Influence of pressure-gradient and shear on ballooning stability in stellarators

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A simple, semi-analytic method for expressing the ballooning growth rate as functions of the pressure-gradient and the averaged magnetic shear is introduced. The simplicity of the expressions allows for interpretation of the important physical effects at work in determining instability thresholds and determines whether or not a given stellarator configuration will possess a region of second stability.

For a given configuration, the the ballooning equation is written in the Sturm-Liouville form $[\partial_\eta P \partial_\eta + Q - \lambda \sqrt{g}^2 P]\xi = 0$, where the ballooning coefficients $P = B^2/g^{\psi\psi} + g^{\psi\psi} L^2$ and $Q = 2p' \sqrt{g}(G + \iota\!\!\!-I)(\kappa_n + \kappa_g L)$. Here L is the integrated local shear, $L = \int_{\eta_k}^{\eta} s(\eta') d\eta'$, where $s = \iota\!\!\!-' + \tilde{s}$ is the local shear, and κ_n, κ_g represent the normal and geodesic curvatures. This is an eigenvalue equation and for realistic geometry must be solved numerically.

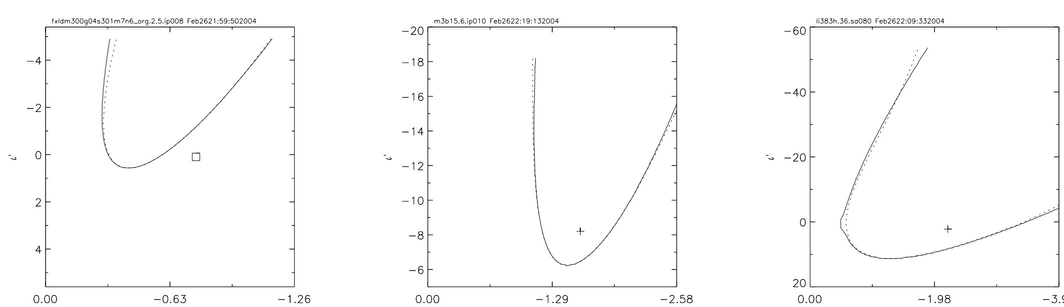
To determine the effect of variations in the pressure-gradient and average shear we employ the method of profile variations. An analytic variation in the pressure-gradient, $p' \to p' + \delta p'$, and average shear, $\iota\!\!\!-' \to \iota\!\!\!-' + \delta\iota\!\!\!-'$, is imposed at an arbitrary flux surface of an MHD equilibrium. The relevant equilibrium quantities are then adjusted to preserve force balance.

The variations $(\delta p', \delta\iota\!\!\!-')$ alter the ballooning coefficients, $P \to P + \delta P$, $Q \to Q + \delta Q$, and the impact of the variations on the ballooning growth rate may be determined using eigenvalue perturbation theory. An expression for the change in the ballooning eigenvalue can be derived

$$\lambda(\delta p', \delta\iota\!\!\!-') = \lambda_o + \frac{\partial\lambda}{\partial p'}\delta p' + \frac{\partial\lambda}{\partial\iota\!\!\!-'}\delta\iota\!\!\!-' + \frac{\partial^2\lambda}{\partial p'^2}\delta p'^2 + \frac{\partial^2\lambda}{\partial p'\partial\iota\!\!\!-'}\delta p'\delta\iota\!\!\!-' + \frac{\partial^2\lambda}{\partial\iota\!\!\!-'^2}\delta\iota\!\!\!-'^2 + \ldots, \qquad (1)$$

where λ_0 is the eigenvalue of the original equilibrium and explicit expressions for the derivatives, up to arbitrary order, are determined from a *single eigenfunction calculation.* Using only this information, whether increased pressure-gradient is stabilizing or de-stabilizing, and the existence of a second stable region, can be determined.

Marginal stability diagrams for an LHD-like configuration (left), a quasi-poloidal configuration (center) and an NCSX-like quasi-symmetric configuration(right) are presented in the figure below. The solid line shows the exact calculation: that is, the eigenvalue equation is resolved numerically for each variation $(\delta p', \delta\iota\!\!\!-')$ on a grid of 200×200 points; and the dotted line shows the approximation provided by Eqn.(1). Also shown is the location in $(p', \iota\!\!\!-')$ space of the original equilibrium, indicated with a + if that equilibrium is unstable or a $\square$ if it is stable.



The analysis presented here provides both a numerically efficient and physically insightful approach to determining second stability in stellarators.