Boundary modulation effects on MHD instabilities in Heliotrons

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Introduction (discrepancy on MHD stabilities)

- **High-β ($\gtrsim 3$%) plasmas** are established in **inward-shifted** LHD configurations

  [MOTOJIMA, O., et al., Nucl.Fusion $\mathbf{43}$ (2003) 1674.]

  $\Updownarrow$

  Discrepancy on MHD stabilities

  $\Updownarrow$

  Theoretical analyses indicete **strong MHD instabilities**.

  [NAKAJIMA, N., et al., J.Plasma Fusion and Res. SER. $\mathbf{6}$ (2004).]

- Factors resolving this discrepancy
  - **Stability analyses** for given MHD equilibria
    1. *Nonlinear saturation of MHD instabilities*

       [ICHIGUCHI, K., et al., 19thIAEA.]

       [MIURA, H., et al., this IAEA (TH/2-3).]
    2. *Two-fluids effects (simple FLR effects), diamagnetic rotation, Hall-effects*
  - Reconsideration of **MHD equilibria** themselves

    1. *Pressure profile*, 2. *Net current*, 3. **Boundary**

Introduction (discrepancy on MHD equilibria)

Theoretically used MHD equilibria

- **Fixed boundary MHD equilibria**
  boundary from a **clear Last Close Flux Surface (LCFS)** of the vacuum
- **Free boundary MHD equilibria with a limiter where $P = 0$**
  virtual material limiter at the outboard of the horizontally elongated clear LCFS

$$\Downarrow$$

strong MHD instabilities in inward-shifted configurations

background theoretical conjecture

Plasmas do not expand so much beyond the clear LCFS of the vacuum

$$\Updownarrow$$

discrepancy on MHD equilibria

$$\Updownarrow$$

experimental observations

∇P exists beyond a clear LCFS of vacuum up to a position with a fairly long connection length L_c.

[MORISAKI, T., et al., J.Nucl.Materials **313-316** (2003) 548.]

Introduction (purposes)

1. **By reducing the discrepancy on MHD equilibria, the MHD stability is reconsidered.**
   For such a purpose, we will take the theoretical standpoint that
   (a) **an experimentally observed $\nabla P(\neq 0)$ beyond a clear LCSF is governed by a transport in a stochastic magnetic field region without clear flux surfaces**,
   (b) so that **averaged flux surfaces** or **nested flux surfaces** with
      i. a **small thermal conductivity** χ_e
      ii. a **long connection lengh** L_c
      iii. a **definite rational transform** $\iota\!\!\!-$

      should be considered to **exist even in such a stochastic magnetic field region.**
   (c) In such a region, **free boundary motions of MHD equilibria** will be **allowable.**
2. **To show the boundary modulation of MHD equilibria brought by such the free boundary motion can significantly improve MHD stability**, **leading to better consistency between experiment and theory.**

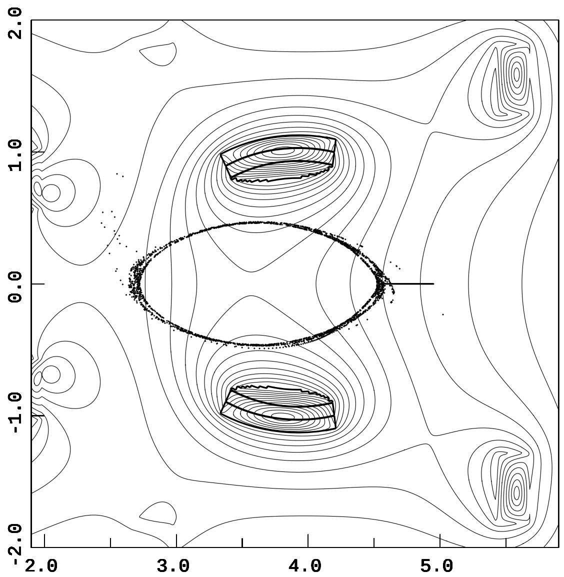
Properties of peripheral magnetic field (vacuum)

To estimate the region where the averaged flux surfaces can exist, peripheral magnetic field is examined.

1.1 **field line tracing (Poincare plots)**

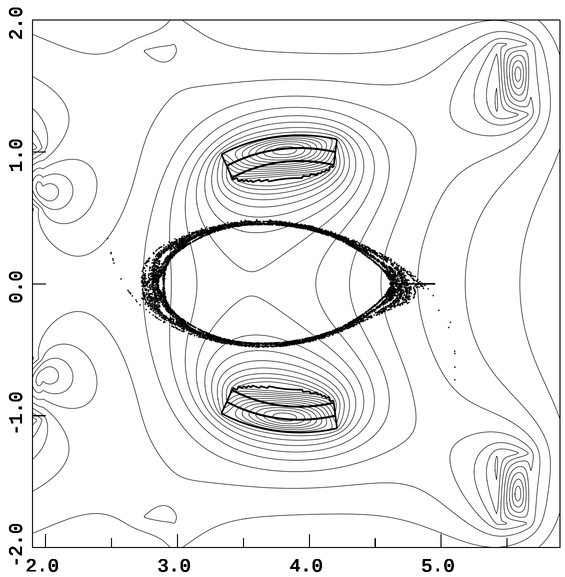
Inward-shifted

($R_{va} = 3.60$m)



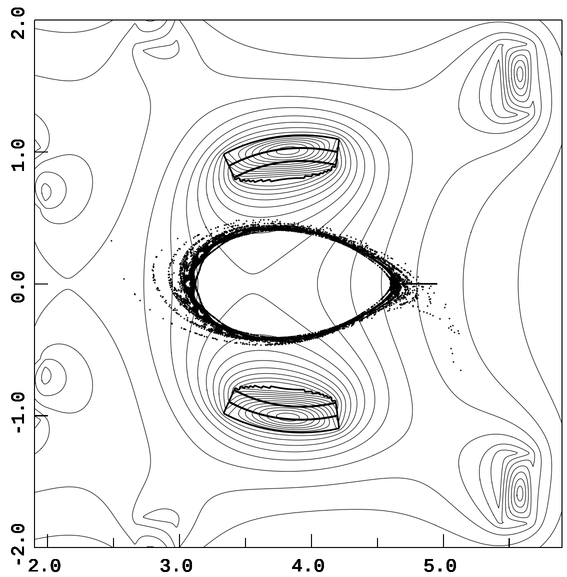
standard

($R_{va} = 3.75$m)



outward-shifted

($R_{va} = 3.90$m)



Inward-shifted configurations are characterized as those **with the most thin width of the stochastic region.**

Properties of peripheral magnetic field (vacuum)

L_c : connection length of magnetic field lines up to the wall

Region with quite short L_c : direct loss region
Region with relatively long $L_c \gg \lambda_{mfp}$: anomalous transport region

Electron thermal transport coefficient χ_e :

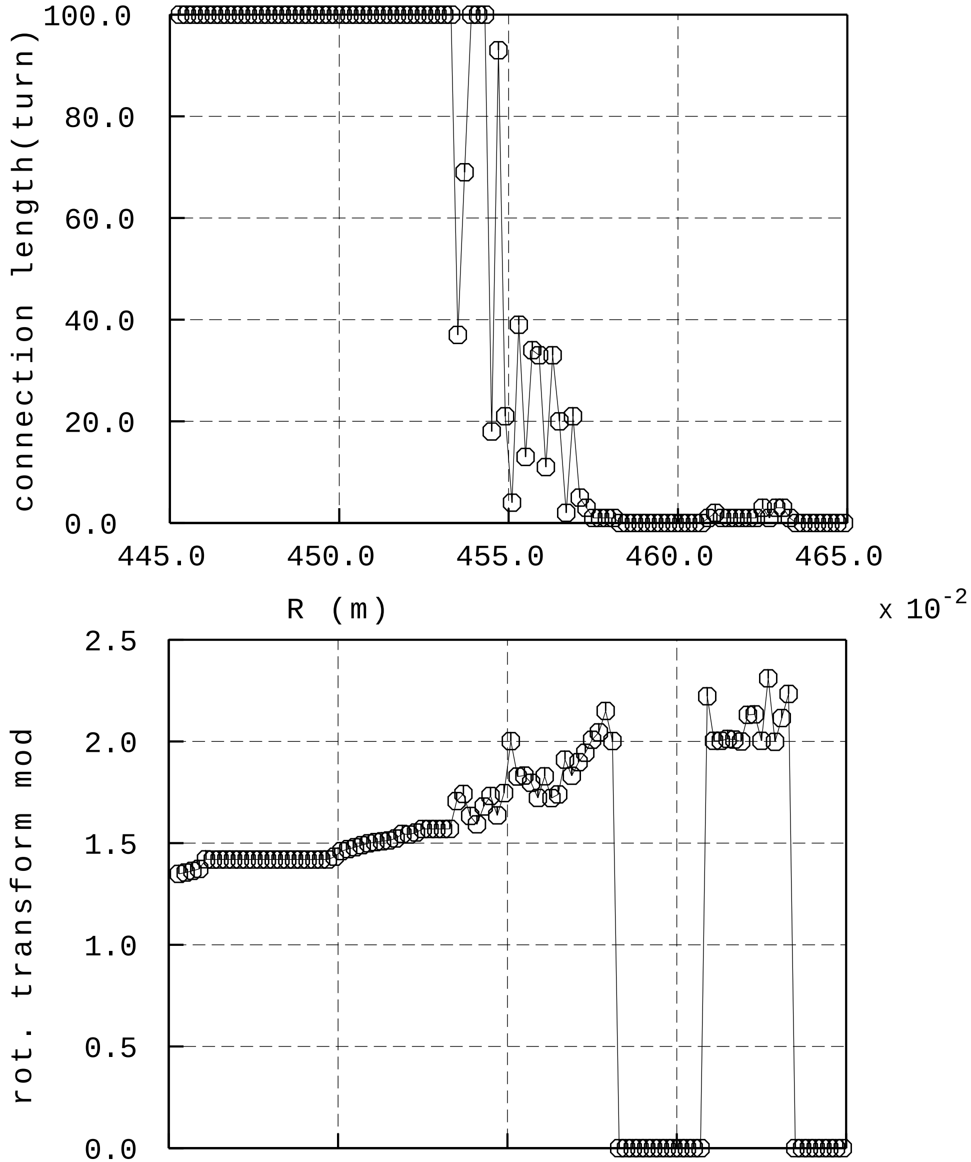
$$\chi_e = \frac{4 v_{Te} \delta k_{||}}{\pi^{3/2} \overline{k}_r^2} \begin{cases} \mathcal{R}^2 & \text{for } \mathcal{R} \leq 1 \\ \mathcal{R} & \text{for } \mathcal{R} \geq 1, \end{cases} \qquad \mathcal{R} \equiv \left[\frac{\pi}{8} \frac{L_{||} \overline{k}_r^2}{\delta k_{||}} \sum_m \left\langle \left(\frac{\delta B_{rmk_{||}}}{B} \right)^2 \right\rangle_{k_{||}} \right]^{1/2}$$

where $\boldsymbol{B}$: averaged regular magnetic field, $\delta \boldsymbol{B}$: fluctuating magnetic field

In inward-shifted configurations with most thin stochastic layer, χ_e, brought by the stochastic magnetic field, might be considered to be **small.**

Properties of peripheral magnetic field (vacuum)

L_c & ι_v **(Inward-shifted configuration)** $(R_{wall} = 1.8\text{m}, \quad 1 \text{ toroidal turn} \sim 22\text{m})$



- **clear** flux surfcace **up to** $\iota_v \sim 1.48$
- **LCFS up to** $\iota_v \sim 1.58 = 30/19$

range of ι_v	character
$\iota_v \lesssim 1.48$	**clear flux surfaces**
$1.48 < \iota_v \lesssim 1.58$	**long** L_c
$1.58 < \iota_v \lesssim 2.00$	**short** L_c
$2.00 < \iota_v$	very short L_c

Averaged flux surfaces are assumed to exist up to $\iota_v \sim 2.0$

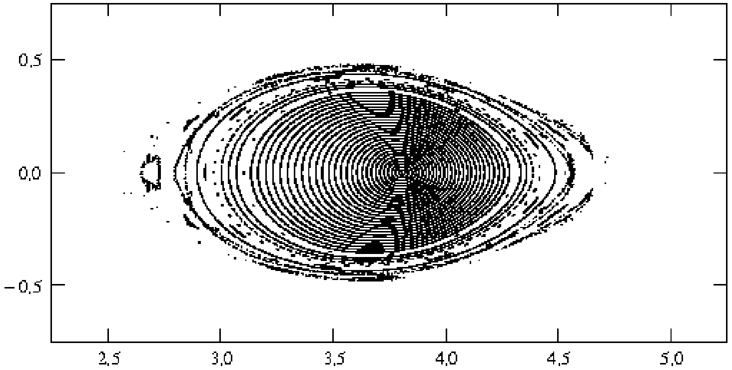
Selected vacuum boundary

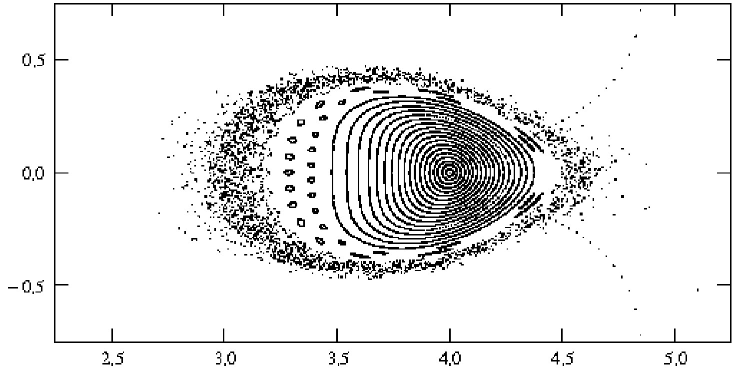
- $\iota_v = 1.48$ **(clear surface)**
- $\iota_v = 1.58$ **(near LCFS)**
- $\iota_v = 1.72$ **(outside of LCFS)**

Properties of peripheral magnetic field (finite-β)

Inward-shifted configuration

Poincare plots(HINT) $\langle\beta\rangle = 1.4\%$ $\langle\beta\rangle = 3.7\%$





- **Width** of peripheral magnetic islands becomes **wide**, as β increases.
- **Stochastic field created near the plasma periphery through island-overlapping penetrates into core region,** as β increases.
- ∇P **still exists in the stochastic magnetic region**.

Averaged flux surfaces holding ∇P should be considered to exist in the stochastic region.

Selected free boundary MHD equilibria

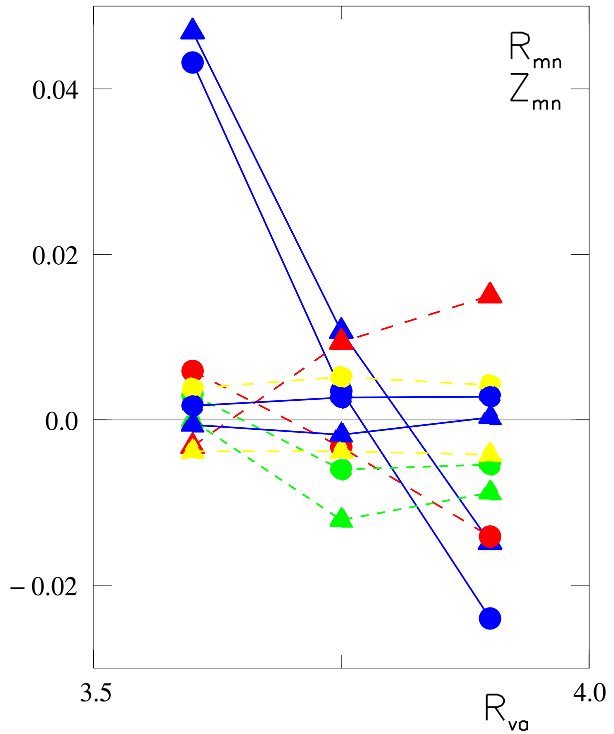
To study the effect of the boundary modulation brought by the free motion of an MHD equilibrium,

- **free boundary MHD equilibria** with keeping total toroidal flux inside of the plasma constant **(VMEC)**, which correspond to
  $\iota_v = 1.48$ **: (clear flux surface)**
  $\iota_v = 1.58$ **: (near LCFS)**
  $\iota_v = 1.72$ **: (outside of LCFS)**
- $I(s) = 0$: currentless
- $P(s) = P_0(1-s)(1-s^9)$ with changeable P_0 and $s = \rho^2, \rho$: normalized minor radius.
- Stability analyses **(CAS3D3)**: **free boundary ($\xi \cdot \nabla s \neq 0$)** incompressible or compressible perturbations
- Resultant MHD equilibria might be **valid for the stability analyses for low-n modes with $n < 10$**, because the minimum toroidal mode number of the magnetic islands is $n = 10$.
- High-mode-number modes with $n \gg 10$ will be affected by fine structure of magnetic islands.

Properties of boundary spectra (vacuum)

Vacuum axis R_{va} dependence of dominant boundary spectra

Dominant boundary spectra except for $(m, n) = (0, 0)$ and $(m, n) = (1, 0)$ for three vacuum configurations : **inward-shifted**, **standard**, **outward-shifted** configurations

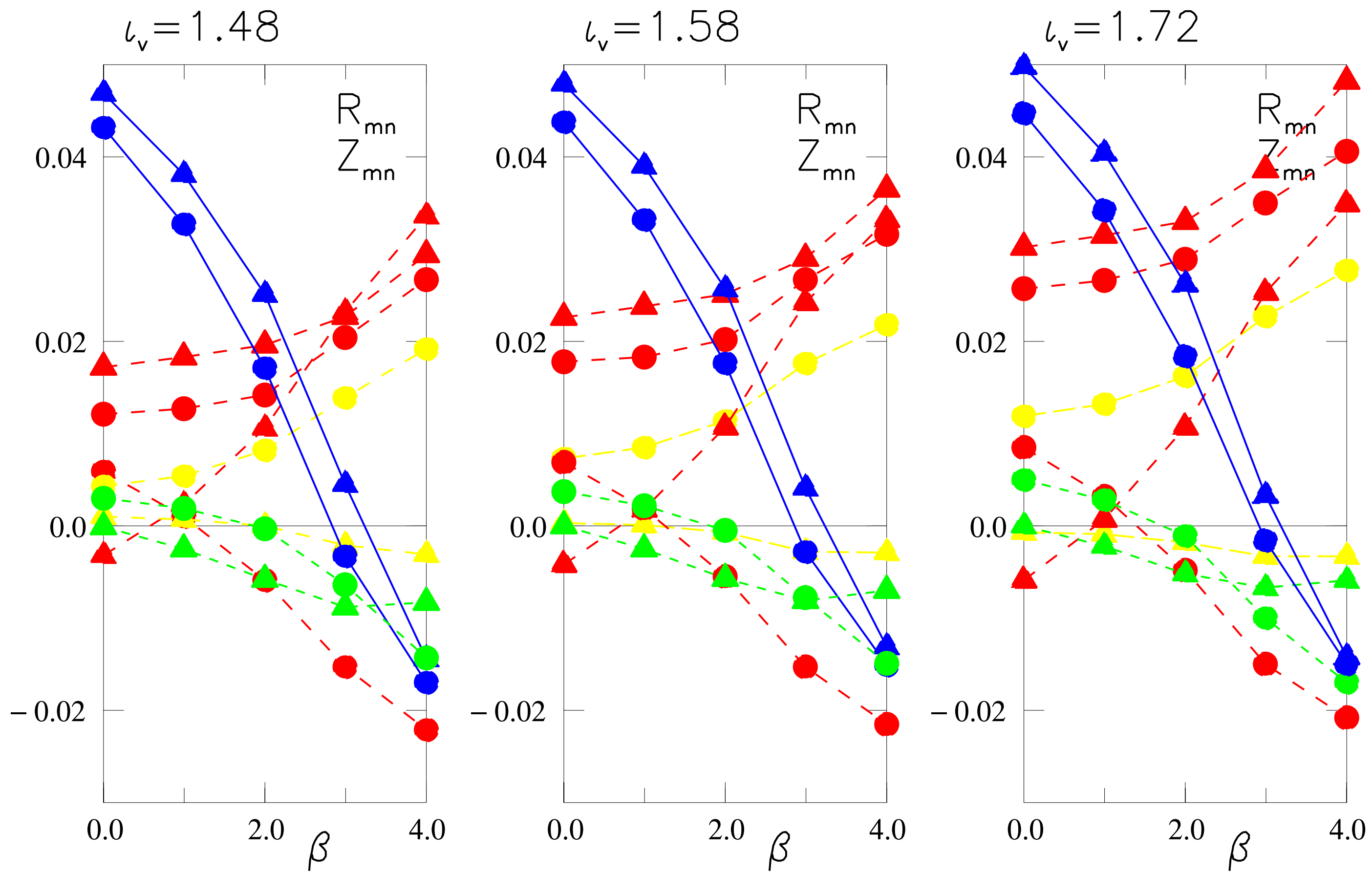


inward-shifted	:	$R_{va} = 3.6$ m
standard	:	$R_{va} = 3.75$ m
outward-shifted	:	$R_{va} = 3.9$ m
triangles	:	Z_{mn}
circles	:	R_{mn}
simbols	:	$(m, n) = (0, \neq 0)$ **: bumpy component**
simbols	:	$n = 10$
simbols	:	$(m, n) = (\neq 0, 0)$ **: axisymmetric**
simbols	:	$n = 20$

1. **Significant changes** appear in **bumpy components** $(m, n) = (0, 10)$
2. **These change** may be related to the **Mercier stability (worse for inward-shifted and better for outward-shifted configurations)**
3. **Same change will be brought by a large Shafranov shift in inward-shifted.**

Properties of boundary spectra (finite-β)

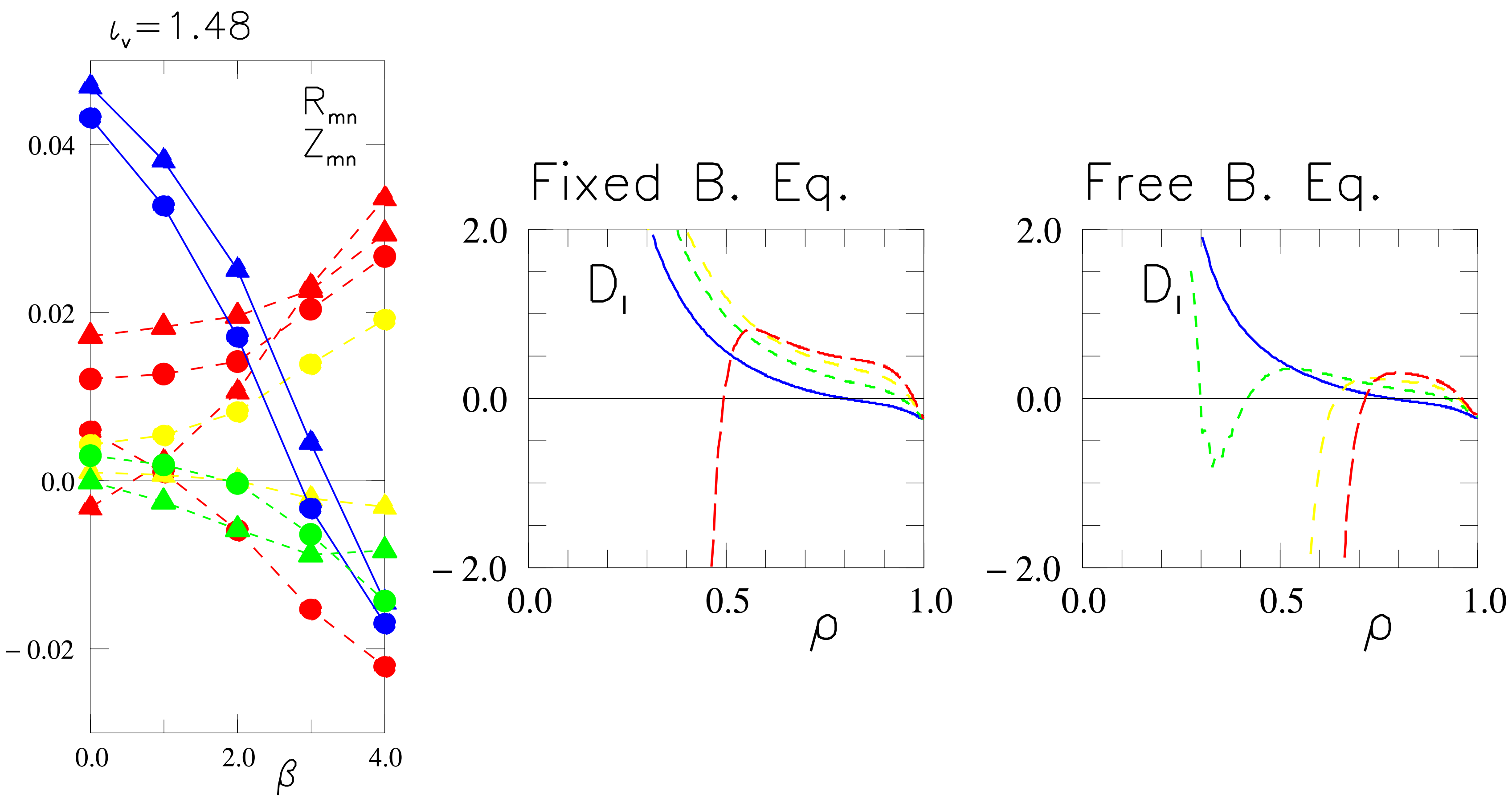
Similar changes of bumpy components are brought by the Shafranov shift of a whole plasma in inward-shifted configurations



- **Change of bumpy components by β does not depend on the choice of vacuum boundary or ι_v.**

MHD stability analyses

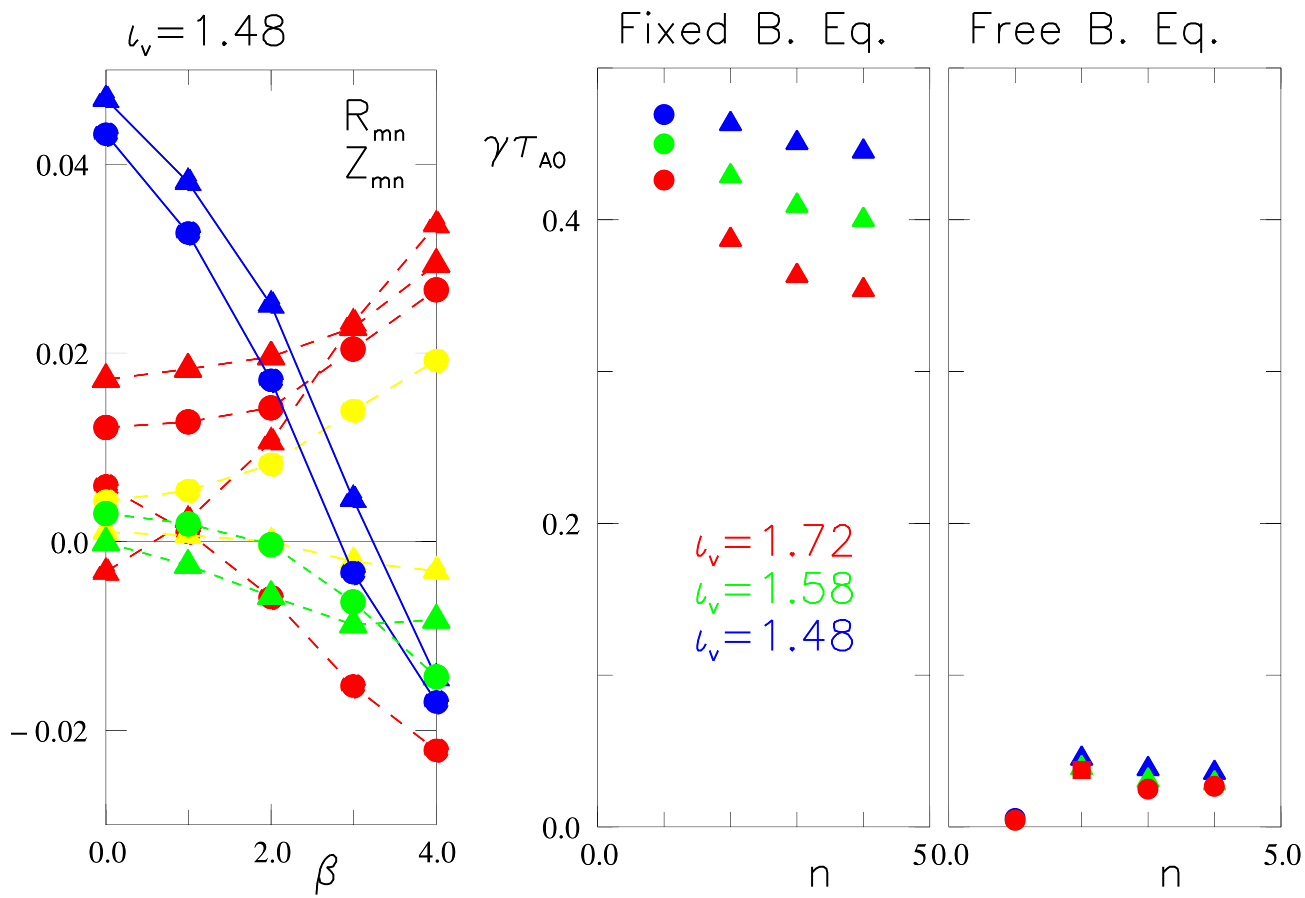
Relation between bumpy components and Mercier criterion D_I



- **Magnetic well** formation by **only a Shafranov shift of the magnetic axis (fixed boundary)** is **not effective.**
- **Change of the plasma boundary; reduction of bumpy components,** brought by the free boundary motion of MHD equilibrium **significantly improves** D_I**.**

MHD stability analyses

Relation between bumpy components and growth rate γ (incompressible perturbations)

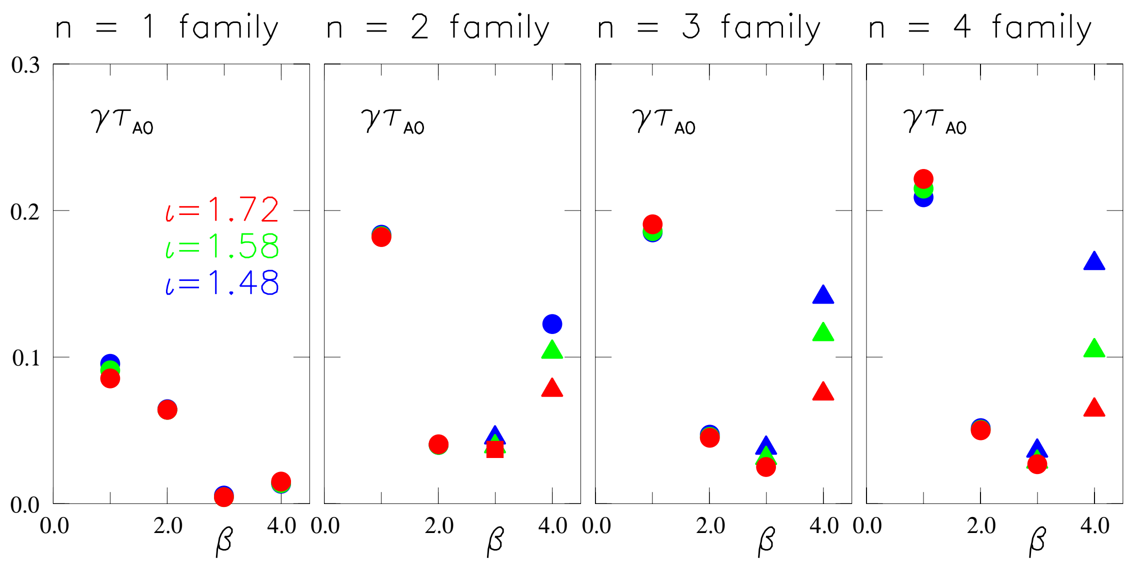


$\langle\beta\rangle = 3\%$,
$\gamma\tau_{A0} = 0.1 : 40\mu$sec,
$\gamma_{incomp} \sim \gamma_{comp} \times 3$

- **Significant reduction of the growth rates** is brought by **change of the plasma boundary; reduction of bumpy components.**
- **Change of γ has good correlation to that of D_I.**

MHD stability analyses

β-dependences of γ (incompressible perturbations)



$\iota_v = 1.48$
$\iota_v = 1.58$
$\iota_v = 1.72$
$\gamma\tau_{A0} = 0.1 : 40\mu$sec,
$\gamma_{incomp} \sim \gamma_{comp} \times 3$

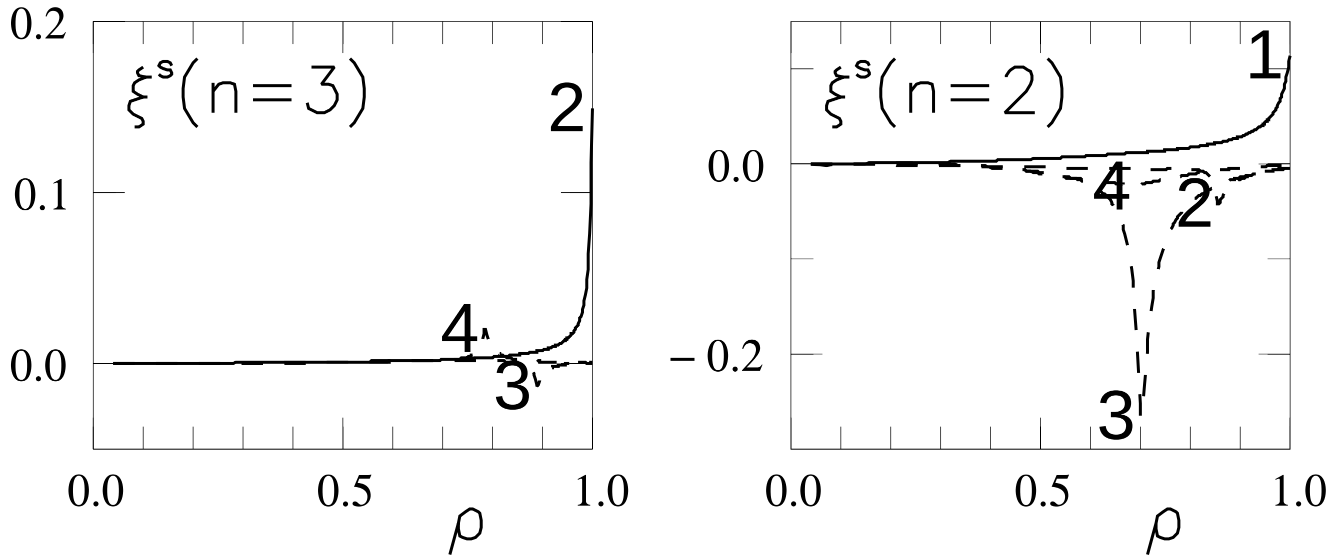
- **Up to $\langle\beta\rangle \sim 3\%$ (interchange regime)**, γ **reduces independent of plasma vacuum boundary or ι_v.**
- **Above $\langle\beta\rangle \sim 3\%$ (ballooning regime)**, γ **increases.**
- **In ballooning regime, larger plasma has smaller γ.**
- γ **is the range of** ω_i^* except for $\langle\beta\rangle \sim 1\%$, **so that instabilities might be harmless** (ω_i^* is evaluated from the dominant poloidal mode numbers).

MHD stability analyses

Excitation of eigenmodes with external components : $(m,n) = (2,3)$ **and** $(m,n) = (1,2)$ (incompressible perturbations)

$t_v = 1.48$
$\gamma\tau_{A0} = 2.89 \times 10^{-2}$

$t_v = 1.72$
$\gamma\tau_{A0} = 3.72 \times 10^{-2}$



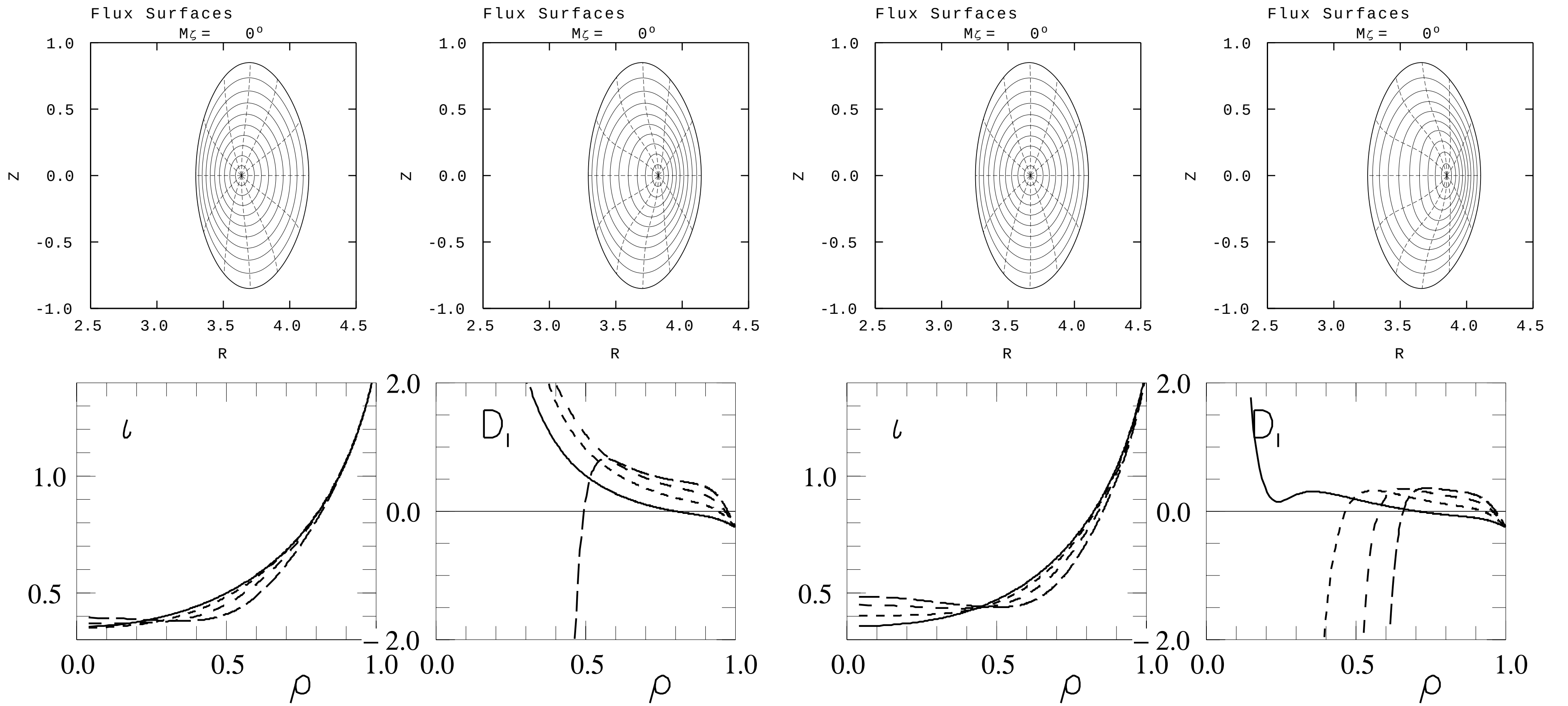
$\langle\beta\rangle = 3\%$,
$\gamma\tau_{A0} = 0.1 : 40\mu$sec,
$\gamma_{incomp} \sim \gamma_{comp} \times 3$

- $(m,n) = (2,3)$ **and** $(m,n) = (1,2)$ **Fourier components may be mainly observed by the magnetic probes for these unstable modes.**
- **Magnetic signals with** $(m,n) = (2,3)$ **are usually observed experimentally.** [SAKAKIBARA, S., et al., Plasma Phys. Control. Fusion **44** (2002) A217.]
- **Magnetic signals with** $(m,n) = (1,2)$ **are observed in high-**β **operations.** [SAKAKIBARA, S., et al., EPS]

Check (MHD equilibria under modulated fixed boundary)

Bumpy components with $(m, n) = (0, \neq 0)$ are eliminated from vacuum boundary spectrum

original vacuum boundary $(\iota_v = 1.48)$ **modulated boundary** $(\iota_v = 1.48)$

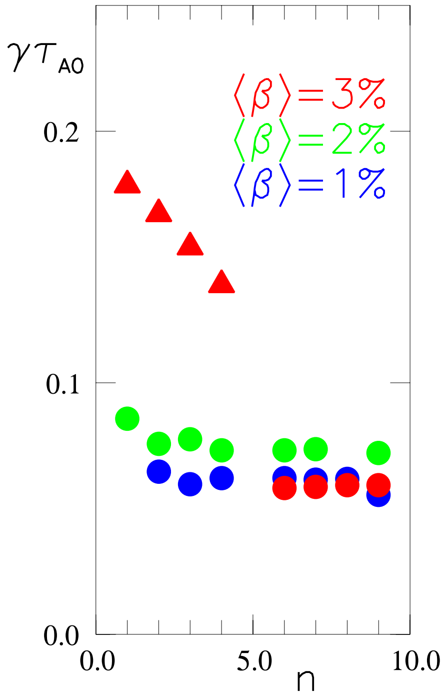


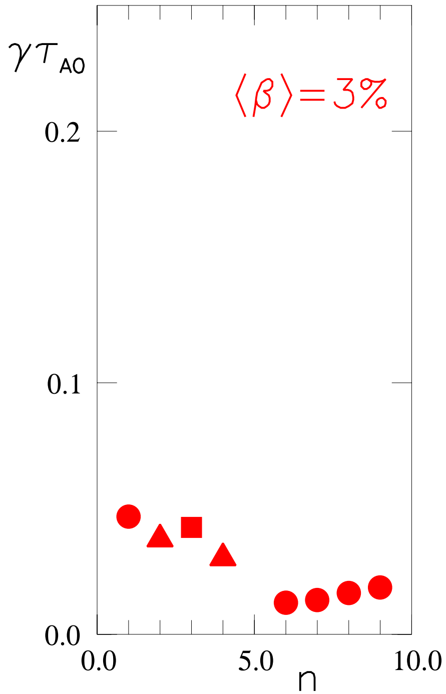
- **Slight boundary modulation leads to significant change of MHD stability.**

Check (MHD equilibria under modulated fixed boundary)

Similar reduction of γ observed (compressible perturbations)

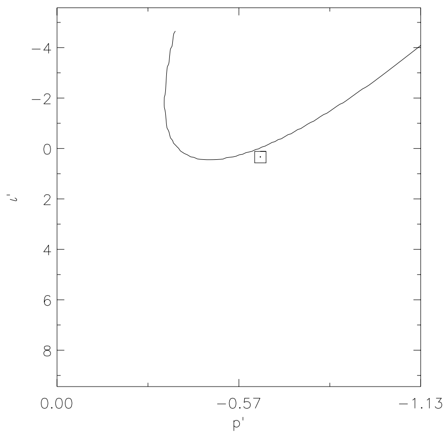
original boundary **modulated boundary**





$\iota_v = 1.48$

$\gamma\tau_{A0} = 0.1 : 40\mu$sec



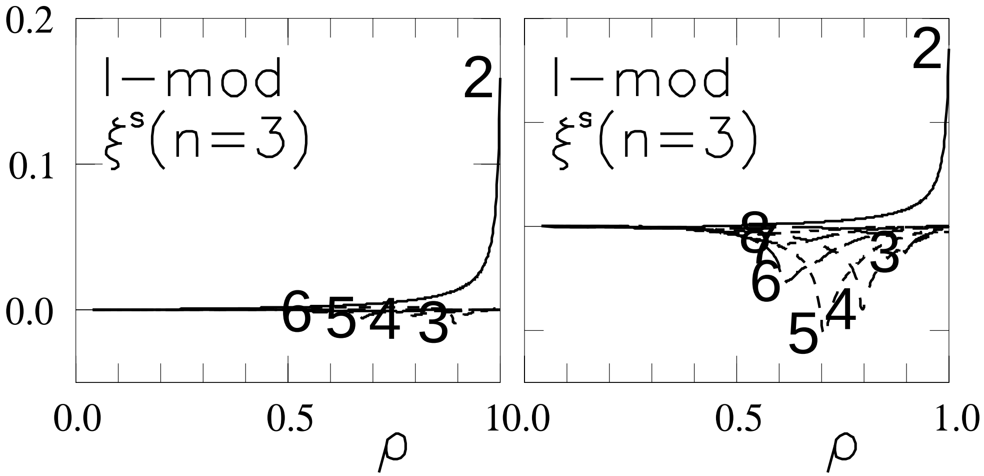
- **Core region stays in the 2nd stability of ballooning modes**, in high-β equilibria.
  [HUDSON,S. et al., this IAEA (TH/P2-24)]

Check (MHD equilibria under modulated fixed boundary)

Similar excitation of eigenmodes with an external component : $(m, n) = (2, 3)$
(compressible perturbations)

$\langle\beta\rangle = 2\%$ $\gamma\tau_{A0} = 1.82 \times 10^{-2}$

$\langle\beta\rangle = 3\%$ $\gamma\tau_{A0} = 4.24 \times 10^{-2}$



$t_v = 1.48$
$\gamma\tau_{A0} = 0.1\ :\ 40\mu$sec

Conclusions and Discussions

- **Boundary modulation by a free motion of MHD equilibrium has significant stabilizing effects for ideal pressure-driven modes**, where **essential modulation is reduction of the bumpy components of the plasma boundary.**
- **These stabilizing effects do not depend on the choice of the averaged flux surfaces up to $\langle\beta\rangle \sim 3\%$ (interchange regime).**
- **In the ballooning regime (above $\langle\beta\rangle \sim 3\%$), wider plasmas are more stable.**
- **In any cases**, $\gamma \sim \omega_i^*$ **(except for $\langle\beta\rangle \sim 1\%$), so that instabilities may be harmless.**
- Depending on the chosen boundary, **various external modes**, **which have same Fourier spectrum as those experimentally observed, are excited.**
- In experiments, both the plasma boundary and the pressure profile will change in β ramp-up phases, according to the heating and the density control. **Proper choice of MHD equilibrium might lead to better coincidences between theory and experiment.**
- Linearized ideal MHD analyses are still useful.
- Stability analyses will be performed for other inward-shifted configuration, where highest-β ($\langle\beta\rangle \sim 4\%$) plasmas are achieved.