How NCSX was designed to suppress magnetic islands.

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Motivation and Outline.

1) Stellarators will in general have islands and, unless avoided or suppressed, islands and associated regions of chaos lead to poor plasma confinement.

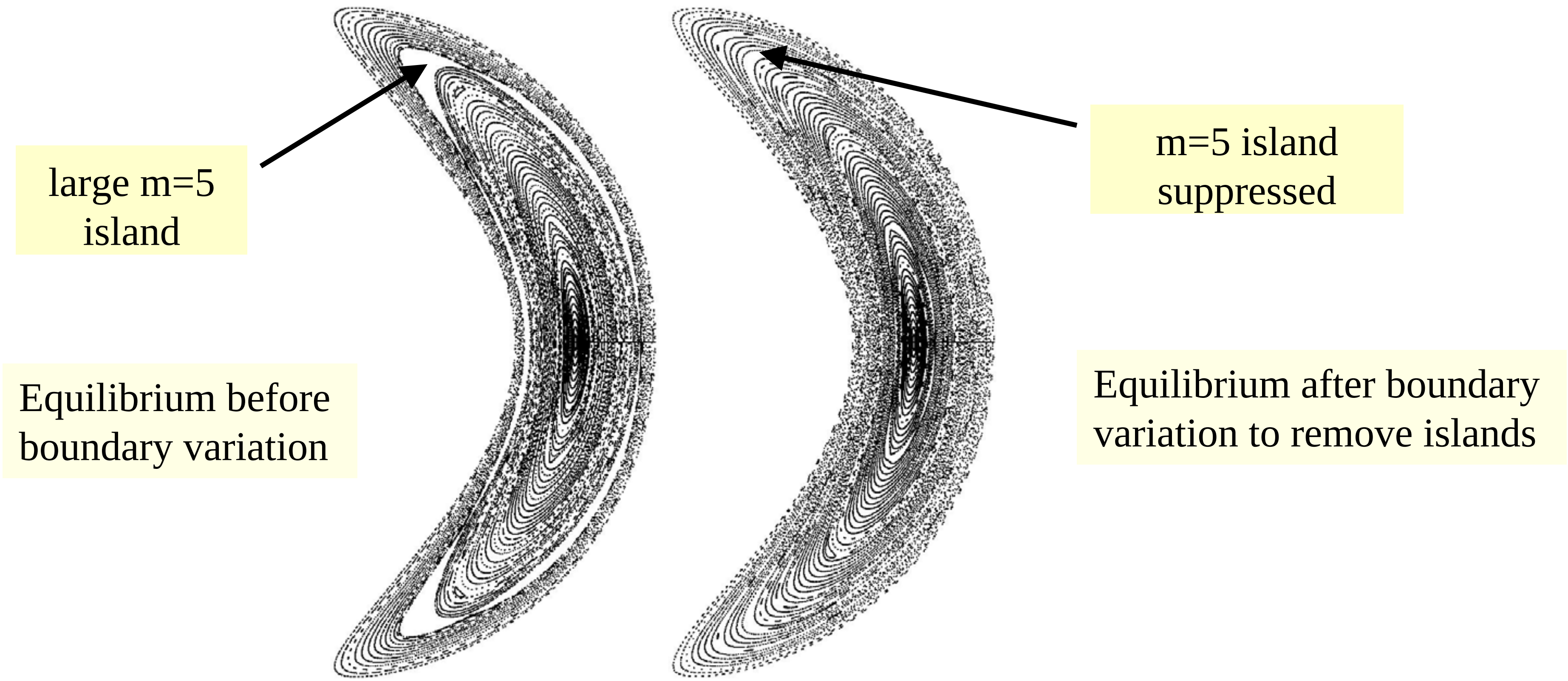
2) Islands in free-boundary NCSX equilibria were eliminated (coil-healing) as a final step in the coil design, after the coil-plasma optimization.

3) Details of the coil healing procedure are described.

4) Results for NCSX are presented showing a stable stellarator equilibrium, with build-able coils, with “good-flux-surfaces”.

Islands may be removed by boundary variation.

1) Equilibrium *(including islands & resonant fields as calculated by PIES)* is determined by plasma boundary.



, . . . but PIES is too slow for optimization.

Coil healing is required because …

1) The plasma and coils are designed using optimization routines that shape plasma boundary to achieve desired physics, while satisfying engineering constraints on coil geometry.
(see Strickler IAEA-CN-94/FT/P2-06)

2) All fast equilibrium codes (in particular VMEC) presuppose perfect nested magnetic surfaces $\rightarrow$ existence & size of magnetic islands cannot be addressed.

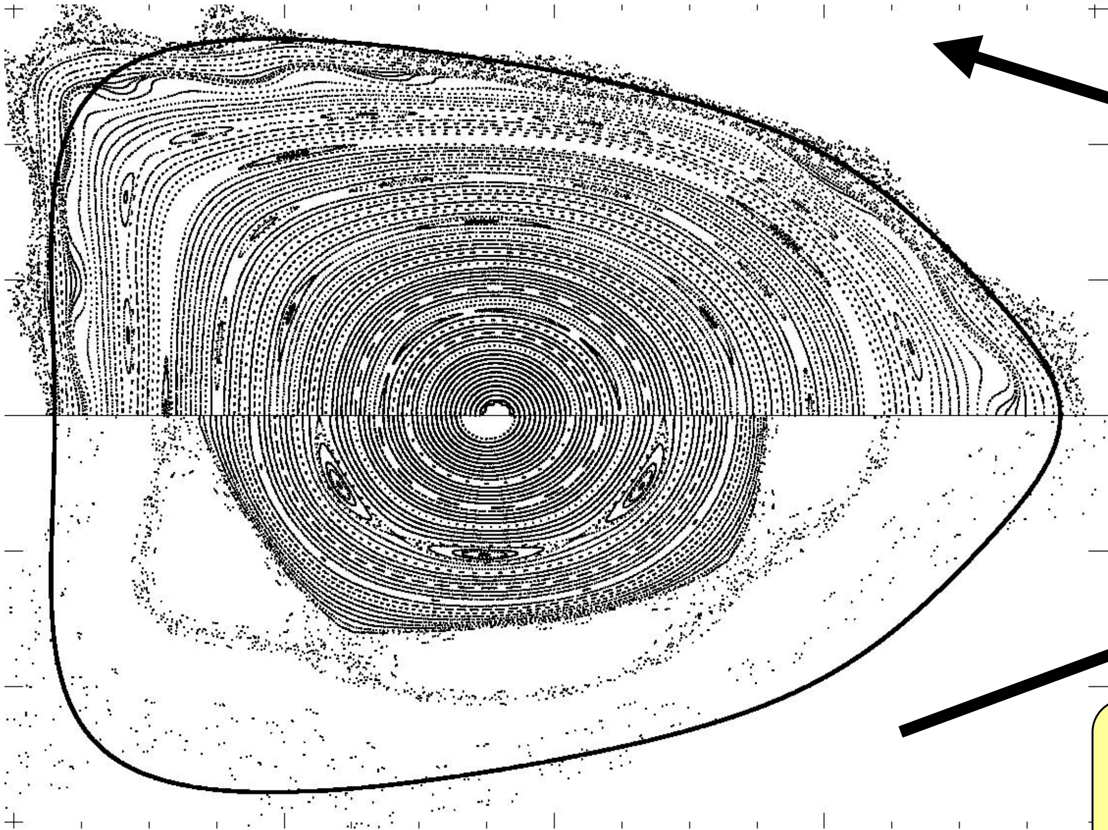
3) After the plasma and coil optimization, the coil geometry is modified to suppress the spectrum of ***B.n*** relevant to island formation $\rightarrow$ this procedure is called *coil-healing*.

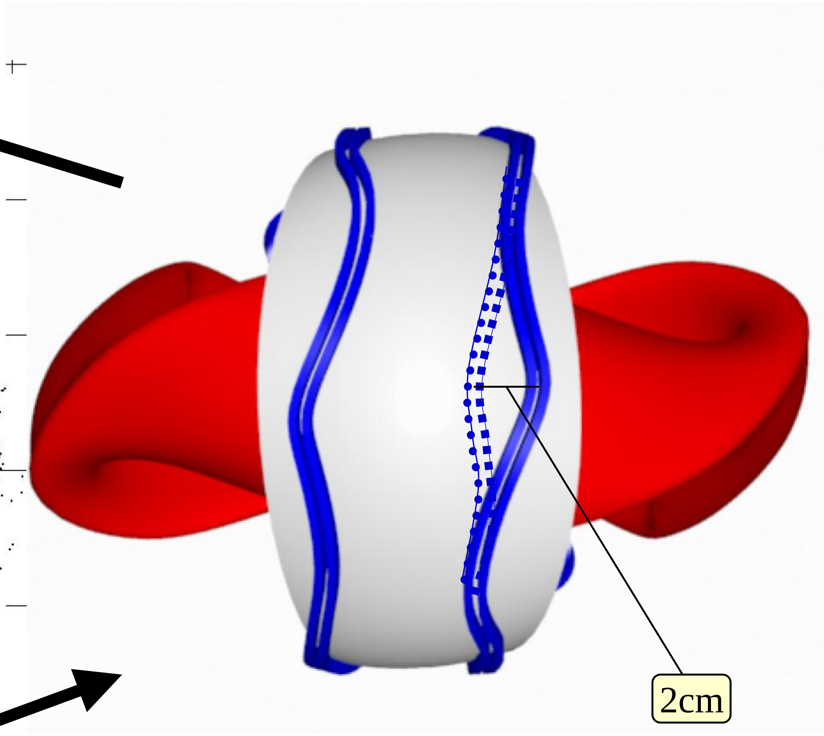
4) Coil-healing must not degrade the optimized plasma properties (ideal stability, quasi-axisymmetry, aspect ratio, . . .).

5) Coil-healing must not violate engineering constraints.

Coils are modified to remove islands.

`healed’ equilibrium
with good surfaces





coil adjustment
to remove resonant fields
(not to scale)

original equilibrium
with islands and chaos

The equilibrium calculation and coil healing proceed simultaneously via an iterative approach.

Based on the PIES code

A single PIES/healing iteration is shown

1) $\mathbf{B}^n = \mathbf{B}_P{}^n + \mathbf{B}_C(\xi^n)$ — total field = plasma + coil field

2) $\nabla p = \mathbf{J}^{(n+1)} \times \mathbf{B}^n$ — calculate plasma current

3) $\mathbf{J}^{(n+1)} = \nabla \times \mathbf{B}_P$ — calculate plasma field

4) $\mathbf{B}_P{}^{(n+1)} = \alpha \mathbf{B}_P{}^n + (1-\alpha)\mathbf{B}_P$ — blending for numerical stability

5) $\underline{\mathbf{B}} = \mathbf{B}_P{}^{(n+1)} + \mathbf{B}_C(\xi^n)$ — nearly integrable field

6) $-R_i = \nabla R_{Cij} \bullet \delta\xi^n{}_j$ — resonant fields function of ξ

7) $\xi_j{}^{(n+1)} = \xi_j{}^n + \delta\xi_j{}^n$ — coil geometry adjusted

At each iteration, the coil geometry is adjusted to cancel the resonant fields

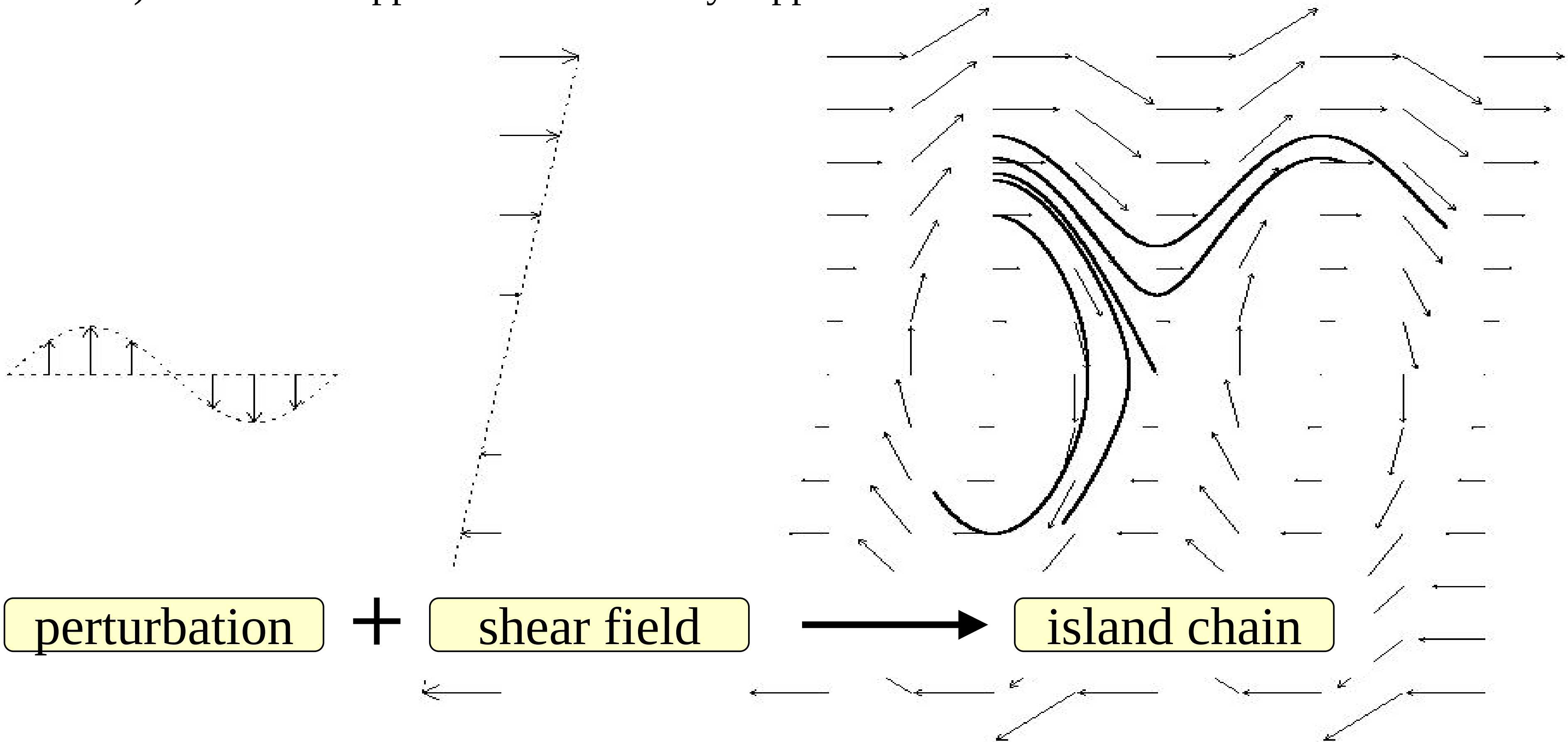
n iteration index ; $\mathbf{B}_P$ plasma field ; $\mathbf{B}_C$ coil field ; ξ coil geometry harmonics ;
α=0.99 blending parameter ; ∇R_{Cij} coupling matrix.

Rational surfaces are located.

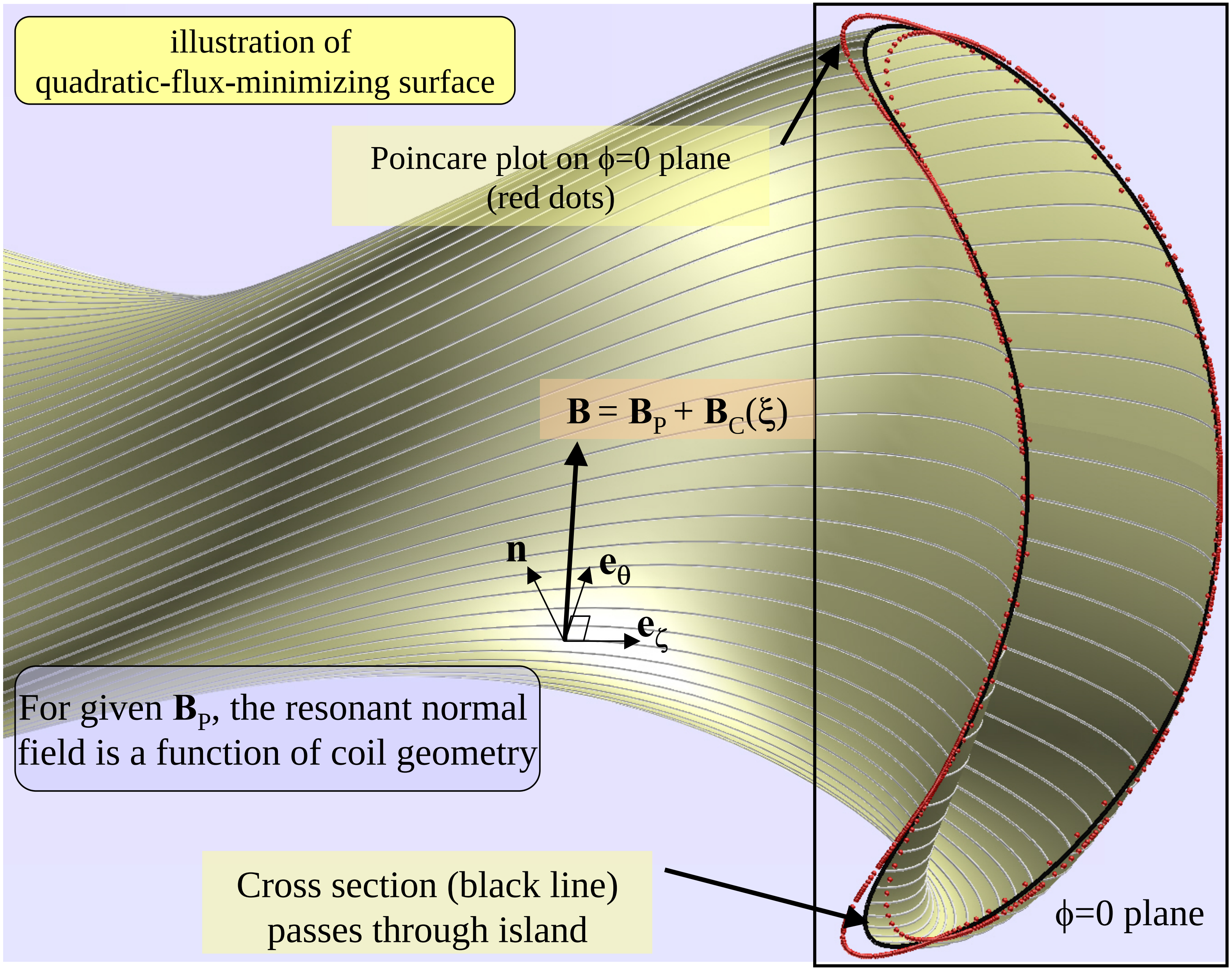
1) Quadratic-flux minimizing surfaces may be thought of as rational rotational transform flux surfaces of a nearby field with good flux surfaces.
Dewar, Hudson & Price.Physics Letters A, 194:49, 1994
Hudson & Dewar, Physics of Plasmas 6(5):1532,1999.

2) Resonant normal fields at the rational surfaces form islands, and island overlap causes chaos.

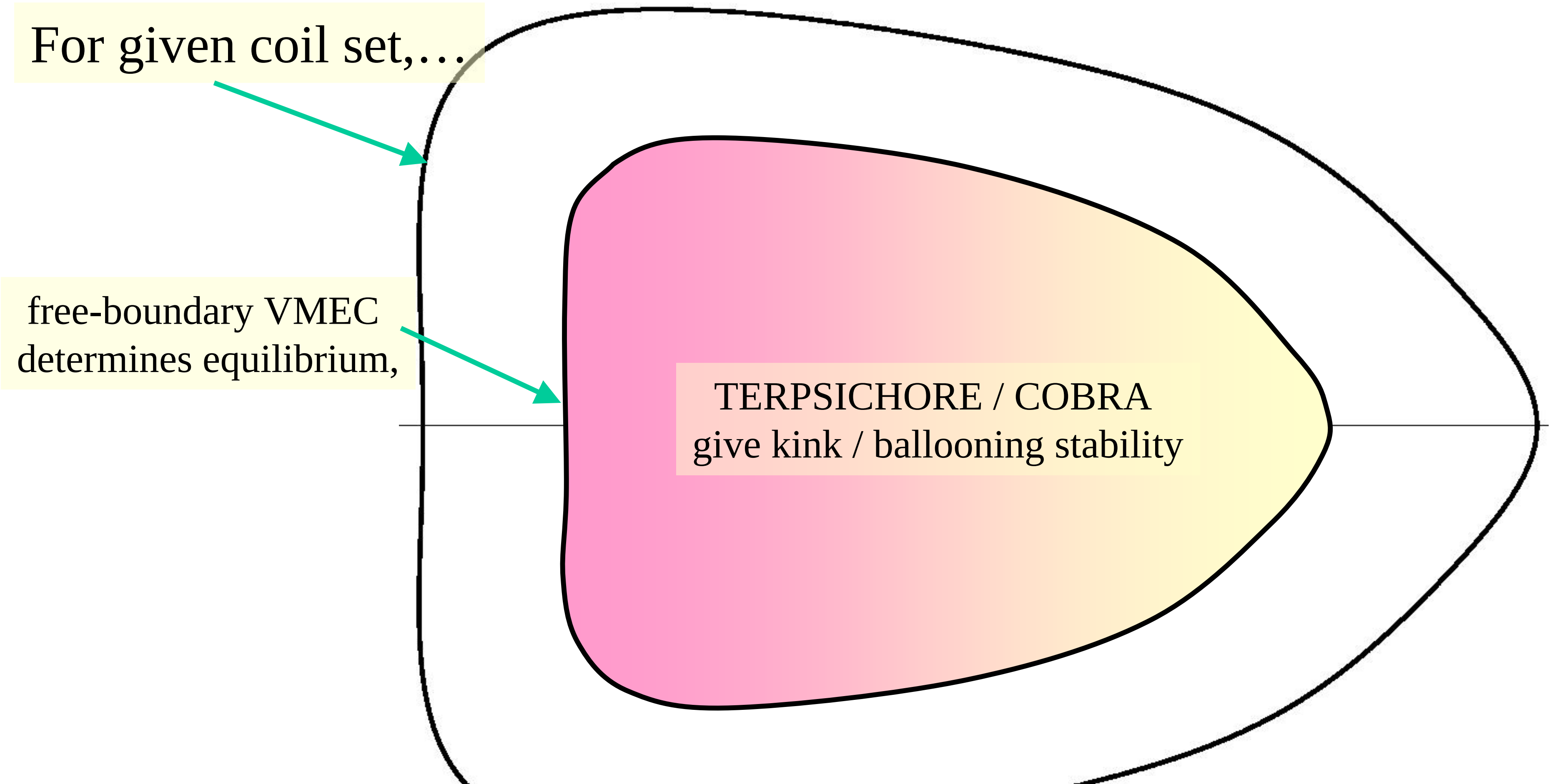
3) Island suppression achieved by suppression of resonant fields.



The field normal to the rational surface is calculated.



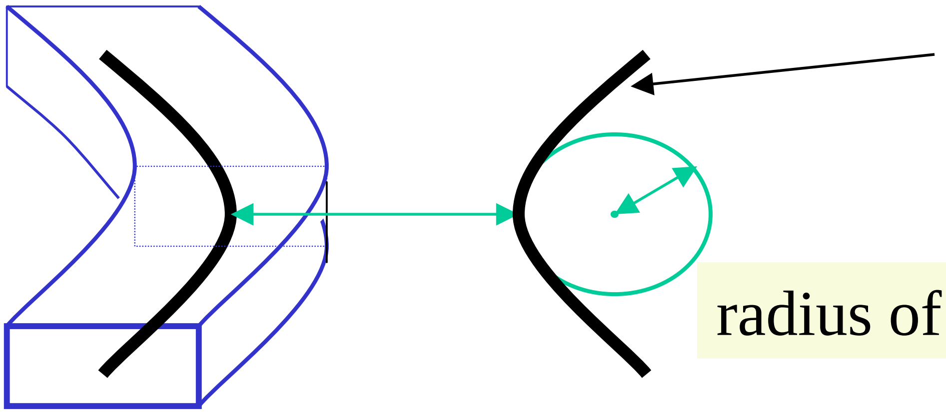
Plasma stability is calculated.



Kink stability and ballooning stability
expressed as functions of coil geometry

Engineering constraints are calculated.

1) To be “build-able”, the coils must satisfy engineering requirements.
2) Engineering constraints are calculated by the COILOPT code.
3) In this application, the coil-coil separation and coil minimum radius of curvature are considered.



Coil-coil separation and minimum radius of curvature expressed as functions of coil geometry

Resonant fields are cancelled by coil-adjustment; engineering constraints and plasma stability is preserved.

1) The coil-coil separation and coil minimum radius of curvature are functions of coil-geometry ξ.
2) Ideal stability (kink, ballooning) are functions of free-boundary equilibrium, which in turn is a function of coil-geometry ξ.

$$\mathbf{R} = \mathbf{R}(\xi) = \begin{pmatrix} B^{C}_{n,m} - B^{P}_{n,m} \\ \Delta - \Delta_0 \\ \lambda - \lambda_0 \end{pmatrix}$$

coil/plasma resonant fields

engineering constraints

ideal stability

initial values

3) The desired solution is **R = 0.**

Standard numerical methods find solution **R**=0.

1) First order expansion for small changes in ξ

$$\mathbf{R}(\xi) = \mathbf{R}(\xi_0) + \nabla\mathbf{R}_C \cdot \delta\xi.$$

2) A multi-dimensional Newton method solves for the coil correction to cancel the resonant fields at the rational surface

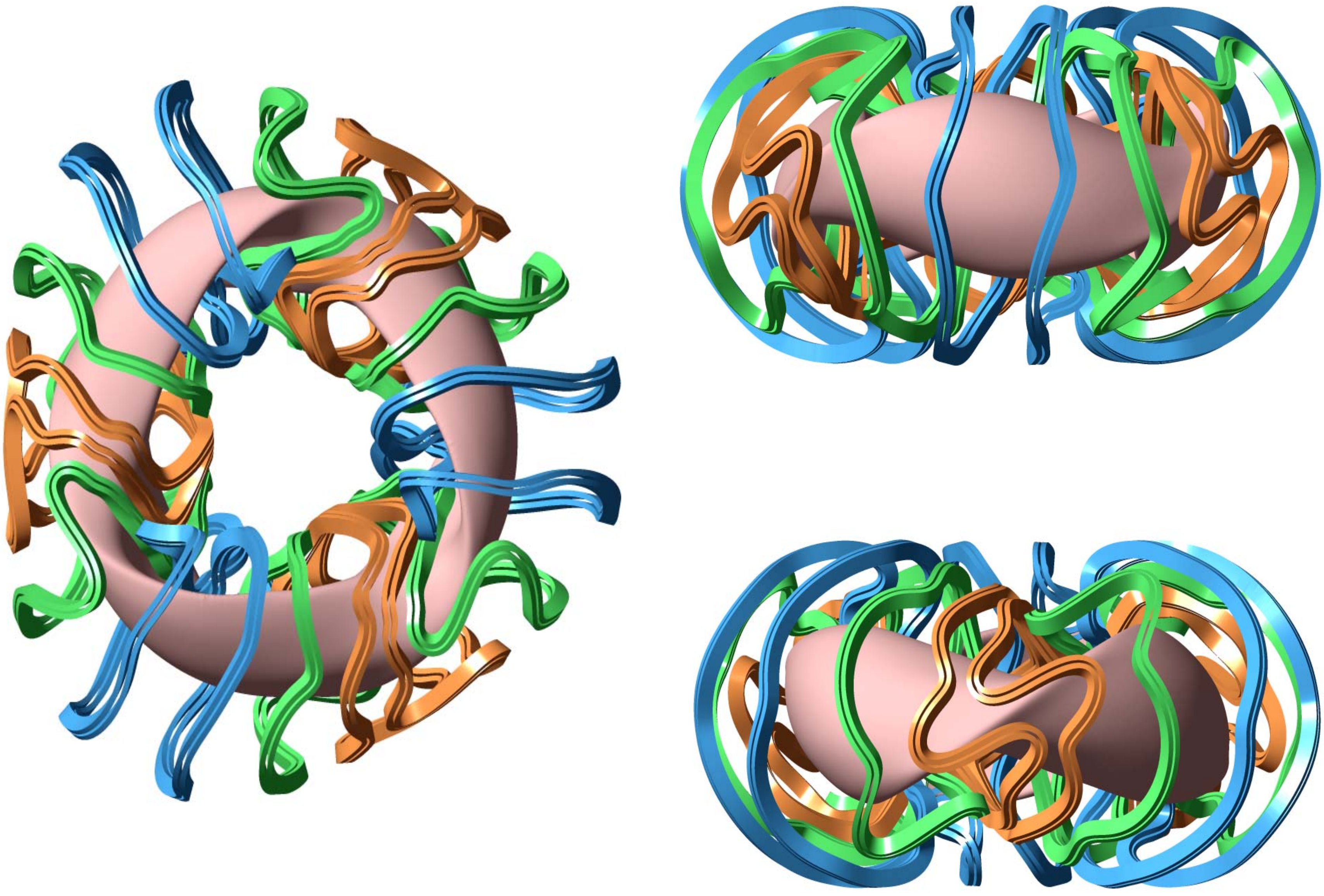
$$\sum_j \nabla R_{Cij} \cdot \delta\xi_j = -R_i.$$

3) The coil geometry is adjusted $\xi \rightarrow \xi + \delta\xi.$

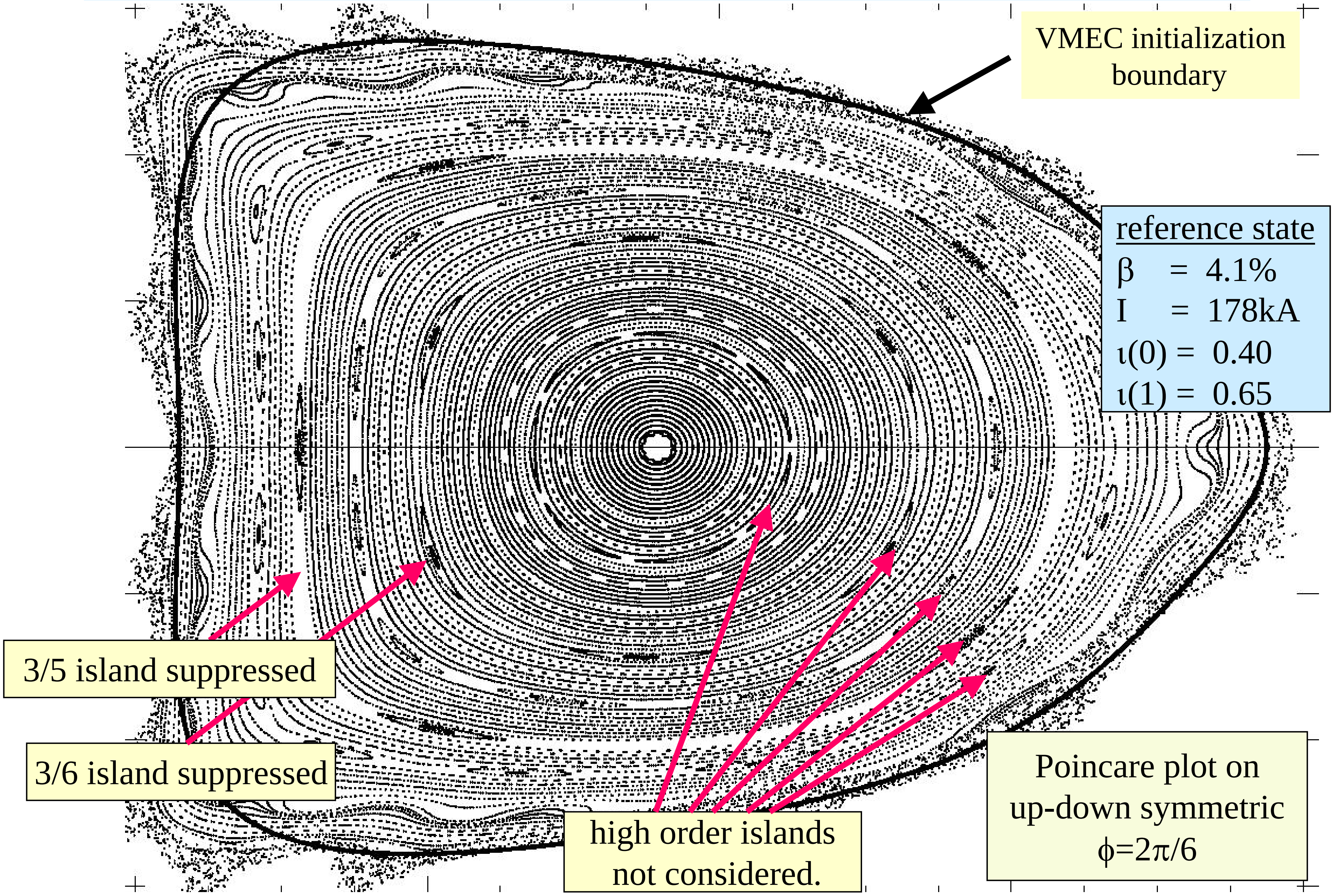
4) At each PIES iteration, the coils are adjusted to remove resonant fields.

5) As the iterations proceed, the coil field and plasma field converge to an equilibrium with selected islands suppressed.

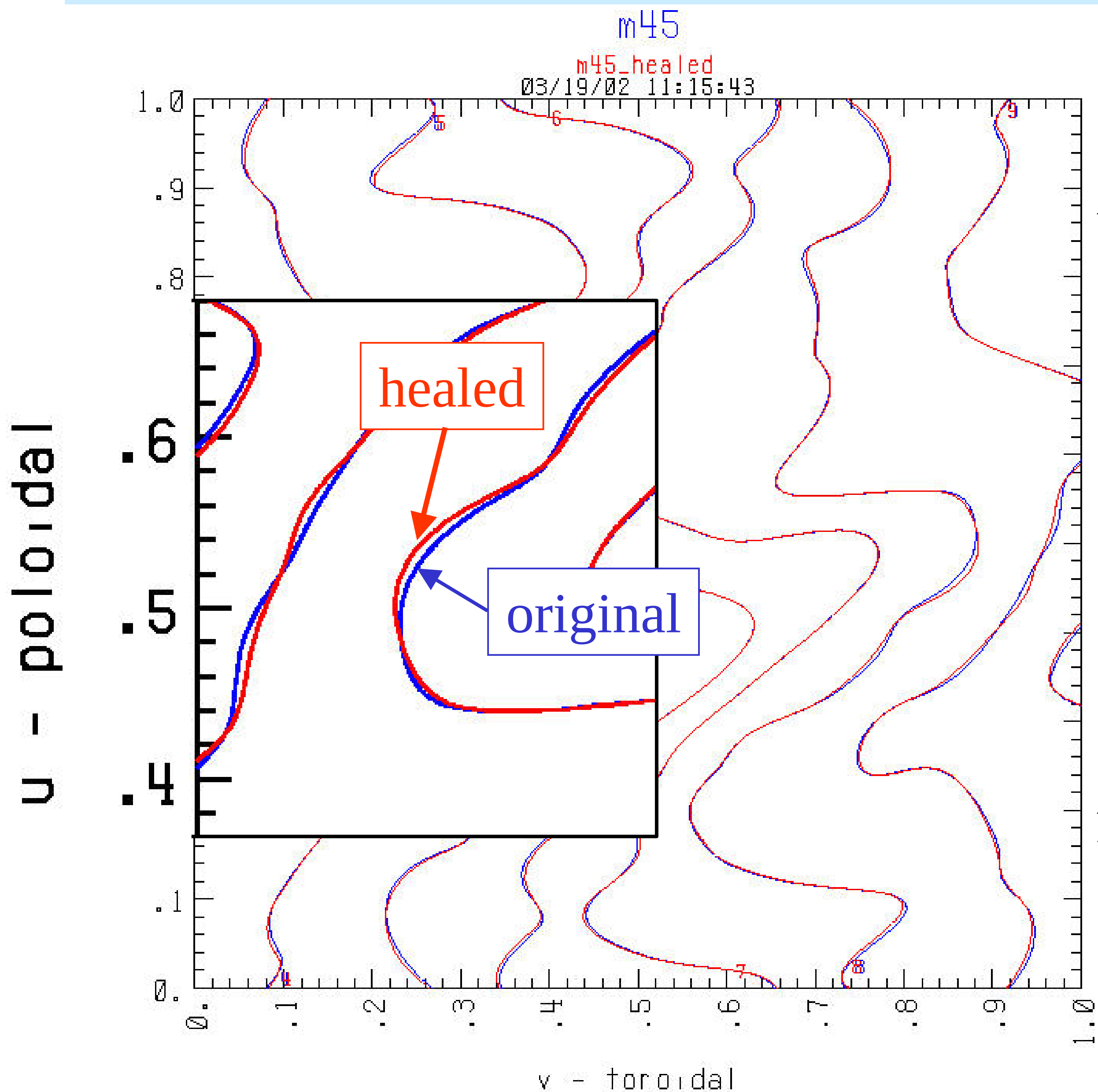
The NCSX modular coil geometry will be adjusted.



The healed configuration has good-flux-surfaces.



The magnitude of the coil change is acceptable.



- plot shows coils on toroidal winding surface

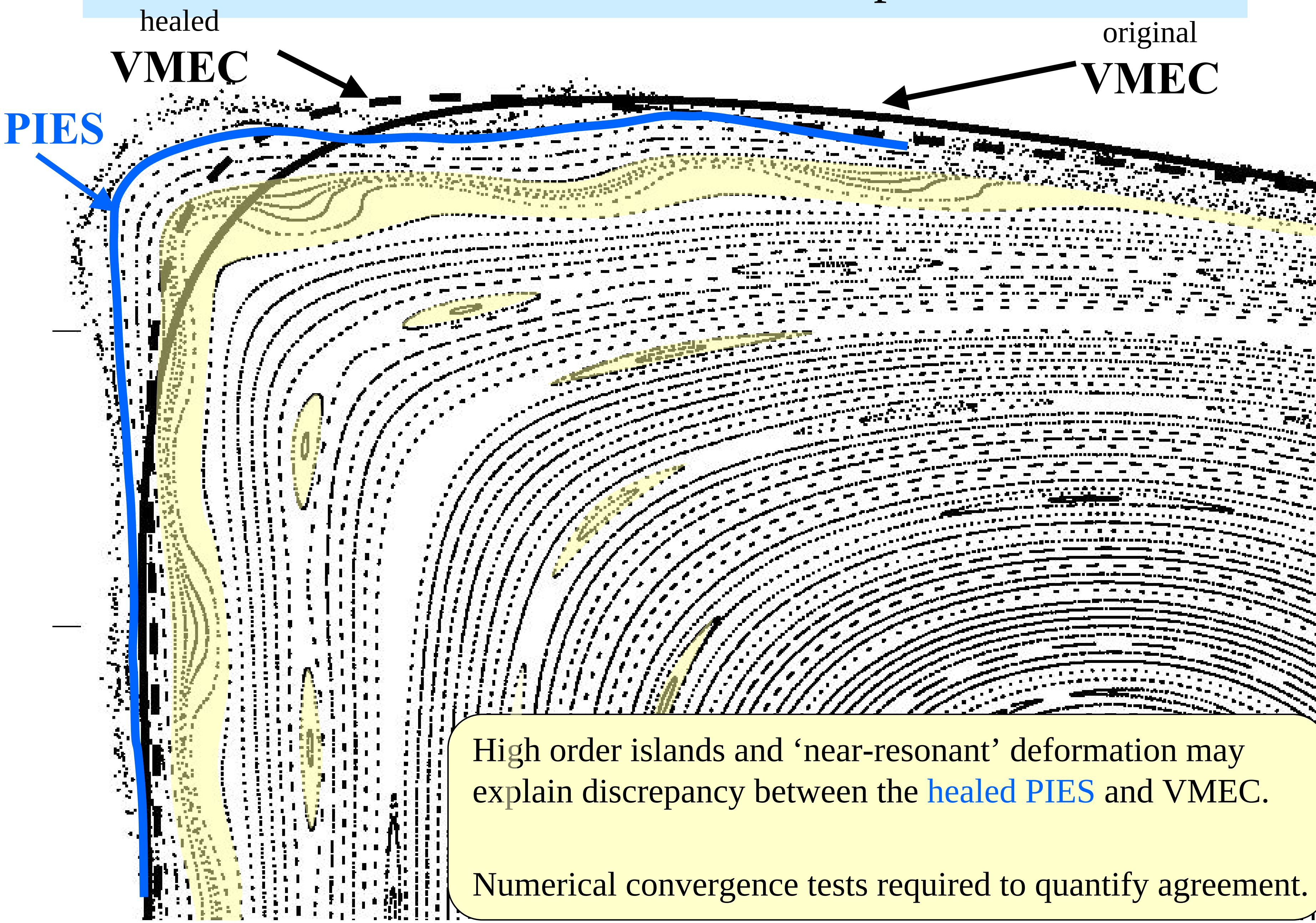
- coil change ≈ 2cm

- coil change exceeds construction tolerances

- does not impact machine design (diagnostic, NBI access still ok)

* the *resonant* harmonics have been adjusted

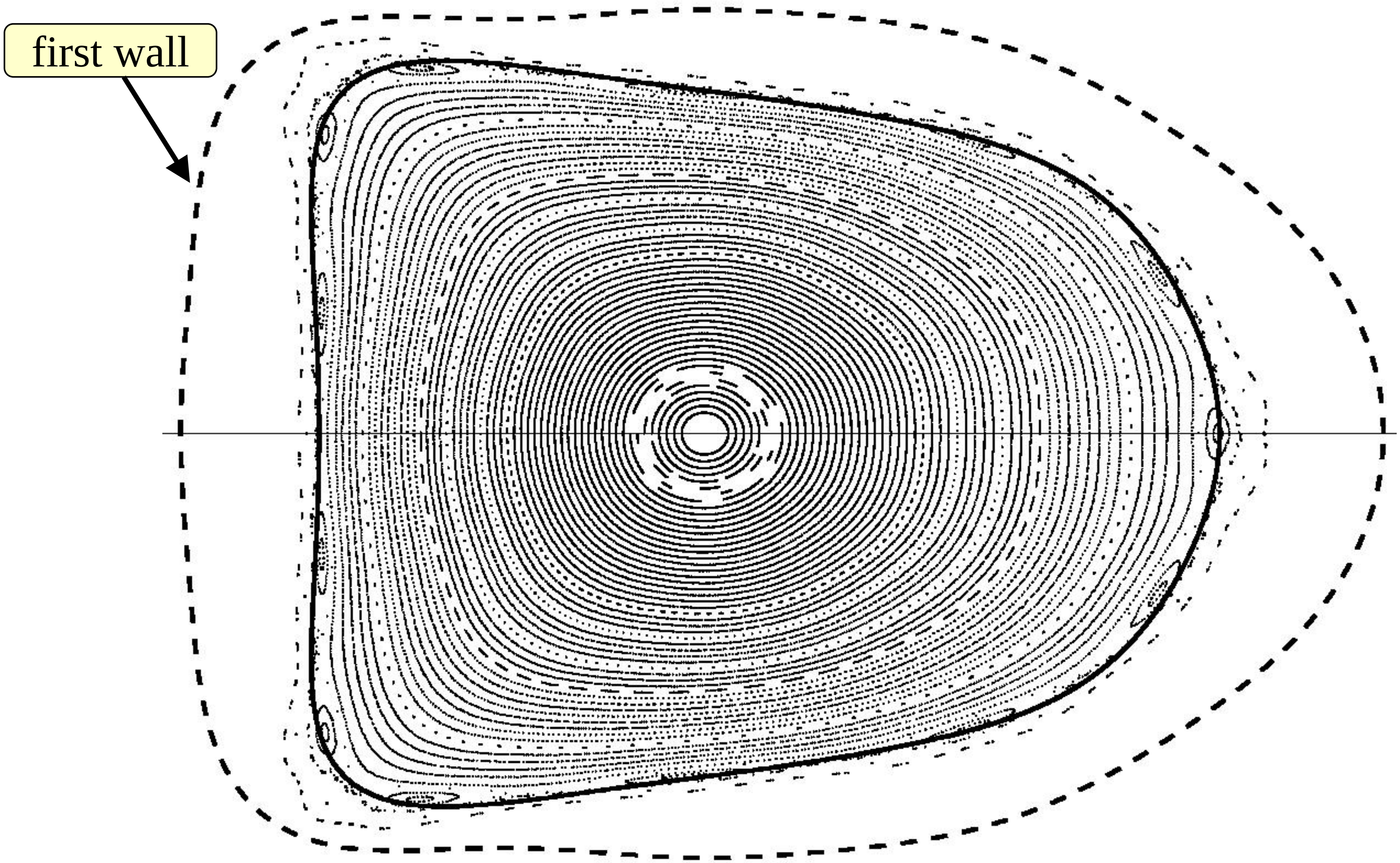
Healed PIES / VMEC comparison



High order islands and 'near-resonant' deformation may explain discrepancy between the healed PIES and VMEC.

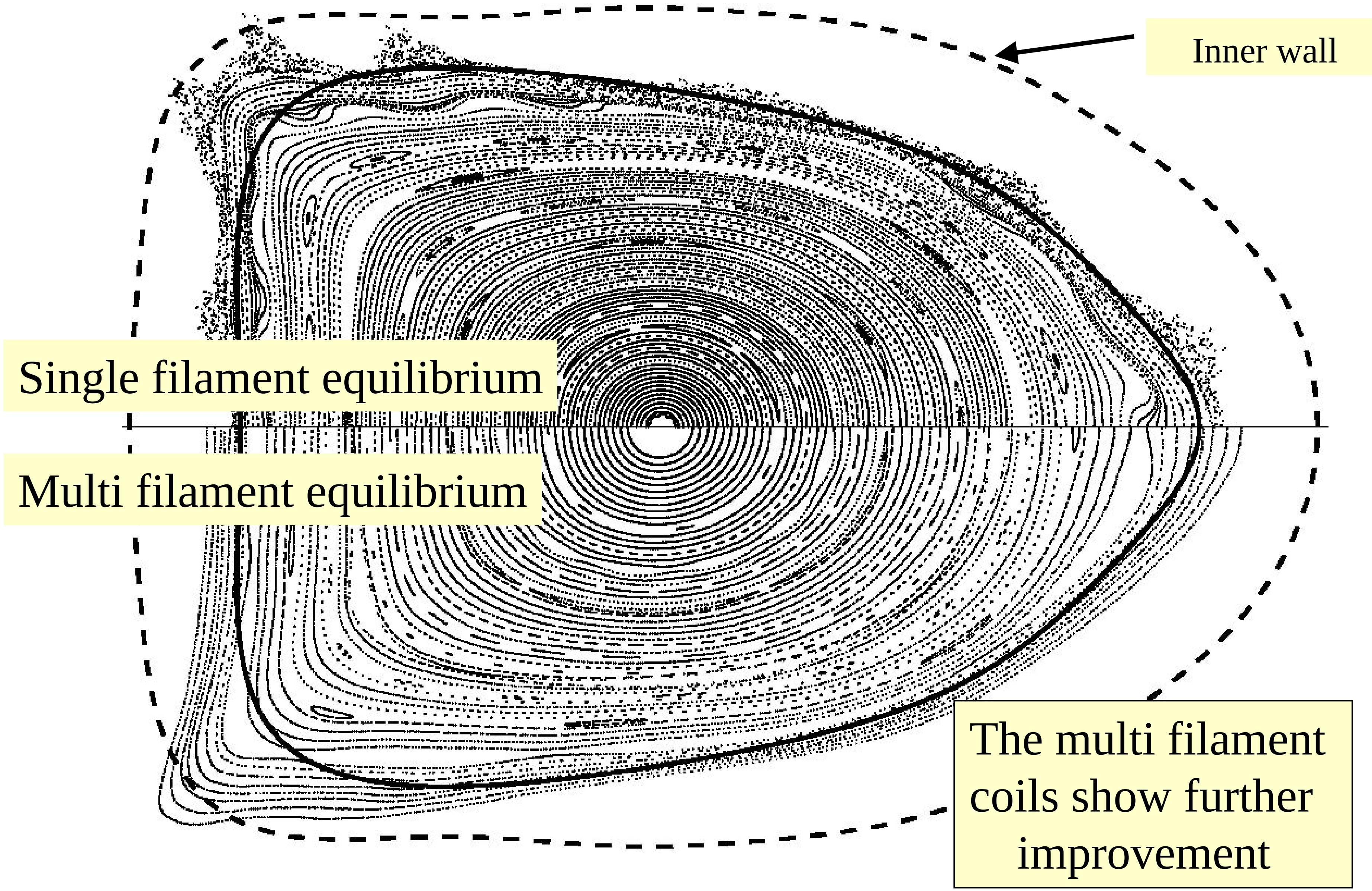
Numerical convergence tests required to quantify agreement.

The healed coils support good vacuum states.



The healed coils maintain good vacuum states

Finite thickness coils show further improvement.



Robustness of healed coils

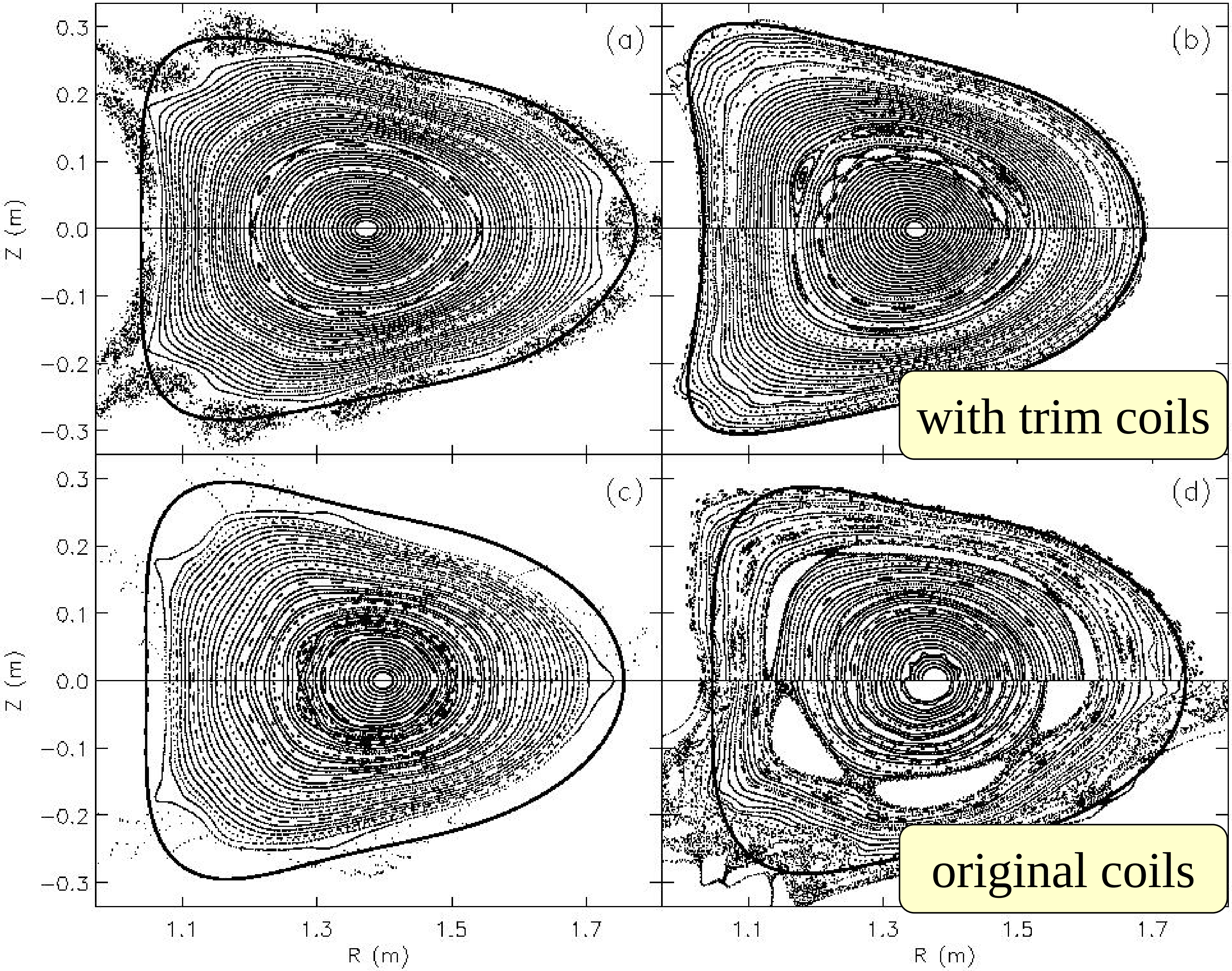
1) The discharge scenario is a sequence of equilibria, with increasing β, that evolves the current profile in time self-consistently from the discharge initiation to the high β state.

time	a	I (A)	β (%)	axis ι	edge ι
050	4.415	5.34e4	1.22	0.443	0.543
100	4.390	8.16e4	3.38	0.427	0.511
116	4.383	9.02e4	3.67	0.442	0.598
139	4.427	9.97e4	3.93	0.358	0.585
303	4.466	1.32e5	4.58	0.307	0.655

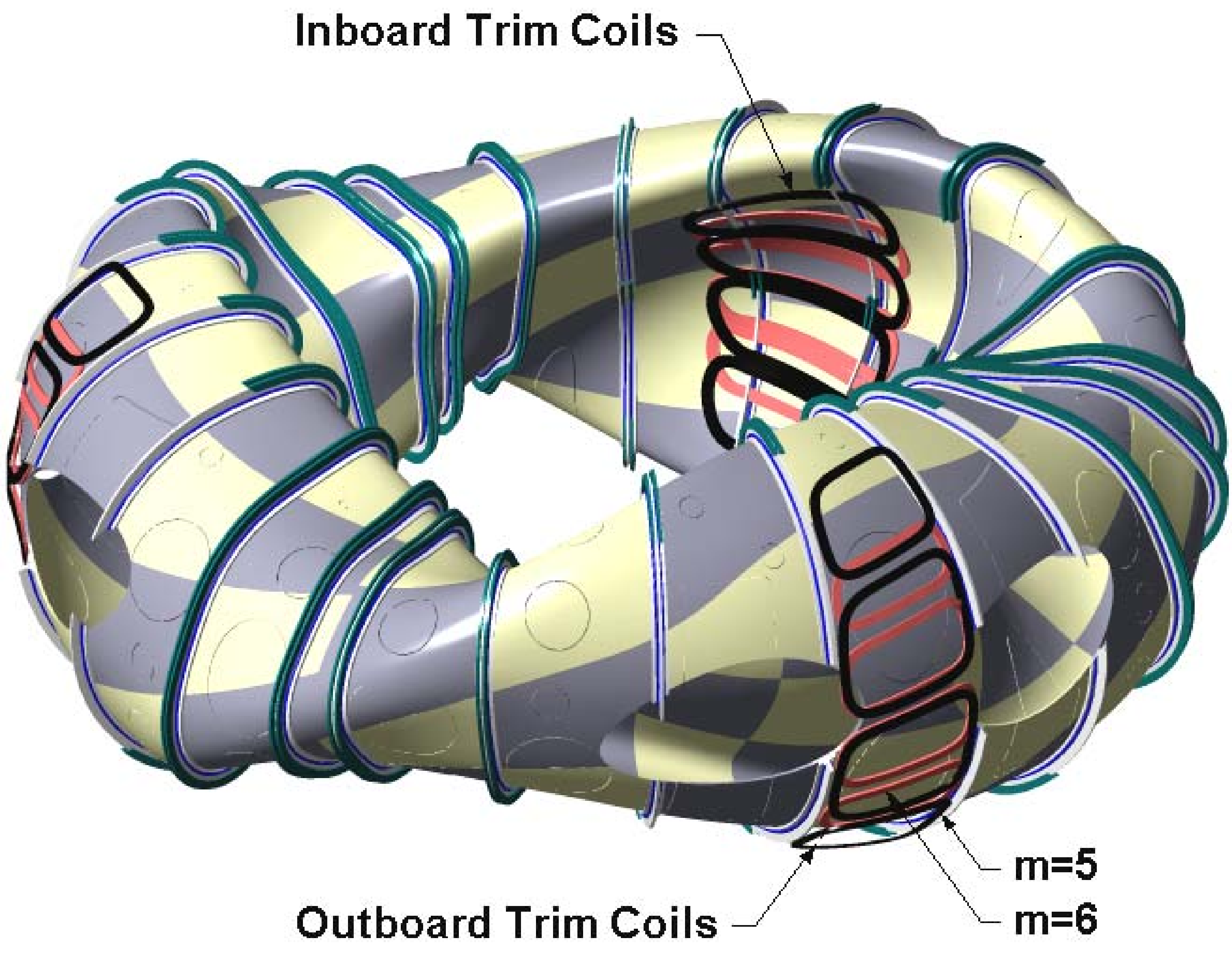
2) The healed coils show improved configurations for this sequence.

3) NOTE : this sequence has in no way been optimized for surface quality.

Healed coils are improved for discharge seq.



Trim Coils are included in the NCSX design.

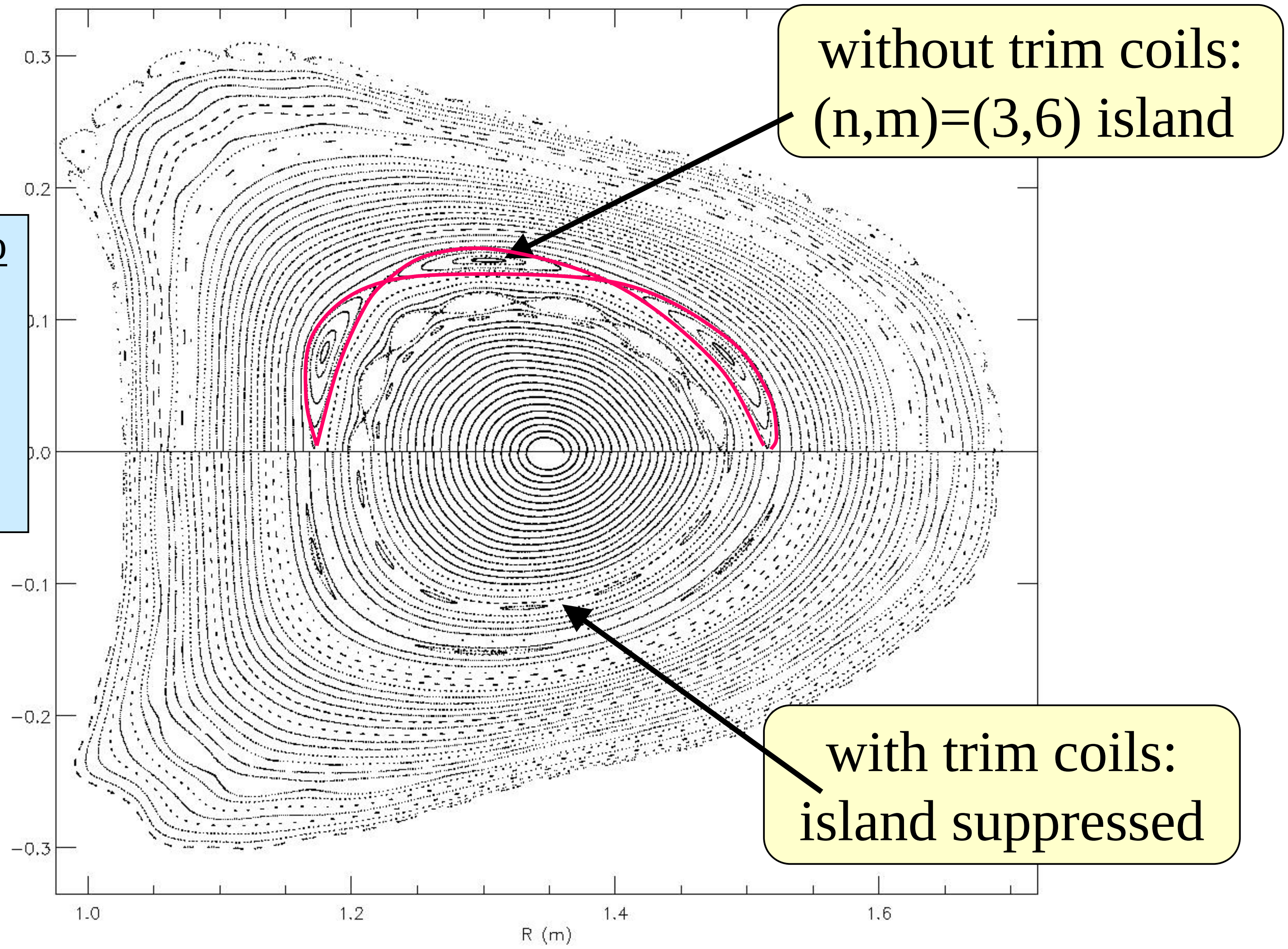


Trim Coils provide additional island control

the same procedure determines the trim coil currents which suppress islands

discharge scenario

t = 100ms
β = 3.38%
I = 81.6kA
ι(0) = 0.427
ι(1) = 0.511



Summary

1) The plasma and coils converge *simultaneously* to a free-boundary equilibrium with selected islands suppressed, while preserving engineering constraints and plasma stability.

2) Adjusting the coil geometry at every PIES iteration enables effective control of non-linearity of the plasma response to changes in the external field.

3) Coils support a variety of equilibria with good flux surface quality, and trim coils provide control of islands.