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FREE-BOUNDARY FULL-PRESSURE ISLAND HEALING IN A STELLARATOR : COIL-HEALING.

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Abstract

A procedure for modifying stellarator coil designs to eliminate magnetic islands in free-boundary full-pressure magneto-hydro-dynamic (MHD) equilibria is presented. Islands may be removed by making small changes to the coil geometry and also by variation of trim coil currents. Configurations and a coil design relevant to the National Compact Stellarator Experiment (NCSX) is used to illustrate the technique.

I Introduction

The NCSX design adopted the ‘reverse-engineering’ technique [1]. The plasma boundary is designed to achieve desired physics properties. Subsequently, coils are designed to produce the optimal boundary. This process does not guarantee good flux surfaces in the plasma interior, and further adjustment of the coil design is needed. This paper describes a procedure that adjusts the coil shapes to produce good flux surfaces.

The NCSX plasma design study considered compact stellarator configurations with good transport and stability properties [2]. Quasi-axi-symmetry is used to obtain good drift trajectories which in turn provide good transport properties. Good ballooning stability is produced by imposing a strong component of axi-symmetric shaping, with advanced tokamak designs used as a guide, and the rotational-transform profile is constrained to be monontonically increasing for neo-classical tearing stability. Kink stability is produced by a combination of shear and a stabilizing three-dimensional shaping of the boundary. The three-dimensional shaping is determined using a Levenberg-Marquardt optimizer which incorporates the various physics requirements, doing so by adjusting the non-axi-symmetric components of the boundary to produce the desired rotational-transform profile, to ensure kink and ballooning stability, and to optimize quasi-axisymmetry.

Two different families of quasi-axi-symmetric configurations with attractive stability and transport properties have been found. One has small externally generated transform on axis and a large externally generated shear. This type of configuration requires an externally driven seed current on axis. The other has a substantial externally generated transform on axis, allowing it to have a fully bootstrap-consistent current profile, with the current density going to zero on axis. The larger externally generated transform in the interior allows the vaccuum field to have more favorable magnetic well properties, and PIES calculations indicate improved flux surface quality. The NCSX reference plasma configuration, named li383, is a 3 field period configuration of the second type. The nominal design β

is 4% and the rotational-transform profile is shown in Fig. 1.

After a satisfactory boundary is determined, a set of coils that match the boundary, and satisfy certain engineering constraints, is designed. To correct potential construction errors and to assist elimination of dangerous resonances, trim coils are included in the design.

The three-dimensional nature of stellarators guarantees that magnetic islands will in general exist. Sufficiently large magnetic islands will result in loss of confinement. 'Island-healing' techniques have been applied to stellarator vacuum fields [3]. To heal finite-β configurations a method of computing full-pressure free-boundary equilibria is required. The MHD equilibria used in the optimization process are computed using the VMEC code [4] which assumes that nested magnetic surfaces exist everywhere; therefore, additional analysis is required to address the existence of magnetic islands and such analysis is performed with the PIES code [5]. An earlier paper [6] considered the island content of fixed-boundary li383 equilibrium and presented a method to eliminate magnetic islands by making small changes to the boundary. This article extends the analysis to free-boundary equilibria and makes small variations to the coil geometry to remove magnetic islands in full-pressure equilibria. Additionally, trim coil currents are varied to heal an intermediate plasma state.

In Sec. II, the method by which coil sets are derived and the iterative procedure of PIES code is described. The island elimination procedure and results for a NCSX relevant coil set are presented in Sec. III, and in Sec. IV an extension of the method is used to determine optimal trim-coil currents for healing intermediate plasma states.

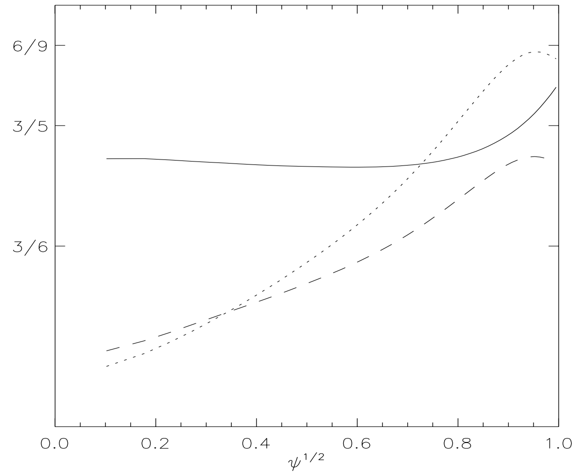
II COILOPT and PIES

The coils are designed to minimize the magnetic field normal to the reference plasma surface subject to various constraints on the coil lengths, minimum coil radius of curvature, coil separation, current density, and engineering access. To do this, the COILOPT code [7] uses a parametric representation of the coils placed on a winding surface $R = \sum_i R_i \cos(m_i\theta + n_i N\phi), Z = \sum_i Z_i \sin(m_i\theta + n_i N\phi)$, with each coil having a toroidal variation

$$\phi_i = \sum_k \left[\phi_{i,k,c}\cos(k\theta') + \phi_{i,k,s}\sin(k\theta')\right], \tag{1}$$

and $\theta' = \theta + \sum_j \theta'_j \sin(j\theta)$. A coil set with 7 coils per period, named 0907, is derived and used in the island elimination procedure below.

Figure 1: Rotational-transform of vacuum (solid), $\beta \sim 3\%$ case (dashed) and li383 (dotted) plotted against the square root of toroidal flux.



To calculate free-boundary equilibria for a given coil set, the PIES fixed-boundary solver has been combined with the NESTOR[8] vacuum code to create a free-boundary finite-pressure MHD equilibrium solver for general stellarator magnetic fields. The fixed-boundary and vacuum solutions are interfaced on a boundary outside the plasma with the requirement that the fields be continuous. PIES iteratively finds solutions to $\nabla p = \mathbf{J} \times \mathbf{B}$, starting from a VMEC initialization.

Calculating the equilibrium consistent with the 0907 coils shows significant island chains Fig. 2. In this plot, about 100 iterations have been performed; as the PIES iterations continue, this equilibrium further degrades. In all other Poincaré plots shown, PIES has been iterated to convergence which typically requires 300-400 iterations. The convergence properties of the free-boundary PIES calculation depend on the pressure and current profiles, the coil field, and the field initialization. If the coil field matches a boundary that is consistent for an equilibrium without islands, then PIES will rapidly converge if it is initialized by that fixed boundary VMEC equilibrium.

The dangerous islands for li383 are the $(3,6)$ and $(3,5)$. These may be removed by making small variations to the coil geometry as described in the following section.

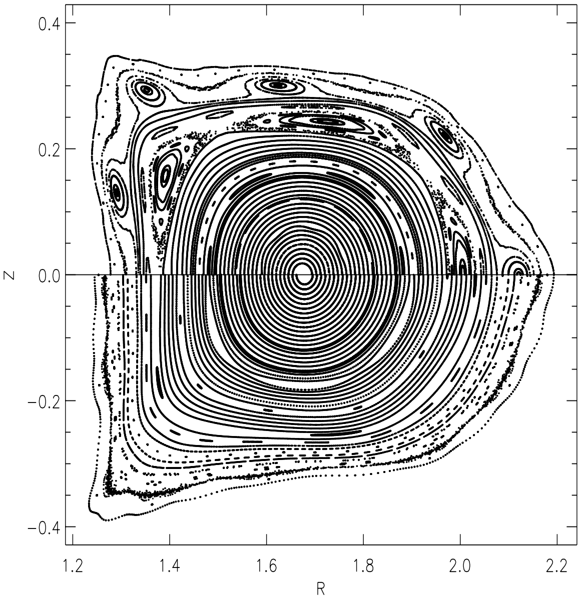
III Island healing.

Magnetic islands are caused by resonant radial magnetic fields where the rotational-transform is rational. The resonant fields, $\mathbf{B}$, at selected rational surfaces are considered a vector function of the coil geometry parameters, $\mathbf{r}$, and related via a coupling matrix $\mathbf{C}$

$$\mathbf{B}(\mathbf{r}_0+\delta\mathbf{r}) = \mathbf{B}(\mathbf{r}_0) + \mathbf{C}\cdot\delta\mathbf{r} + \ldots \qquad (2)$$

The coupling matrix is simply the matrix of first partial derivatives (computed numerically) of the resonant fields, which are calculated via the construction of quadratic-flux minimizing surfaces [3]. The singular value representation $\mathbf{C}=\mathbf{U}\mathbf{w}\mathbf{V}^T$ enables $\mathbf{C}$ to be inverted and an iterative Newton procedure will find the parameter set eliminating resonances : $\delta\mathbf{r}_{i+1} = -\mathbf{V}\mathbf{w}^{-1}\mathbf{U}^T\mathbf{B}_i$. Ideally, each PIES calculation would be iterated to convergence, but this requires excessive computational time and a fixed number, N, of iterations is performed. Applying the method to the coil set 0907, and referring to Eqn(1), the set $\{\phi_{i,k,c}, \phi_{i,k,s} : \forall i; k=5,6,10\}$ is chosen to the independent variables set. This amounts to 21 independent variables. The resonant fields $\mathbf{B}_{3,5}, \mathbf{B}_{3,6}$ are selected. Even though Fig. 2 shows the $(3,6)$ island to be small, if it is not included in the resonance elimination, the changes made to the coils may cause this island to grow. In addition, a parameter, $\delta=\sum_i(\rho_i-\rho_0)^2$, which represents the variation of the radial coordinate, ρ_i, along a magnetic field line from its starting location, ρ_0, is included to minimize the distortion of the edge with respect to the reference boundary. Note that this parameter is strictly non-negative. Its inclusion complicates the Newton procedure as the minimum of δ must in general be located. Care must be taken to update the coupling matrix as the iterations proceed, achieved using Broyden's method, and small singular values must be deleted. The resonant fields and δ are scaled by their initial values and PIES is terminated after $N=20$ iterations. The following reduction is observed, where δr is the magnitude of the total change made to the independent variables.

Figure 2: The li383 equilibrium with the original 0907 coils (above) and with the 'healed' 0907h coils (below).



	δ	$B_{3,5}$	$B_{3,6}$	δr
0	1.0000	1.00000	1.00000	0.00000
1	0.3067	0.09258	−1.10797	0.00603
2	0.2779	0.04495	0.01287	0.00649
3	0.2481	0.00060	−0.05112	0.00748

Subsequent iterations fail to reduce the magnitude of the target vector; nevertheless, the reduction is satisfactory. Using the 'healed' coil set, 0907h, and iterating PIES to convergence, an equilibrium with greatly improved flux surface is obtained Fig. 2. The total change made to the coils in real space is about 1.7cm.

IV Trim Coils

The island-elimination method described above does not guarantee elimination of islands at plasma states connecting the vacuum to the full-pressure state. For this, trim coils, designed to couple to selected resonances on interior plasma surfaces, are used. Sets of 4 m=5 coils and 4 m=6 coils are designed to provide effective control of the m=5 and m=6 resonances.

A plasma state intermediate to the vacuum and full-pressure state, with 3% β and rotational-transform profile shown in Fig. 1, is considered. A large $(3, 6)$ island exists Fig. 3. Considering now the coil geometry fixed and varying the 4 m=6 trim coil currents, the currents required to cancel the $(3, 6)$ and $(6, 12)$ resonant fields are iteratively determined as

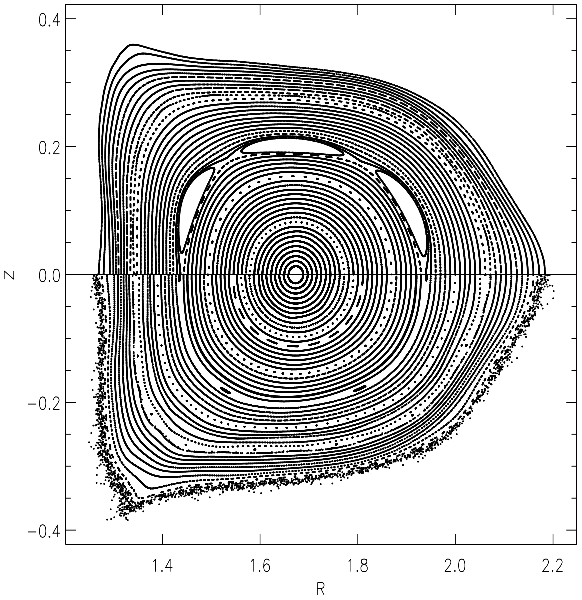
	$B_{3,6}$	$B_{6,12}$	I
0	-0.99910	0.04249	0
1	-0.01181	0.00077	3606
2	-0.00019	0.00001	3651
3	0.00000	0.00000	3652

The resonant fields are normalized so initially the squares summed is equal to unity, and I is the magnitude of the total current vector in the m=6 trim coils. The resonant fields are reduced by 10^8! The island content in the converged equilibrium is very small Fig. 3. The trim coil currents required are $2.4kA, -1.5kA, 2.3kA$ and $-34A$.

V Comments

A practical method to design and ‘heal’ coils has been presented. Islands have been dramatically reduced in size. Since this work was performed, improved coil sets have been developed to which the coil-healing method has been applied, but with mixed success. Reduction of resonances is typically achieved at N iterations; however, there is no guarantee that PIES has converged at this point nor that the configuration remains ‘healed’ as the iterations continue. Nevertheless, healed coil sets have been constructed for the full pressure, intermediate pressure and vacuum (not shown) and work on this topic is continuing, as is work investigating the physics of island formation as discussed by [9].

Figure 3: The $\beta = 3\%$ equilibrium without (above) and with trim coils (below).



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