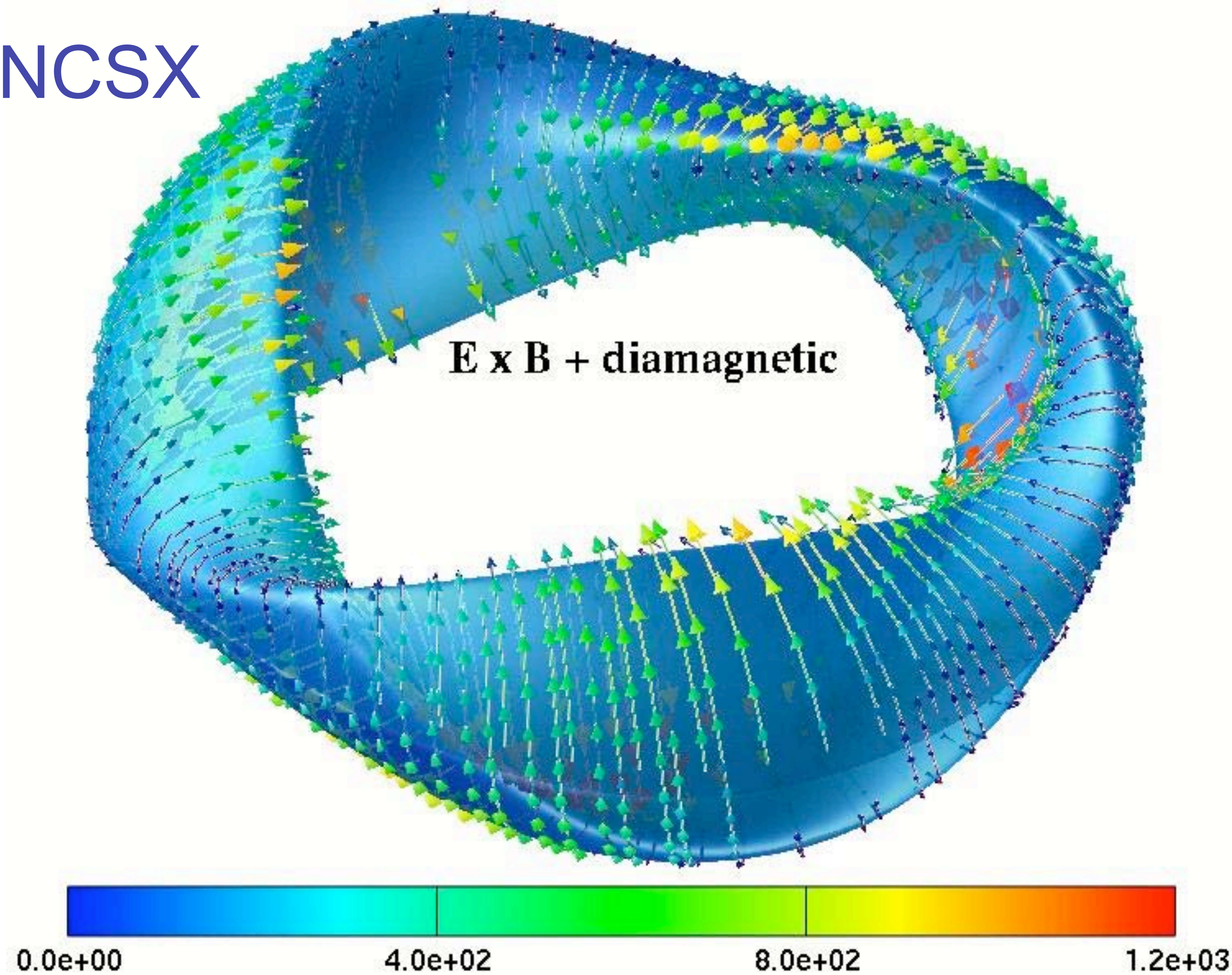
HSX

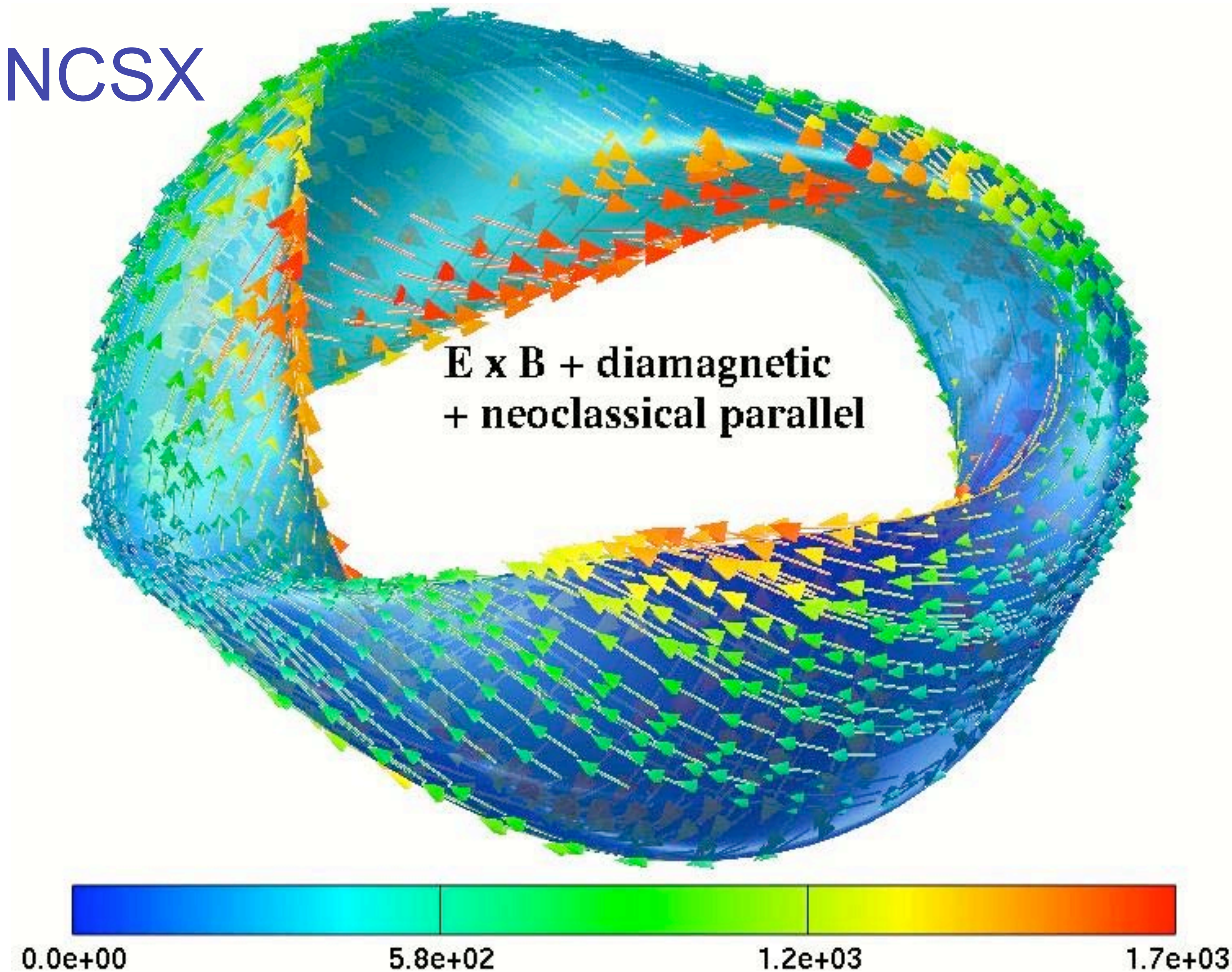

E x B + diamagnetic
+ neoclassical parallel
0.0e+00
5.5e+03
1.1e+04
1.7e+04

HSX

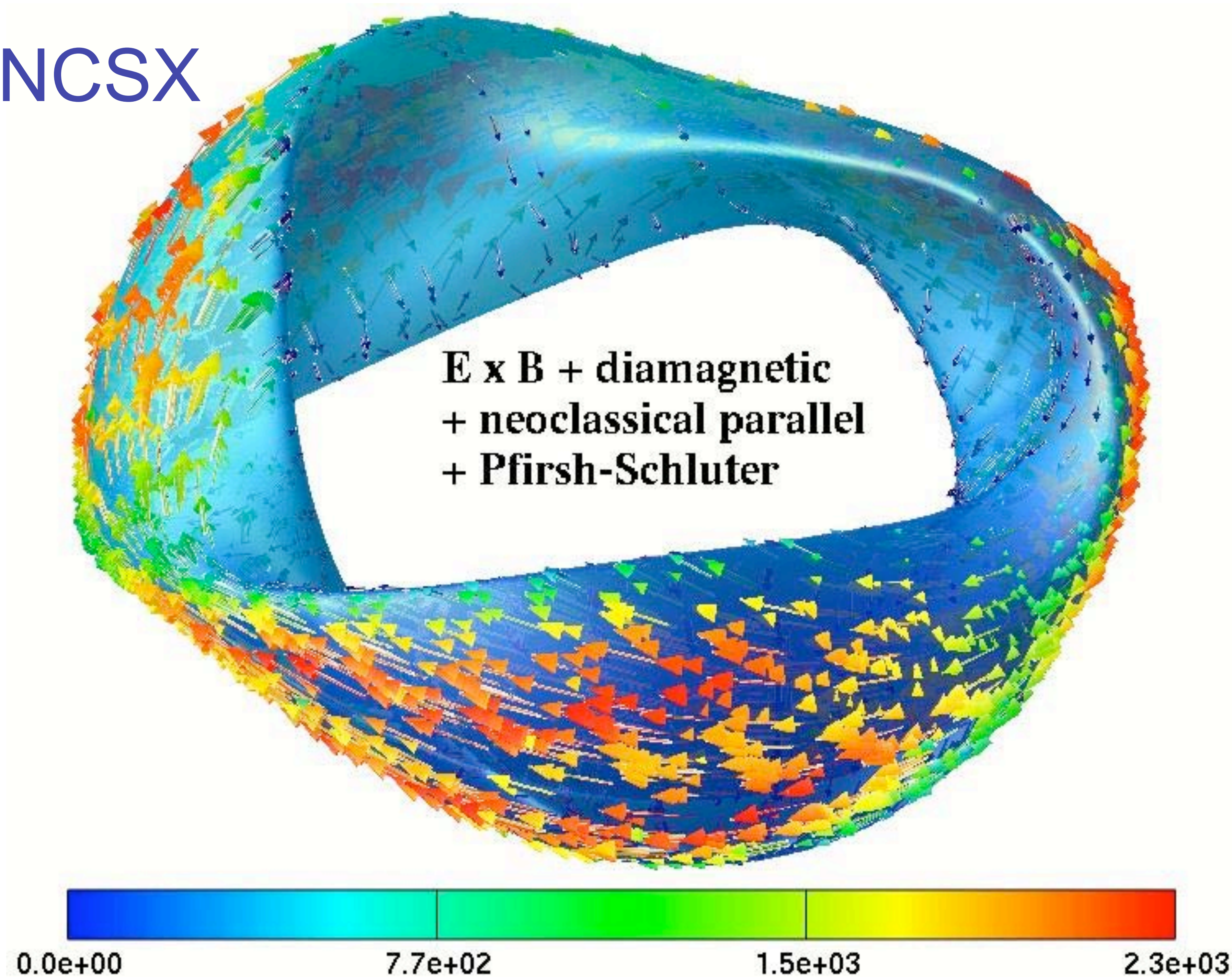

E x B + diamagnetic
+ neoclassical parallel
+ Pfirsch-Schluter
0.0e+00
5.7e+03
1.1e+04
1.7e+04

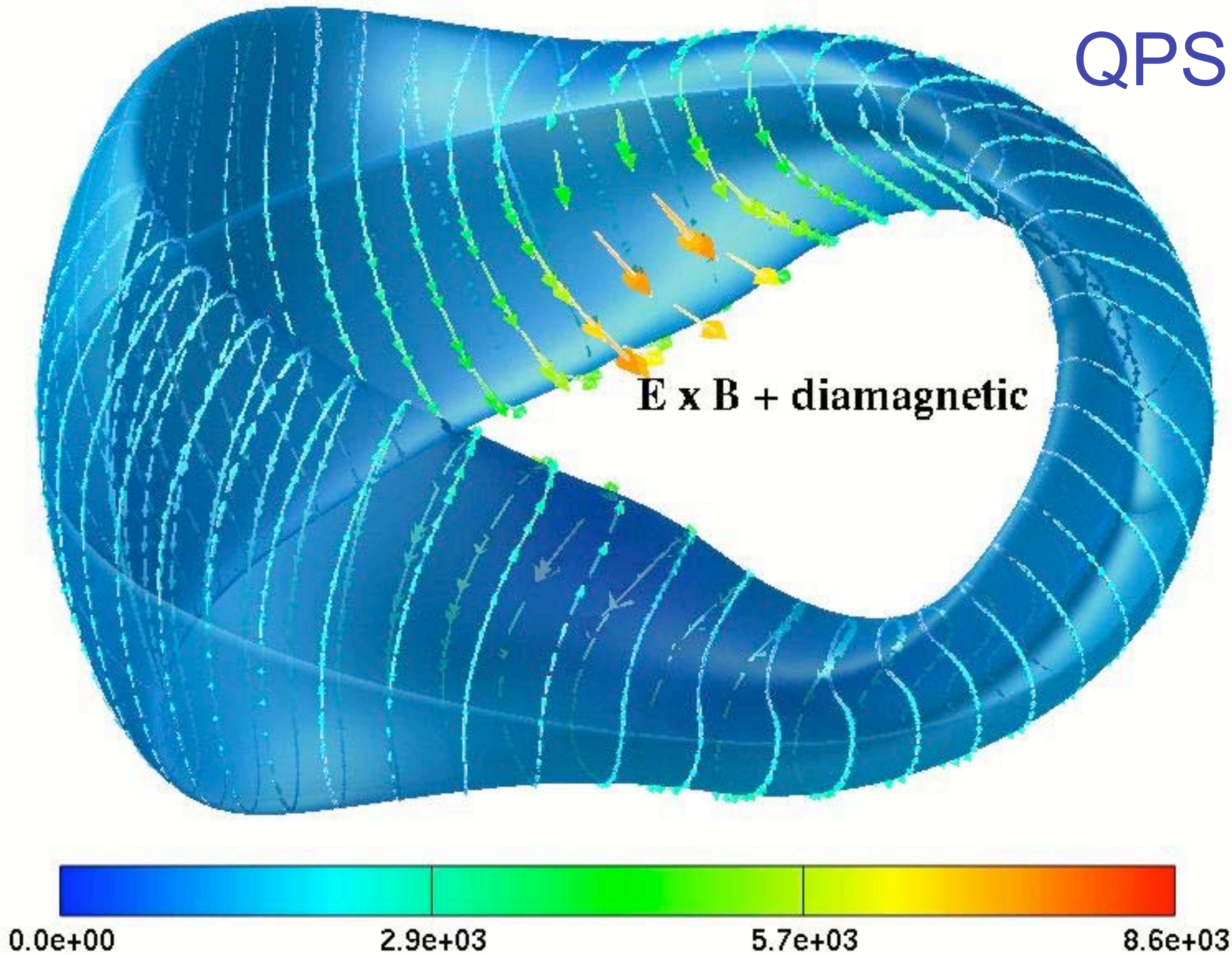
NCSX

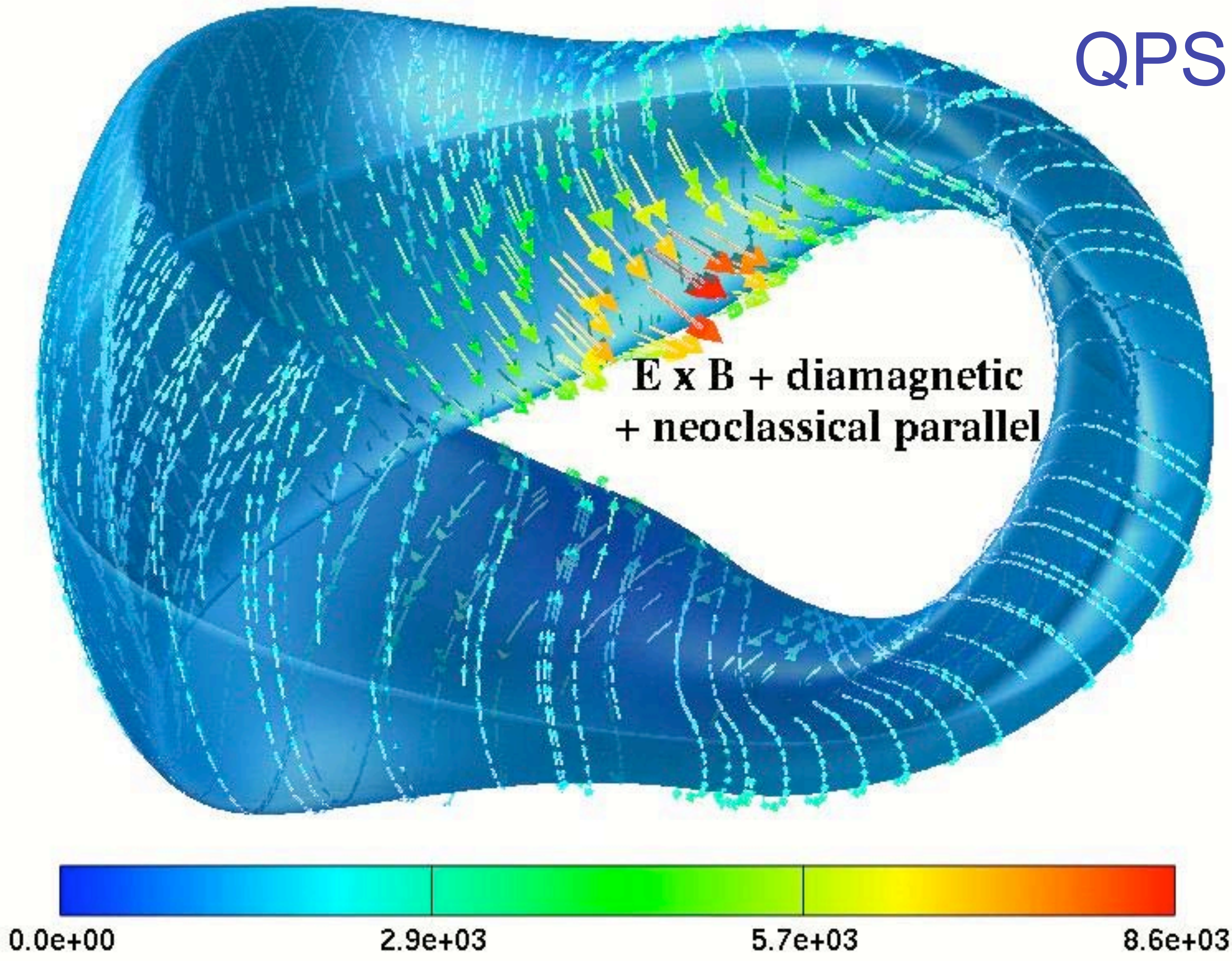


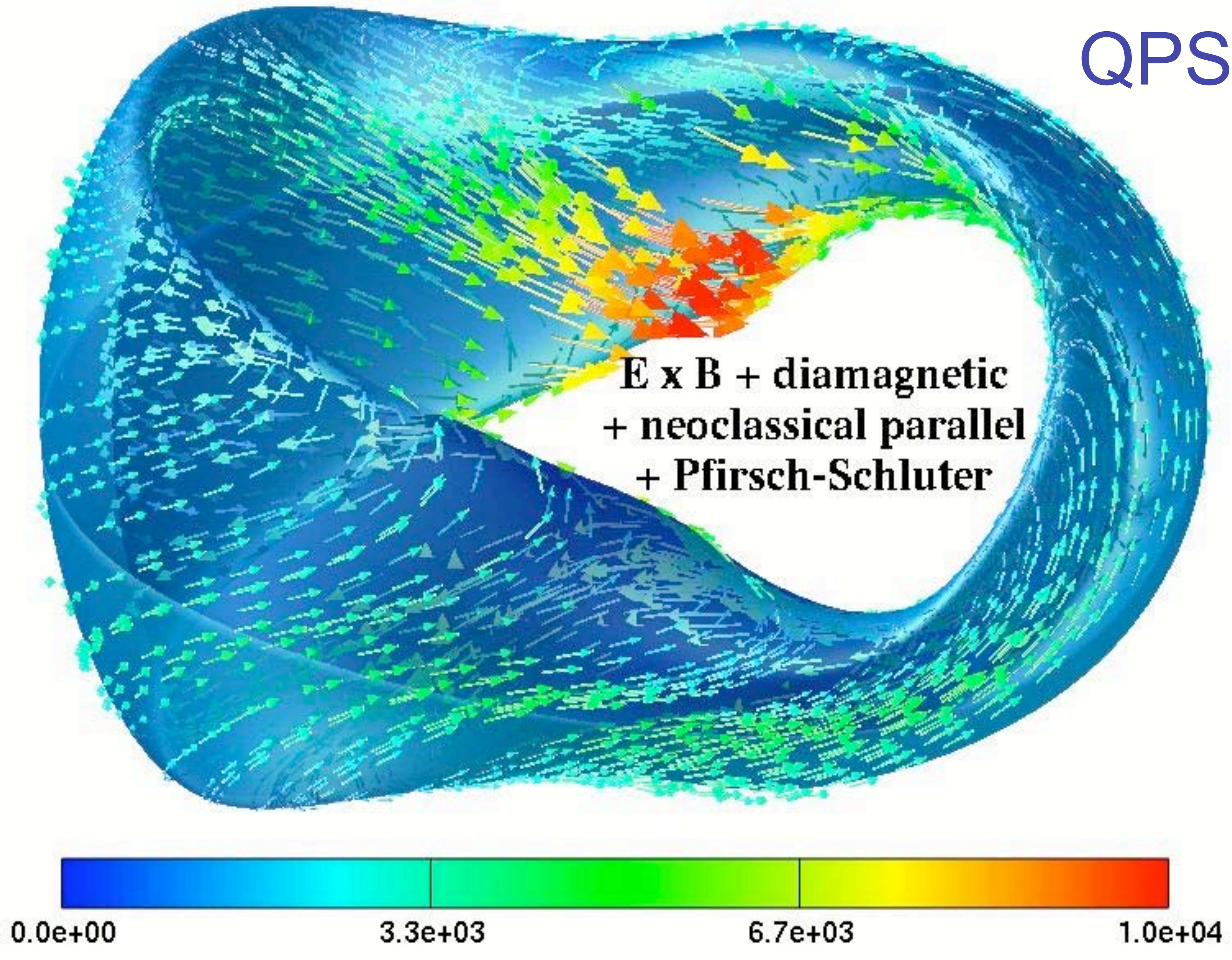

NCSX
E x B + diamagnetic
+ neoclassical parallel
0.0e+00
5.8e+02
1.2e+03
1.7e+03

NCSX


E x B + diamagnetic
+ neoclassical parallel
+ Pfirsh-Schluter
0.0e+00
7.7e+02
1.5e+03
2.3e+03


QPS
E x B + diamagnetic
0.0e+00
2.9e+03
5.7e+03
8.6e+03


QPS
E x B + diamagnetic
+ neoclassical parallel
0.0e+00
2.9e+03
5.7e+03
8.6e+03


QPS
E x B + diamagnetic
+ neoclassical parallel
+ Pfirsch-Schluter
0.0e+00
3.3e+03
6.7e+03
1.0e+04

Summary

- Both neoclassical and anomalous flow damping are important in stellarators.
- The neoclassical flow damping can be reduced to the level where anomalous damping dominates.
- Two components of the momentum balance equation must be solved to yield the stellarator flows.
  - Toroidal balance leads to the ambipolarity constraint→electric field.
  - Parallel momentum balance specifies the neoclassical parallel flows.
- Underlying flow pattern reflect the symmetry of the device.